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Citation for published version (APA):

Polyuga, R. V. (2010). *Discussion on: "Passivity and structure preserving order reduction of linear port-Hamiltonian systems using Krylov subspaces"*. (CASA-report; Vol. 1052). Technische Universiteit Eindhoven.

Document status and date:

Published: 01/01/2010

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

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EINDHOVEN UNIVERSITY OF TECHNOLOGY
Department of Mathematics and Computer Science

CASA-Report 10-52
September 2010

Discussion on: “Passivity and structure preserving
order reduction of linear port-Hamiltonian
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ISSN: 0926-4507

Discussion on: "Passivity and Structure Preserving Order Reduction of Linear Port-Hamiltonian Systems using Krylov subspaces"

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The port-Hamiltonian approach to modeling and control of complex physical systems has arisen as a systematic and unifying framework during the last twenty years, see [11], [7], [12], [2] and the references therein. The port-Hamiltonian modeling captures the physical properties of the considered system including the energy dissipation, stability and passivity properties as well as the presence of conservation laws. Another important issue the port-Hamiltonian approach deals with is the interconnection of the physical system with other physical systems creating the so-called physical network. In real applications the dimensions of such interconnected port-Hamiltonian state-space systems rapidly grow both for lumped- and (spatially discretized) distributed-parameter models. Therefore an important issue concerns (structure preserving) *model reduction* of these high-dimensional models for further analysis and control.

There are a variety of methods and algorithm for model reduction of linear systems serving different purposes. An excellent overview of model reduction theory for linear input-state-output systems can be found in [1], [10].

The paper by Wolf., et. al. [14] presents a new scheme for model reduction of linear port-Hamiltonian systems with dissipation using Krylov subspaces. The scheme preserves the port-Hamiltonian structure and passivity property. Furthermore, the scheme is moment matching and computationally efficient and therefore can be applied to reduce systems with extremely large dimension (i.e. larger than 1000).

The authors start with the description of linear port-Hamiltonian systems, which, in the absence of algebraic constraints, take the following form ([11]):

$$\begin{cases} \dot{x} &= (J - R)\nabla H(x) + Gu, \\ y &= G^T \nabla H(x), \end{cases} \quad (1)$$

with $H(x) = \frac{1}{2}x^T Qx$ the *total energy* (Hamiltonian), $Q \in \mathbb{R}^{n \times n}$ positive definite symmetric *energy* matrix and $R \in \mathbb{R}^{n \times n}$ positive semi-definite symmetric *dissipation* matrix. The skew-symmetric matrix $J \in \mathbb{R}^{n \times n}$ and the input matrix $G \in \mathbb{R}^{n \times m}$ specify the *interconnection* structure. Since J is skew-symmetric and R is positive semidefinite it immediately follows that $\frac{d}{dt} \frac{1}{2}x^T Qx = u^T y - x^T Q R Q x \leq u^T y$. Thus if $Q \geq 0$ (and the Hamiltonian is non-negative) any port-Hamiltonian system is *passive* (see also [13],[11]).

In this paper the authors concentrate on the single-input

single-output (SISO) port-Hamiltonian systems which can be written (since $\nabla H(x) = Qx$) as

$$\begin{cases} \dot{x} &= (J - R)Qx + gu, \\ y &= g^T Qx, \end{cases} \quad (2)$$

with $g \in \mathbb{R}^n$.

The authors revisit the model reduction problem for linear systems by moment matching. The system (2) can be treated as a linear time-invariant state space model of the form

$$\begin{cases} \dot{x} &= Ax + bu, \\ y &= cx, \end{cases} \quad (3)$$

having the transfer function $G(s) = c^T(sI - A)^{-1}b$ with the Taylor series expansion

$$G(s) = -m_0 - m_1(s - s_0) - \dots - m_i(s - s_0)^i - \dots \quad (4)$$

around an arbitrary expansion point s_0 . The coefficients m_i are the moments of the full order model (3).

