Microstructure characterisation and homogenisation of acoustic polyurethane foams: measurements and simulations
Gao, K.; van Dommelen, J.A.W.; Geers, M.G.D.

Published in:
International Journal of Solids and Structures

DOI:
10.1016/j.ijsolstr.2016.09.024

Published: 01/01/2016

Document Version
Author's version before peer-review

Please check the document version of this publication:

• A submitted manuscript is the author's version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

Citation for published version (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.
• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal?

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Download date: 15. Dec. 2018
Microstructure characterization and homogenization of acoustic polyurethane foams: measurements and simulations

K. Gao\textsuperscript{a}, J.A.W. van Dommelen\textsuperscript{a,*}, M.G.D. Geers\textsuperscript{a}

\textsuperscript{a}Department of Mechanical Engineering, Eindhoven University of Technology, Eindhoven, The Netherlands

Abstract

The sound absorption ability of porous materials is strongly related to the underlying microstructure. In this paper, acoustic properties of a polyurethane (PU) foam are determined from its microstructure with a computational homogenization method. The foam is analyzed using X-ray computed tomography (CT) and scanning electron microscopy (SEM). Based on the obtained microstructure information, a parallel model of a fully-open and a partially-open Kelvin cell with thin membranes is built to represent the foam. The corresponding effective material parameters, including the dynamic density and the stiffness tensor, are obtained by applying a computational homogenization approach. Numerical simulations of an impedance tube test based on Biot’s equations with parameters obtained from the homogenization are compared with the measured sound absorption coefficients. Considering the limitations of the simplified microscopic model, a good agreement between the measurements and the simulation results for the PU foam is found.

*Corresponding author

Email address: J.A.W.v.Dommelen@tue.nl (J.A.W. van Dommelen)

1. Introduction

Acoustic foams are widely used for sound absorption purposes, whereby their performance strongly depends on the underlying complex microstructure. The sound propagation problem in porous materials can be solved using multiscale models. For example, the asymptotic homogenization method has been applied to a porous material including an elastic solid skeleton and a compressible viscous gaseous fluid [1, 2]. Moreover, the mixture theory was used to obtain a set of macroscopic constitutive equations by applying volume integrations on the microscopic mass and momentum conservation equations [3, 4]. Recently, the authors proposed a computational homogenization approach for acoustic porous materials [5, 6]. It straightforwardly assesses the macroscopic influence of the microstructure and a comparison with direct numerical simulations (DNS) illustrated the feasibility of this approach for a simple cubic microstructure. In this paper, the homogenization approach is exploited and assessed by comparing the measurements of impedance tube tests on a real polyurethane (PU) foam with simulations based on a representative microstructural volume element of this foam.

The homogenization method relies on a statistically representative microstructural description for the foam. A unit cell model can capture the essential microstructural features of a foam if it is identified properly. For example, the effective elastic properties of open-cell foams were predicted by using a 2D honeycomb [7] and a 3D cubic cell [8]. A widely used geometry is the Kelvin cell, a tetrakaidecahedron cell that can fully fill the space and nearly minimizes the
surface energy [9]. In open-cell foams, the effective elastic properties are related to the struts and the vertices. Ample work has been done by focusing on these two aspects of the Kelvin cell. Various shapes of cross-sections such as circular, square, equilateral triangular and Plateau borders have been studied in [10, 11, 12] and the influence of the material in the vertex has been investigated in [13]. It has been shown that the effective elastic properties predicted by these Kelvin cell models agree well with experimental data [12, 13]. Although random multi-cell models and Weaire-Phelan cell models can be more accurate, the Kelvin cell still shows a satisfactory performance in terms of predicting linear elastic properties [14, 15, 16]. Many acoustic foams are partially-open foams in which cell faces are partially covered by thin membranes. The membranes can influence not only the viscous-thermal dissipation in foams [17, 18, 19, 20, 21] but also the elastic properties [21, 22]. Therefore, membranes have been implemented in the Kelvin cell model resulting in good predictions of both the acoustic properties [20, 23, 24] and the elastic properties [22], considering the simplicity of this model.

To define a representative microscopic model, besides the mechanical properties of the material, a proper understanding of the microstructure is required. Besides the strut length and the strut thickness, information on the cell volume and the cell orientation can be important. X-ray computed-tomography (CT) may be used to obtain a 3D image of the foam and to characterize its microstructure. For example, PU foam samples were investigated and information regarding the strut length, the face area and the cell volume has been quantified from stick figure images processed by volume thinning in Montminy et al. [25]. The characterized results were used for building a Kelvin cell model [26] and multi-cell models with
perturbed Kelvin cells [14, 15]. Scanning electric microscopy (SEM) is widely used to observe detailed features of the microstructure. For instance, the geometry of the struts and the cells of PU foams has been extracted from SEM images and used for further modelling in [13, 16]. Moreover, ratios of open and partially-open faces of PU foams were estimated by counting the number of various types of faces in Doutres et al. [19] and Zhang et al. [21].

In this paper, the PU foam is characterized by using both X-ray CT and SEM. The X-ray CT is used for the characterization of the skeleton of the foam including information of the struts, the faces and the cells. The opening and the distribution of thin membranes are studied on the basis of SEM images. In this paper, a relatively simple microscopic representative model, which requires an affordable computational power and time, is used. Therefore, a parallel model of a fully-open Kelvin cell and a partially-open Kelvin cell is built based on the geometrical characterization results. Next, the computational homogenization approach is applied and the effective material parameters of the foam are obtained. The homogenization approach is assessed by comparing the results of the simulations at the homogenized macro scale with experimental measurements.

