Simulation of inductive heating

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THEORY

Our main assumptions will be that the sources of the magnetic field have a sinusoidal time dependence, that the magnetic permeability does not depend on the magnetic field and that the geometry is two dimensional (translational or rotational symmetry). Material properties are allowed to depend on temperature and spatial coordinates, thus making the system of equations non-linear. Furthermore, we will assume that the quasi-static approximation is valid, that is, effects due to displacement currents (electromagnetic radiation) are neglected. In axisymmetry a special transformed formulation was used for the eddy currents to avoid inaccuracies around the Z-axis. A description will be given of an integrated simulation environment for the solution of coupled eddy current and heat dissipation problems. The software has been constructed using the high level language PDL and the package generator Mammy ([8,9]).

VARIATIONAL FORMULATION

A finite element discretization of the differential equations is obtained by writing \( A = (0, 0, A(x, y, t)) \), together with the gradient of an electric scalar potential \( V = (0, 0, V(T)) \). The external current which may depend on spatial coordinates.

\[ \text{Mammy} \]
To allow the possibility of enforced current conservation we assume a partition of \( \Omega \) in current conservation domains \( \Omega_k \) and write \( V = \sum \nu_k \Phi_k \) (\( \Phi_k \) being the characteristic function of \( \Omega_k \)). On each \( \Omega_k \) we have an additional equation for the unknown domain constant \( V_k \):

\[
F_k = -i \omega \sum \sigma \nu_k \Phi_k + V_k \sum \nu_k d\nu_k - I_k^{\text{red}} = 0 \tag{6}
\]

For the heat transfer equation we approximate \( T \) by \( T(x, y, t) = \sum T_j \phi_j(x, y) \). The discretized heat transfer equation is now given by \( G_k(T)T' = g_k(T, T) \) where the matrix \( G_k \) and the right-hand side vector \( g_k \) are given by

\[
(G_k)_{ij} = \int \phi_i \phi_j \, d\Omega \tag{7}
\]

\[
(g_k)_j = \int (-\phi_j \nabla T + \phi_j) \, d\Omega - \int \lambda \nabla \phi_j \cdot \nu \, d\Omega + \int \Phi_{\text{heat}} \phi_j \, d\Omega \tag{8}
\]

### NUMERICAL ALGORITHMS

The equations can be solved for transient and steady state situations. In both cases the solution algorithm is based on a sequential iteration process, because of the different time scales of the two equations: First the eddy current equation is solved, then the temperature equation call for an update of the eddy current equation. For the time integration use is made of a Gear type variable order, variable step-size Backwards Differencing algorithm for stiff ordinary differential equations. The steady state algorithm is based on a sequential Newton-Raphson approach.

The finite element discretization uses a triangular mesh with linear elements. In the heat equation Lobatto quadrature is used for integration of the \( \phi \) and \( J \cdot \nu \) terms. This implies that the matrix \( G_k \) in (7) is diagonal. The linearized systems for the eddy current equation are complex and symmetric (if no current conservation is applied and \( v = 0 \)). They are treated as real nonsymmetric systems. In the case of the heat equation, the linear systems are only symmetric if no velocity effects are considered and if the thermal conductivity \( \lambda \) is independent of temperature. The resulting linear systems are solved using a non-symmetric sparse preconditioned Bi-Conjugate Gradient iterative method or a symmetric ICCG ([1]). For large velocities a special upwind scheme is used to deal with the singularly perturbed character of the differential equations (see [2]). The use of this upwind scheme results in better accuracy for the same mesh sizes. The method consists of replacing the weighting functions \( \phi_j \) by \( \phi_j + p_j \), where \( p_j \) is a function defined by

\[
p_j = \frac{1}{2} \left( \coth(x) - \frac{1}{x} \right) (\nu \cdot \nabla \phi_j) \beta_j, \quad x = \frac{\sigma c}{2 \lambda} \left\| \nu \right\| \beta_j,
\]

where

\[
\beta_j = \frac{3}{2} l_j(v)
\]

and \( l_j(v) \) is the length of the line segment obtained by intersecting the line through the barycenter of the \( j \)-th triangle in the direction of \( \nu \) with this triangle. This means that for each element \( \Omega_k \) the matrix \( G_k \) and the right-hand side \( g_k \) in (7) and (8) are augmented by

\[
\int_{\Omega_k} \sigma \nu_k \phi_j \, d\Omega
\]

and

\[
\int_{\Gamma_k} \lambda \nabla \phi_j \cdot \nu \, d\Omega - \int_{\Omega_k} \sigma c \nu \cdot \nabla T \phi_j \, d\Omega
\]

respectively.

### RZ COORDINATES

An RZ coordinate system can be used for problems that are invariant under rotations around the \( Z \)-axis. A consequence of the continuity of the potential \( A \) is that the boundary condition \( A = 0 \) for \( r = 0 \) should be satisfied.

