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NONLINEAR OSCILLATIONS IN MEMS RESONATORS

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Abstract: Nonlinearities in MEMS silicon resonators are caused by different effects. Depending on the resonator layout, different nonlinearities may be dominant in the resonator response. A model for a clamped-clamped beam resonator is derived, which contains both a mechanical and an electrical part. The dynamic behaviour of the resonator is investigated both numerically and experimentally in order to obtain a qualitative match.

Keywords: MEMS resonator, nonlinearities, dynamic behaviour.

1 INTRODUCTION

Micro-electromechanical silicon resonators provide an interesting alternative for quartz crystals as accurate timing devices in oscillators for modern data and communication applications. Their compact size, feasibility of integration with IC technology and low cost are major advantages. However, as the resonators are small in size, they have to be driven close to nonlinear regimes in order to store enough energy for a sufficient signal to noise ratio [1]. Depending on the specific resonator layout, different nonlinearities may be dominant in the resonator dynamic behaviour.

In order to determine the influence of resonator nonlinearities on the performance of oscillators, the dynamic behaviour of resonators has to be understood. Clamped-clamped beam resonators have been realised and measured by various research groups, see for instance [2]. However, measurements and simulations of the nonlinear behaviour have not been addressed. In this paper, a qualitative match will be established between numerical and experimental investigations of nonlinear dynamics in a clamped-clamped beam resonator. The long-term goal of this project is to derive guidelines for optimal resonator design, based on a quantitative match.

The outline for this paper is as follows. First, in sections 2 and 3, models for a clamped-clamped beam resonator and the measurement circuit will be derived, respectively. These two will be combined into a simulation model in section 4. Experiments and results will be discussed in section 5. Finally, in section 6, some conclusions will be drawn.

2 RESONATOR MODEL

An example of a clamped-clamped beam resonator is depicted in Fig. 1. Its characteristic vibration shape is shown in Fig. 1(a). Due to out-of-plane vibration, the resonator is often called a flexural resonator. The actuation of the resonator is realised by means of a dc and an ac voltage component, see Fig. 1(b).

A single degree-of-freedom (1DOF) lumped model, in line with [1, 3], describing the flexural displacement $x$ of the middle of the beam resonator is the following:

$$m\ddot{x} + b\dot{x} + k(x)x = F_e(x),$$

where $m$, $b$ and $k(x)$ are the lumped mass, damping and nonlinear stiffness of the system, respectively. Furthermore, $x$ and $\ddot{x}$ denote the first and second time derivative of $x$, respectively. The electrostatic force $F_e(x)$ is given by:

$$F_e(x) = \frac{1}{2} \frac{C_0 d_0}{(d_0 - x)^2} V^2,$$

where $C_0$ is the capacitance over the gap when $x = 0$ and $d_0$ is the corresponding initial gap width. The capacitance for arbitrary $x$ is given by $C(x) = \frac{C_0 d_0}{d_0 - x}$. $V$ is the excitation voltage, consisting of a dc and an ac component: $V = V_{dc} + V_{ac}\sin(2\pi ft)$. $V_{dc}$ is the so-called bias voltage, and $V_{ac}$ and $f$ are the amplitude and frequency of the ac voltage, respectively.

Taylor expansion of the electrostatic force:

$$F_e(x) = \frac{1}{2} \frac{C_0}{d_0} V^2 \left(1 + 2 \frac{x}{d_0} + 3 \frac{x^2}{d_0^2} + 4 \frac{x^3}{d_0^3} + \text{h.o.t.}\right)$$

reveals that this force introduces softening non-linear behaviour. In (3), h.o.t. denotes higher-order terms. The non-linearity in spring stiffness results from silicon material nonlinearities (higher-order elastic effects, see [1, 4]) or from geometric nonlinearities (mid-plane stretching of a clamped-clamped beam, see for instance [5, 6]) and can be written as:

$$k(x) = k_0 + k_1 x + k_2 x^2,$$
where \( k_0 \) is the linear stiffness and \( k_1 \) and \( k_2 \) denote the quadratic and cubic stiffness parameters, respectively. Depending on the specific layout of the clamped-clamped silicon beam, \( k_1 \) and \( k_2 \) may become positive or negative. In [3], a softening spring stiffness was observed for a longitudinal mode beam resonator.

