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Citation for published version (APA):

Rotman, R., Raz, O., & Tur, M. (2003). The conversion matrix for optical filters with arbitrary transfer function. In *Proceedings of the International Topical Meeting on Microwave Photonics, 2003, MWP 2003, 10-12 September 2003, Budapest, Hungary* (pp. 247-250). Institute of Electrical and Electronics Engineers.
<https://doi.org/10.1109/MWP.2003.1422891>

DOI:

[10.1109/MWP.2003.1422891](https://doi.org/10.1109/MWP.2003.1422891)

Document status and date:

Published: 01/01/2003

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
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The conversion matrix for optical filters with arbitrary transfer function

Ruth Rotman, Oded Raz and Moshe Tur

Faculty of Engineering, Tel-Aviv University, Tel Aviv, Israel 69978
e-mail: rotman@eng.tau.ac.il, Tel: 972-3-6407374, Fax: 972-3-6410189

Abstract

Full analytical expressions for the effect of an arbitrary optical transfer function on small signal RF modulations are presented. Simulations of a device, having both amplitude and phase variations, were performed to prove its validity.

Introduction

Today's modern radar systems, employing true time delay beamformers, are implemented using dispersive devices such as chirped Bragg gratings [1], as well as other dispersion compensating modules. These devices are characterized by a transfer function of the form [2] $H(\omega) = A(\omega) \exp[-j\varphi(\omega)]$.

Future wide-bandwidth RF systems will have bandwidths representing a large fraction of the center frequency. Under these circumstances, the traditional method of analysis [3,4,5], which assumes a frequency independent $A(\omega)$ and expands $\varphi(\omega)$ around ω_0 , may not provide the system designer with the required accuracy. In a previous paper [6], we generalized the conversion matrix, [3,4,5], relating the optical input intensity and phase modulations to their corresponding output values, to include any arbitrary form of $\varphi(\omega)$, using a fairly simple small signal analysis. However, these analyses ignore the frequency-dependence of the amplitude of the transfer function, $A(\omega)$. In this paper, we further generalize the conversion matrix to also take into account the

ever present frequency-dependent variations of the magnitude of $H(\omega)$.

Theory

The field of an input optical signal of the form

$$E_{in}(t) = \sqrt{I_{in}(t)} \exp[j\phi_{in}(t)] \exp[j\omega_0 t],$$

characterized by small intensity and phase variations, can be approximately expressed by

$$E_{in}(t) = \sqrt{\langle I \rangle} \cdot [1 + \Delta I_{in}(t)/2\langle I \rangle + j\phi_{in}(t)] \cdot \exp[j\omega_0 t] \quad (1)$$

Assuming single tones for the intensity $[\Delta I(t)/2\langle I \rangle]_{in} = m/2 \cdot \cos(\omega_{RF}t + \phi_{RF})$ and phase $\phi_{in}(t) = s/2 \cdot \cos(\omega'_{RF}t + \phi'_{RF})$ variations, the input field can be written as

$$E_{in}(t) = \sqrt{\langle I \rangle} \left\{ 1 + \frac{m}{4} [\exp[j(\omega_{RF}t + \phi_{RF})]] + \frac{m}{4} [\exp[-j(\omega_{RF}t + \phi_{RF})]] + j \frac{s}{4} [\exp[j(\omega'_{RF}t + \phi'_{RF})]] + j \frac{s}{4} [\exp[-j(\omega'_{RF}t + \phi'_{RF})]] \right\} \cdot \exp[j\omega_0 t] \quad (2)$$

This optical signal comprises five optical frequencies $\omega_0, \omega_0 \pm \omega_{RF}, \omega_0 \pm \omega'_{RF}$. The propagation of the electromagnetic field through a linear optical medium is described by $E_{out}(\omega) = H(\omega)E_{in}(\omega)$, where $E_{in}(\omega)$ and $E_{out}(\omega)$ are respectively, the input and output complex amplitudes of the electric field at the optical frequency ω , and $H(\omega) = A(\omega) \exp[-j\varphi(\omega)]$ is the optical transfer function of the

medium. Thus, at the output of the dispersive medium we get:

$$E_{out}(t) / \sqrt{\langle I \rangle} = \left\{ 1 + \frac{m}{4} \left[\exp[-\gamma_+ + j(\omega_{RF}t - \Delta\phi^+(\omega_{RF}) + \phi_{RF})] + \frac{m}{4} \left[\exp[-\gamma_- - j(\omega_{RF}t - \Delta\phi^-(\omega_{RF}) + \phi_{RF})] + j \frac{s}{4} \left[\exp[-\gamma'_+ + j(\omega'_{RF}t - \Delta\phi^+(\omega'_{RF}) + \phi'_{RF})] + j \frac{s}{4} \left[\exp[-\gamma'_- - j(\omega'_{RF}t - \Delta\phi^-(\omega'_{RF}) + \phi'_{RF})] \right] \right] \right] \right\} \cdot A(\omega_0) \exp[j(\omega_0t - \varphi(\omega_0))] \quad (3)$$

