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Citation for published version (APA):

Aartrijk, van, M., & Clercx, H. J. H. (2008). Preferential concentration of heavy particles in stably stratified turbulence. *Physical Review Letters*, 100(25), 254501-1/4. [254501].
<https://doi.org/10.1103/PhysRevLett.100.254501>

DOI:

[10.1103/PhysRevLett.100.254501](https://doi.org/10.1103/PhysRevLett.100.254501)

Document status and date:

Published: 01/01/2008

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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- The final published version features the final layout of the paper including the volume, issue and page numbers.

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Preferential Concentration of Heavy Particles in Stably Stratified Turbulence

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(Received 11 December 2007; published 26 June 2008)

The effect of preferential concentration of heavy particles in a homogeneous stably stratified turbulent flow is studied by means of direct numerical simulations. Particle distributions show different clustering patterns in horizontal and vertical directions, thereby representing the anisotropy of the flow. Preferential concentration in stably stratified turbulence can be quantified using 2D and 3D radial distribution functions and the correlation dimension $\mathcal{D}_2$. With increasing stratification strength, the effect of preferential concentration decreases. Furthermore, it is found that in stably stratified turbulence preferential accumulation is enhanced when gravitational forces act on heavy particles.

DOI: 10.1103/PhysRevLett.100.254501

PACS numbers: 47.55.-t

Dispersion of particles plays an important role in both industrial processes and geophysical environments. Turbulence is often considered to be a good particle mixer. However, for inertial particles in a turbulent flow local particle accumulation is observed, a phenomenon that is called preferential concentration. A simplified explanation for this effect is that particles that are heavier than the surrounding fluid are transported out of the cores of vortical structures, due to centrifugal forces. This leads to highly nonuniform particle distributions, where particles collect in regions of high strain rate and low vorticity [1]. A more detailed explanation is given by Goto and Vassilicos [2], who introduce the sweep-stick mechanism. The local high and low particle concentrations can have an enormous influence on mixing and clustering and can affect, for example, rain initiation in clouds [3].

The effect of local particle accumulation was first observed by Maxey and Corrsin [4] in cellular flow fields. A clear overview of the effect of preferential concentration in several types of flows is given by Eaton and Fessler [1]. The topic is elaborately studied in isotropic turbulence (2D and 3D), theoretically, numerically, and experimentally [2,5–11]. In this work we study the effect of small-scale preferential concentration of heavy particles in homogeneous stably stratified turbulence. Turbulent flows displaying stable density stratification are often encountered in nature, for example, the nocturnal atmospheric boundary layer, coastal areas, and lakes. In stably stratified flows a negative vertical density gradient is present; the average density of the fluid is decreasing with height. Strongly stratified flows consist of two types of motions: gravity waves and nonpropagating quasihorizontal vortical motions [12]. These flows typically display a layered structure with strong shearing between these layers [13]. The nonlinear vortex mode is responsible for this anisotropy. The linear wave mode acts in the vertical and causes suppressed vertical particle dispersion, fluid particles remain within a layer around their equilibrium height [12,14].

This study is performed by means of direct numerical simulations. A pseudospectral code is used that solves the full Navier-Stokes equations with Boussinesq approximation on a triple-periodic domain [14,15]. In a precomputation a divergence-free homogeneous isotropic turbulent initial velocity field is created. At $t = 0$ a linear stable background density stratification is switched on which is kept constant throughout the simulation. Density fluctuations are present on top of the linear profile. The total density is given by $\rho_f = \rho_0 + \bar{\rho}(z) + \rho'(x, y, z, t)$ with ρ_0 a reference value, $\bar{\rho}$ the time-independent linear background profile, and ρ' the fluctuations. The relative importance of the stratification can be expressed by the Froude number $Fr = u'/(L_h N)$, with u' the rms velocity, L_h the horizontal integral length scale, and $N^2 = -\frac{g}{\rho_0} \frac{\partial \bar{\rho}}{\partial z}$ the buoyancy frequency. Here, g is the gravitational acceleration. A homogeneously stratified turbulent flow will develop and large scale forcing is applied in the horizontal direction (velocity components u and v , with vertical wave mode $k_z = 0$) to reach a quasisteady state [16]. Some properties of the flow are given in Table I; an elaborate description of the resulting flow plus a justification of the forcing method can be found in Ref. [14].

TABLE I. Three different flows are studied. One isotropic (case N0) and two with moderate (N10) and strong (N100) stratification. The ratio of the horizontal and vertical length scale L_z/L_h gives an impression of the anisotropy of the flow. Note that although turbulence is suppressed by stable stratification, $Re_\lambda = u'\lambda/\nu$ (λ the Taylor microscale) increases with increasing N due to an increase of the horizontal length scales.

