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Semi-intrusive quantification of uncertainties in stochastic electromagnetic interactions: Analysis of a spectral formulation

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Abstract — A stochastic approach is presented to statistically characterise uncertainties in electromagnetic interactions. A spectral definition of the observables allows for the assessment of the effect of a randomly deformed device independent of the incident field.

1 Introduction

In computational electromagnetics, many deterministic algorithms have been developed to model the interaction between a material system and an incident field for antenna design, scattering, and electromagnetic compatibility (EMC) purposes. However, in practice, the knowledge of the actual configuration can be affected by uncertainties concerning the features of the scatterer or the incident field, which can lead to significant deviations between simulations and measurements.

In these cases, rather than aiming for an exhaustive study of all the possible situations, which can be extremely tedious, stochastic rationales postulate that the unknown parameters of the configuration vary randomly. Probability theory is then employed to propagate this initial randomness through the model and to measure the randomness induced on its output, or observable, via statistical moments, that can be determined by a limited number of computations.

The propagation of the randomness can be performed *non-intrusively* by viewing the deterministic model as a black box, having the

scattering device and the incident field as input, and providing the value of observables such as the induced voltage or current at a given port region [1, 2]. However, evaluating the effect of a given geometrical randomness on different excitations requires as many statistical studies of the entire configuration, as there are incident fields. Alternatively, a *semi-intrusive* approach can be adopted to characterize the randomness of the receiving device independent of the incident field. This strategy has been applied by Michielsen in [3] with the aid of a Taylor expansion of the incident field, and by Brown in [4], where asymptotic relations are employed to simplify the formulation of the problem in the presence of an unbounded rough surface.

This paper proposes a semi-intrusive method based on a spectral reformulation of the observable, chosen as the voltage V_e induced at the port of a receiving device. This spectral definition is introduced in Section 2. The deterministic model is randomized in Section 3 to express the average and the variance of V_e in terms of the statistical moments of a current distribution depending solely on the scatterer. Results are then provided in Section 4 for the example of a transversely undulating wire.

2 Deterministic parametrization

A device enclosed in the perfectly conducting surface S_α and containing a port is considered. The geometry of S_α is explicitly described and controlled by a vector $\alpha = (\alpha_1, \dots, \alpha_n)$, which belongs to the fixed domain $\mathcal{A} \subset \mathbb{R}^n$. External electromagnetic sources radiate the field $\mathbf{E}^i$, which induces the voltage V_e at the port, i.e.

$$V_e = - \int_{r \in S_\alpha} \mathbf{J}_\alpha(\mathbf{r}) \cdot \mathbf{E}^i(\mathbf{r}). \quad (1)$$

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The normalized distribution $\mathbf{J}_\alpha$ depends solely on S_α as it is induced, in the absence of $\mathbf{E}^i$, by a unit current source applied to the port. The solution of a frequency-domain electric-field integral equation to obtain $\mathbf{J}_\alpha$ is the major numerical effort involved in this model.

Geometrical modifications of the setup are obtained by varying α in $\mathcal{A}$, which in turn infers changes in $\mathbf{J}_\alpha$ and in V_e . Further, although $\mathbf{E}^i$ is assumed to be independent of the scattering device, it still needs to be evaluated at points $\mathbf{r} \in S_\alpha$, thereby complicating the task of characterizing the variations of S_α independent of $\mathbf{E}^i$. This limitation is sidestepped by applying Plancherel's theorem [5, p. 186] to recast Equation (1) in

$$V_e = - \int_{\mathbf{k} \in \mathbb{R}^3} \widetilde{\mathbf{J}}_\alpha(\mathbf{k}) \cdot \widetilde{\mathbf{E}}^i(-\mathbf{k}), \quad (2)$$

where $\widetilde{\mathbf{J}}_\alpha$ and $\widetilde{\mathbf{E}}^i$ are the Fourier transforms, with respect to $\mathbf{r}$, of $\mathbf{J}_\alpha$ and $\mathbf{E}^i$ respectively. The key advantage of Equation (2) lies in the fact that the effect of the randomness of S_α is entirely encompassed in $\widetilde{\mathbf{J}}_\alpha$. The computation of $\widetilde{\mathbf{J}}_\alpha$ is eased by the bounded support of $\mathbf{J}_\alpha$, viz S_α . Further, the evaluation of the improper integral in Eq. (2) can be simplified by making assumptions on the type of excitation $\mathbf{E}^i$.

