Model based commutation containing edge coils for a moving magnet planar actuator

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1. Introduction

A planar actuator is an extension of a brushless linear actuator in that it is able to position in a plane instead of over a line. It can replace the use of xy-drives that consist of multiple stacked linear actuators. A challenging application for these planar actuators is the wafer stage used in the semiconductor lithography. Developments in lithographic technologies such as the use of extreme UV, require stages that can operate in vacuum. Therefore, the utilization of planar actuators that have active magnetic bearings [1], i.e. a single stage that has long-stroke capability in the xy-plane with the remaining four degrees magnetically borne, is being researched for these applications.

At Eindhoven University of Technology a moving magnet planar actuator has been realised [2]. It is named Herringbone Pattern Planar Actuator (HPPA) after its coil topology. This actuator is shown in Figure 1. The translator consists of an aluminum plate and a Halbach permanent magnet array. The translator has a size of 300 mm x 300 mm. The stator consists of 84 coreless coils in a herringbone pattern arrangement (406 mm x 400 mm).

Although the actuator can be classified as an AC synchronous motor, the DQ0-decomposition cannot be used for the decoupling of flux and current and consequently of force, torque and current. Therefore, new decoupling and commutation methods have been created and every coil is separately excited with a single phase current amplifier. It needs emphasising that the current waveforms are non-sinusoidal.

The forces and torques among the permanent magnet array on the translator and the stator coils beneath the magnet array can be accurately modeled by an analytical model [3]. This harmonic model assumes an infinitely large permanent magnet array as it is based on a harmonic description of the magnetic flux density of the permanent magnets and the Lorentz force and torque calculation. In the commutation algorithm that calculates the coil currents [4] only the first harmonic is taken into account.

Only coils below and near the edge of the magnet array have significant contribution to the force and torque acting on the translator. Therefore, depending on the translator position a limited set of coils is simultaneously actuated. The harmonic model based commutation method omits the coils near the edge of the translator which are influenced by the end effect of the magnet array (edge coils) from the commutation to prevent modeling errors. This results in a maximum of 24 coils which are active simultaneously.

This paper will deal with adding the edge coils to the commutation, while keeping it model based. Increasing the number of coils that are active will decrease the power dissipation in the coils because of a lower current density needed for levitation and propulsion of the translator. It also allows for a higher acceleration with the same maximum current. In section 2, the model based commutation is explained, and the method to incorporate the edge coils in the commutation is proposed and presented in detail. Sections 3 and 4 show simulation results and measurements that verify the anticipated results. Finally, conclusions are formulated.

Figure 1: (a): position of the magnet plate $d_p$, (b): the magnet plate, (c): HPPA at Eindhoven Univ. of Techn.

2. Planar commutation method

Firstly, the commutation method, based on the harmonic model, which switches smoothly among the active coil set is presented [4][5]. Secondly, the method for including edge coils is explained, it uses a weighting based on the model error of the edge coils. Finally, the manner in which the weighting is obtained is presented.

2.1. Model based commutation

The commutation that is used in the HPPA is based on a harmonic model that gives the relation between the current through a coil and the wrench, the vector containing forces and torques: $\dot{\omega} = [\rho, \tau, \tau, \tau]^T$, acting on the translator depending on its relative position. The wrench contribution of coil current $i_j$ is denoted by,

$$\dot{\omega}_j(d_p) = \Gamma(d_p - \tilde{d}_e) i_j,$$

where $d_e$ is the position of coil $j$, and $d_p$ the translator position (defined in Figure 1a). The matrix $\Gamma(d_p) \in \mathbb{R}^{6 \times 84}$ is composed with evaluations of the harmonic model at the position $d_p$ by,

$$\Gamma(d_p) = \begin{bmatrix}
\tilde{\rho}(\tilde{d}_p - \tilde{d}_1) & \cdots & \tilde{\rho}(\tilde{d}_p - \tilde{d}_{84})
\end{bmatrix}.$$
Figure 2: The weighting \( \delta_p(q) \). The dashed lines indicate the centre positions of the coils.

