Sliding window recursive DFT with dyadic downsampling: a new strategy for time-varying power harmonic decomposition

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Abstract — The main objective of this paper is to present a time-varying harmonic decomposition using the sliding-window Discrete Fourier Transform including a dyadic downsampling. The method has been conceived to decompose and to visualize time-varying harmonics up to 15th order, with a low computational effort. The downsampling must be done carefully in order to impose no aliasing error to the lower order harmonics. The algorithm has been very efficient and helpful when tracking time-varying power harmonic and it has been applied to track harmonic distortion and understand time-dependent power quality events.

The method presents potential for other applications such as control and protection.

Index Terms—Harmonic distortion, harmonic decomposition, recursive DFT, time-varying harmonics.

I. INTRODUCTION

Signals decomposition techniques are concerned with the way that the original signal can be split into individual components, including harmonics, interharmonics, subharmonics, etc. Normally, signal decomposition is carried out in the time-domain, such that the time-varying behavior of each harmonic component can be observable. This subject is an important issue in power quality analysis for different reasons, including analysis of loads behavior, failure detection, pattern recognition of events, etc.

There are different techniques that can be used to separate frequency components; among them the most used have been Short Time Fourier Transform (STFT) and Wavelets Transforms [1, 2, 3].

Unfortunately, the structures using wavelet are not able to decouple the frequencies completely [2].

Other techniques have been proposed when the fundamental frequency is time varying and the sampling frequency is not synchronous, such as: adaptive notch filter [4], Phase-Locked Loop (PLL) [5, 6], resonator-in-a-loop filter [7] and a multistage implementation of narrow low-pass digital filters valid to extract stationary harmonic components [8].

For most of the applications in power quality one can work with a synchronous sampling frequency, as well as consider the fundamental frequency practically constant with no inter-harmonic. In these cases, other approaches can be used, such as [9] and [10].

In [9] the authors presented a new methodology to separate the harmonic components until the 15th harmonic using multirate and filter bank approach. The method is able to track time-varying power harmonic frequencies without frequency spillover. An alternative to this approach is to use a Sliding-Window Recursive-DFT (SWR-DFT) [10], which presents a low computation burden, no phase delay and a short transient time.

This paper presents an improvement to be inserted to the SWR-DFT method presented in [10], including a dyadic downsampling before each group of harmonics to be extracted and tracked.

The advantage of this new strategy compared to a previous one is the reduced processing time and decreasing of computational effort without loss of information.

II. SLIDING WINDOW RECURSIVE DFT

In Fourier series theory, (1) and (2) are well known for real periodic signals. The second one (rectangular form) is, of course, related to the first one through (3) and (4).

\[
x(k) = a_0 + 2\sum A_n \cos(w_n k + \theta_n) \tag{1}
\]

\[
x(k) = a_0 + 2\sum Y_{c_n}(k) \cos(w_n k) - Y_{s_n}(k) \sin(w_n k) \tag{2}
\]

\[
A_n = \sqrt{(Y_{c_n})^2 + (Y_{s_n})^2} \tag{3}
\]

\[
\theta_n = \arctan \left( -\frac{Y_{s_n}}{Y_{c_n}} \right) \tag{4}
\]

In being so, the rectangular (quadrature) terms \( Y_{c_n} \) and \( Y_{s_n} \) can be obtained by using the expressions in (5),
$$Y^k_{c_n} = \frac{2}{N} \sum_{t=0}^{N-1} x_{(k-N)t} \cos \left( \frac{2\pi h l}{N} \right) \quad (5-a)$$
$$Y^k_{s_n} = \frac{2}{N} \sum_{t=0}^{N-1} x_{(k-N)t} \sin \left( \frac{2\pi h l}{N} \right); \quad (5-b)$$

where $N$ is the number of samples per cycle and $k$ is the actual sample.

These expressions are very common in algorithms of protection numerical relay and normally are performed just to extract the fundamental component phasor ($h = 1$). The moving or sliding window concept is then applied, that is, as a new sample becomes available, the oldest is discarded and the new one is included in the calculation, in such a way that $N$ is always the same during the processing task. The sine and cosine coefficients are defined as function of $N$ for each component $h$. For $h = 1$ the algorithm using (5) is known as full-cycle DFT [11], which can also be represented as a difference equation in its general form (6).

$$y[k] = \sum_{m=0}^{M} b_m x[k-m] - \sum_{n=1}^{N} a_n y[k-n] \quad (6)$$

Adopting $a_n = 0$ and $a_0 = 1$, (6) becomes a non-recursive numerical filter, whose frequency response can be easily found from a difference equation, in Z-domain, designed for 16 samples/cycle, as illustrated in Fig. 1.