Given a finite set of fixed directions of incidence $\{\mathbf{u}_1, \dots, \mathbf{u}_M\}$, together with the wave-vectors $\mathcal{K}_M = \{\mathbf{k}_l = (2\pi/\lambda)\mathbf{u}_l, l = 1, \dots, M\}$, where λ denotes the wavelength, $\mathbf{E}^i$ can be constructed as the superposition of plane waves propagating along the directions $\mathcal{K}_M$ as follows

$$\mathbf{E}^i : \mathbb{R}^3 \ni \mathbf{r} \mapsto \sum_{l=1}^M \mathbf{e}_l \exp(-j\mathbf{k}_l \cdot \mathbf{r}) \in \mathbb{C}^3. \quad (3)$$

The polarization vectors $\{\mathbf{e}_l \in \mathbb{R}^3, \mathbf{e}_l \cdot \mathbf{k}_l = 0, l = 1, \dots, M\}$ of the plane waves represent a degree of freedom allowing for the construction of a variety of incident fields. The contribution of a given direction $\mathbf{k}_l$ can, for instance, be “switched off” simply by setting $\mathbf{e}_l = \mathbf{0}$. Conversely, a single plane wave propagating in the direction $\mathbf{k}_l \in \mathcal{K}_M$ is obtained by canceling all the but the l -th polarization vector.

Interestingly, the Fourier counterpart of $\mathbf{E}^i$ becomes a combination of Dirac distributions and $\widetilde{\mathbf{J}}_\alpha$ needs to be evaluated only for $\mathbf{k} \in \mathcal{K}_M$ to deduce the voltage V_e as follows

$$V_e(\alpha) = - \sum_{l=1}^M \widetilde{\mathbf{J}}_\alpha(\mathbf{k}_l) \cdot \mathbf{e}_l \quad (4)$$

A duality exists between the Eqs (1) and (4): in the spatial domain, the support of $\mathbf{J}_\alpha$, viz S_α , determines the points at which $\mathbf{E}_\beta^i$ is evaluated, whereas in the spectral domain, the support of $\widetilde{\mathbf{E}}_\beta^i$, viz $\mathcal{K}_M$, dictates the directions in which $\widetilde{\mathbf{J}}_\alpha$ is calculated.

3 Randomization

The effect of uncertainties concerning the geometry of S_α is now investigated. To begin with, the variations of α in $\mathcal{A}$ are assumed to be random according to a known probability density function f_α , which is chosen a priori. In a non-intrusive approach, the statistical moments of V_e , such as its mean are obtained as

$$\mathbb{E}[V_e] = \int_{\alpha' \in \mathcal{A}} V_e(\alpha') f_\alpha(\alpha') d\alpha', \quad (5)$$

where the integral is approximated by a quadrature rule adapted to the dimension of $\mathcal{A}$ [2]. On the other hand, with the representation of Equation (4) the randomness of α will affect V_e only via $\widetilde{\mathbf{J}}_\alpha$, thereby enabling the definition of the mean and the variance of V_e in terms of the average and the covariance of $\widetilde{\mathbf{J}}_\alpha$,

$$\mathbb{E}[V_e] = - \sum_{l=1}^M \mathbb{E}[\widetilde{\mathbf{J}}_\alpha](\mathbf{k}_l) \cdot \mathbf{e}_l, \quad (6)$$

$$\text{var}[V_e] = \sum_{l_1=1}^M \sum_{l_2=1}^M \mathbf{e}_{l_1} \cdot \mathcal{C}[\widetilde{\mathbf{J}}_\alpha](\mathbf{k}_{l_1}, \mathbf{k}_{l_2}) \cdot \mathbf{e}_{l_2}^t, \quad (7)$$

in which the superscript “t” indicates the vectorial transposition. Due to the fact that $\widetilde{\mathbf{J}}_\alpha = (\widetilde{\mathbf{J}}_\alpha^x, \widetilde{\mathbf{J}}_\alpha^y, \widetilde{\mathbf{J}}_\alpha^z)$ is a complex-valued vector, the covariance matrix $\mathcal{C}[\widetilde{\mathbf{J}}_\alpha]$ will be Hermitian and consist of 3×3 sub-matrices assessing the mutual correlation between the components of $\widetilde{\mathbf{J}}_\alpha$.

The statistics of $\mathbf{J}_\alpha$ are also defined as integrals similar to Equation (5), which are computed by a quadrature rule suited to the dimension of $\mathcal{A}$. These statistics of $\mathbf{J}_\alpha$ need to be computed only once and can then be employed to deduce the average and the variance of the voltage induced by any excitation resulting from a combination of plane waves associated to $\mathcal{K}_M$.

4 Results

This semi-intrusive rationale is illustrated through the example of a perfectly electrically conducting (PEC) thin-wire structure, which can be regarded as a transmission line. As shown in Figure. 1, the wire lies over an infinite PEC plane to which it is connected via two vertical wires, one of which contains a port.

Figure 1: Thin-wire setup.