The matrix \( \Gamma(q_p) \) gives the relation between the active coil currents and the wrench vector acting on the mass centre point of the translator. In the next few equations the dependency on \( q \) is omitted for notational ease. To obtain the currents that produce a certain desired wrench vector, the generalised inverse of \( \Gamma \),

\[
\Gamma^{-} = \Delta \Gamma^T \left( \Gamma \Delta \Gamma^T \right)^{-1},
\]

is calculated. Where \( \Delta \) can be an arbitrary weighting such that \( \Gamma \Delta \Gamma \) remains non-singular. The currents that will render a desired wrench vector are obtained by,

\[
\vec{i} = \Gamma^{-} \vec{w}_{des}.
\]

The harmonic model, however, is only valid for the coils that are directly beneath the magnet array. To limit the generalised inverse to only those coils, the weighting \( \Delta(q_p) = \text{diag}(\delta_1(q_p-\tilde{q}_{1}), \delta_2(q_p-\tilde{q}_{1}), \ldots, \delta_5(q_p-\tilde{q}_{5})) \) is selected as a raised cosine for both directions (x and y). The weighting function \( \delta_5 \) is shown in Figure 2. It weights the coils that are outside the area covered by the magnet plate with zero and the coils that are correctly modeled by the harmonic model with one. It also forces the transition from active to non-active and vice versa to be smooth. This smooth transition is necessary to avoid discontinuous currents, which are not allowed for an inductive load. A lot of coils are weighted with zero and, therefore, they are omitted from the commutation. \( \Delta_r \) and \( \Gamma_r \) are reduced forms obtained by eliminating from \( \Delta \) and \( \Gamma \) all columns and rows corresponding to zeros on the diagonal of \( \Delta \). This results in the commutation for the active coils,

\[
\Gamma_r^{-} = \Delta_r \Gamma_r^T \left( \Gamma_r \Delta_r \Gamma_r^T \right)^{-1}.
\]

The repeated pattern of the stator coils makes \( \Gamma_r^{-} \) change periodically over the stroke, but the active coil set varies over the total stroke. Switching among the coils that are active is allowed because the commutation method has forced the use of the coils that enter and leave the active set to zero.

### 2.2. Adding edge coils

If coils effected by the end effect of the magnet array (edge coils) are added to the active coil set, an error will occur in the commutation because the harmonic model is used. Simulation results obtained from a model which includes the end effects, indicate that this error is partly a multiplicative error, which is illustrated in Figure 3. This multiplicative error can be corrected by incorporating it in the weights of the commutation. Hence, making the weights based on physics rather than an arbitrary smooth function.

New denote the relation between coil currents and the wrench based upon the harmonic model by \( \Gamma_H \) and the multiplicative error for the coil currents by the diagonal edge matrix \( E \). We can express \( \Gamma_e \), the relation between coil currents and the wrench for active coils including edge coils as,

\[
\Gamma_e = \Gamma_H E.
\]

\( E(q) \) is obtained similar to \( \Delta(q) \), but with weighting function \( e(q) \) instead of \( \delta(q) \). Using the same pseudo inverse to calculate the commutation as in (3) gives,

\[
\Gamma_e^{-} = \Delta_e^{-} \left( \Gamma_e \Delta_e \Gamma_e^T \right)^{-1}.
\]

This defines the commutation of an active coil set including edge coils for a desired wrench vector \( \vec{w}_{des} \),

\[
\vec{i} = \Gamma_e^{-} \vec{w}_{des}.
\]

By introducing a new weighting function \( \Delta^* = E \Delta \Gamma^T \), the edge effects are taken into the weighting function. The calculation of the commutation with correction for the edge effects becomes similar to (3). This gives,

\[
\Gamma_{H_e}^{-} = \Delta^* \Gamma_{H_e} \left( \Gamma_{H_e} \Delta_e \Gamma_e^T \right)^{-1} = E \Gamma_H^{-},
\]

resulting in the relation,

\[
E \vec{i} = \Gamma_{H_e}^{-} \vec{w}_{des}.
\]

The weighting \( E \), \( \Gamma_e \) and \( \Gamma_{H_e} \) can be reduced similar to \( \Delta_r \), \( \Gamma_r \) and \( \Gamma^{-} \).