### 3 MEASUREMENT CIRCUIT

Both the response measurement and the actuation of the resonator takes place electrically. The dc and ac voltages are applied to the electrodes of the resonator by means of bias tees. The dc voltage is applied to the electrodes on both sides of the clamped-clamped beam, which is grounded itself (see Fig. 1(b)). The ac component is applied to one electrode, whereas the other electrode is used for measuring the output \( V_{out} \), which is a measure for beam motion. In order to be able to measure the ac component of \( V_{out} \), dc decoupling is present by means of an additional decoupling capacitor \( C_{dec} \). This is depicted in Fig. 2(a). The resonator output voltage is measured on a 50 Ω resistor (\( R_2 \)).

![Resonator schematic](image)

**Fig. 2. Measurement configuration.**

In order to derive an expression for the output voltage measured on \( R_2 \), consider Fig. 2(b), which contains only the output loop of the electrical circuit (grey part in Fig. 2(a)). \( C(x) \) denotes the variable resonator capacitance. The differential equations in the two unknowns \( v \) and \( i \) for this electrical circuit are determined from node analysis. At node \( A \), Kirchhoff’s current law reads:

\[
i_0 = i + i_{res}, \tag{5}\]

where \( i_{res} \) denotes the current through the resonator. Furthermore, the current through the decoupling capacitor is given by:

\[
i = C_{dec} \frac{dv}{dt} - R_2 \frac{di}{dt}. \tag{6}\]

The currents in (5) are given by:

\[
i_0 = \frac{V_{dc} - v}{R_1}, \quad \text{and} \quad i_{res} = \frac{d(C(x)v)}{dt}. \tag{7}\]

By substituting (7) into (5) and using \( V_{out} = iR_2 \), two coupled differential equations for current \( i \) and voltage \( v \) are obtained:

\[
C(x) \frac{dv}{dt} = - \left( \frac{1}{R_1} + \frac{\partial C(x)}{\partial x} \frac{\dot{x}}{R_1} \right) v - i + \frac{V_{dc}}{R_1} \tag{8}
\]

\[
C_{dec} \left( \frac{dv}{dt} - R_2 \frac{di}{dt} \right) = i, \tag{9}
\]

where, in (8), the time derivative of the resonator capacitance \( C(x) \) is elaborated as \( \frac{dC(x)}{dt} = \frac{\partial C(x)}{\partial x} \dot{x} \).

### 4 SIMULATION MODEL

The total model for the clamped-clamped beam resonator and the measurement circuit consists of a state space description of the 1DOF resonator model (1), together with the equations for the measurement circuit (8)–(9). In this way, a model with four states \((x, \dot{x}, v, i)\) is obtained. The measured output equals \( V_{out} = iR_2 \).

In order to investigate the dynamic behaviour of the total model, a nonlinear dynamic analysis is performed, for which continuation techniques are already available. The numerical package AUTO [7] is applied to the model for determining periodic solutions (resonator oscillations) for varying excitation frequency \( f \). Results will be discussed in section 5.

### 5 EXPERIMENT AND RESULTS

Resonators are fabricated using Silicon-On-Insulator (SOI) wafers. First, aluminum bondpads are defined on the wafer surface. Next, aluminum bondpads and electrical lines. A microscope image of the clamped-clamped beam resonator can be seen in Fig. 3. Here, the gray material is silicon (Si), thin dark lines are lithography etch gaps and the white, grainy material corresponds to the aluminum (Al) bond pads and electrical lines.
Six aluminum bond pads can be distinguished. The outer four bond pads are connected to ‘ground’, such that the beam itself is grounded. The middle two bond pads are used for actuation and measurement purposes. The measurement circuit in the experiments is the same as depicted in Fig. 2(a).