![Fig. 1 – Response frequency full-cycle DFT filter](image)

DFT calculations in (5) and (6) represent more calculations than are actually necessary in practice [11] and by simple adjustment the full-cycle window can become a recursive form of a full-cycle algorithm to compute the rectangular terms $Y^k_{c}$ and $Y^k_{s}$, as the structure shown in Fig. 2.

If the same structure is applied for each integer $h \neq 1$, the phasors of each harmonic are then obtained, according to the recursive equations (7):

$$Y^k_{c} = Y^k_{c} + (x_k - x_{k-N}) \cos \left( \frac{2\pi h}{N} k \right) \quad (7-a)$$
$$Y^k_{s} = Y^k_{s} + (x_k - x_{k-N}) \sin \left( \frac{2\pi h}{N} k \right) \quad (7-b)$$

where $x_k$ is the newest sample corresponding to $N$ and $x_{k-N}$ is the oldest sample corresponding to a fundamental full cycle earlier.

### III. The Decomposition Structure

Normally, the DFT recursive algorithm has been used to extract and compute the amplitude and phase of the fundamental for protections purpose [11], but not the waveform. Nevertheless, the main objective of this work is, in fact, to obtain the fundamental waveform, as well as the waveform of each individual harmonic. This task can be performed by considering and using the rectangular form (2) that has become possible from all the methodology based on Fourier Theory. The implementation of this approach can be accomplished in two ways: (a) using the sine and cosine coefficients previously calculated and stored. In this case, the algorithm must perform an internal product using a vector of coefficients in each observable window. (b) Using a digital sine-cosine generator. This second way is more effective and has been adopted to decompose and analyze some signals from power systems events, as will be demonstrated.

A digital sine-cosine generator is presented in [12], but it can be implemented with some minor modifications according to the following matrix equation:

$$\begin{bmatrix}
    s_1(n) \\
    s_2(n)
\end{bmatrix} =
\begin{bmatrix}
    \cos(w_k) & \sin(w_k) \\
    -\sin(w_k) & \cos(w_k)
\end{bmatrix}
\begin{bmatrix}
    s_1(n-1) \\
    s_2(n-1)
\end{bmatrix} \quad (8)$$

where $s_1(n)$ is a sine function and $s_2(n)$ is a cosine function.

In adopting this sine-cosine generator, both, the decomposition and the reconstruction tasks can run parallel to each other, according to Fig. 3.

For extracting $N$ harmonics it is necessary to employ a $N$ structure as shown in this Fig. 3, but there are some advantages when using it, such as:

i) low computational effort, suitable for real time decomposition implementation;
ii) no phase delay;
iii) transient time equal to the sliding window width.

Window of one cycle, the convergence is reached after one cycle.

On the other hand, the disadvantages of the method are related to the limitations of the DFT:

i) a synchronous sampling is needed;
ii) interharmonics are a source of error to the process.
IV. DYA DIC DOWNSAMPLING

The number of mathematical operations necessary to track time-varying harmonics can be substantially reduced if a reduced sampling rate is used. Therefore, the Sliding Window Recursive DFT has been implemented for different groups of harmonics and, for each group, a different number of samples is used.

Suppose a signal whose sampling frequency is 15,360 Hz or 256 samples/cycle of 60 Hz. Of course, this sampling rate, according to Shannon Theory (Nyquist criteria), is more than enough to compute and visualize up to the 15th harmonic. Thus, why not reduce the sampling rate according to the desired harmonic with a desired resolution?

To answer this question an experimental algorithm has been implemented using dyadic downsampling [13], according to Fig. 4.

In the SWR-DFT algorithm each time-step needs just one addition, one subtraction and one multiplication to perform a complete cycle for each rectangular term, according to (7). This is repeated for all harmonic components. If there are \( N \) samples per cycle, all the operation must be multiplied by \( N \) in order to accomplish a complete time period of 60 Hz. Considering, for example, a signal with \( N = 256 \), the number of operations to accomplish a complete cycle can be calculated as: \( 3 \) (operators + - *) x 256 (samples) x 16 (components) x 2 (rectangular terms \( Y_s \) and \( Y_c \)) resulting in 24,576 operations. However, adopting the downsampling strategy, the operation number is reduced to a half in each subsequent level.

In the Fig. 4, four groups of harmonics are represented, including the fundamental and the dc component. The dyadic (2\(^n\)) downsampling up to 8 is adopted to reduce the computational effort. In this case, the number of operations per cycle is 11,520, representing a reduction of 53% in operations numbers.

