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Flames in context of thermo-acoustic stability bounds

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Abstract

Bounds are derived for the acoustic losses such that a thermoacoustic system with a given flame can be guaranteed to be stable. The analysis is based on the flame's acoustic input-to-output properties represented by its scattering matrix. The developed analytical and numerical techniques allow estimating the maximum reflection coefficients (equivalently – acoustic losses) which are sufficient to ensure stable operation of a given burner. It is shown that the calculated numerical upper-bound is less conservative than the analytical one. The frequency dependence of the required acoustic losses provides (i) a thermo-acoustic signature of the flame and (ii) guidelines for the proper design of the up- and downstream acoustics from the flame. The method is illustrated on two burners/flames of premixed multiple Bunsen type. The frequency dependence of the upper bounds allows to identify those frequency ranges where the flame is more likely to cause instability of the complete system.

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1. Introduction

The susceptibility of a combustion process to thermoacoustic instability is a widely known practically relevant problem which also draws abundant academic attention [1–5]. In general, thermoacoustic instabilities are induced by a positive unstable feedback loop between the unsteady

heat release and the acoustic response of the surrounding system. In order to prevent the instability, the thermo-acoustic properties of the burner and the acoustic response of the surrounding system, e.g. the combustor plenum, should be matched in a suitable manner. Typical parameters of interest include the dimension of the in- and exhaust ducts, the presence and location of acoustical dampers, and the burners flow settings.

There is usually a complex interplay between the various system parameters and the overall system stability, and accordingly it is far from trivial to derive general design rules. As a consequence, the problem is usually solved by an ‘educated’ trial and error approach, either directly on the system at hand, or on an extensive thermo-acoustic

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model. The design of a stable combustor is therefore typically a resource consuming stage of development.

Generally speaking, the design problem is often encountered in either one of the following forms: (i) the burner design itself is fixed and (part of) the acoustic response, e.g. the in- and exhaust ducts, can be modified, or (ii) the acoustic response is fixed and the burner should be modified. Naturally, in both situations the goal is to achieve a stable overall system within the constraints posed by other desired system properties, e.g. thermal efficiency and emissions.

Within the present contribution, we focus on the first case. By definition, we will therefore suppose that the design of the flame/burner is known and fixed. As a consequence, the thermoacoustic response of the burner to acoustic excitation from the acoustic part of the system is also fixed. Within this context, the goal of the current paper is to explicitly translate the response of the burner to limitations on the design of the acoustic response. In particular, the approach considers the minimal required acoustic losses to achieve stable operation. This not only provides guidelines for the acoustic design, it will also yield a stability signature for the given flame. For example, if one given burner will require significantly more losses to achieve stable operation than another, the latter one can be generally considered more stable.

To the best of our knowledge, this formulation was not extensively treated in the literature. A recent paper [6] assesses damper performance based on their effect on the growth rate, but does not provide explicit bounds to guarantee stability. The work presented in [7] is strongly related to the current contribution, because the instability potential of the flame was characterized based on its maximum possible acoustic energy amplification. Such approach is complementary to the one presented here, with the difference that we explicitly consider the implications of a given flame response in terms of acoustic design limitations. From a similar perspective, in [8–10] the intrinsic (in)stability property of a flame as an acoustically isolated element was studied. The present contribution is inspired by the results obtained in [7,11,8].

The remaining paper is then build up as follows. The first section shortly summarizes all required theoretical concepts. In Section 2, we formally derive the analytical and numerical methods required to formulate the sought upper bounds on the system's acoustic losses in order to guarantee stable operation. Next, in Section 3, the developed methods will be illustrated on two particular burner cases with multiple Bunsen-type premixed flames. These examples are selected because their thermo-acoustics is extensively studied by many research groups, see for instance [3,5,12].

Furthermore this flame type is practically relevant for multiple small-scale applications, e.g. domestic boilers. Finally, Section 5 discusses additional applications of the method, and considers avenues for further research.

2. System theoretical context

The problem described in the introduction can be formulated conveniently in the context of acoustical network modelling. Within a standard set of assumptions, each element in the thermoacoustic system can be described by an acoustic scattering matrix $\mathbf{S}$ [13–15]. Using the convention of harmonic time dependence e^{st} , with $s = j\omega + \sigma$, the input output relation of an element is given by

$$\begin{bmatrix} g_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} S_{11}(s) & S_{12}(s) \\ S_{21}(s) & S_{22}(s) \end{bmatrix} \begin{bmatrix} f_1 \\ g_2 \end{bmatrix}, \quad (1)$$

where the ingoing waves (f_1, g_2) are considered the inputs and the outgoing waves (g_1, f_2) as outputs [16].

