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Analytical and numerical solution for a rigid liquid-column moving in a pipe with fluctuating reservoir-head and venting entrapped-gas

by

A.S. Tijseling, Q. Hou, Z. Boskuş
ANALYTICAL AND NUMERICAL SOLUTION FOR A RIGID LIQUID-COLUMN MOVING IN A PIPE WITH FLUCTUATING RESERVOIR-HEAD AND VENTING ENTRAPPED-GAS

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ABSTRACT

The motion of liquid filling a pipeline is impeded when the gas ahead of it cannot escape freely. Trapped gas will lead to a significant pressure build-up in front of the liquid column, which slows down the column and eventually bounces it back. This paper is an extension of previous work by the authors in the sense that the trapped gas can escape through a vent. Another addition is that the driving pressure is not kept constant but fluctuating. The obtained analytical and numerical solutions are utilized in parameter variation studies that give deeper insight in the system’s behavior.

Key words  
mass oscillation, nonlinear spring, entrapped gas, venting gas, orifice, rigid column, pipe filling, analytical solution

INTRODUCTION

The presence of gas in liquid-conveying pipelines may make the system’s dynamic behavior unpredictable. Entrapped gas pockets may store and release energy in response to unsteady liquid flow. In particular for rapid events, caused by fast valve maneuvers and pump starting, the system pressures and temperatures may rise to unacceptably high levels. The subject has received much attention with pioneering work by C.S. Martin [1a-e] and E. Cabrera [2a-e] and their co-workers. Many relevant papers have appeared since, up to the latest publications [3a-k]. The authors contributed to the subject by providing analytical expressions that give insight in the damped oscillation of a liquid-column/gas-pocket system [4a]. This modest contribution is extended herein by letting the driving pressure fluctuate and by allowing the entrapped gas to escape through an orifice. Martin’s seminal paper [1a] is revisited, exactly forty years after its publication, with a slightly improved model. This study is intended to form a theoretical basis for the analysis of rapid pipe filling [4b], accelerating liquid slugs [4c], and the start-up of liquid flow in undulating pipelines [3e].

C.S. MARTIN’S TEST PROBLEM

The test problem is a reservoir-pipe-orifice system where gas at atmospheric pressure is trapped near the orifice. Figure 1 is a simple sketch of the situation. Sudden opening of an
upstream valve connected to the reservoir (not shown in Fig. 1) makes the liquid move towards the gas pocket. Depending on the size of the orifice, the liquid column will bounce back or not. It is to be noted that slow valve opening or large orifices will not lead to pressures higher than the reservoir pressure. The data are taken from the water-air system described in [1a]. The initial length of the water column is $L_0 = 100$ m and the diameter of the pipe is $D = 0.20$ m with corresponding flow area $A$. The air occupies an initial length $L_{\text{gas},0} = 12.73$ m with an initial absolute pressure head $H_{\text{gas},0} = 10.4$ m; the pipe length thus is $L = L_0 + L_{\text{gas},0} = 112.73$ m. The absolute pressure head of the reservoir is $H_R,0 = 31.1$ m, so that $H_R,0 / H_{\text{gas},0} = 2.99$. The specific gas constant for air is $R = 287 \text{ m}^2/(\text{s}^2 \text{K})$ and the polytropic coefficient $n$ is either 1.2 or 1.4 herein. The skin friction coefficient for the liquid flow is $f = 0.02$, but valve resistance and entrance head losses are ignored by setting $K_v = 0$ and $K_e + 1 = 0$, where the “1” represents reservoir head transferred to kinetic flow energy at the pipe entrance. The pipe is horizontal with $\theta = 0$. The orifice has the varied diameter $D_{\text{or}}$ with corresponding area $A_{\text{or}}$ and discharge coefficient $C_d = 1$.

The liquid column and air pocket have initial masses of 3142 kg and 0.48 kg, respectively. The gravitational acceleration is $g = 9.81 \text{ m/s}^2$, the mass density of the water is $\rho = 1000 \text{ kg/m}^3$, and the initial air density is taken equal to $\rho_{\text{gas},0} = 1.2 \text{ kg/m}^3$ which corresponds to an initial temperature $T_{\text{gas},0} = 296.2 \text{ K}$ (these values were not specified in [1a]).

When the reservoir head is kept constant, by a control device, we simply take $\alpha = 0$.

**GOVERNING EQUATIONS**

For filling liquid from an open reservoir with fluctuating head $H_R(L)$ into a pipe which contains a perfect gas and a downstream vent (Fig. 1), the governing equations for velocity $v$ and length $L$ of the liquid column, and pressure head $H_{\text{gas}}$ and mass $m_{\text{gas}}$ of the entrapped gas, are

\[
\frac{dv}{dt}(t) = \frac{g (H_{R,0} + \alpha L_0 - H_{\text{gas}}(t))}{L(t)} - g \alpha + g \sin \theta - \frac{f}{2D} [v(t)] - \frac{K_v}{2L(t)} [v(t)] [v(t)]
\]

\[
\frac{dL}{dt}(t) = v(t)
\]

\[
\frac{dH_{\text{gas}}}{dt}(t) = \frac{n H_{\text{gas}}(t) v(t)}{x_L - L(t)} + \frac{n H_{\text{gas}}(t)}{m_{\text{gas}}(t)} \frac{dm_{\text{gas}}}{dt}(t)
\]