2. Homogenization approach

When an acoustic wave is propagating in a porous material, there are two coupled problems at different scales: at the macroscopic scale, the porous material is considered as homogeneous, whereas at the microscopic pore scale the material is intrinsically inhomogeneous. In the homogenization approach, the macroscopic characteristic length related to the external excitation is assumed to be much larger than the microscopic characteristic length, i.e. the two problems at the macro- and
micro-scales can be separated. The two problems are studied in the frequency domain and the time derivative is replaced by \( j \omega \) where \( j \) is the imaginary unit and \( \omega \) is the angular frequency. At the macroscopic scale, the solid displacement \( u^s_M \) and the fluid pressure \( p^f_M \) are chosen as the macroscopic field variables. Momentum conservation of the solid and mass conservation of the fluid govern the macroscopic behaviour:

\[
f^s_M = \nabla_M \cdot (\sigma^s_M)^\top, \quad \text{and} \quad \varepsilon^f_M - \nabla_M \cdot u^f_M = 0.
\]  

(1)

Here, the operator \( \nabla_M \) represents the spatial gradient at the macroscopic scale. In the first equation, \( \sigma^s_M \) is the macroscopic Cauchy stress of the solid and \( f^s_M \) is the inertial force exerted on the solid. In the second equation, \( \varepsilon^f_M \) is the macroscopic volumetric change of the fluid and \( u^f_M \) is the fluid displacement.

In the microscopic representative volume element (RVE), the solid phase and the fluid phase are coupled through a continuous interface condition. In the solid, the displacements \( u^s_m \) are governed by conservation of linear momentum and Fourier’s law is adopted for the thermal diffusion:

\[
-\omega^2 \rho^s_0 u^s_m = \nabla_m \cdot \sigma^s_m
\]

\[
\rho^s_0 C^p_s j \omega \theta^s_m = -\nabla_m \cdot (-k^s \nabla_m \theta^s_m).
\]

(2)

Here, \( \rho^s_0 \) is the static density of the solid; \( \sigma^s_m \) is the microscopic Cauchy stress of the solid; \( C^p_s \) is the thermal capacity at constant pressure; \( \theta^s_m \) is the temperature of the solid; and \( k^s \) is the thermal conductivity of the solid. The isotropic linear elastic constitutive law is applied by ignoring thermal expansion:

\[
\sigma^s_m = \left( K^s - \frac{2}{3} G^s \right) \text{tr}(\varepsilon^s_m) I + 2 G^s \varepsilon^s_m,
\]

(3)

with \( K^s \) and \( G^s \) the bulk and shear moduli of the solid. The linear strain in the solid is given by \( \varepsilon^s_m = \frac{1}{2} \left[ \nabla_m u^s_m + (\nabla_m u^s_m)^\top \right] \). The fluid is governed by the linearized
Navier-Stokes-Fourier equations:

\[-\omega^2 \rho_f^0 u_m^f = - \nabla_m p_m^f + \nabla_m \left[ j \omega \mu_f^f \left( \nabla_m u_m^f + (\nabla_m u_m^f)^T - \frac{2}{3}(\nabla_m \cdot u_m^f)I \right) \right] \]

\[p_{0}^f C_p^f j \omega \theta_m^f = j \omega p_m^f - \nabla_m \cdot (-k_f^f \nabla_m \theta_m^f) \] (4)

\[\frac{p_m^f}{P_0} = \frac{\theta_m^f}{T_0} - \nabla_m \cdot u_m^f, \]

where \(\rho_f^0\) is the static density of the fluid; \(u_m^f\) is the microscopic fluid displacement; \(p_m^f\) is the fluid pressure; \(\mu_f^f\) is the viscosity of the fluid; \(C_p^f\) is the thermal capacity of the fluid at constant pressure; \(\theta_m^f\) is the temperature of the fluid; \(k_f^f\) is the thermal conductivity of the fluid; and \(P_0 = 0.101\) MPa and \(T_0 = 293\) K are the ambient pressure and temperature. A periodic boundary condition is adopted on the solid external surface based on the macroscopic solid deformation:

\[u_m^{+} - u_m^{-} = (\nabla_M u_M^f)^T \cdot (x_m^{+} - x_m^{-}) \] (5)

where \(^+\) and \(^-\) denote opposite boundary points. For the fluid, the pressure on the boundary is assumed to depend on the macroscopic fluid pressure and its gradient

\[p_m^f = p_M^f + \nabla_M p_M^f \cdot x_m^f \] (6)

and the boundary viscous traction is ignored. Furthermore, a periodic boundary condition for the thermal flux is applied, so that the total thermal flux leaving the RVE vanishes.

The microscopic RVE problem is solved using the finite element method. Based on the microscopic response, the condition of energy consistency is enforced to retrieve the work-conjugate macroscopic solid stress and macroscopic
fluid displacement:

\[(1 - \phi)f_M^f = \frac{1}{V} \int_{S_e} (n \cdot \sigma_M^s) dA, \quad (1 - \phi)\sigma_M^s = \frac{1}{V} \int_{S_e} (n \cdot \sigma_M^s) x_m dA,\]

\[\phi \epsilon_M^f = \frac{1}{V} \int_{S_e} n \cdot u_m^f dA, \quad \phi u_M^f = \frac{1}{V} \int_{S_e} (n \cdot u_m^f) x_m dA,\]

(7)

where \(S_e^s\) and \(S_e^f\) are the external solid surface and the external fluid surface respectively. Since the microscopic RVE problem is linear, only effective material parameters are required. Note that when the microscopic fluctuation of the solid and the average of the macroscopic fluid deformation are small, Biot’s poroelastic theory is recovered at the macroscopic scale:

\[(1 - \phi)f_M^f - \phi \nabla_M p_M^f = -\omega^2 (1 - \phi) \rho_0^s u_M^f - \omega^2 \phi \rho_0^f u_M^f,\]

\[-\phi \nabla_M p_M^f + \omega^2 \phi \rho_0^s u_M^s = -\omega^2 \rho^e \cdot (u_M^f - u_M^s),\]

(8)

with a linear stress-strain relation:

\[(1 - \phi)\sigma_M^s = 4D : \epsilon_M^s + Q \epsilon_M^f,\]

\[-\phi p_M^f = Q : \nabla_M u_M^s + R \epsilon_M^f.\]

(9)

Here, Biot’s effective parameters \(\rho^e\), \(4D\), \(Q\) and \(R\) are obtained numerically from the microscopic response.