2.1. Thermoacoustic feedbackloop

Within the context of thermoacoustic systems, one can generally distinguish two types of scattering elements (i) the active elements e.g. the burner, and (ii) the passive acoustic elements. Without further loss of generality, a typical thermoacoustic feedbackloop can be represented as shown in Fig. 1.

Here, $\mathbf{S}_f$ is the flame or burner scattering matrix which is assumed to be known, either from measurements or modelling. The matrix $\mathbf{R}$ on the other hand contains the lumped acoustic response up- and downstream from the flame,

$$\mathbf{R} = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}, \quad (2)$$

where R_1 and R_2 are the up- and downstream reflections with respect to the flame element. Note that in general the entries of the acoustic two-port matrices are complex and dependent on the complex frequency s .

Fig. 1. System representation of a basic thermoacoustic system in terms of scattering matrices, $\mathbf{S}_f$ the flame scattering matrix and $\mathbf{R}$ the acoustic reflection matrix. For the purpose of analysis, additional input and outputs are provided by $\mathbf{u}$ and $\mathbf{y}$.

2.2. Characteristic system equation

Figure 1 has close analogies with a basic feedback loop encountered in control theory, see for example [17]. In particular, one can recognize the thermoacoustic feedback loop as a positive interconnection between two Multiple Input Multiple Output (MIMO) systems. As such, one can define an open loop system $\mathbf{L} = \mathbf{R}\mathbf{S}_f$. In order to understand the closed loop dynamics, it is convenient to introduce an explicit input vector $\mathbf{u}$ and output vector $\mathbf{y}$, see again Fig. 1. It is straightforward to derive that in this case, the closed loop transfer $\mathbf{CL}(s)$ can be written as [17,18]:

$$\mathbf{CL}(s) = [\mathbf{I} - \mathbf{L}(s)]^{-1} \mathbf{L}(s), \quad (3)$$

$$\mathbf{CL}(s) = \frac{1}{\det(\mathbf{I} - \mathbf{L}(s))} \text{adj}(\mathbf{I} - \mathbf{L}(s)) \mathbf{L}(s), \quad (4)$$

where $\det$ and adj refer to the determinant and the adjoint of a matrix. It can be shown that any possible closed loop transfer function will contain the term $[\mathbf{I} - \mathbf{L}(s)]^{-1}$ [17,18]. Henceforth, the poles of the closed loop system are given by the solution of the characteristic equation,

$$\det(\mathbf{I} - \mathbf{L}(s)) = 0. \quad (5)$$

Note again that $s = j\omega + \sigma$ is the Laplace variable. Therefore, the real part of the solution determines the stability of the pole, and hence the closed loop system, with $\sigma > 0$ being unstable. Furthermore, it is important to realize that even when there is no explicit input $\mathbf{u}$ present, the response of the closed loop system to a non-zero initial condition is still determined by the stability of the closed loop poles.

3. Bounds on the acoustic scattering matrix

The characteristic Eq. (5) provides a convenient starting point for determining conditions on the matrix $\mathbf{R}$, and by extension on the up- and downstream reflection coefficients R_1, R_2 such that the closed loop system can be guaranteed stable.

Given the constraint that the open loop system description is stable, it can be shown that closed loop stability can be guaranteed if the maximum eigenvalue of the open loop frequency response satisfies [17],

$$\max_i |\lambda_i(\mathbf{L}(j\omega))| < 1, \forall \omega, \quad (6)$$

where $\rho(\mathbf{L}(j\omega)) \triangleq \max_i |\lambda_i(\mathbf{L}(j\omega))|$ is called the spectral radius. The general proof of this statement is beyond the scope of this paper. However some intuitive understanding can be obtained from the fact that in case of a Single Input Single Output system (SISO) with loop transfer L the condition means that $|L| < 1$ and hence the Nyquist curve can never encircle the point -1 . Because this poses a limit on the maximum

amplification factor of the loop, this is also called the small-gain theorem.

In order to apply (6), the open loop system $\mathbf{L} = \mathbf{R}\mathbf{S}_f$ needs to be stable. Here, note that $\mathbf{R}$ is only composed out of acoustically passive reflection coefficients, and as such cannot produce any unstable open loop poles. In contrast, under some conditions on the flame transfer function, it is possible that the flame scattering matrix introduces unstable poles [8–10]. Nevertheless, for many practical flames the MIMO system represented by $\mathbf{S}_f$ is in fact stable and hence the product $\mathbf{R}\mathbf{S}_f$ will be stable.

