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High Performance Stationary Frame Filters for Symmetrical Sequences or Harmonics Separation Under a Variety of Grid Conditions

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Abstract—This paper proposes a group of high performance filters for fundamental positive / negative sequences and harmonics detection under varied grid conditions based on a basic filter cell. The filter cell is demonstrated to be equivalent to a band-pass filter in the stationary frame, and is easily implemented using a multi-state-variable structure. To achieve high performance in different grid conditions, cascaded filters are developed for distorted and unbalanced grids. This paper also analyzes the robustness of the filter for small frequency variations, and improves its frequency-adaptive ability for large frequency changes. Furthermore, it is proved that this filter can also be applied for the synchronization in a single-phase system. Considering the digital implementation of the filter, four discretization methods and the resulting limitations are investigated. The effectiveness of the presented filters is verified by experiments.

I. INTRODUCTION

For the control of power electronics-based grid-interfacing systems, synchronization with the utility grid is essential. Obviously, the detection of the fundamental positive-sequence components should be accurate under unbalanced and/or distorted conditions. Furthermore, in order to deal with power flow control or power quality improvement (like active power filtering, voltage dips compensation, etc.), the detection of negative-sequence components or harmonics is also always needed [1]-[3]. Although in the utility grid the frequency is usually very stable, frequency fluctuations sometimes can be caused by transient faults on the grid, or frequently occur in weak small-scale networks. This problem will result in system trips.

Many interfacing methods that have been presented in the literature for varied grid conditions, which are either limited to the purpose of synchronization, or for symmetrical-sequence detection under unbalanced conditions. To synchronize with the grid, different closed-loop control algorithms are developed based on a conventional phase-locked-loop (PLL) structure [4]-[6], or a clean grid signal is generated before using the PLL [7]. Alternatively, in the manner of open-loop control, fundamental-sequence separation can be directly achieved based on signals estimation or calculation [8]-[10]. However, these methods are usually sensitive to the grid frequency. In addition, some robust methods were proposed to deal with unbalanced, distorted, and variable-frequency grid conditions. For instance, a scheme based on a decoupled double synchronous reference frame (SRF) PLL in [11] eliminates the detection errors of a conventional PLL by separating the positive-sequence and negative-sequence components in the double SRF. Although a higher performance PLL is achieved, it needs a high amount of computation time due to doing the transformation and inverse transformation of reference frames twice.

Therefore, this paper proposes an alternative stationary frame method with a group of filters used for a variety of grid conditions. These filters are developed step by step based on a basic filter cell. First, the principle of the basic filter cell is presented. Following that, cascaded filters and frequency-adaptive filters are derived for the application in fixed-frequency and variable-frequency conditions. Next, the applicability in single-phase system is analytically proved, and limitations of the digital implementation are investigated. Finally, experiments are carried out to verify the effectiveness of the proposed filters.

II. BASIC FILTER CELL

This section presents the principle of the basic filter cell that will be used to develop high performance filters later on. The implementation structure of the basic filter was introduced in [7] to build a robust PLL by separating the fundamental positive-sequence component from unbalanced and/or distorted grids for the PLL. By utilizing a multi-state-variable structure, this cell can be easily implemented to achieve the function of a second-order band-pass filter in a stationary frame. To extend the application of this idea, an improved filter for fundamental positive and negative sequence voltage detection was described in [3], where detailed design formulas were given. Similarly, a generalized selective-harmonic band-pass filter cell can be derived.

For unbalanced distorted voltages, the positive- and negative-sequence components in the $\alpha$ – $\beta$ frame are ex-
pressed by
\[ v_{o\beta}(t) = v_{o\alpha}(t) + jv_{o\beta}(t) = \sum_{n=1}^{\infty} (V_{o+}^n e^{j\omega_n t} + V_{o-}^n e^{-j\omega_n t}), \]
where \( n \) is the harmonic number, \( \omega_1 \) the fundamental radian frequency; the superscript symbol "o" denotes conjugate, and complex numbers are denoted with a bar subscript.