Case	N (s^{-1})	Fr	M^3	$k_{\max}\eta$	L_z/L_h	Re_λ
N0	0	...	128 ³	1.13	1.0	85
N0_256	0	...	256 ³	1.16	0.98	135
N10	0.31	0.11	128 ³	1.53	0.16	100
N100	0.98	0.04	128 ³	1.27	0.08	170
N100_256	0.98	0.06	256 ³	1.41	0.11	200

When a statistically stationary flow field is obtained, particles are released at random positions in the domain. Next, their trajectories are calculated according to $\frac{d\mathbf{x}_p(t)}{dt} = \mathbf{u}_p(t)$ together with

$$\frac{d\mathbf{u}_p(t)}{dt} = \frac{1}{\tau_p}(\mathbf{u}(\mathbf{x}_p, t) - \mathbf{u}_p(t) - w_s \hat{z}). \quad (1)$$

Herein is $\mathbf{x}_p$ the particle position, $\mathbf{u}_p$ the inertial particle velocity, $\mathbf{u}$ the fluid velocity at the position of the particle (derived from the velocity field with use of cubic spline interpolation), and w_s the particle settling velocity in quiescent fluid. The particle response time is $\tau_p = \frac{d_p \rho_p / \rho_0}{18\nu}$, with d_p and ρ_p the particle diameter and density, respectively, and ν the kinematic viscosity. Equation (1) is a simplified version of the Maxey-Riley equation [17] in the limit of small ($d_p < \eta$, η the Kolmogorov length scale) and heavy ($\rho_p \gg \rho_f$) rigid spherical particles with particle Reynolds number $\text{Re}_p = \frac{d_p |\mathbf{u} - \mathbf{u}_p|}{\nu} \ll 1$. It takes into account drag and gravitational forces. The particle settling velocity resulting from gravitational forces is given by $w_s = \tau_p g'$ [7] and will be expressed by $W = w_s / w'$ with w' the rms velocity of the fluid in the vertical direction. A reduced gravitational acceleration g' acting on the particle is introduced to be able to study particles with $\text{St} = \mathcal{O}(1)$ and $W = \mathcal{O}(1)$ simultaneously. The strength of the gravitational forces is denoted by $g^* = g'/g$. Particle-particle interactions and influence of the particles on the flow field are assumed negligible, since we are using small particles ($d_p < \eta$) and low particle volume fractions.

Three different flows are studied: one homogeneous isotropic turbulent flow and two stably stratified flows (one moderately and one strongly stratified). For different particle response times 20 000 particles are tracked in each flow for about 40 eddy turnover times $T_E = L_h / u'$. In the following the particle response time will be expressed using the Stokes number $\text{St} = \tau_p / \tau_K$, with τ_K the Kolmogorov time scale. The range of investigated Stokes numbers is $\mathcal{O}(0.1) - \mathcal{O}(10)$. Both St and W are functions of τ_p and they are related as $W = f(g', \text{St})$. For illustration purposes also some short runs with 10^6 identical particles are performed. The initial velocities of the particles are set equal to the local fluid velocity and after about two T_E the particles are completely adapted and reach a quasisteady distribution.

Qualitatively, the effect of preferential concentration can be seen in a snapshot of the particle positions at a single time step, as shown in Fig. 1(a) for isotropic turbulence. Clearly, the particles are not distributed uniformly over the domain; strong local accumulation is observed. Similar results were found by Refs. [7,10]. For stratified turbulence the particle distribution looks completely different. In Figs. 2(a) and 2(b) the results are shown from a simulation without gravitational forces acting on the particles. The rationale for setting $g' = 0$ in Eq. (1) is to study first the

FIG. 1. (a) Particle positions in a vertical cross section of the domain for case N0, from a run with 10^6 particles with $\text{St} = 0.96$. (b) Mean absolute value of the vorticity ω as a function of the number of particles per bin (bin size 2.5η) for cases N0 and N100 as shown in (a) and Figs. 2(a) and 2(b).

pure effect of inertia on preferential concentration. Also in stratified turbulence preferential concentration is found, but a clear distinction needs to be made between the horizontal and the vertical direction. The particle distribution reflects the anisotropy of the flow. In the horizontal direction particles cluster on larger scales than in isotropic turbulence, whereas in the vertical direction thin, sheared layers are observed. The particle positions are correlated with the vorticity ω of the flow. As can be seen in Fig. 1(b), the average value of ω in regions where large amounts of particles are found is smaller than in void regions.