During the experiments, the excitation frequency \( f \) is slowly increased (sweep up) and decreased (sweep down) around the fundamental frequency of the resonator. At each frequency, \( V_{\text{out}} \) is measured. An amplitude-frequency plot for excitation parameters \( V_{\text{dc}} = 70 \text{ V} \) and \( V_{\text{ac}} = 0.544 \text{ V} \) is depicted in Fig. 4(a).

Here, the peak to peak value \( V_{\text{pp}} \) of the output signal \( V_{\text{out}} \) is plotted versus the excitation frequency \( f \).

From the lower plot of Fig. 4(b), it can be seen that the resonator output voltage has a sawtooth-like shape. This indicates that the resonator behaves non-linearly. Furthermore, it is observed that the response is rather noisy. This noise has a relative large influence on the peak to peak value \( V_{\text{pp}} \) of the output.

In order to understand this, consider Figs. 5 and 6.

In Fig. 5, a part of the Fourier transforms of the responses at point A and B can be seen. Clearly, large peaks are present, which correspond to the excitation frequency and multiples thereof. Furthermore, the time signals at both points can be seen to have the same noise floor.

After selecting only the large peaks in the Fourier transformed signals, an inverse Fourier transform is calculated to obtain signals, which contain much less noise. This is depicted in Fig. 6.

From Fig. 6, it can be seen that the corrected signals will have smaller peak to peak values \( V_{\text{pp}} \) than the original ones. Next, the effect of these corrections on the amplitude-frequency curve is investigated. Results of this correction are depicted in Fig. 7. Correction for noise results in a shift of the whole curve to lower voltages. It can be seen that the influence of the noise is less for higher output voltage amplitudes.
Next, parameters in the simulation model are adjusted such that the simulated response matches the measured response. Initial estimates for the linear parameters have been obtained from the physical dimensions and mass of the resonator. Next, the nonlinear parameters are adjusted to obtain a qualitatively accurate fit. The final result is depicted in Fig. 8(a). For the simulated response (in black), solid lines represent peak to peak values of stable periodic solutions, whereas the dashed lines correspond to unstable periodic solutions (which are not seen in measurements). The transition between stable and unstable periodic solutions is characterised by cyclic fold (cf) bifurcations (see for instance [6]).

From Fig. 8, it becomes clear that a qualitative match is obtained. However, in a quantitative sense, still some discrepancies are present, for instance, the mismatch between the peak to peak values of the numerical and experimental curves. Thermal noise is a possible cause for this mismatch. Namely, thermal noise caused by the motional resistance of the resonator is estimated to be $1.0 \cdot 10^{-3}$ mV. This will be investigated further.

The parameter values for the simulation model are $m = 0.227$ ng, $k_0 = 1.49 \cdot 10^4$ N/m, $k_1 = 0$ N/m², $k_2 = -3.00 \cdot 10^{15}$ N/m³, $b = 1.53 \cdot 10^{-9}$ kg/s, $d_0 = 0.330$ µm, $C_0 = 0.165$ fF, $C_{bias} = 0.082$ µF, $R_1 = 1.00$ MΩ, $R_2 = 50.0$ Ω and $f_0 = \frac{1}{2\pi} \sqrt{\frac{k_0}{m}} = 12.89$ MHz.

6 CONCLUSION

The results presented in this paper provide a very promising starting point for a thorough understanding of MEMS resonator dynamics. It can be concluded that a relatively simple model already predicts the measured response very well, since relevant dynamic aspects near the resonance peak are captured in a qualitative sense.

Future work will incorporate improvement and extension of the numerical model in order to obtain better quantitative predictions. Furthermore, design aspects and the effect of different resonator layouts should be included.

REFERENCES