where

$$\begin{aligned} \Delta\phi^+(\omega_{RF}) &= \varphi(\omega_0 + \omega_{RF}) - \varphi(\omega_0); \\ \Delta\phi^-(\omega_{RF}) &= \varphi(\omega_0) - \varphi(\omega_0 - \omega_{RF}); \\ \exp[-\gamma_{\pm}] &= A(\omega_0 \pm \omega_{RF}) / A(\omega_0) \end{aligned} \quad (4)$$

Using

$$\begin{aligned} \Gamma_+ &= [\exp(-\gamma_+) + \exp(-\gamma_-)] / 2 \\ \Gamma_- &= [\exp(-\gamma_+) - \exp(-\gamma_-)] / 2 \\ \Gamma'_+ &= [\exp(-\gamma'_+) + \exp(-\gamma'_-)] / 2 \\ \Gamma'_- &= [\exp(-\gamma'_+) - \exp(-\gamma'_-)] / 2 \end{aligned} \quad (5)$$

Eq. (3) can be manipulated by dividing the output E field into odd and even harmonic functions to obtain:

$$E_{out}(t) / \sqrt{\langle I \rangle} = \left\{ 1 + \frac{m}{4} \left[\exp[-\gamma_+ + j(\omega_{RF}t - \Delta\phi^+(\omega_{RF}) + \phi_{RF})] + \frac{m}{4} \left[\exp[-\gamma_- - j(\omega_{RF}t - \Delta\phi^-(\omega_{RF}) + \phi_{RF})] + j \frac{s}{4} \left[\exp[-\gamma'_+ + j(\omega'_{RF}t - \Delta\phi^+(\omega'_{RF}) + \phi'_{RF})] + j \frac{s}{4} \left[\exp[-\gamma'_- - j(\omega'_{RF}t - \Delta\phi^-(\omega'_{RF}) + \phi'_{RF})] \right] \right] \right] \right\} \cdot A(\omega_0) \exp[j(\omega_0t - \varphi(\omega_0))] \quad (6)$$

$$E_{out}(t) / \sqrt{\langle I \rangle} = \left\{ 1 + \frac{m}{2} \exp[-j\Phi_{amp}(\omega_{RF})] \cdot \left\{ \Gamma_+ \cos[\omega_{RF}t + \phi_{RF} + \Phi_{phase}(\omega_{RF})] + j\Gamma_- \sin[\omega_{RF}t + \phi_{RF} + \Phi_{phase}(\omega_{RF})] \right\} + \frac{s}{2} \exp[-j\Phi_{amp}(\omega'_{RF})] \cdot \left\{ j\Gamma'_+ \cos(\omega'_{RF}t + \phi'_{RF} + \Phi'_{phase}) - \Gamma'_- \sin(\omega'_{RF}t + \phi'_{RF} + \Phi'_{phase}) \right\} \right\} \cdot A(\omega_0) \exp[j(\omega_0t - \varphi(\omega_0))] \quad (7)$$

where:

$$\begin{aligned} \Phi_{amp}(\omega_{RF}) &= 0.5\varphi[(\omega_0 + \omega_{RF}) + \varphi(\omega_0 - \omega_{RF}) - 2\varphi(\omega_0)] \\ \Phi_{phase}(\omega_{RF}) &= -0.5[\varphi(\omega_0 + \omega_{RF}) - \varphi(\omega_0 - \omega_{RF})] \end{aligned} \quad (8)$$

With similar expressions for $\Phi_{amp}(\omega'_{RF})$ and $\Phi_{phase}(\omega'_{RF})$. Thus, the Fourier transforms of the output intensity and optical phase variations at the single frequency ω_{RF} , are related to their input values by:

$$\begin{aligned} \left[\frac{\Delta I(j\omega) / 2\langle I \rangle}{\phi(j\omega)} \right]_{out} &= T(\omega) \left[\frac{\Delta I(j\omega) / 2\langle I \rangle}{\phi(j\omega)} \right]_{in} \\ A_1 &= \sqrt{\{\Gamma_+ \cos[\Phi_{amp}(\omega)]\}^2 + \{\Gamma_- \sin[\Phi_{amp}(\omega)]\}^2} \\ A_2 &= \sqrt{\{\Gamma_+ \sin[\Phi_{amp}(\omega)]\}^2 + \{\Gamma_- \cos[\Phi_{amp}(\omega)]\}^2} \\ \vartheta_1(\omega) &= -\tan^{-1} \frac{\Gamma_- \sin[\Phi_{amp}(\omega)]}{\Gamma_+ \cos[\Phi_{amp}(\omega)]} \\ \vartheta_2(\omega) &= \tan^{-1} \frac{\Gamma_- \cos[\Phi_{amp}(\omega)]}{\Gamma_+ \sin[\Phi_{amp}(\omega)]} \end{aligned} \quad (9)$$

$$T = \exp[j\Phi_{phase}(\omega)]$$

$$\begin{bmatrix} A_1 \exp[j\vartheta_1(\omega)] & A_2 \exp[j\vartheta_2(\omega)] \\ -A_2 \exp[j\vartheta_1(\omega)] & A_1 \exp[j\vartheta_2(\omega)] \end{bmatrix}$$