In order to quantify the effect of preferential concentration multiple methods exist [10,18,19]. Here we will use

FIG. 2. Particle distributions in horizontal (a),(c) and vertical (b),(d) cross sections for case N100, from runs with 10^6 particles. (a),(b) $\text{St} = 3.1$ and $W = 0$ (no gravity acting on the particles); (c),(d) $\text{St} = 8.1$ and $W = 8.4$.

the correlation dimension $\mathcal{D}_2$ as used by Bec *et al.* [10] and the equivalent radial distribution function as introduced by Sundaram and Collins [18]. The radial distribution function (RDF) gives an impression at what length scales the effect of preferential concentration takes place. It gives the ratio of the number of particle pairs found at a given separation distance to the expected number of pairs if the particles are uniformly distributed [20]. The three-dimensional RDF is defined as

$$g_{3D}(r_i) = \frac{P_i/V_i}{P/V}, \quad (2)$$

where P_i is the number of pairs within a separation distance between $r_i - \Delta r/2$ and $r_i + \Delta r/2$, $P = N_p(N_p - 1)/2$ is the total number of particle pairs, V is the total volume of the domain, and $V_i = (4/3)\pi[(r_i + \Delta r/2)^3 - (r_i - \Delta r/2)^3]$ is the volume of a shell with thickness Δr and nominal separation radius r_i . For a uniform particle distribution $g_{3D} = 1$. The size of the particle clusters can be estimated by looking at the length scale where the RDF becomes less than unity [21]. In order to measure the anisotropy of the particle distribution, a two-dimensional RDF is computed according to $g_{2D}(r_i) = (P_i/A_i)/(P/A)$ [21] using planar slices with thickness $\delta/\mathcal{L} = 0.005$ ($\mathcal{L}$ is the domain size). The total area of a planar slice is denoted by A and $A_i = \pi[(r_i + \Delta r/2)^2 - (r_i - \Delta r/2)^2]$ is the area of the shell associated with the nominal separation distance r_i within the plane.

For isotropic turbulence (case N0, results not shown) the 2D and 3D RDFs are the same in the studied range of r . The point of intersection $g_{3D} = 1$ shifts towards larger scales with increasing St . This is expected, as particles with higher St are less sensible for smaller eddies. For $St = 0.96$ [cf. Fig. 1(a)] this intersection occurs around $r/\eta = 20$, corresponding to structures with a size of about $1/20\mathcal{L}$. For isotropic turbulence we found the largest values for g_{3D} , and thus maximum preferential concentration, for $St \approx 1$.

For case N100 without gravitational forces working on the particles the 2D and 3D RDFs are plotted in Fig. 3 for $St = 3.1$ and $St = 8.1$. The form of the 3D RDFs is similar to those found for isotropic turbulence, though the values are smaller for a given Stokes number. The plot for $St = 3.1$ shows the behavior around maximum preferential concentration for case N100. For both smaller and larger St the values of the 2D and 3D RDFs are smaller. Very small St particles resemble fluid particles and follow the flow without clustering. Larger St particles are less sensible for the smallest scales in the flow and therefore the decrease of the RDF is mainly visible for small values of r/η , as can be seen by comparing the results for $St = 3.1$ and $St = 8.1$ in Fig. 3. For stratified turbulence, the difference between preferential concentration in horizontal and vertical cross sections as seen in Fig. 2 is quantified with the 2D RDF. It can be seen in Fig. 3 that at the smallest scales the effect of

FIG. 3. 2D and 3D radial distribution functions for case N100 with $W = 0$: (a) $St = 3.1$, (b) $St = 8.1$. Note the different range of the vertical axes in both figures.

preferential concentration is stronger in vertical slices, whereas for larger scales it is clearly stronger in horizontal slices. The 3D behavior seems to be determined mainly by the vertical distance between particles. The point of intersection $g_{3D} = 1$ for $St = 3.1$ lies around $r/\eta = 20$. For vertical separation distances the intersection $g_{2D} = 1$ occurs at slightly smaller values, but for horizontal separation distances it is about $r/\eta = 35$.