### 2.3. Fitting a function to the multiplicative error

Only a part of the model error can be seen as a multiplicative error separable in x and y. Moreover, this multiplicative model error is different for each component in \( \vec{y}(q) \). By correcting for the model error in the weighting of the generalised inverse \( \left( \Delta^* \right) \) all wrench components per coil are weighted equally. Therefore, the weighting \( E \) is fitted using an average direction independent measure.

The measurements of the contribution of one coil to the wrench vector acting on the translator are not accurate enough for making a fit of the multiplicative error. Therefore, the fit is made on data obtained from simulations. The simulations are performed with a model which approximates the magnetic field of the magnet array by modeling all the individual permanent magnets as magnetic surface charges and obtains the wrench contribution from the Lorentz forces acting on the coil. This model is presented in detail in [3]. Simulations over the full xy-stroke are given in [6].

To limit the number of parameters that have to be fitted, the weight function \( e(q) \) is chosen as a raised cosine filter, the length of the roll-off, \( L_p \) and the length of the pass band, \( 2L_p \) are both used as optimisation variables,

\[
e(q) = \begin{cases} 
1 & |q| \leq L_p \\
\frac{1}{2} + \frac{1}{2} \cos \left( \frac{\pi (q - L_p)}{L_w} \right) & L_p > |q| > L_w + L_p \\
0 & |q| \geq L_w + L_p \end{cases}
\]

where \( L_w \) is the length of the pass band. The final fit function, \( e(q) = e(q_1) e(q_2) \),

\[
e(q) = e(q_1) e(q_2),
\]
where \( q_i \) is the \( i \)th element of the vector \( q \). The cost, \( c_i \), for the optimisation is given by,

\[
c_i(q) = (\mathbf{f}^T(q) - e_{\text{in}},L_p(q)\mathbf{f}^T(q))e_{\text{in}},L_p(q)
\]  

(13)

Where \( \mathbf{f} \) is the \( i \)th component of the wrench vector per unit current obtained from simulations by the charge model, \( e_{\text{in}},L_p \) is obtained in a similar way from the harmonic model and \( e_{\text{in}},L_p \) denotes the weighting with parameters \( L_p \) and \( L_w \). One can see the criterion is weighted by \( e_{\text{in}},L_p \) that is fitted. This is because the more a current is penalised the less it is present in the commutation. Therefore, its error does not contribute as much as a less penalised current.

The optimisation is performed over discrete grid points. Therefore, \( c_i(q) \) can be represented in a matrix as, \( [C]_{ij} = c_i((x_i, y_k)^T) \), where \( (x_k, y_k) \) is the \((i,k)\)th grid point in a dense grid. The criterion for the fit is given by the sum over the Frobenius norm of all matrices \( C_i \),

\[
J(L_w,L_p) = \sum_{i=1}^{6} \|C_i(L_w,L_p)\|_{\text{Fro}}.
\]  

(14)

Choosing a different or higher order function shape (which can better fit the error at the edge) decreases the error, but increases the computation time of the real-time control system and, hence, limits the sample-time.

Optimisation over the criterion in (14) gives \( L_p = 106.9 \) and \( L_w = 60.3 \). Figure 4 depicts this weighting function.

![Figure 3: Force component \( F_z \) simulated over \( q_2 = 0 \) and \( |q_1| \leq 0.25 \) by a model which includes edge effects (—) and the harmonic model (— —).](image)

![Figure 4: Weighting function \( e(q_j) \) with \( L_p = 106.9 \) and \( L_w = 60.3 \).](image)