3. Foam characterization

In this section, the macroscopic properties of the foam are given and the microstructure of a PU foam is characterized using X-ray CT and SEM. Moreover, the properties of the PU are discussed.
3.1. Macroscopic properties

A 30 cm × 30 cm × 5 cm plate of PU foam was supplied by Huntsman Polyurethanes. In this paper, the 5-cm thickness direction of the plates is referred to as the x axis. The bulk density $\rho_0$ and the loss factor $\eta_x$ of the foam are provided by the supplier. Here, the loss factor is defined as the ratio of the imaginary part and the real part of the stiffness. The Young’s modulus $\bar{E}_x$ (along the x axis) was obtained from the compression curves of 6 specimens (5 cm diameter cylinders with a height of 5 cm) where the engineering strain was ranging from 1.5% to 3.0% with a loading rate during compression of 100 mm/min, which is close to the loading rate used in the following impedance tube measurements. These properties and the corresponding standard deviations are summarized in Table 1.

<table>
<thead>
<tr>
<th>$\rho_0$ [kg m$^{-3}$]</th>
<th>$\bar{E}_x$ [MPa]</th>
<th>$\bar{\phi}$ [-]</th>
<th>$\eta_x$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.018 ± 0.0037</td>
<td>0.970 ± 0.0018</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 1: Bulk density, Young’s modulus, porosity and loss factor of the foam.

3.2. Geometrical characterization of the skeleton

SEM images of the foam are presented in Fig. 1. Fig. 1a shows a cross-section along the x axis, whereas Fig. 1b depicts a cross-section perpendicular to the x axis. Obviously, the structure of this foam is anisotropic. Furthermore, thin membranes are observed in the images.

The X-ray CT measurements are used to characterize the geometry of the skeleton of the foam [25]. Three cylindrical samples (with a diameter of 15-20 mm and a height of 25-30 mm) of the foam were scanned with a spatial res-
olution of 5 µm \(^1\). The samples were aligned such that the height direction corresponds with the \(x\) axis. Moreover, the three samples were taken from different positions of the raw sheets to obtain representative data and reduce the influence of inhomogeneities within a sheet. For each sample, two cubic volumes with 500\(^3\) voxels were taken for analysis and labelled as ‘a’ and ‘b’. The scanning results provide grayscale images, which are converted into 3D binary images by applying Ostu’s threshold method \([27]\). The porosity \(\phi\) was calculated as the fraction of ‘0’ elements in the associated matrix. The average porosity of the six cubes is given in Table 1. The 3D binary images were further processed by using Thinvox \([28, 29]\). The thick struts in the images were thinned into sticks \([25]\), preserving the connectivity. The locations of vertices and the associated connectivity were identified using the processed stick images.

The strut length is taken as the distance between two connected vertices. By

\(^1\)Using a Phoenix X-ray CT scanner with a scanning voltage of 60 kV and a current of 240 µA
making a slice in the middle of a strut, the mid-area of the strut is obtained by counting the number of non-zero pixels in the slice. The thickness depends on the resolution of the scan, which is therefore unreliable for very thin struts [25]. Hence, struts thinner than 5 pixels were ignored. Moreover, struts shorter than 50 µm were also ignored because they were categorized as large vertices. The mean value and the standard deviation of the strut length are given in Table 2. In the analysed results, the struts are classified into different groups according to the strut length. The group width is 25 µm for struts shorter than 300 µm and 50 µm for other struts. The corresponding mean values and the standard deviations of the mid-area are calculated and shown in Fig. 2. The data was fitted by a 2nd-order polynomial with respect to the inverse of the strut length as depicted by the red curve. The coefficients of the polynomial are also listed in Table 2. In Fig. 2, a clear trend emerges: a shorter strut is thicker than a longer one. The cross section of a strut is variable along its length and has the Plateau border characteristic [13, 14]. However, the variable thickness of a strut and the Plateau border only have an influence on the acoustical behaviour, which is significantly smaller than that of the thin membranes [26, 19]. Therefore, a constant triangular cross-section is assumed in the numerical model.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>171</td>
<td>108</td>
<td>335</td>
<td>9.53 × 10^4</td>
<td>−1.92 × 10^7</td>
</tr>
</tbody>
</table>

Table 2: The mean value µ_L and the standard deviation σ_L of the strut length and the three coefficients in the polynomial relation.

The grayscale images were pre-segmented into separate domains by using the watershed algorithm [30]. To avoid an over-segmentation due to imperfections
of the image, an H-minima transform was applied before the segmentation [31]. After the segmentation, small segmented domains were obtained and only the domains including a complete cell were selected for further improvement: the vertices of one cell in the selected domain were identified and the associated faces were determined.

In total, 421 cells were selected and analysed to characterize the morphology of the foam. The number of vertices was counted for each cell and the total average $N_V$ is $20.03 \pm 6.71$. Similarly, the average number of struts $N_S$ is $31.01 \pm 10.47$ and the average number of faces $N_F$ is $13.03 \pm 4.05$. The ratio of the numbers of struts and vertices ($N_S / N_V$) is 1.55, whereas the ratio of the number of struts and faces ($N_S / N_F$) equals 2.38. A graph of the number of struts per face is presented in Fig. 3. It reveals that there are many 4-sided and 5-sided faces in the foam. For a Kelvin cell, the average strut number per vertex is 1.5 and the connectivity of the vertices is well captured by using a Kelvin cell. However, a Kelvin cell only contains six 4-sided faces and eight 6-sided faces. The absence of 5-sided faces in the Kelvin cell may influence both the mechanical and acoustic properties. Since
an average strut number per face of 2.57 in a Kelvin cell is still close to the equivalent number of the foam and since the Kelvin cell presents additional advantages such as simplicity and low computational costs, the Kelvin cell is adopted here.

![Pie chart showing the distribution of strut numbers per face in the foam.](image)

Figure 3: Distribution of the number of struts per face in the foam.