For any $y \in [y_m, y_m]$, the Cartesian coordinates of the axis correspond to

$$x_\alpha(y) = \alpha_1 \sin[2\pi(y - y_m)], \quad (8a)$$

$$z_\alpha(y) = 0.05 + \alpha_2 \sin[2\pi(y - y_m)], \quad (8b)$$

where $\alpha = (\alpha_1, \alpha_2)$ has statistically independent components that are uniformly distributed in $\mathcal{A}_1 = \mathcal{A}_2 = [-0.03, 0.03]$ m. The frequency is chosen as $f = 500$ MHz, and the 50 directions of incidence ($M=50$), from which the set $\mathcal{K}_M$ is obtained, are specified in polar coordinates by $\{\mathbf{u}_l : \theta_l(l) = l\pi/(2M), \phi_l(l) = \pi/4\}$ for $l = 1, \dots, M$.

The amplitude of the covariance matrix of $\tilde{\mathbf{J}}_\alpha$ is plotted in Figure 2. The symmetry in this figure results from the Hermitian nature of $\mathcal{C}[\tilde{\mathbf{J}}_\alpha]$. As mentioned in Section 3, $\mathcal{C}[\tilde{\mathbf{J}}_\alpha]$ has a 3×3 structure where the sub-blocks indicate

the mutual statistical correlation between the Cartesian components of $\tilde{\mathbf{J}}_\alpha$. Each sub-block expresses the intensity of the statistical correlation between the components of $\tilde{\mathbf{J}}_\alpha$ for different directions of incidence in $\mathcal{K}_M$.

Figure 2: Amplitude of the covariance of $\tilde{\mathbf{J}}_\alpha$: $\mathcal{C}[\tilde{\mathbf{J}}_\alpha]_{dB}(\mathbf{k}_{l_1}, \mathbf{k}_{l_2}) = 10 \log \left(\left| \mathcal{C}[\tilde{\mathbf{J}}_\alpha](\mathbf{k}_{l_1}, \mathbf{k}_{l_2}) \right| \right)$.

The component $\tilde{\mathbf{J}}_\alpha^y$ has the largest auto-correlation, with a maximum around $\theta_i = 25^\circ$. This feature stems from the orientation of the wire mainly along the y direction. Although the geometrical undulations of S_α are identical in the x and z directions, the auto- and cross-correlation terms involving $\tilde{\mathbf{J}}_\alpha^z$ outweigh those of $\tilde{\mathbf{J}}_\alpha^x$. This difference is due to the presence of the ground plane, which enhances the effect of the vertical geometrical variations. Further, the very low statistical coupling between $\tilde{\mathbf{J}}_\alpha^x$ and $\tilde{\mathbf{J}}_\alpha^z$ is worth mentioning. This lack of correlation can be related to the independence of α_1 and α_2 , by using transmission-line theory.

The tensors $\mathbb{E}[\tilde{\mathbf{J}}_\alpha]$ and $\mathcal{C}[\tilde{\mathbf{J}}_\alpha]$ are now successively tested by M plane waves, each one associated with a given wave vector in $\mathcal{K}_M$, with a unit amplitude and a θ polarization. For each of these plane wave, the mean and the variance of V_e are computed both non-intrusively using Equation (5), and semi-intrusively via Equations (6) and (7). The results, displayed in Figure 3 highlight the match between both results. The module $|\mathbb{E}[V_e]|$ oscillates between 60 mV and 260 mV as θ_i varies in $[0^\circ, 90^\circ]$, whereas $\sigma[V_e]$ has a maximum value of 65 mV

for $\theta_i = 25^\circ$, as expected from the analysis of Figure 2.

Figure 3: $|E[V_e]|$ and $\sigma[V_e] = \sqrt{\text{var}[V_e]}$ computed non-intrusively and a semi-intrusively.

The main asset of the semi-intrusive method appears when considering the cumulative computation time depicted in Figure 4. The pre-computation of the statistical moments of $\tilde{\mathbf{J}}_\alpha$ amounts to a duration of 49 seconds, after which the evaluations of $E[V_e]$ and $\sigma[V_e]$ are performed very rapidly in 5 ms. Comparatively, via the non-intrusive approach, for each incident field, the computation of $E[V_e]$ and $\sigma[V_e]$ amounts to approximately 8 seconds.

Figure 4: Cumulative computation time.

5 Conclusions

The semi-intrusive approach presented in this paper hinges on a spectral definition of the ob-

servables via Plancherel's theorem. The resulting representation allows for a refined statistical characterization of the randomness of a receiving device independent of the incident field. This feature is particularly appreciable in EMC when a given device needs to be tested under various types of excitations. Moreover, significant gains in computation time are achieved when multiple incident fields need to be considered, as demonstrated in the case of the thin wire. The semi-intrusive rationale also permits the study of random incident fields resulting from combinations of plane waves. The extension of this method to handle more general types of excitations is depends on the use of an efficient method to interpolate the spectrum of the incident field.

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