In the current setting, we would like to consider the burner, and thus $\mathbf{S}_f$, as a fixed (but frequency dependent) entity. In order to guarantee the stability of the thermoacoustic system, the goal is to infer some requirements on the acoustic properties of the remaining part of the combustor, captured in $\mathbf{R}$, such that $\rho(\mathbf{R}\mathbf{S}_f) < 1, \forall \omega$.

In the current contribution this problem is solved using both an analytical approach, making use of well known matrix properties, as well as using a numerical method. In the following, it is shown that the analytical approach yields an elegant, but conservative upper bound on the minimum acoustic losses, whereas the numerical approach yields much less conservative constraints.

3.1. Analytical bounds on $\mathbf{R}$ using matrix norms

In order to separate the influence of $\mathbf{S}_f$ and $\mathbf{R}$ on the constraint $\rho(\mathbf{R}\mathbf{S}_f) < 1$, it is desirable to reformulate the spectral radius in terms of bounds on the norms of the individual system matrices. One possibility is to use the maximum singular value $\bar{\sigma}$ of the matrix as an upper-bound for its maximum eigenvalue. By definition, the maximum eigenvalue specifies the maximum amplification in case the input and output directions are the same. In essence, the maximum singular value is the maximum amplification factor irrespective of the in- and output directions. In addition, as a norm, the maximum singular value satisfies the Cauchy inequality $\|\mathbf{A}\mathbf{B}\| \leq \|\mathbf{A}\| \cdot \|\mathbf{B}\|$ [17]. Therefore, one finds an upper-bound for the spectral radius of the thermoacoustic interconnection as,

$$\rho(\mathbf{R}\mathbf{S}_f) \leq \bar{\sigma}(\mathbf{R}\mathbf{S}_f) \leq \bar{\sigma}(\mathbf{R})\bar{\sigma}(\mathbf{S}_f). \quad (7)$$

As a consequence, Eq. (6) is satisfied when,

$$\bar{\sigma}(\mathbf{R}) < \frac{1}{\bar{\sigma}(\mathbf{S}_f)}, \quad \forall \omega. \quad (8)$$

And since $\mathbf{R}$ is diagonal, one can write,

$$\max(|R_1|, |R_2|) < \frac{1}{\bar{\sigma}(\mathbf{S}_f)}, \quad \forall \omega. \quad (9)$$

Equation (9) provides a frequency dependent maximum of the magnitude of the up- and downstream reflection coefficients. In other words, the

bound specifies (conservatively) the minimum acoustic losses required for a stable thermoacoustic system, and hence it can be used as a design constraint.

Note that compared to Eq. (6) two steps of conservatism are introduced, (i) by using the maximum singular value, and (ii) by the triangle inequality. In addition, the small gain theorem itself is conservative because the phase of R_1 and R_2 is excluded. However, in the current setting this is exactly what is needed, because in many practical systems the phase of R_1 and R_2 is mainly determined by the lengths of the inlet and exhaust ducts.

3.2. Numerically determined bounds

The goal of this section is to obtain a less conservative bound than (9) by means of a numerical optimization problem. Consider again Eq. (7). Note that the frequency dependence of S_r is fixed. Next, let us write $\mathbf{R}$ at a single frequency as,

$$\mathbf{R} = \begin{bmatrix} |R_1|e^{j\alpha_1} & 0 \\ 0 & |R_2|e^{j\alpha_2} \end{bmatrix}, \quad (10)$$

and hence for a given S_r at some frequency ω , the $\rho(\mathbf{R}S_r)$ is determined by the absolute values $|R_1|$, $|R_2|$ and the phases α_1 , α_2 .

In close analogy with the analytical bound (9), let us consider the situation that one wishes to obtain an upper-bound on the absolute value of the combination $(|R_1|, |R_2|)$ irrespective of α_1 and α_2 . At some frequency ω , and a fixed combination $[|R_1|, |R_2|]$, the maximum eigenvalue is,

$$C(|R_1|, |R_2|, j\omega) = \max_{\alpha_1, \alpha_2} \rho(\mathbf{R}S_r) \quad (11)$$

With the maximum possible spectral radius at a fixed $(|R_1|, |R_2|)$ known, the next task is to determine the set of allowable combinations $\{|R_1|, |R_2|\}$ such that $C(|R_1|, |R_2|, j\omega) < 1$. Note that since we consider R_1, R_2 to be passive systems, i.e. they do not provide any amplification, it is clear that $0 \leq |R_1|, |R_2| \leq 1$, $\forall \omega$. Given such range, Fig. 2 provides a typical example of the complete function (11) at a fixed frequency. Note that the function is monotonically increasing from (0,0) and the black line provides the border for which $C(|R_1|, |R_2|, j\omega) = 1$.