After the identification of one cell, the cell volume was calculated as the volume of the convex hull of all vertices in the cell. The mean value $\mu_V$ and the standard deviation $\sigma_V$ of all samples are listed in Table 3. Moreover, all data points of a single cell in the binary image were mapped on the best matching ellipsoid. The corresponding principal axes of this ellipsoid were used to extract the general shape and orientations of the cells. The geometrical anisotropy ratio $e$ is defined as the length of the 1st principal axis divided by the average length of the other two principal axes, which have nearly the same length for all cells. The mean values $\bar{e}$ of all samples are also listed in Table 3. The average geometrical anisotropy ratio is 2.27, indicating that the cells in the foam are elongated.

An equal-area pole figure of the 1st principal axis is given in Fig. 4, where the projection direction is the $x$ axis, showing the preferred elongated direction in the foam. Note that the cells are typically oriented with their long axes along the $x$ axis.
<table>
<thead>
<tr>
<th></th>
<th>1a</th>
<th>1b</th>
<th>2a</th>
<th>2b</th>
<th>3a</th>
<th>3b</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>62</td>
<td>80</td>
<td>34</td>
<td>69</td>
<td>74</td>
<td>102</td>
<td>421</td>
</tr>
<tr>
<td>$\sum V$ [mm$^3$]</td>
<td>3.163</td>
<td>4.241</td>
<td>1.560</td>
<td>3.878</td>
<td>2.756</td>
<td>5.237</td>
<td>20.834</td>
</tr>
<tr>
<td>$\mu_V$ [mm$^3$]</td>
<td>0.051</td>
<td>0.053</td>
<td>0.046</td>
<td>0.056</td>
<td>0.037</td>
<td>0.051</td>
<td>0.050</td>
</tr>
<tr>
<td>$\sigma_V$ [mm$^3$]</td>
<td>0.051</td>
<td>0.052</td>
<td>0.047</td>
<td>0.064</td>
<td>0.037</td>
<td>0.062</td>
<td>0.054</td>
</tr>
<tr>
<td>$\bar{e}$ [-]</td>
<td>2.29</td>
<td>2.41</td>
<td>2.06</td>
<td>1.90</td>
<td>2.83</td>
<td>2.15</td>
<td>2.27</td>
</tr>
</tbody>
</table>

Table 3: Cell size and geometrical anisotropy ratios of all samples.

Figure 4: Orientations of the 1st principal axes of the cells. The projection direction is the $x$ axis.

3.3. Membrane characterization

In this section, the thin membranes are characterized using SEM images. For the samples, 2 blocks were taken from different positions in the raw sheets. To capture the anisotropy, two samples with different cross sections were taken from each block as shown in Fig. 1. Then, SEM images were made for every sample and direction. Faces that are normal to the viewing direction in the SEM images were selected and the number of fully-closed faces was counted. The total number of analysed faces $N_{\text{total}}$ and the number of fully-closed faces $N_c$ are given in Table 4. The area of one face was calculated by counting the pixels inside the face. As
shown in Table 4, the fraction of fully-closed faces in the $x$ slice is much smaller than the one in the $yz$ slice. Although the inhomogeneity of the foam may also result in a difference of the two ratios, the major difference originates from the non-uniform distribution of the fully-closed faces in the foam, which preferably reside in the $yz$ plane. Furthermore, the significant difference in the area fractions of the fully-closed face also supports this result. Note that faces which are neither parallel with the $x$ axis nor with the $yz$ plane cannot be taken into account because the measurement of the area is inaccurate.

<table>
<thead>
<tr>
<th>Slice</th>
<th>$x$</th>
<th>$yz$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{total}$</td>
<td>127</td>
<td>68</td>
</tr>
<tr>
<td>$N_c$</td>
<td>28</td>
<td>49</td>
</tr>
<tr>
<td>$N_c/N_{total}$</td>
<td>0.22</td>
<td>0.72</td>
</tr>
<tr>
<td>$A_{total}$ [mm$^2$]</td>
<td>6.83</td>
<td>4.26</td>
</tr>
<tr>
<td>$\sum A_c/A_{total}$</td>
<td>0.14</td>
<td>0.73</td>
</tr>
</tbody>
</table>

Table 4: Information on the selected faces and distribution of the fully-closed faces. $A_{total}$ is the total area of the measured faces. $\sum A_c$ is the total area of the fully-closed faces.

In Fig. 1, there are partially-open and fully-open faces. The opening in a face is identified by checking the contour of the grayscale level in the local region of the face. To distinguish between partially-open and fully-open faces, a two-component Gaussian mixture model was used to cluster the measured data by using an expectation-maximization algorithm [32]. It was assumed that the opening area $A_o$ and the face area $A_f$ follow a multivariate Gaussian distribution. The result of the Gaussian mixture model is given in Table 5. The proportion
\( w_i \) represents the probability of the associated group \( i \) and is determined by the expectation-maximization algorithm [32]. Table 5 shows that the two groups of open faces have significantly different opening ratios, representing the area fraction of the opening in the face. A low opening ratio \((= A_o/A_f)\) means that there is more membrane in this face and the opening ratio of a fully-closed face is 0. In this paper, the area of a face was measured as the one enclosed by the middle lines of the struts. Due to the thickness of the struts, the opening ratio is always smaller than 1 and the one for a fully-open face in the foam ranges between 0.7 and 0.9. A scatter plot of the face area and the opening ratio is shown in Fig. 5, where the two groups are indicated by colours. The corresponding Gaussian distributions are also plotted as a contour graph in Fig. 5. The filling colours indicate the probability density and a higher value suggests a higher probability density. The majority of the faces in group 1 are partially-open. Note that the average opening ratio of group 2 as given in Table 5 is close to the one of a fully-open face. Therefore, group 1 is considered as a group of partially-open faces and group 2 is fully-open.

