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Voltage control in distribution networks using fast model predictive control

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Abstract—Voltage control is a growing issue in distribution networks operation, due to the increase in load and also distributed generation. In this work model predictive control (MPC) is proposed as a better solution as compared to conventional ones to deal with this problem. A fast MPC algorithm is used to allow online optimization of voltage profile. the algorithm is tested on a 20 kV distribution network.

Index Terms—voltage control, distribution networks, model predictive control.

I. INTRODUCTION

The expected change in the use and design of the electric power system is urging for new control and operation strategies of the networks. The main cause for this is the massive introduction of renewable energy sources leading to large amounts of distributed generation in the grid. It has been shown that distributed generation (DG) has some challenging impacts on the grid. Among these are voltage variations, power quality and protection issues [1], [2]. These generation units are connected most of the time to the distribution (MV) due to their relatively smaller size when compared to conventional power production units [3]. As a result, distribution networks are becoming active networks, having power flows in both directions. This ultimately means that new control applications are needed in these networks in order to ensure a proper and reliable operation of the distribution grid [4]. New control means that control strategies that have been used in the past to control the transmission grids, are not necessarily and exactly applicable for distribution grids. This is particularly true in the case of voltage control, where conventionally reactive power was used. But this might not be efficient in distribution grids, due to the difference in the impedance of the two networks.

Traditionally, power systems were controlled using linear controllers, based on a simplified, linearized models [5]. These controllers are easy to design, but have limited performances. This performance limitation is mostly seen when the operating conditions in the power systems are changing, and the controller have been tuned in the neighborhood of a predefined operating point [6].

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Today however, power electronics is offering more sophisticated control algorithms to be implemented. Constrained optimal control is one class of modern control theory that could be used to deal with the voltage control problem in distribution networks. If an online optimization process of the network is feasible, then the result would be a much better control quality compared to linear controllers. Starting from this motivation, a novel voltage control methodology based on online optimization of the voltage at the point of common coupling is introduced in this paper. Specifically, we demonstrate how model predictive control (MPC) [7] can be applied to achieve a faster control, and also a control whose performance is not altered by a change in the operating conditions of the power network.

This paper is organized as follows, in section II, the voltage control problem in distribution networks is discussed. In section III the general theory of Model Predictive Control, and a fast MPC algorithm are presented. In section IV, a case study based on model predictive is given. A conclusion and the future research are presented at the end of this paper.

II. VOLTAGE CONTROL PROBLEM

Voltage control is traditionally done via reactive power control, using synchronous generators, capacitors and recently FACTS devices. Reactive power was used from these devices as the only control parameter. Voltage control strategies in distribution networks are based on changing substation transformer tap changers, besides using reactive power compensation devices [8], [9], and possibly, using active and reactive power from DGs. However, controlling voltage through only reactive power may not be efficient because of the low X/R ratio of distribution networks, compared to transmission networks.

One way of dealing with this problem, is to do an online estimation of the X/R ratio and then to use the so-called modified active and reactive power to determine the amounts of active and reactive power to be injected into the grid [10]. The active and reactive power needed for voltage control in the distribution networks is nothing more than the losses in the network. So because the X/R is comparable to one, the active and reactive losses have the same order of magnitude. Another way of solving this problem is by using a full model of the network to be controlled. Combining this full model with a constrained control strategy is potentially very effective in achieving the aforementioned control objective.

III. MODEL PREDICTIVE CONTROL

A. General theory

Standard MPC (also referred to as moving horizon or receding horizon control) is essentially a class of computer control algorithms to control the future behavior of a system through the use of an explicit model of the latter [11] [12]. At each control step the MPC algorithm computes an open-loop sequence of controls in order to optimize the future system behavior.

The optimal policy is found by minimizing the cost J given by:

$$J(x, u) = \sum_{j=k}^T x_{j|k}^T Q x_{j|k} + u_{j|k}^T R u_{j|k} \quad (1)$$

s.t.

$$x_{\min} \leq x_{j+1|k} \leq x_{\max}, k \leq j$$

$$u_{\min} \leq u_{j|k} \leq u_{\max}, k \leq j$$

$$x(k+1) = Ax(k) + Bu(k) + d(k)$$

With variables

$$x(k+1), \dots, x(k+T-1), u(k), \dots, u(k+T-1),$$

T is a parameter, called the time horizon.

If $u^*(k), \dots, u^*(k+T-1), x^*(k+1), \dots, x^*(k+T-1)$, is the optimal solution for the problem (1), then at each time instant k the MPC strategy takes $u(k) = u^*(k)$.

Model predictive control performs then an online optimization process to find the optimal policy to be applied. However, in the case where the system is to be controlled has a rather high dimension (more than 10 states), the optimization process require high computational time and may be very slow.

B. Fast Model Predictive control

Referring to the work of Boyd *et al* [13], a fast MPC algorithm is introduced to in the aim of reducing the computational time needed in the online optimization process. The algorithm is particularly interesting as it is capable of handling systems of higher orders.