When looking for a transfer function which can separately derive \( k^{th} \) harmonic components from the input \( \alpha, \beta \) signals in the stationary frame, two selective-harmonic filters, named \( G_k^+(s) \) and \( G_k^-(s) \), are found to achieve this purpose in terms of positive and negative sequences. The filter actions are expressed with
\[ \frac{V_o^+(s)}{V_{o\alpha}(s)} = G_k^+(s), \]
\[ \frac{V_o^-(s)}{V_{o\alpha}(s)} = G_k^-(s). \]

These equations can be easily implemented in the \( \alpha - \beta \) frame by time domain digital techniques. More consideration on digital implementation will be presented in a following section. Fig. 1 shows the implementation diagram for positive-sequence components. The filter for negative-sequence components is identical but changes the central frequency to \(-k\omega_1\). Note that two internally derived variables, \( v_{o\alpha} - v_{o\beta} \) and \( v_{o\beta} - v_{o\beta} \), are taken out from the filter. These represent the residues of the two input signals minus the extracted components. In summary, the detection of \( v_{o\alpha}^k = v_{o\alpha}^+ + jv_{o\beta}^+ \) and \( v_{o\beta}^k = v_{o\alpha}^- + jv_{o\beta}^- \) is achieved with (3) and (4), since \( v_{o\alpha}^k \approx v_{o\alpha}^+ \) and \( v_{o\beta}^k \approx v_{o\beta}^- \).

III. Operation Under Fixed-Frequency Conditions

Based on the basic filter cell derived above, cascaded filters are constructed to output fundamental positive-sequence com-
ponents, fundamental negative-sequence components, and harmonics. These filters are categorized into fixed- and variable-frequency classes. This section focuses on filter design limited to fixed-frequency conditions.

**A. Positive- and Negative-sequence Detection**

Theoretically, when inputting unbalanced and distorted signals, fundamental positive- and negative-sequence components can be directly filtered out with the above proposed filter cells by setting the index \( k \) to 1 and -1 in Fig. 1. For this case, a frequency domain plot of the basic filter is drawn in Fig. 2. It can be seen that both positive- and negative-sequence filters have unity gain and zero phase-shift at the central frequency. By decreasing the bandwidth parameter \( \omega_b \), the damping ratio for other frequency components is increased, however, at the price of increased response time. This will be a compromise in a practical design. Unfortunately, in practical applications, input signals usually involve a large proportion of positive-sequence components which are difficult to damp totally. Therefore, the negative-sequence component is too small to be detected accurately by using only a basic filter cell. As a consequence, for the basic negative-sequence filter a set of input signals is required with the fundamental dominant positive sequence already removed. Thanks to the implementation structure, these signals are exactly the two variables \( v_{\alpha} - v_{\alpha k} \) and \( v_{\beta} - v_{\beta k} \) in Fig. 1 when \( k = 1 \).

It follows that a cascaded filter is constructed for the separation of two fundamental sequences. Fig. 3(a) illustrates the implementation diagram based on the filter cell, where the negative-sequence component is removed. Note that the residue of harmonics \( v_{\alpha h} \) and \( v_{\beta h} \), that is the total of other harmonics, are output if there exist other components in the input signals other than the fundamental-frequency ones.

**B. Harmonics Separation**

The filter described above deriving negative-sequence component and total harmonics (Fig. 3(a)) can be used by active power filters, for instance, compensating for three-phase unbalanced nonlinear loads. Nevertheless, there are other applications for which it is desirable to detect a specific harmonic, e.g. in the application of selective harmonic compensation. Similarly, a cascaded selective-harmonic filter can be constructed to separate harmonics, as shown in 3(b). For each individual filter cell, the bandwidth should be fine tuned based on the actually present distortion.

It is pointed out that, for a three-phase system with a symmetric distortion, harmonics can be divided into two groups in terms of positive and negative sequences. In other words, harmonics \( v_{n \beta k} \) only exist in terms of positive sequences when \( k = 6m + 1 \) (\( m = 1, 2, 3... \)), or exists in terms of negative sequences when \( k = 6m + 1 \). This helps to make the implementation easier since each individual harmonic needs one either positive or negative filter cell. Otherwise, twice the number of filter cells are needed and therefore the computation time is doubled. A frequency domain plot for the
The cascaded filter is drawn in Fig. 4, illustrating the frequency response of the filters of Fig. 3 (a) and (b). It can be seen that the filter operates equivalently to a notch filter at the positive fundamental frequency, hence improving the filter's effectiveness. In fact, more structures can be constructed in a similar manner as those cascaded filters presented in this section, depending on practical necessities.

IV. OPERATION UNDER VARIABLE-FREQUENCY CONDITIONS

The previously designed filters were developed for a fixed-frequency situation. It is quite relevant to investigate and improve their frequency insensitivity when applied to variable-frequency conditions.