Finally, the thickness of the membranes was measured directly from the SEM images. To measure the thickness, the cross section of a membrane should be parallel with the viewing direction. An average thickness of the membranes of \( 1 \pm 0.331 \, \mu m \) was obtained based on 10 different membranes. Considering that the membrane is much thinner than the strut, the thickness of the membrane is further assumed to be uniform in this paper.

### 3.4. Solid properties

Mechanical properties of the solid have a minor influence on the sound absorption performance of the porous material. The mechanical properties of the PU are
Table 5: Information on partially-open and fully-open faces in the foam. Group 1 represents the partially-open faces and group 2 captures the fully-open faces.

<table>
<thead>
<tr>
<th>Group</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_i [-] )</td>
<td>0.82</td>
<td>0.18</td>
</tr>
<tr>
<td>( N_{\text{open}}^i )</td>
<td>99</td>
<td>19</td>
</tr>
<tr>
<td>( \Sigma A_o [\text{mm}^2] )</td>
<td>1.62</td>
<td>1.92</td>
</tr>
<tr>
<td>( \Sigma A_f [\text{mm}^2] )</td>
<td>4.28</td>
<td>2.75</td>
</tr>
<tr>
<td>( \Sigma A_o / \Sigma A_f )</td>
<td>0.38</td>
<td>0.70</td>
</tr>
<tr>
<td>( \frac{1}{N} \Sigma (A_o/A_f) )</td>
<td>0.36</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Figure 5: Relation of the opening ratio and the face area. The contour graph shows the probability density of the corresponding Gaussian distributions.

Figure 5: Relation of the opening ratio and the face area. The contour graph shows the probability density of the corresponding Gaussian distributions.

estimated based on the bulk density, the measured Young’s modulus and the corresponding numerical results. It will be presented in the next section. Moreover, the
Poisson’s ratio $\nu = 0.4$ is estimated from Tsukinovsky et al. [33] and the thermal properties $C_p = 1800 \text{ J kg}^{-1} \text{ K}^{-1}$ and $k_s = 0.022 \text{ W m}^{-1} \text{ K}^{-1}$ are estimated from the common thermal properties of PU [34].

4. Parallel model of Kelvin cells

Based on the microstructural information obtained in the previous section, a parallel model of two Kelvin cells is built for numerical analysis in this section. The Kelvin cells are described in the first part of this section. Thereafter, the homogenization approach is applied and the obtained effective material parameters are discussed.

4.1. Description of Kelvin cells

The volume of the Kelvin cell is chosen as the average cell volume equalling $0.050 \text{ mm}^3$ as given in Table 3. To construct an anisotropic Kelvin cell, the vertices of an isotropic cell with a volume of $0.050/2.27 \text{ mm}^3$ are stretched along the $x$ axis by a factor of 2.27 as given in Table 3. For each strut, the corresponding thickness is obtained according to the polynomial relation in Table 2. The cross-section of the strut is assumed to be triangular with vertices lying in the three associated faces. The resulting transversely isotropic Kelvin cell without membrane (i.e. a fully-open cell) is depicted in Fig. 6a.

There are three types of faces observed in the foam: a fully-closed face, a fully-open face and a partially-open face. Here, the fully-open cell is used to represent the fully-open faces and its fraction $\varphi_1$ is therefore $0.109 (= w_2 \times (1 - N_c/N_{total}))$ such that the fraction of the fully-open faces agrees with the statistical result for the foam. The remaining partially-open faces and the fully-closed faces are represented by another Kelvin cell with membranes as shown in Fig. 6b. Since
the fully-closed faces in the foam mostly appear in the $yz$ plane, the faces normal to the $x$ axis are taken fully-closed in the cell model. The opening ratio of the faces normal to the $yz$ plane are set to 0.38 according to Table 5. As discussed in the previous section, the obtained distribution of the membranes was only based on faces parallel with the $x$ axis and the $yz$ plane. Since the wave propagates along the $x$ axis in the analyses, the opening ratio of the six-sided faces is set to 0.07 so that the projection of the Kelvin cell on the $yz$ plane agrees with the $yz$ slice in the SEM images.

![Figure 6](image)

(a) Fully-open, $\varphi_1 = 0.109$  
(b) Partially-open, $\varphi_2 = 0.891$

Figure 6: Two Kelvin cells and their fractions based on the identified geometrical microstructure data.

In Figure 6, neither the fully-open cell nor the partially-open cell can fully represent the microstructure of the foam alone. Hence, the combination of the two cells is required. In this paper, a parallel connection of the two cells is assumed and the effective material parameters of the foam will be obtained based on this parallel-cell model in the next section. The porosity of the fully-open cell is about 0.9682 and the one of the partially-open cell is 0.9638. The weighted average porosity of the parallel model is therefore 0.9643, which is close to the estimated
foam porosity of 0.970 in Table 1.

4.2. Homogenization of the Kelvin cell model

Considering the anisotropy of the Kelvin cell, nine simulations with different loading condition as listed in Table 6 are used to calculate Biot’s parameters. COMSOL 4.3b is used to solve the microscopic RVE problem. Because the membranes are much thinner than the struts, shell elements are adopted to account for the thin membranes. Continuity of the displacements is used to couple the shell elements, the fluid elements and the solid elements. The rotation between the shell elements and the solid elements is suppressed. As discussed in the previous section, the solid density is taken as 700.28 kg m$^{-3}$ corresponding to a bulk density of 25 kg m$^{-3}$. The Young’s modulus of the solid $E_s$ is 0.7411 MPa. The effective Young’s modulus of the parallel model in the $x$ direction at 50 Hz, which is close to the loading rate in the measurement, by using a characteristic deformation of $10^{-4}$, then equals 18 kPa, which is consistent with Table 1. In the following discussion, for comparison, the homogenization results of the fully-open cell and the partially-open cell are also considered in the absence of the parallel connection.