Several other downsampling strategies can be adopted, depending on the desired resolution for each harmonic. Figure 5 is an example. Also, no dyadic downsampling may be performed. Nevertheless, it is important to take care with the aliasing error.

By adopting the scheme in Fig. 5, for example, the spectrum of the 15th harmonic will superimpose to the fundamental component and, consequently, the amplitude and phase of the fundamental will be affected.

One strategy to avoid this error is to implement an anti-aliasing filter before SWR-DFT\(_{16}\) or to subtract the 15th harmonic from the original signal as soon it has been extracted.

V. SIMULATION RESULTS

A structure similar to the Fig. 4, however including one more level of downsampling has been used to track time-varying harmonic signals generated by simulations. Two examples are shown:
A. Synthetic Signal

These kinds of hypothetical signals, generated using a mathematical model, are important to test these classes of algorithms because the content of the signals is known. Thus, the process results can be compared and analyzed to observe the errors. Thus, several synthetic signals have been generated in Matlab™ to test the structure presented in this work. For example, the signal shown in Fig. 6 has 16 components (dc up to 15th). The signal is portioned in four different segments in such way that the result is distorted with some harmonics in steady-state and others time-varying modulated by a constant or by a exponential functions (crescent or de-crescent) or simply with abrupt changes of magnitude and phase, as well as a dc component.

![Image](https://via.placeholder.com/150)

Fig. 6 – Synthetic signal.

Figure 7 shows some components that are present in the signal from dc up to 15th harmonic. The left column represents the original components and the right column the same components obtained through the SWR-DFT with a dyadic downsampling in five levels, according to the following scheme: 256 samples/ cycle for 12th to 15th harmonic; 128 for 8th to 11th; 64 for 4th to 7th; 32 for 2nd to 3rd and, finally, 16 for dc and 1st component.

By reasons of simplicity and space limitation not all components are shown in Fig. 7. For example, the 5th harmonic was generated with a dc component (a small dc step). Although it is not shown in the figure, it will appear exactly as it is, when the dc component is extracted. It is important to remark that all waveforms of the time-varying harmonics that are contained in the signal have been extracted with efficiency and good accuracy. Naturally, the effect of the downsampling can be observed in the different outputs: they have different numbers of samples (in the right column).

An important observation is in regard to the 15th harmonic in the adopted scheme. As said, this component will interfere in the fundamental value. Figure 8 illustrates the result of another simulation, in which the 15th harmonic jumps from 0 to 0.2 up causing error to the fundamental, with the same value, when this component is extracted.

![Image](https://via.placeholder.com/150)

Fig. 7 – First column: original components, second column: decomposed signals.

Fig. 8 – Alias error due to 15th harmonic.

B. Simulated Signal

Simulated signals can be obtained from different “Electromagnetic Transient Programs”, such as ATP, SimPower-Matlab, PSCAD™, etc. These signals, depending on the precision of the models, will represent the real world with great fidelity. Therefore, it is very important to use them to test any kind of algorithm to be implemented in Intelligent Electronic Devices (IEDs).
Figure 9 shows a piece of a system that has been modeled in PSCAD, which contains a source and two section of a transmission line feeding transformers, linear and non-linear loads, like a six pulses bridge. Some disturbances are provoked in this system, such as load imbalance, load rejection and failures in the converters pulse system though the control interface.

The signals captured during the simulations have served to analyze and understand some time-varying harmonics that appears during these events.

Figure 10 is an example of a current signal that has been decomposed and whose results can be seen in Fig. 11. The odd harmonics will vary during a load imbalance disturbance associated with an angle shooting variation. These harmonics can be tracked and observed using the SWR-DFT proposed.

Fig. 9 – Modeled system in PSCAD.

Fig. 10 – Variation of current caused by variable load

VI. CONCLUSIONS

This paper presents a method for time-varying harmonic decomposition based on sliding-window recursive-DFT using a dyadic downsampling strategy.

From the results presented in this paper, as well as several analyses with other signals, it is possible to conclude that the combined techniques of recursive DFT and downsampling strategy bring some advantages to decompose non-stationary signals, when compared with other methodologies previously cited. Lower computation burden, no phase delay and a short transient time are important aspects to be taking into account when implementing, this tool in futures IEDs for real time applications. On the other hand, the disadvantages are inherent to all DTF based algorithms, i.e., the need for synchronous sampling and the influence by the presence of interharmonics. However, other strategies to solve the problems of interharmonics and synchronized time-step have been studied and will be presented opportually.

VII. REFERENCES

VI. BIOGRAPHY

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