A. Robustness for Small Frequency Variations

It is worth noticing that the proposed filters are robust to small frequency variations. This can be explicitly analyzed from the frequency response characteristics, as shown in Fig. 5, where the central frequency of the filter is fixed at 50 Hz and the frequency of the input signals vary between 47 Hz and 53 Hz. It can be seen that the magnitude change and phase shift are small within 49 Hz to 51 Hz, i.e., ±2% deviation from the central frequency, and the effects decrease when extending the bandwidth. In general, 2% of frequency tolerance in the grid is big enough. For higher performance, the robustness of filters can be improved by increasing the bandwidth \( \omega_b \) slightly at the cost of slower response.

B. Adapt to Large Frequency Changes

However, it is illustrated clearly in Fig. 5 that a large frequency deviation from the central frequency can lead to a serious phase shift and magnitude damping. Therefore, a frequency updating scheme should be added to the filter cell. A widely implemented PLL structure can be used to update the value of \( \omega_1 \). The implementation structure is shown in Fig. 6, where an input variable \( \omega_{\text{in}} \) should be given as the initial value around the central frequency, and a low pass filter is used to eliminate the ripple on \( \omega_1 \) introduced by the PLL regulator. On the other hand, note that the PLL also benefits from the filter because the derived signal is separated from noises or harmonics, although its magnitude and phase are influenced at the moment of frequency change.

V. FURTHER CONSIDERATION

A. Application for Single Phase

The filter cell was introduced in the \( \alpha - \beta \) frame in Section II, apparently, for a three-phase system, but it can also be used for single phase applications. To help understanding, a single-phase system can be regarded as an extreme case of three-phase unbalance. By transforming the single phase signal, denoted by \( v_{1\phi} \), to the \( \alpha - \beta \) frame, we obtain

\[
\begin{bmatrix}
v_{\alpha}
v_{\beta}
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2}
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}
\end{bmatrix} \begin{bmatrix}
v_{1\phi}
0
0
\end{bmatrix} = \begin{bmatrix}
\frac{2v_{1\phi}}{3}
0
0
\end{bmatrix}.
\]

(5)

It means that a set of signals from (5) can be used as the input signal of the filter. On the other hand, the single-phase signal can be composed in terms of symmetrical sequences. In phasor notation it can be expressed as

\[
\begin{bmatrix}
\frac{V_{1\phi}}{0}
0
0
\end{bmatrix} = \mathbf{A} \begin{bmatrix}
\frac{V_0}{V_{1\phi}}
\frac{V_+}{V_{1\phi}}
\frac{V_-}{V_{1\phi}}
\end{bmatrix},
\]

(6)

where

\[
\mathbf{A} = \begin{bmatrix}
1 & 1 & 1
1 & a & a^2
1 & a^2 & a
\end{bmatrix}, \quad a = e^{-j \frac{2\pi}{3}},
\]

complex numbers are denoted with a bar, and the subscripts \( \alpha, \beta \), \( \alpha^2 \), and \( 0 \) denote the positive, negative and zero sequences, respectively. After manipulation, the positive- and negative-sequence components can be calculated by

\[
V_{1\phi}^+ = V_{1\phi}^- = \frac{1}{3} V_{1\phi}.
\]

(7)
Therefore, if the filter is configured with its fundamental frequency at the central frequency, the two signals derived from the filter will be two signals in the $\alpha - \beta$ frame, which represent the positive-sequence component of the extremely unbalanced three-phase signals. One is in phase with the input signal, the other orthogonal, and both have one third the amplitude of the single-phase signal.

It is remarked that the bandwidth for this application should be lower than for the application in a three-phase system in order to get good results. This is because the amplitudes of the positive-sequence and the negative-sequence components are equal when the single-phase system is regarded as an extremely unbalanced three-phase one.

### B. Digital Implementation and Limitation

To implement the filters in a digital way, different methods can be used for the discretization of the integrator ($\frac{1}{s}$) in the filter cell. Several typical methods that are investigated in the $z$-domain are:

a) Forward Euler:

$$\frac{1}{s} \rightarrow T_s \frac{z}{z - 1},$$

b) Backward Euler:

$$\frac{1}{s} \rightarrow T_s \frac{1}{z - 1},$$

c) Trapezoidal (or Tustin):

$$\frac{1}{s} \rightarrow T_s \frac{z + 1}{2(z - 1)},$$

d) Two-step Adams-Bashforth:

$$\frac{1}{s} \rightarrow T_s \frac{3z - 1}{2z^2 - z},$$

where $T_s$ denotes the discrete time step.