The standard weak Galerkin formulation in RZ coordinates implicitly assumes that the potential \( A \) can be properly approximated by piecewise linear elements (to lowest order). However the analytical solution \( A \) of the homogeneous magnetostatics equations can be written as a linear combination of the functions \( r \) and \( 1/r \), so clearly erroneous results can be expected near the \( Z \)-axis when using the standard linear elements (see e.g. [7]). A horrifying example was shown in [10]. This effect is encountered in particular when there is an interface with a high permeability jump very near the \( Z \)-axis. One dimensional analysis of this phenomenon for \( A \) and for \( r \) formulation can be found in [11]. In that report upper bounds are derived for the spatial step size \( h \) which ensures that the relative error in \( H(\text{axis}) \) or the relative magnitude of the spurious current \( I = -2\pi \int_{r_1}^{r_2} \frac{\partial \theta}{\partial r} \, dr \) are less than 1%. We recall parts of the results in the following table. It is assumed that \( r_1 \ll r_2 \).

<table>
<thead>
<tr>
<th>Situation description</th>
<th>Upper bounds for ( h )</th>
<th>( \mu_1 )</th>
<th>( \mu_2 )</th>
<th>( I )</th>
<th>( H(\text{axis}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air on axis; ( \mu = \mu_1/\mu_2 \approx 1 )</td>
<td>( \frac{\mu_1}{2 \sqrt{\lambda_1}} )</td>
<td>( \frac{\mu_2}{2 \sqrt{\lambda_2}} )</td>
<td>( \frac{\mu_1}{2 \sqrt{\lambda_2}} )</td>
<td>( \frac{1}{2 \lambda_1} )</td>
<td></td>
</tr>
<tr>
<td>Metal on axis; ( \mu_1 \gg \mu_2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In [10] a novel remedy was proposed that solves the general approximation problem near the \( Z \)-axis, thus allowing metal-air interfaces close to the axis. The new unknown \( F(s,t) = \sqrt{H(s,z)} \) (with \( s = r^2 \)) one obtains a reformulation of the original equation which can be approximated reasonably well by piecewise linear elements. This approach combines the approximation properties of the standard \( A \) method (giving good results near the axis for problems with air on the axis) and those of another conventional approach, using \( rA \) as unknown, which is known to give better results for large \( r \) and also near the axis with metal on the axis. In the same situations this new method gives more accurate results near the axis than both the \( A \) method and the standard \( A \) method.

In our situation, where we have to deal with eddy currents as well, we can show that this method is also advantageous. Study of the 1D equation in RZ shows that after a coordinate transformation the solution \( G(y) = A(y/\sqrt{\omega \sigma_0}) \) of the homogeneous equation satisfies a Bessel equation:

\[
y^2 \hat{G}'' + y \hat{G} + (y^2 - 1) \hat{G} = 0,
\]

where the argument \( y = \sqrt{\omega \sigma_0} r = r \sqrt{2} / \delta \) is complex. Here \( \delta = \sqrt{2(\omega \mu_0)} \) is the skin depth ([13] p. 301, 488). The asymptotic behavior for small complex arguments \( y \) can be shown to be the same as in the magnetostatic case. The same transformation will therefore be useful for the eddy current situation as well.

The method has been implemented in terms of the original \( A \) (where \( s = r^2 \)), although the linear systems are solved using \( F \) as unknown, for reasons of symmetry (if \( v = 0 \)) and because it gives better conditioned matrices. The following representations were used:

\[
w_j = \frac{1}{\sqrt{\gamma}} b_j(s, z)
\]
The abrupt changes in material properties mean that special
lows the modelling of proximity effects and temperature effects
The possibility of imposing a non-zero applied current
generated by these eddy currents decreases.
about the permeability of vacuum. The skin depth varies with
the effects of the varying external magnetic field. Although eddy
currents flow in a larger region, this actually means that the heat
care has to be taken to guide the algorithm across such a tran-
sition. To this end an automated control mechanism has been
employed. Use is made of the extra gauge unknown \( V \) which can
be modelled in the way we described, ferromagnetic materials
are nonlinear in general, so the applicability will be limited.