In the parallel-cell model, according to the acoustic-electromagnetic analogy, the same macroscopic pressure gradients (as given in loading sets 1 and 2 in Table 6) are applied to the two Kelvin cells and the corresponding macroscopic fluid displacements of the two cells $u^{f}_{M1}$ and $u^{f}_{M2}$ are used to calculate the volume weighted-average displacement $u^{f}_{M}$:

$$u^{f}_{M} = \varphi_1 u^{f}_{M1} + \varphi_2 u^{f}_{M2}.$$  \hspace{1cm} (10)

The weighted-average displacement is then used to calculate the coupling density tensor $\rho^c$. Furthermore, for the calculation of $^dD$, $Q$ and $R$, the same macroscopic
solid deformation and the macroscopic fluid pressure are applied to each Kelvin cell as given in loading sets 3 to 9 in Table 6. The corresponding parameters are calculated for each cell and the macroscopic properties are defined as their volume weighted-averages:

\[ \mathbf{D} = \varphi_1 \mathbf{D}_1 + \varphi_2 \mathbf{D}_2 \quad \mathbf{Q} = \varphi_1 \mathbf{Q}_1 + \varphi_2 \mathbf{Q}_2 \quad \text{and} \quad \mathbf{R} = \varphi_1 \mathbf{R}_1 + \varphi_2 \mathbf{R}_2. \] (11)

The obtained coupling density tensor is diagonal and the 11 component is shown in Fig. 7. For comparison, \( \rho'_{11} \) calculated from the fully-open cell and the partially-open cell are given by red dashed and blue dashed-dotted curves, respectively. The result of the parallel-cell model is close to the one of the partially-open cell because the latter has a high fraction in the parallel-cell model. Note that the

<table>
<thead>
<tr>
<th>Set</th>
<th>( \nabla_M p_M^f ) [Pa/m]</th>
<th>( \nabla_M u_M^f ) [-]</th>
<th>( p_M^f ) [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 10^3 e_1 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>( 10^3 e_2 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>( 10^{-3} e_1 e_1 )</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>( 10^{-3} e_1 e_2 )</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>( 10^{-3} e_1 e_3 )</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>( 10^{-3} e_2 e_2 )</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>( 10^{-3} e_2 e_3 )</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>( 10^{-3} e_3 e_3 )</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6: Loading conditions used to calculate Biot’s parameters. The index (1, 2, 3) corresponds to the \((x, y, z)\) system.
membranes significantly increase both the real and the imaginary parts of the coupling density tensor, indicating an increase of the viscous dissipation.

Figure 7: 11 component of the coupling density tensor $\rho^c$.

The stiffness tensor $^4D$ is not symmetric because of the inertial effects [5]. However, since the asymmetry is quite small in the studied frequency range, a macroscopically symmetric solid stiffness is used instead, i.e. $D_{ijkl} = D_{klij}$. Moreover, the stiffness tensor is transversely cubic-symmetric with respect to the $yz$ plane. The coefficients at 100 Hz are listed in Table 7, showing that the loss factors are of the order of $10^{-3}$. The small loss factors are mainly due to the interaction between the air and the solid since the solid is purely elastic. When the frequency increases from 100 Hz to 900 Hz, the order of the loss factors of $^4D$ increase from $10^{-3}$ to $10^{-2}$ (the highest loss factor is 0.0548 for $D_{2233}$ at 900 Hz). However, they are still considerably smaller than the measured loss factor of 0.19 in Table 1. Therefore, for this foam, to account for the measured loss factor, the viscoelasticity of the PU should be considered. Since the microscopic problem is solved in the frequency domain, viscoelasticity may be incorporated by using a complex Young’s modulus $E^\ast$. However, experimental information on the com-
plex Young’s modulus is not available, and the estimated elastic properties of the PU had to be used instead in this paper.

<table>
<thead>
<tr>
<th>$D_{1111}$</th>
<th>$D_{1122}$</th>
<th>$D_{2222}$</th>
<th>$D_{2233}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.10 + 0.05j</td>
<td>5.37 + 0.04j</td>
<td>4.31 + 0.04j</td>
<td>3.23 + 0.04j</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D_{1212}$</th>
<th>$D_{2323}$</th>
<th>$Q_{11}$</th>
<th>$Q_{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.21 + 0.00j</td>
<td>1.16 + 0.00j</td>
<td>15.77 + 0.42j</td>
<td>13.77 + 0.37j</td>
</tr>
</tbody>
</table>

Table 7: Effective parameters at 100 Hz of the parallel model [kPa].

5. Macroscopic experiments and numerical simulations

In this section, the homogenization approach is assessed by comparing measurements in an impedance tube with corresponding numerical simulations. First, experimental conditions of the impedance tube and corresponding model assumptions are described. Thereafter, the measurements and the numerical results are compared and discussed.

5.1. Impedance tube measurements

The set-up of the impedance tube is shown in Fig. 8. The length of the tube is 1 m and the internal diameter is 50 mm. Six microphones are distributed uniformly along the tube and the distance between two microphones is 175 mm. Four cylindrical samples with a thickness of 40 mm were prepared for the measurements. During the measurement, the sample was put in a metallic sample holder with an internal diameter of 50 mm. The impedance tube and the holder were connected tightly and a thin rubber ring was used between the connection to
avoid air leakage. Furthermore, a solid metallic disk with the same dimension as the holder was assembled behind the holder and it was considered as a rigid back plate in the measurement. Another rubber ring was used in the connection of the holder and the rigid back plate. More details of this set-up can be found in Temiz et al. [35].