To characterize the spatial distribution of particles in clusters with sizes of $\mathcal{O}(\eta)$, the correlation dimension $\mathcal{D}_2$ is computed. When the particles are distributed uniformly, $\mathcal{D}_2$ equals the space dimension d ($d = 3$ in our case). Deviation from a uniform distribution yields $\mathcal{D}_2 < d$ and indicates preferential concentration. $\mathcal{D}_2$ is obtained from the slope of the cumulative RDF and it is estimated through the small-scale algebraic behavior of the probability to find two particles at a distance less than a given r : $P_2(r) \sim r^{\mathcal{D}_2}$ [10]. The dependence of $\mathcal{D}_2$ on St and N is shown in Fig. 4(a). A distinction is made between the results derived from simulations with resolutions of $M^3 = 128^3$ and $M^3 = 256^3$. They follow the same pattern and it can thus be concluded that the sensitivity to changes of Re_λ is weak, which was previously found for isotropic turbulence [10,19,20]. The results found for case N0 are in agreement

FIG. 4. (a) Correlation dimension $\mathcal{D}_2$ as a function of St for three different stratification strengths. A distinction is made between results derived from 128^3 and 256^3 runs. (b) $\mathcal{D}_2$ as a function of St for case N100 with different values of g^* . The error in computing $\mathcal{D}_2$ from a fit of $P_2(r) \sim r^{\mathcal{D}_2}$ to the slope of the cumulative 3D RDF is about 1.5%.

with the results by Bec *et al.* [10]. The effect of preferential concentration peaks around $St = 1$ with a corresponding value of $\mathcal{D}_2 \approx 2.3$ and $\mathcal{D}_2 \uparrow 3$ for $St \rightarrow 0$ and $St \rightarrow \infty$. Looking at the values of $\mathcal{D}_2$ for cases N10 and N100 [Fig. 4(a)], it is seen that with increasing stratification the minimum value of $\mathcal{D}_2$ increases, indicating that the effect of preferential concentration decreases. Furthermore, this minimum shifts to a higher St number. The decrease of preferential concentration with increasing N is related to the intensity of the vortical structures in the flow. We found that when N increases the vorticity ω becomes less intense.

Since in stably stratified turbulence gravity acts on the fluid, it is relevant to study the effect of gravitational forces on the particles too [last term on the right-hand side in Eq. (1)]. Contrary to what happens in isotropic turbulence [7], here we will show that a net particle settling velocity enhances the effect of preferential concentration in stably stratified turbulence. Qualitatively this can be seen in Figs. 2(c) and 2(d) where the particle positions are plotted in a horizontal and vertical cross section of the domain. In vertical direction it can be seen that particles collect on paths while moving on average in negative vertical direction. Particles are transported out of the large horizontal vortical structures and while sinking they form zigzag routes in between these structures [see, for example, left-most path in Fig. 2(d)]. These localized vertical paths result in stronger preferential concentration in horizontal slices too. In the horizontal direction the effect of preferential concentration is visibly stronger than in Fig. 2(a). Although the void regions are slightly smaller than in Fig. 2(a), the particles cluster in sharper streaks.

Quantitatively the enhanced effect of preferential concentration means that with increasing g^* the values of the RDFs increase, even though the points of intersection $g_{3D} = 1$ and $g_{2D} = 1$ remain more or less the same for all g^* in the studied range. The distinction between horizontal and vertical 2D RDFs reduces. The values of $\mathcal{D}_2$ decrease with increasing g^* , as can be seen in Fig. 4(b). For $g^* = 2.0 \times 10^{-5}$ this effect is only visible for the largest studied Stokes numbers, where $W > 1$. For larger g^* ($g^* = 1.6 \times 10^{-4}$) lower values of $\mathcal{D}_2$ compared to $g^* = 0$ are already found for smaller St . It can be concluded that enhanced preferential concentration is only found for large sinking velocities; up to about $W = 1$ no effect of gravity on particle accumulation is observed.

In some of the runs (mainly the high τ_p runs), some of the particles had particle Reynolds numbers of order 1. As the requirement $Re_p \ll 1$ is not fulfilled here, a separate run is performed where nonlinear drag effects are taken into account. The first two terms on the right in Eq. (1) are replaced by $\frac{1}{\tau_p}(1 + 0.15Re_p^{0.687})(\mathbf{u}(\mathbf{x}_p, t) - \mathbf{u}_p(t))$ [7]. The influence of this nonlinear drag on preferential concentration in stratified turbulence as expressed by the 2D and 3D RDFs is found to be negligible.

In conclusion, we have shown that the effect of preferential concentration in stably stratified turbulence decreases with increasing stratification. Differences are exhibited between the clustering in the horizontal and the vertical direction, that can be quantified using the two-dimensional radial distribution function. Besides, when gravitational forces are taken into account for the particle motion, preferential accumulation of particles is enhanced in both the horizontal and the vertical direction.

This program is funded by the Netherlands Organisation for Scientific Research (NWO) and Technology Foundation (STW) under Grant No. ESF.6239 and sponsored by NWO-NCF for the use of supercomputer facilities.

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