The QP problem defined in (1) is rewritten in a compact form, where a new variable z is introduced:

$$z = (u(k), x(k+1), u(k+1), \dots, x(k+T-1), u(k+T-1))$$

And the QP becomes:

$$\begin{aligned} & \text{minimize } z^T H z \\ & \text{subject } Pz \leq h, \quad Cz = b \end{aligned} \quad (2)$$

Where

$$H = \begin{bmatrix} R & 0 & \dots & 0 & 0 \\ 0 & Q & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & Q & 0 \\ 0 & 0 & \dots & 0 & R \end{bmatrix}, P = \begin{bmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \\ -1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & -1 \end{bmatrix}, h = \begin{bmatrix} u_{\min} \\ x_{\min} \\ \vdots \\ -u_{\max} \\ -x_{\max} \end{bmatrix}$$

$$C = \begin{bmatrix} -B & I & 0 & 0 & \dots & 0 & 0 \\ 0 & -A & -B & I & \dots & 0 & 0 \\ 0 & 0 & 0 & -A & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & I & 0 \\ 0 & 0 & 0 & 0 & \dots & -A & B \end{bmatrix}, b = \begin{bmatrix} Ax(k) + \bar{d} \\ \bar{d} \\ \bar{d} \\ \vdots \\ \bar{d} \\ \bar{d} \end{bmatrix}$$

Where $\bar{d}$ represents the value of the disturbance at each sampling period and is assumed to be constant during the prediction horizon.

In order to solve the QP defined in (2), the so called *Primal barrier method* [14] is used. The inequality constraints in QP (2) are replaced by a barrier term which is then inserted in the objective function.

The problem is then approximated as follows:

$$\begin{aligned} & \text{minimize } z^T H z + \kappa \phi(z) \\ & \text{subject } Cz = b \end{aligned} \quad (3)$$

Where $\kappa > 0$ is a barrier parameter and ϕ is the log barrier associated with the inequality constraints, given by:

$$\phi(z) = \sum_{i=1}^{IT} -\log(h_i - p_i^T z), \quad (4)$$

where $p_1^T \dots p_{IT}^T$ are the rows of P .

Problem (3) is an equality constrained QP and can be solved by the Newton method. As mentioned earlier, if using a conventional Newton method, like interior point method, the calculation time may prohibit the use of Model predictive control for online applications.

The special structure of the QP involved in MPC could be exploited to speed up to the optimization process [13]. This approach is based on the infeasible start Newton method, where the initial point can be chosen outside the feasible set (equality constraint) [14].

A dual variable v is associated with the equality constraint $Cz = b$. The optimality condition for (3) is then:

$$\begin{aligned} r_d &= 2Hz + g + \kappa P^T d + C^T v = 0 \\ r_p &= Cz - b = 0 \end{aligned} \quad (5)$$

where $d_i = 1/(h_i - p_i^T z)$ and p_i^T represents the i th row of P . The term $\kappa P^T d$ is the derivative of $\kappa \phi(z)$. r_p is called the primal residual, and r_d is called the dual residual. The residual vector $r = [r_d^T \ r_p^T]^T$ is then used to express the optimality condition, $r = 0$, for the problem (3).

Applying linearization theory on the optimality conditions (5), the Newton step equations are given by:

$$\begin{bmatrix} 2H + \kappa P^T(d)^2 P & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \Delta z \\ \Delta v \end{bmatrix} = - \begin{bmatrix} r_d \\ r_p \end{bmatrix} \quad (6)$$

Once the steps Δz and Δv are found, the step size $s \in [0,1]$ is found using a backtracking line search on the norm of the residual r , while maintaining $Pz < h$ for each updated point

[13]. Once the step size is found, the primal and dual variables are then updated by:

$$z = z + s\Delta z, \quad v = v + s\Delta v$$

The algorithm terminates when the residual r is below a predefined limit. The matrix $2H + \kappa P^T(d)^2 P$ is block diagonal and therefore block elimination can be applied to solve problem (6). Block elimination proceeds first by finding the Schur complement $Y = C\Phi^{-1}C^T$ and $\beta = -r_p + C\Phi^{-1}r_d$, and then solving $Y\Delta v = -\beta$ and $\Phi\Delta z = -r_d - C^T v$, to get Δv and Δz respectively.

IV. CASE STUDY: VOLTAGE CONTROL IN DISTRIBUTION NETWORKS

The case study under consideration is a 20kV distribution network depicted in Fig. 1. It consists of a two feeder network, having 7 nodes. The impedance between each two nodes in this network is assumed to be the same, with 10 km length. The controller is a PWM converter, having an active power source. The active power source is added because, as mentioned in the introduction, the low X/R ratio of distribution grids.