Extending the commutation to include all coils that contribute to the wrench introduces errors, due to previously stated issues with incorporating the modelling error in the weighting of the commutation. By choosing \( \Delta \) smaller than \( E \), the model error is decreased because the amount of coils affected by the edge effect that are added to the active coils set is smaller. The selection of \( \Delta \) is an engineering process where a compromise has to be found between the amount of model error that is allowed by elongating \( \Delta \) and the decrease in dissipation and maximum coil current that is desired. Current implementation uses a \( \Delta \) that has the same function form given in (11), but has different parameter values, \( L_\Delta p = L_\Delta P \) and \( L_\Delta w = 17 \) mm. Increasing the length of \( \Delta \) will result in a lower power dissipation, but will also increase the error in the commutation. This is illustrated in Figure 5 for a cross section of the full \( xy \)-stroke (\( y = 0 \)), for a desired wrench of \([-8.25, 8.25, 8.25, 0, 0, 0]^T \) [N,N,m]. Three cases are shown. Firstly, the extended commutation with the multiplicative error reduction weighting \( E \). Secondly, the commutation of the same amount of coils but without the weighting \( E \). Thirdly, the extended commutation with weighting \( E \) and a weighting \( \Delta \) using \( L_\Delta w = 35 \) mm instead of \( L_\Delta w = 17 \) mm.

3. Simulations

The wrench can not directly be measured on the actuator. Open-loop errors in the commutation can only be observed in simulations. These errors will influence the tracking error of the controller. The currents produced by the commutation are used in the surface charge based model of the actuator to predict the resulting wrench. Based on these simulations the length of \( \Delta \) is determined.

The predicted coil currents are used to estimate the ohmic losses of the planar actuator. In Figure 6, simulations for a wrench \( \mathbf{w} = [0, 0, 0, 0, 0]^T \) with \( F_z = mg = 80 \) N (\( m = 8.225 \) kg) are given over the total \( xy \)-stroke. Because the active coil set is reduced when the translator partly exceeds the stator edge, both simulations show a higher dissipation near the edge of the \( xy \)-range. The extended commutation has a more smooth power dissipation, and the highest current value over the total stroke is reduced compared to the commutation without edge coils. The mean value of the power dissipation is decreased by 23.25%.

![Figure 5: Resulting force \( F_z \) for the commutation with ( | ) and without ( — — ) multiplicative error reduction (using \( L_\Delta w = 17 \)mm) and with a wider weighting \( \Delta \) \( (L_\Delta w = 35 \)mm) (— —).](image)

4. Measurements

Open-loop characteristics have been obtained by levitating the carrier at a constant position, thus having to excite a constant force in the \( z \)-direction. As the controller is an integrating controller, the tracking error will go to zero, resulting in an almost constant controller output. The mass is known, hence, the gravitation force is also known. Comparing this force to the output of the controller for \( F_z \) gives the error in the commutation. In Figure 8 the measurement over the region indicated in Figure 7 is shown. A trend can be seen in both measurement errors, which is observed in all measurements performed on this setup, and is caused by misalignment of the measurement frame, and the actuator. The rms value of the detrended errors in \( F_z \) are 0.397N for the improved commutation and 0.247N for the unmodified commutation (defined in [4]). The error variance is

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higher for the extended commutation, but the maximum of the absolute error, compared to an excitation of 80.6N, is still small, 2.04N for the improved commutation and 1.73N for the original.

Figure 6: Predicted ohmic losses for the full xy-stroke, with an airgap of 1.5mm.

Figure 7: The corner point positions of the measured area enclosed by the white line. The dark coils are fully used in the commutation, the light ones are not and the medium tones are weighted.

5. Conclusion

In this paper a model based method to include the end effects of the magnet array in the commutation of a magnetically levitated planar actuator with moving magnets has been presented. The end effects have been included in the commutation algorithm by modeling them as a multiplicative error. They have been corrected for by an extra weighting in the generalised inverse.

Simulations have confirmed the decrease in error by including the end effects in the commutation algorithm. The use of more coils in the commutation shows also a significant decrease in power dissipation. The measurements verify the error in the commutation can be suppressed by the controller.

Expanding the commutation with the coils at the edges of the translator will improve the actuator performance. Furthermore, it increases the feasibility region for the commutation algorithm, allowing for a more energy efficient solution. The maximum power and the maximum coil current for a given acceleration profile will decrease and, therefore, higher accelerations can be attained with the same amplifiers. Moreover, it creates more freedom in the design of a new actuator.

References