![Figure 8: Set-up of the impedance tube (right) and the metallic sample holder (left). The sound source is placed on the right side of the tube and the sample is on the left.](image)

The diameters of the samples were slightly larger than 50 mm to avoid air leakage between the rigid wall and the samples. This implies that the samples were slightly pre-compressed in the radial direction during the measurements. The influence of this factor is not considered because the deformation of the foam in the radial direction is smaller than about 3 %. The test frequency ranges from 100 Hz to 900 Hz with a step size of 20 Hz. According to the pressure values of the microphones along the impedance tube, the normal incident sound absorption coefficients can be calculated based on the reflective pressure $p_{ref}$ and the incident pressure $p_{inc}$ by using a plane wave assumption:

$$\alpha = 1 - \left| \frac{p_{ref}}{p_{inc}} \right|^2.$$  \hspace{1cm} (12)

5.2. Model assumptions and numerical solution

The macroscopic configuration of the impedance tube simulations is shown in Fig. 9. The behaviour of the air is governed by the Helmholtz equation with
a sound speed $c_{\text{air}} = 343 \text{ m/s}$, and a given incident plane wave $p_{\text{inc}} = e^{-jkx} \text{ [Pa]}$, where $k$ is the wave number and $x$ is the horizontal distance from the air-porous interface with the acoustic-poroelastic coupling condition [36]. In the measurements, the foam samples were not glued to the solid disk and the holder. Hence, a 1-mm air gap is introduced between the foam and the rigid back plate (i.e. the solid disk). Considering the pre-compression of the samples, the wall is considered as motionless and impenetrable for the air. The discretization is based on 648 triangular-prism second-order elements in the foam layer and the simulations are conducted in 3D using COMSOL 4.3b. In the simulations, sound absorption coefficients are calculated according to Eq. (12) and are averaged over the interface between the foam and the air.

![Figure 9: Macroscopic configuration of the impedance-tube simulations.](image)

### 5.3. Results and discussions

A comparison of the sound absorption coefficients is plotted in Fig. 10. In the 200-400 Hz regime, the resonance of the foam results in peaks in the numerical results and plateaus in the measurements. When the frequency becomes higher than 400 Hz, the parallel-cell model and the partially-open cell model agree much better with the measurements than the fully-open cell without the membranes,
showing that the membranes increase the dissipation (mainly the viscous dissipation [22]) in the foam.

Figure 10: Comparison between the measurements and the numerical simulations. The markers are the results of the measurements and different symbols represent different samples.

The different behaviour in the resonance regime between simulations and measurements mainly originates from the deviation between the corresponding loss factors. As discussed in the previous section, the measured loss factor of the foam is 0.19, whereas the loss factors obtained in the simulations, ranging from $10^{-3}$ to $10^{-2}$, are much smaller than this value. To study the influence of the loss factor, an additional test simulation is conducted by increasing the loss factor of $D_{1111}$ to 0.19 without affecting its real part, which increases the magnitude of $D_{1111}$. The sound absorption coefficients of the two simulations are shown in Fig. 11a. Because $D_{1111}$ is larger in this simulation (i.e. $\eta_{1111} = 0.19$), at low frequencies, the corresponding deformation of the foam is smaller than the one of the parallel-cell model when the same pressure is applied and therefore, the stored energy of the solid is also less. As shown in Fig. 11b, the fraction of the total energy stored in
the solid $E_s^i$ in the simulation with an increased loss factor is smaller than the one in the parallel-cell model, particularly in the 200-400 Hz range. This suggests that the resonance of a foam with a higher loss factor is less efficient in terms of the sound absorption performance. Therefore, the resonance peak is smoother in the test simulation. Here, the total stored energy of the solid $E_s^i$ and the total stored energy of the fluid $E_f^i$ are calculated by

$$E_s^i = (1 - \phi) \int_{V_p} \left[ \sigma_M^s : \epsilon_M^s + f_M^s \cdot u_M^s \right] dV \quad \text{and} \quad E_f^i = -\phi \int_{V_p} \left( p_M^f \epsilon_M^f + u_M^f \cdot \nabla_M p_M^f \right) dV,$$

where $V_p$ is the domain of the porous material.

![Figure 11: Comparison of simulations with different loss factors.](image)

(a) Sound absorption coefficients

(b) $|E_s^i/(E_s^i + E_f^i)|$

For 500-900 Hz, the partially-open cell and the parallel models underestimate the absorption coefficients. At high frequencies, the main absorption mechanism is the viscous dissipation between the fluid and the solid, which is mainly affected by the thin membranes. According to the SEM images, more fully-closed membranes are observed near the top and the bottom of the cell (along the $x$ direction). This characteristic is not fully captured in a single Kelvin cell because of the lim-
ited number of faces. Secondly, the opening of the 6-sided faces in the Kelvin cell does not exactly agree with the statistical analysis because the membrane information mainly focuses on the faces parallel with the viewing direction in the SEM images. Thirdly, although the connections between different types of cells is simply described by the parallel-cell model, the effective fluid path in reality is still considerably more complicated. For example, the coupling between two Kelvin cells, which can increase the viscous dissipation, is not included in a parallel-cell model. Furthermore, the differences among the samples indicate that the spatial inhomogeneity of the foam within the sheet may be significant. Hence, the samples tested in the impedance tube may have a somewhat different microstructure, particularly the thin membranes, than the ones analysed in the SEM images. Finally, also note the intrinsic limitations due to the use of a Kelvin cell such as the uniform cell size, the absence of 5-sided faces, etc.

The comparison with experimental results shows that the presented homogenization approach can be used to predict the absorption performance of the studied acoustic foam when the microscopic RVE is properly identified. Because the fraction of the fully-open faces is small in the studied foam, the numerical results of the partially-open cell model and the parallel-cell model are similar and they are close to the experimental results. The small difference between the two models indicates that the fully-open Kelvin cell has a limited effect on the sound absorption performance of this foam. The results show that the implementation of the thin membranes is essential for the foam to recover a correct sound absorption performance at 500-900 Hz.

The parallel model of single Kelvin cells is expected to be applicable for both partially-open and fully-open foams since it includes the corresponding partially-
open and fully-open cells. Obviously, for foams with many fully-closed faces, the parallel model cannot properly represent the material. An example of such a foam is shown in Fig. 12. This foam has a uniform distribution of fully-closed faces and almost 60% of the faces are fully-closed. Assuming that the fully-closed faces are represented by a fully-closed Kelvin cell (with all faces fully-closed), and adopting the same method presented earlier, the homogenization approach is applied next. For this type of foam, the simulation results fail to capture the characteristics of the measurements (see Fig. 12). This is can be attributed to the deficient description of fully-closed faces in the numerical model. Although the fraction of the fully-closed faces in this type of foam is quite high, it is not realistic to attribute all the fully-closed faces to a single fully-closed Kelvin cell because a fully-closed cell is rare in real foams. To describe the high fraction of the fully-closed faces, a multi-cell model may be used, however it entails a high computational cost, and is therefore not considered in this paper. More details of the microstructure and the numerical simulations of the foam with many fully-closed faces can be found in the Appendix.