Fig. 1. Network under study

Voltage rating: 20 kV

Converter rating: 1 MW

Line section impedance: $R=1.5\Omega$, $X=0.3mH$

$R_{Load1} = R_{Load5} = 120 \Omega$, $R_{Load2} = R_{Load6} = 150\Omega$

$R_{Load3} = R_{Load4} = 180\Omega$, $X_{Load4} = 200mH$

A. Modeling and problem formulation

A detailed modeling approach is adopted to model the network under study. The state space model is given by:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Fd(k) \\ y(k+1) &= Cx(k+1) \end{aligned} \quad (7)$$

Where $x(k)$ represents the currents in each line section of the feeder and also, the currents injected by the converter, and y represents the voltage to be controlled (voltage at node 7).

u , the control input, represents the voltage to be applied by the converter. d the disturbance, represents the voltage at the substation, and is assumed to be known at each sampling period.

All the network elements are modeled in DQ frame, with a synchronously (50 Hz) rotating reference frame. This is so to have DC values which are easier to handle, than AC values,

and also, to have a reduced number of state variables (zero sequence currents and voltages are equal to zero), compared with three phase system.

This model (7), is reformulated such that the fast MPC algorithm described earlier can be applied. The model is augmented by three more state variables: the reference $r(k)$, the disturbance $d(k)$, and the integrated error $v(k)$.

$$\begin{aligned} \begin{bmatrix} x(k+1) \\ r(k+1) \\ d(k+1) \\ v(k+1) \end{bmatrix} &= \begin{bmatrix} A & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ -CA & -CF & I & I \end{bmatrix} \begin{bmatrix} x(k) \\ r(k) \\ d(k) \\ v(k) \end{bmatrix} \\ &+ \begin{bmatrix} B \\ 0 \\ 0 \\ -CB \end{bmatrix} u(k) \end{aligned} \quad (8)$$

The output of the augmented system is taken to be the integrated error. The state weight matrix Q in the MPC formulation is chosen in such a way to minimize the integrated error $v(k)$. In this application, the constraints are imposed on the voltage at the inverter input (before the filter) and also the current injected by the latter.

B. Simulations

Simulations were carried out on Matlab 7.9.0. machine. The Fast MPC algorithm was written under C code, and then converted to MEX file to be used by Matlab. To cover all possible contingencies that can happen in a power system operation, four scenarios were evaluated. Four models were created, where each model represents one scenario.

a) Full model

This model represents the base case, the full model as represented in Fig. 1. The voltage drop at bus at bus 7 is 0.75 pu, which is considered as the worst case scenario in the network under study.

b) Scenario 1: Voltage dip at substation

A voltage dip of 1s representing a short circuit at the higher voltage side is simulated. The simulation results are given in Fig. 2.

Fig. 2. Controlled voltage, injected currents and inverter voltage for scenario 1.

c) Scenario 2: Loss of a feeder

The feeder connected at bus 2, is disconnected at time $t = 2$ s. The simulation results are given in Fig. 3.

Fig. 3. Controlled voltage, injected currents and inverter voltage for scenario 2.

d) Scenario 3: Sudden disconnection of a load

A voltage dip of 1s representing a short circuit at the higher voltage side is simulated. The simulation results are given in figure 2.

C. Discussion

From the simulation results, the proposed control strategy works well under different operating conditions of the grid. The sampling period is 1ms. In the three scenarios, the control is so fast, this is because of the fast MPC algorithm, i.e., the optimization of the error $e = V - V_{ref}$ is quiet efficient. The currents injected by the inverter have the same order of magnitude, this means that the active and reactive power injected have comparable values. This is in line with the fact that X/R ratio on medium voltage grids is the around one.

Fig. 4. Controlled voltage, injected currents and inverter voltage for scenario 3.

V. CONCLUSION AND FUTURE WORK

In this paper we have considered a special type of optimal constrained control for the voltage control problem in the distribution networks. Particularly, it was demonstrated that model predictive control using an algorithm allowing online optimization to be applied for systems with higher dimensions yields satisfactory results. Several operational scenarios were tested, and simulations illustrate the potential of this approach. Future research will focus on extending the test network, in terms of size (more feeders and loads), and also generators like the Doubly Fed Induction Generator (DFIG), to deal with the case of voltage rise caused by these generators. In this work, the time at which each contingency was known beforehand; it would be more realistic to automate the process of contingency detection. This could be achieved using

Bayesian theory, where each model is assigned a weight based on the output of the real system measurements.

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VII. BIOGRAPHIES

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Wil L. Kling received the M.Sc. degree in electrical engineering from the Eindhoven University of Technology, The Netherlands, in 1978. From 1978 to 1983 he worked with Kema and from 1983 to 1998 with Sep. Since then he was with TenneT, the Dutch Transmission System Operator, as senior engineer for network planning and network strategy. Since 1993 he is a part-time Professor at the Delft University of Technology and since 2000 he is also a part-time Professor in the Electric Power Systems Group at the Eindhoven University of Technology, The Netherlands. From December 2008 he is appointed as a full-time professor and as chair of EPS group at the Eindhoven University of Technology. He is leading research programs on distributed generation, integration of wind power, network concepts and reliability. Prof. Kling is involved in scientific organizations such as Cigre and IEEE. He is the Dutch Representative in the Cigre Study Committee C6 Distribution Systems and Dispersed Generation.