By explicitly calculating a similar map as provided in Fig. 2 at each frequency, one has sufficient information to determine if a given set of frequency dependent up- and downstream reflection coefficients satisfies the constraint $C(|R_1(j\omega)|, |R_2(j\omega)|, j\omega) < 1, \forall \omega$ and hence if the system can be guaranteed to be stable.

However, in order to keep the close analogy with the analytical result in Eq. (9), it is desirable to calculate a single frequency dependent (1D) upper-bound on the magnitude of the reflection coefficients. Therefore, let us consider the largest

Fig. 2. The constraint function $C(|R_1|, |R_2|, \omega)$. (—) $C = 1$, (---) largest circular region $\bar{r}$ where $C(|R_1|, |R_2|, j\omega) < 1$.

possible circle $\bar{r} = \sqrt{|R_1|^2 + |R_2|^2}$ which fits into the region where $C(|R_1|, |R_2|) < 1$, also illustrated in the figure. Such region can be conveniently calculated numerically by solving the following constrained minimization problem,

$$\bar{r}(j\omega) = \min_{|R_1|, |R_2|} \sqrt{|R_1|^2 + |R_2|^2}, \quad (12)$$

subject to

$$C(|R_1|, |R_2|, j\omega) = 1; \quad (13)$$

Note that depending on the shape of the black line in Fig. 2, this problem may be non-convex. This has to be taken into account into the implementation, which discussion is out of the scope of this paper. It is clear that the resulting bounds on the up- and downstream reflection coefficients can then be written as,

$$\sqrt{|R_1(j\omega)|^2 + |R_2(j\omega)|^2} \leq \bar{r}(j\omega), \quad \forall \omega. \quad (14)$$

It is evident that Eq. (14) gives an upper-bound for the combination of the up- and downstream losses. For example, one may compensate a lack of downstream losses by increased acoustic losses on the upstream side. Therefore, this bound gives added design flexibility over the analytical bound based on Eq. (9).

4. A case study

4.1. Burner with multiple Bunsen flames

In order to apply the methods presented in the last section, one needs to obtain the scattering matrix response of the flame or burner of interest. As an example, we will consider perforated plate type burners with multiple laminar Bunsen flames. The plate diameter is 5 [cm] and the thickness is 1 [mm]. The hexagonal perforation pattern is

determined by the diameter of the holes D and the pitch between them P in [mm]. In addition, we characterize the operating conditions by the methane to air equivalence ratio Φ and the mean velocity through the open hole area V . Two particular combinations of flow settings and burners will be presented below, namely: (1) D3-P6 at $\Phi = 0.95$ and $V = 120$ [cm/s], and (2) D2-P3.5 at $\Phi = 0.95$ and $V = 75$ [cm/s]. Similar burners and flames are widely studied [19–21].

It is generally accepted that for the burners considered here, the linear flame-acoustic behaviour can be characterized by the flame transfer function $\mathcal{F}$, defined as, $\mathcal{F}(\omega) = (\mathcal{Q}'/\bar{Q})/(\bar{u}'/\bar{u})$, where $\mathcal{Q}'$, u' are the fluctuating and $\bar{Q}$, $\bar{u}$ are the mean heat release rate and upstream velocity respectively. The flame transfer functions were measured using the standard experimental techniques described in [19,20]. Next, the flame scattering matrix can be constructed using the measured transfer function and the linearized Rankine-Hugoniot relations [22]. In the limit of zero Mach number, the flame scattering matrix can be written as,

$$\mathbf{S}_f = \frac{1}{2} \frac{1}{T_{f11}} \begin{bmatrix} -2T_{f12} & 4 \\ T_{f11}^2 - T_{f12}^2 & 2T_{f12} \end{bmatrix}, \quad (15)$$

where $T_{f11} = \epsilon + 1 + \theta\mathcal{F}$, $T_{f12} = \epsilon - 1 - \theta\mathcal{F}$ denote the relative flame transfer matrix elements. Here, $\theta = T_2/T_1 - 1$ represents the temperature ratio, and $\epsilon = (\rho_1 c_1)/(\rho_2 c_2)$, is the relative jump in specific acoustic impedance across the flame. For the results discussed below, these constants were taken as $T_1 = 293$ [K], and $T_2 = 2193$ [K], the adiabatic flame temperature at $\Phi = 0.95$, giving $\theta \approx 6.49$ and $\epsilon \approx 2.83$.