These methods only approximate an ideal integrator when transforming from the time domain to the discrete domain, so the accuracy of the approximation does influence the effects of the filter. The frequency characteristics of the four methods are shown in Fig. 7, where $T_s = 125\mu s$. Compared with an ideal integrator, methods a) and b) have the worst phase-shift starting at around $100$ Hz, d) is much better, and method c) is the best one. However, method c) has an “algebraic-loop” issue due to the implementation structure of the filter cell. A solution for that is to use the closed-loop transfer function of the filter instead, but then the explicit advantage of easy implementation of the filter cell is lost. Certainly, the accuracy of the approximation can also be improved when sampling quicker. In this paper, method d) is selected as a compromise.

Next, a further study was carried out to check the limitation on the effects of the filter when using method d). As an example, a filter cell is investigated, which set a fundamental frequency of $50$ Hz, bandwidth $100$ rad/s, and sampling frequency $8khz$. The bode plots are drawn in Fig. 8, where filters applied for positive-sequence components at the fundamental frequency and low-order harmonics are displayed. It is shown that the filters applied for the harmonics above $7^{th}$ are not correct any more. A possible improvement is to decrease $T_s$, but this should be compromised in practice because of other limitations, for example, the minimum computation time for the control loop.

VI. EXPERIMENTAL RESULTS

Experiments have been carried out for verification purposes. A three-phase programmable AC power source was used to emulate various grid conditions. The controller is built with a dSPACE DS1104 setup. Considering its application for power
electronic converters, which usually have 5kHz to 20kHz switching frequency, a sampling frequency of 8kHz was used to implement the digital filters. A two-step Adams-Bashforth discretization method was used.

Fig. 9 shows the results for a single phase system. The \( \omega_b \) is set to 60\( \text{rad/s} \). It can be seen that a set of sinusoidal signals with a fundamental frequency at 50Hz are derived from the proposed filter when used for a single-phase distorted grid. According to (7), the output positive-sequence components are exactly one third the amplitude of the input single-phase signal.

Next, the filters are verified for the application in three-phase system. The signals coming out of the filters are shown in the \( \alpha-\beta \) frame. In the following experiments, \( \omega_b \) is set to 100 \( \text{rad/s} \) for the fundamental positive-sequence filter and 80 \( \text{rad/s} \) for others.

As shown in Fig. 10, a cascaded filter as in Fig. 3(b) is designed for the harmonics separation from a set of balanced distorted signals. Because of the limitation on the filter for high-order harmonics in the case of 8kHz sampling frequency, only 5\( ^{th} \) and 7\( ^{th} \) harmonics are emulated for the verification.

Fig. 11 (i) and (ii) show the behavior of the filters for symmetrical sequence detection under fixed-frequency and variable-frequency grids, respectively. They show good performance for symmetrical sequence detection and the ability of adapting to frequency changes dynamically.

**VII. CONCLUSION**

This paper introduced a group of high performance filters for fundamental positive sequence, fundamental negative sequence, or harmonics detection, in a polluted grid. The basic
Fig. 11. Experimental waveforms from case (i) a cascaded filter where the balanced grid voltage getting 40% magnitude dips in phase A and B, and from case (ii) a frequency adaptive filter with the grid frequency changing from 50 Hz to 60 Hz at time t. The waveforms from (a) to (d) are: grid voltages in the a-b-c frame, grid voltages in the a-β frame, fundamental positive-sequence and negative-sequence voltages.

filter cell is demonstrated to be equivalent to a band-pass filter in the stationary frame, and can be easily implemented using a multi-state-variable structure. Based on the filter cell, cascaded filters are developed to achieve high accuracy and high performance under unbalanced, distorted, and variable-frequency conditions. By assuming a single-phase system to be an extremely unbalanced three-phase system, the filter is proved to be effective also for single-phase applications. In addition, digital implementation and its limitation were further considered. It is concluded that the proposed filters are appropriate for fundamental and low-order harmonics, and must be improved for high-order harmonics by making the sampling frequency high enough. Finally, the effectiveness of the proposed filters is verified by experiments.

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