Since Eq. (1) is linear in both the intensity and optical phase fluctuations, and complex excitation may be Fourier decomposed and treated, frequency by frequency, by Eq. (9). To compare Eq. (9) with previously obtained results, we expand $\varphi(\omega)$ in a Taylor series around ω_0 :

$$\begin{aligned} \varphi(\omega_0 + \omega') &= \varphi(\omega_0) + \left. \frac{d\varphi}{d\omega} \right|_{\omega_0} \omega' + \left. \frac{1}{2} \frac{d^2\varphi}{d\omega^2} \right|_{\omega_0} (\omega')^2 + \\ &\quad \left. \frac{1}{6} \frac{d^3\varphi}{d\omega^3} \right|_{\omega_0} (\omega')^3 + \dots \end{aligned} \quad (10)$$

It is then easily shown that if only the first and second derivatives of $\varphi(\omega)$ are non-zero (i.e., constant dispersion), Eq. (9) reduces to that of [3], and to

that of [2] when the third-order derivative is not neglected.

Discussion

For RF CW modulation, the phase of the intensity transfer function will not affect transmission. However, in the case of a broadband RF signal, both the phase and magnitude of the RF frequency transfer function (the top left element of the conversion matrix, $T(\omega)$, Eq. (9)) will affect the output intensity. For a wideband RF pulse, slow fluctuations of the phase error (with respect to the pulse bandwidth) will broaden the response in time, while fast fluctuations will raise its time side-lobes [5]. Using small signal analysis for an optical device with a known optical transfer function, the RF phase and amplitude of the transfer function can be quickly found from Eq. (9) and the output signal in the time domain accurately recovered. From Eqs. (9-10), when the initial excitation is pure intensity modulation, the even derivatives of the optical phase of the transfer function affect the amplitude of the intensity, while odd derivatives of the optical phase of the transfer function affect its phase. Attenuation ripples will also deteriorate the output signal, as shown below.

Fig. 1: Intensity/phase modulated optical system with a dispersive optical filter.

Simulation

Signal propagation through the schematics of Fig. 1 was numerically simulated for an input RF intensity modulated optical field of the form:

$$E_{in}(t) = \sqrt{I} \left(1 + \frac{m}{2} \cos(\omega_{RF} t + \phi_{RF}) \right) \exp[j\omega_0 t]$$

To calculate the effect of the optical filter, the Fourier transform of $E_{in}(t)$ (excluding $\exp[j\omega_0 t]$), obtained via an FFT, was multiplied by the filter transfer function: $A(\omega)\exp[-j\varphi(\omega)]$. Finally, using the inverse FFT, the output optical field $E_{out}(t)$ is evaluated, from which, we numerically deduce the RF output intensity at ω_{RF} and its corresponding RF phase. These numerical values were compared with an analytical calculation based on Eq. (9). Our simulated optical device had a sinusoidal frequency-dependent phase, $\varphi(\omega) = 40^\circ \sin[10^{-10}(\omega - \omega_0) + \pi/4]$, with an amplitude of 40 degrees over a period of 10 GHz. As for the magnitude of the filter transfer function, several cases were examined, where the lower sideband of double sideband modulation (with $m=0.3$) was progressively attenuated by 6 and 20 dB (optical power) up to complete attenuation, namely: single sideband modulation (SSB), see Fig. 2. As expected, the results show that as the asymmetry grows larger, the resulting phase and amplitude approach that of SSB modulation [7]. Indiscernible in the figures is the fact that both numerical simulation and calculation

Fig. 2: Magnitude (a) and phase (b) of the RF power transfer function of the sinusoidal phase filter, as a function of the input RF frequency for various attenuation values of the lower sideband for double sideband modulation.

overlap to yield the same results (within less than 1%). These results are independent of m for small enough values of this parameter.

Conclusion

A general expression for the effect of an optical device with arbitrary optical magnitude and phase responses was found under small signal modulation conditions. Using this expression we analyzed the consequences of phase and amplitude ripples of the optical filter on the RF transfer function at high modulation frequencies. Implications to wideband radar signals, were also discussed.

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