4.2. Comparison of analytical and numerical upper-bounds

Given the flame scattering matrix, the analytical and numerical upper-bounds can be readily calculated using Eqs. (9) and (14). Figure 3 shows the flame frequency response, the maximum $\bar{\sigma}(\mathbf{R})$ and numerically calculated upper-bound $\bar{r}$, all for the first case. For the sake of comparison, $\bar{r}$ is normalized with $\sqrt{2}$; the maximum value for a passive acoustic system.

Examination of Fig. 3 shows that the analytical bound $\bar{\sigma}(\mathbf{R})$ is more conservative than the numerically calculated bound $\bar{r}$. This is especially critical in the low frequency range, where high acoustic losses are difficult to realize in practice. Both bounds have distinct minima and maxima signifying the frequency bands for which the required acoustic losses are more or less demanding. In some sense, the line $\bar{r}$ can be interpreted as a ‘figure of merit’, because it quantifies the difficulty of designing a stable acoustic environment for the flame. Note that the relation between the

Fig. 3. Flame frequency response, maximum allowable $\bar{\sigma}(\mathbf{R})$ and $\bar{r}$ for case (1): D3-P6 at $\Phi = 0.95$ and $V = 120$ [cm/s].

frequency dependence of the bounds and the flame frequency response is non-trivial. For example, the minimum values of $\bar{\sigma}(\mathbf{R}) = (\bar{\sigma}(\mathbf{S}_f))^{-1}$ and $\bar{r}$ are determined by the location of the zeros of T_{f11} . Further inspection of Eq. (15) shows that this occurs at frequencies where $|\mathcal{F}| = (\epsilon + 1)/\theta$ and $\angle(\mathcal{F}) = -\pi$. Indeed, the minima of the bounds depicted in Fig. 3 occur where the phase of the transfer function equals $-\pi$ and the gain is close to $|\mathcal{F}| \approx 0.84$. It was found earlier that this condition also determines the maximum possible energy amplification by the flame [11,23]. Note that for this flame, the phase of $\mathcal{F}(j\omega)$ crosses $-\pi$ a number of times. As such, there are also multiple dips in the maximum allowed acoustic losses.

In contrast, the second case, shown in Fig. 4, has a relatively small phase lag. As a consequence, there is only one shallow minima of $\bar{r}$ in the middle frequency range. In general, to guarantee stability of this burner one would need to provide only moderate acoustic losses, but in a very wide frequency range. Again, note that $\bar{\sigma}(\mathbf{R})$ is significantly more conservative than $\bar{r}$.

Fig. 4. Flame frequency response, maximum allowable $\bar{\sigma}(\mathbf{R})$ and $\bar{r}$ for case (2): D2-P3.5 at $\Phi = 0.95$ and $V = 75$ [cm/s].

It is interesting to observe that the relatively small quantitative differences between the flame transfer functions of case 1 and 2, lead to large qualitative differences in the calculated bounds $\bar{r}$ and $\bar{\sigma}(\mathbf{R})$.

5. Discussion and conclusions

In essence, the presented approach allows to extract from the flame scattering matrix these properties of the flame which are relevant for overall thermo-acoustic system stability. In particular, the characterization of the flame response in terms of requirements on the acoustic losses can be used for two purposes. On the one hand, the upper bound on the losses may serve as a measure for the flame instability potential, i.e. as a figure of merit. On the other hand, the bounds can directly serve as guidelines for the design of an appropriate acoustic surrounding such that stable operation can be guaranteed.

A number of interesting extensions to the current approach can be worthwhile to explore. First of all, the results can be verified by direct numerical simulation and experiment. The already performed numerical modelling confirms the validity of the proposed approach. The presentation of these results would add significant length and is out of the scope of the current contribution.

From a fundamental viewpoint, it is essential to further investigate how the upper bounds are affected by the flame properties such as the transfer function and the temperature jump. This may also allow to clarify the influence of the physical flame properties on its thermo-acoustic stability behaviour.

Another important aspect is that the developed method is not limited to a flame scattering matrix $\mathbf{S}_f$ as the only fixed element. In principle, any part of the system, which is adjacent to the flame, and may not be altered in the design, can be included inside a cumulative scattering matrix $\mathbf{S}$. The exact same approach can then be followed to derive design guidelines for the remaining part of the system.

Similarly, it is also possible to consider, using the same method, the inverse problem: a fixed acoustics with the flame as the design variable.

From the practical perspective, the method provides a design goal for the acoustic subsystem. In this context, it should be clear that the revealing of, possibly less conservative, analyti-

cally or numerically determined bounds will be advantageous during the design for stability of a complete combustion appliance.

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