CURIE TEMPERATURE
Near the Curie temperature \( T_{\text{Curie}} \) it is well known that the mag-
netic permeability changes strongly with \( T \) ([13] p. 341). For
\( T \) below the Curie point, \( \mu \) is relatively high, above \( T_{\text{Curie}} \) the
material loses its ferromagnetic properties and acts as a para-
magnetic material. This means that the permeability drops to
about the permeability of vacuum. The skin depth varies with
\( 1/\sqrt{\mu} \). A greater skin depth means less eddy currents to oppose
the effects of the varying external magnetic field. Although eddy
currents flow in a larger region, this actually means that the heat
generated by these eddy currents decreases.
The abrupt changes in material properties mean that special
care has to be taken to guide the algorithm across such a transi-
tion. To this end an automated control mechanism has been
provided in the program which monitors the temperature profile
such that the eddy current equation will be updated as soon as
some critical temperature value is exceeded. In this way a zone
that has relapsed to mild temperature behavior will be treated in the usual way.
It should be noted that, although Curie temperature transitions
can be modelled in the way we described, ferromagnetic materials
are nonlinear in general, so the applicability will be limited.

CURRENT CONSERVATION DOMAINS
In order to simulate objects with a finite structure in the third
dimension, the concept of current conservation domains is em-
ployed. Use is made of the extra gauge unknown \( V \) which can be
a piecewise constant function where the constant may be dif-
ferent for distinct connected components of the workpiece. A
proper nonzero value for \( V \) will allow the specification of applied
currents as well. Each current conservation domain \( \Omega \) may be
thought of as a collection of infinitely long bar conductors which
are connected at infinity. \( \text{We will require that on } \Omega \)
\[ I^{\text{pe}} = \int_A \sigma \mathbf{E} \, d\Omega = -i\omega \int_A \sigma \mathbf{A} \, d\Omega + V \int_A \sigma \, d\Omega \]
so
\[ V = i\omega \int_A \sigma \mathbf{A} \, d\Omega + \frac{\int A \sigma \, d\Omega}{\int \sigma \, d\Omega} \]  
(9)
The possibility of imposing a non-zero applied current \( I^{\text{pe}} \) al-
lows the modelling of proximity effects and temperature effects
in current carrying coils.
Per current conservation domain one additional 'gauge' unknown \( V \) is introduced instead of eliminating \( V \) from (1) by using (9).
Each unknown \( A \) at a node inside such a domain is then coupled
to this unknown \( V \) by means of (1). This will result in a sparse
functional matrix with some additional full columns and rows.

EFFICIENCY CALCULATIONS
The following two efficiency quantifiers can be calculated:
\[ \int_{t_0}^{t_f} \int_{\Omega_{\text{work}}} < \mathbf{J} \cdot \mathbf{E} > \, d\Omega_{\text{work}} \, dt \]  
(10)
\[ \int_{t_0}^{T(t_f)} \int_{\Omega_{\text{work}}} \rho(T) c(T) \, d\Omega_{\text{work}} \, dt \]  
(11)
The first quantity indicates the time integrated energy used in
heating the workpiece. The second integral is the time integrated
thermal energy that is actually contained within the workpiece.
It will be clear, therefore, that the first quantity is always larger
than the second (if \( v=0 \)). The second integral divided by the
first is an efficiency indicator.

INTEGRATED SIMULATION ENVIRONMENT
The Eddy/Heat software package has been developed using the high
level language PDL (Package Designer Language). The database structure, the mathematical formulas and the numerical
algorithms are all described in PDL. A library interface allows a
symbolic reference to existing (Fortran) facilities. The PDL for-
mulation is compiled by Mammy, a Philips' proprietary package
generator, resulting in the source code of a Fortran package. This
code is linked with auxiliary libraries.
This approach proved to be a powerful method for the creation of
high-level flexible engineering software. In the Eddy/Heat
package for instance, material properties can be defined as con-
stants, as expressions, in the form of tables, or as subroutines.
The program then automatically decides which terms contribute to
the Newton matrices.
The analysis module of the package is used in conjunction with the
pre- and postprocessor PE2D ([12]). The description of geo-
metry and magnetic data from PE2D are complemented by an
Attribute File, containing additional material properties and
boundary conditions required for the computation of the heat
transfer. Postprocessing can be done with PE2D and GRAPHS
(Vector Fields Ltd., Oxford).
The package is currently in use within Philips ([5,6]) and operates
under VAX/VMS and UNIX (SUN, Apollo).

RESULTS
Figure 1 shows a simple example of the proximity and skin effects
in a current carrying coil (right). Eddy currents in the conduct-
ing region (left) result in heating of the material. Figure 2 shows

Figure 1: Eddy currents showing proximity and skin effects
in the coil (right) and the heated metal (left).
CONCLUSIONS

A description has been given of a software package for the simultaneous solution of the eddy current and the heat transfer equation for the simulation of inductive heating. Aspects like velocity effects, Curie temperature transitions, RZ coordinates and enforced current conservation have been taken into account. The use of PDL (Package Designer Language) in the definition phase has proved to be a very flexible way to structure the complex combinatorics of several specialized options and to handle redefinition of the algorithms, because time consuming items like adapting datastructures are handled via PDL. This also resulted in an enormous reduction in development time required for the package.

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