The goal of the model reduction by moment matching is to find the reduced model of order $q < n$, such that its first moments are equal to those of the full order model around a chosen point s_0 . The reduced order model is of the form

$$\begin{cases} W^T V \dot{x}_r &= W^T A V x_r + W^T b u, \\ \hat{y} &= c^T V x_r, \end{cases} \quad (5)$$

where $V, W \in \mathbb{R}^{n \times q}$ are the projection maps, which are chosen using Krylov subspaces, generally defined as $K_q(M, v) = \text{span}\{v, Mv, \dots, M^{q-1}v\}$. If the columns of V are chosen in such a way that they are the basis for the shifted input Krylov subspace $K_q((A - s_0 I)^{-1}, (A - s_0 I)^{-1}b)$, and W is arbitrary, then the reduced order model (5) preserves q moments of the full order model (3) around s_0 [3].

For the numerical calculation of V , the known Arnoldi algorithm [?] can be used, which returns an orthonormal basis V (with $V^T V = I$) of the required Krylov subspace. The widely used choice $W = V$ is referred to as a one-sided method.

The model reduction by moment matching is applied to linear port-Hamiltonian systems (2) in the following way. Firstly, the projection map V is constructed on the basis of the shifted Krylov subspace $K_q = \text{span}\{(J - R)Q - s_0 I)^{-1}g, \dots, ((J - R)Q - s_0 I)^{-q}g\}$. Secondly, using V , the full order port-Hamiltonian system (2) is projected resulting in the reduced order model

$$\begin{cases} V^T QV \dot{z}_r &= V^T Q(J - R)QV z_r + V^T Qbu, \\ \hat{y} &= g^T QV z_r \end{cases} \quad (6)$$

Finally, the state space transformation $z_r = (V^T QV)^{-1} x_r$ (for nonsingular Q) gives the reduced order model

$$\begin{cases} \dot{x}_r &= V^T Q(J - R)QV(V^T QV)^{-1} x_r + V^T Qbu, \\ \hat{y} &= g^T QV(V^T QV)^{-1} x_r. \end{cases} \quad (7)$$

It is proven in the paper by Wolf et.al. [14] that the reduced order model (7) is moment matching and port-Hamiltonian.

Note that the scheme described above can be seen as a two-sided model reduction scheme with the projection map W given as $W = QV$. The authors also mention that in a similar way moments at infinity (Markov parameters) can be matched.

Model reduction of port-Hamiltonian systems by moment matching at infinity was considered in [9], where the energy matrix Q is transformed to the identity matrix. One of the attractive advantages of the method in the paper by Wolf et.al. [14], as compared to that of [9], is that there is no need for a computationally expensive coordinate transformation. Indeed, in general it is cheaper to compute the inverse of the reduced order matrix $V^T QV$ than to transform a full order matrix Q to the identity matrix. In fact, it is shown in [8] that both methods yield equivalent reduced order moment matching port-Hamiltonian models, in the sense of sharing the same transfer function.

Other structure preserving port-Hamiltonian reduction methods can be found in [8],[9],[4],[6],[5].

At the end of the paper the method is illustrated by a numerical example. A full order port-Hamiltonian model, corresponding to a clamped beam, is reduced by the one-sided Krylov method and the port-Hamiltonian method, presented in this paper. The behaviors of the full and reduced order models are compared. It is seen that the one-sided Krylov method fails to preserve stability, while the port-Hamiltonian method produces a stable reduced order port-Hamiltonian model.

The paper by Wolf et.al. [14] provides a computationally efficient moment matching method for structure preserving model reduction of port-Hamiltonian systems. The reduced order models, apart from matching moments of the full order model, inherit the port-Hamiltonian structure and thus passivity and stability. The presented method offers in fact a solution to the general stability preservation problem in Krylov subspace methods for linear dynamical systems, since every stable linear system $\dot{x} = Ax + bu$ can be written (provided that the computational cost is acceptable) in the port-Hamiltonian form $\dot{x} = (J - R)Qx + gu$. The contribution offers one more reason to model large real world problems in the port-Hamiltonian form.

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