6. Conclusions

The microstructure of an acoustic PU foam has been investigated to construct a microscopic RVE. The connectivity of the vertices, the strut thickness and the statistical cell morphology were analysed through X-ray CT. It has been found that a transversely isotropic Kelvin structure can represent the skeleton structure of this foam. The opening and the distribution of the thin membranes were studied based on SEM images. Three types of faces were observed: fully-closed faces, fully-open faces and partially-open faces. The fully-closed faces are distributed
Figure 12: Microstructure of a foam with many fully-closed faces and comparison of the measurements and the simulation.

non-uniformly in the foam and there are more partially-open faces than fully-open faces. To obtain a relatively simple but representative microstructure, two single Kelvin cells are built based on the characterization results. According to the membrane information, non-uniformly distributed membranes were incorporated in one single Kelvin cell to represent the fully-closed faces and the partially-open faces. Another Kelvin cell without membranes was considered to include the fully-open faces. The two Kelvin cells have different fractions and a parallel arrangement was adopted to include the effects of both cells.

Next, a computational homogenization approach was applied to the Kelvin cells and the effective material parameters of the foam were obtained. This homogenization approach was assessed by comparing experimental measurements and simulations. Considering the limitations of the simplified microscopic models, the simulation results of the impedance tube agree with the experimental measurements well, showing the feasibility of the homogenization approach for the
studied foam. The partially-open cell model and the parallel-cell model both give similar and good descriptions. A difference between the models with membranes and the one without membranes mainly occurs when the frequency is higher than 400 Hz. It reveals that the thin membranes are important for the sound absorption performance of this foam type at mid-high frequencies. The opening and the distribution of the membranes together determine the viscous effects at the pore scale and are essential to correctly capture the sound absorption performance of a foam. Nevertheless, the presented approach still has limitations for foams with a high fraction of fully-closed faces.

Appendix A. Modelling of the foam with many fully-closed faces

Comparing with the foam studied in the body of the paper, this foam with many fully-closed faces has a smaller density of 15 kg·m$^{-3}$ and a higher Young’s modulus of 0.21 MPa. The porosity estimated based on X-ray CT images is 0.985. By following the same characterization procedures, the result of the analysis of the X-ray CT images is given in Table A.8. Cells in this foam are also elongated along the $x$ axis and a transversely-isotropic Kelvin structure should be considered.

<table>
<thead>
<tr>
<th>$a$ [$\mu m^2$]</th>
<th>$b$ [$\mu m^3$]</th>
<th>$c$ [$\mu m^4$]</th>
<th>$\mu V$ [mm$^3$]</th>
<th>$\bar{\epsilon}$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>505</td>
<td>$-2.65 \times 10^4$</td>
<td>$3.83 \times 10^7$</td>
<td>0.092</td>
<td>1.40</td>
</tr>
</tbody>
</table>

Table A.8: Geometrical data extracted to build a transversely isotropic Kelvin structure.

The distribution of the fully-closed faces is uniform as shown in Table A.9, i.e. there is no significant difference between the $x$ slice and the $yz$ slice. Moreover, Table A.10 shows that the fraction of partially-open faces is larger than the fraction of fully-open faces.
<table>
<thead>
<tr>
<th>Slice</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\text{total}}$</td>
<td>202</td>
<td>141</td>
</tr>
<tr>
<td>$N_c/N_{\text{total}}$</td>
<td>0.61</td>
<td>0.56</td>
</tr>
<tr>
<td>$A_{\text{total}}$ [mm$^2$]</td>
<td>5.36</td>
<td>3.75</td>
</tr>
<tr>
<td>$\sum A_c/A_{\text{total}}$</td>
<td>0.61</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Table A.9: Data on the selected faces and distribution of the fully-closed faces.

<table>
<thead>
<tr>
<th>Group $i$</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_i$ [-]</td>
<td>0.69</td>
<td>0.31</td>
</tr>
<tr>
<td>$N_{\text{open}}^i$</td>
<td>105</td>
<td>35</td>
</tr>
<tr>
<td>$\sum A_o$ [mm$^2$]</td>
<td>0.63</td>
<td>0.82</td>
</tr>
<tr>
<td>$\sum A_f$ [mm$^2$]</td>
<td>2.78</td>
<td>1.45</td>
</tr>
<tr>
<td>$\sum A_o/\sum A_f$</td>
<td>0.23</td>
<td>0.56</td>
</tr>
<tr>
<td>$\frac{1}{N} \sum (A_o/A_f)$</td>
<td>0.25</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table A.10: Data on partially-open and fully-open faces in the foam with many fully-closed faces.

Based on the above information, three types of Kelvin cells shown in Fig. A.13 are built in the parallel model: a fully-open cell, a partially-open cell with a uniform opening ratio of 0.25, and a fully-closed cell with all fully-closed faces (i.e. the opening ratio is zero).
Figure A.13: Three Kelvin cells and their corresponding fractions.

Acknowledgement

This research was supported by the Dutch Technology Foundation STW, applied science division of NWO, and the Technology Program of the Ministry of Economic Affairs (under grant number 10811). The foam samples were supplied by Huntsman Polyurethanes. The authors thank Muttalip Temiz for the measurement of the sound absorption coefficients in the impedance tube, Chaowei Du for the SEM images, and Marc van Maris for the guidance with the X-ray CT.

References


URL http://www.cs.princeton.edu/~min/thinvox/


URL http://www.engineeringtoolbox.com/