\[
\frac{dm_{\text{gas}}}{dt}(t) = \begin{cases} -C_d A_{\text{or}} Y(H_{\text{gas}}(t)) \sqrt{2 \rho g} \left( m_{\text{gas}}(t) \left( H_{\text{gas}}(t) - H_{\text{atm}} \right) / A (x_L - L(t)) \right) & \text{if } H_{\text{gas}}(t) / H_{\text{gas},0} \leq 1.89 \\ -C_d A_{\text{or}} \sqrt{\rho g} \left( k \left( \frac{2}{k+1} \right) \nabla k \left( \frac{k-1}{k} \right) m_{\text{gas}}(t) H_{\text{gas}}(t) / A (x_L - L(t)) \right) & \text{if } H_{\text{gas}}(t) / H_{\text{gas},0} > 1.89 \end{cases}
\]

with

\[
Y(H_{\text{gas}}) = \begin{cases} 1 & \text{if } H_{\text{gas}} = H_{\text{gas},0} \\ \left[ \frac{k}{k-1} \left( \frac{H_{\text{gas},0}}{H_{\text{gas}}} \right) \right] \frac{1}{k} & \text{if } 1 - \left( \frac{H_{\text{gas},0}}{H_{\text{gas}}} \right) \frac{k-1}{k} \end{cases}
\]

\[
(5a)
\]
Equations (2) and (3) are Eqs. (2b) and (2a) in Ref. [4a], with the driving head \( H_R \) according to Eq. (1b) herein. Equations (4) and (5) combine Eqs. (25-28) in Ref. [1a], with the auxiliary variables

\[
\frac{\rho_{\text{gas}}(t)}{\rho_{\text{gas}}(t_0)} = \left( \frac{p_{\text{gas}}(t)}{p_{\text{gas}}(t_0)} \right) \frac{\rho_{\text{gas}}(t_0)}{\rho_{\text{gas}}(t_0)}
\]

\[
T_{\text{gas}}(t) = \frac{\rho g H_{\text{gas}}(t)}{R \rho_{\text{gas}}(t)}
\]

\[
\Delta P_{\text{gas}} = \rho g (H_{\text{gas}}(t) - H_{\text{atm}})
\]

All symbols are specified in the Nomenclature. The event starts by instantaneously opening an upstream valve that separates liquid and gas. The volume of the gas pocket \( V_{\text{gas}} \) is determined by the length \( L(t) \) of the liquid column, from which gas density \( \rho_{\text{gas}} \) and temperature \( T_{\text{gas}} \) follow. Of course, all these quantities are coupled through the governing equations (2-5). The pressure difference across the orifice is \( \Delta P_{\text{gas}} \) and back flow of the gas is not allowed herein (\( H_{\text{gas}} > H_{\text{atm}} \)). Choking flow is assumed to occur for \( H_{\text{gas}} > 1.89 H_{\text{atm}} \). It is noted that for small volumes, the gas pocket cannot be regarded as a "reservoir" in the derivation of the orifice equation (5); the kinetic energy of the upstream gas is shown in Fig. 2 in its dependence of \( H_{\text{gas}} \) and temperature.

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The analytical expression for \( v(L) \) derived in Ref. [4a] for a closed system (\( A_{or} = 0 \)) and constant reservoir head (\( \alpha = 0 \)) is extended herein for a fluctuating reservoir head (\( \alpha \geq 0 \)) and reads

\[
v = L \frac{K+1}{2} e^{-C_2 L} \sqrt{2 g \left( H_{R,0} + \alpha L_0 \right) \int_{L_0}^{L} L^K e^{2 C_2 L} \, dL^L - C_1 \int_{L_0}^{L} \frac{L^K e^{2 C_2 L}}{(x_L - L^L)^n} \, dL^L + \left( \sin \theta - \alpha \right) \int_{L_0}^{L} L^K e^{2 C_2 L} \, dL^L}
\]

where \( C_1 := L_{\text{gas}}(t_0) H_{\text{gas}}(t_0), \quad C_2 := \frac{f}{2D}, \quad K := K_c + K_v \) and \( \nu(L_0) = 0 \). Local losses are absent in the limit case \( K = -1 \). Fully symbolic solutions can be found for special cases only [4a]. Approximate symbolic solutions can be found – for example – by taking \( L^K = L_0^K \) and \( (x_L - L^K)^n = (x_L - L_0)^n \) in the integrands.

### NUMERICAL SOLUTION

The governing equations (2-5), with (5) substituted in (4), can be casted in the autonomous form:

\[
\frac{dy}{dt} = f(y), \quad \text{with} \quad y := \begin{pmatrix} v \\ L \\ \frac{H_{\text{gas}}}{m_{\text{gas}}} \end{pmatrix}
\]

The explicit Euler method has been used to solve Eq. (8) with a numerical time step \( \Delta t = 1 \) ms for all simulations herein.

### RESULTS

#### Constant reservoir-head and closed end

The confined system of Martin (1976) [1a] is simulated to verify the numerical solutions for the case \( A_{or} = 0, n = 1.2 \) and \( \alpha = 0 \). Martin’s results in Fig. 3 and the current results in Fig. 4 are consistent. Maximum values and their timings are compared in Table 1. Martin (unnecessarily) used \( L_0 \) instead of \( L(t) \) in the first term on the right-hand side of Eq. (2), which explains the small differences. Another source of discrepancy is numerical error. The maximum column length is \( L_{\text{max}} = 110.5 \) m, which means a growth of 11% compared to Martin’s results.

<table>
<thead>
<tr>
<th>Table 1 Comparison of maximum velocities and pressures.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_{\text{max}} ) (m/s)</td>
</tr>
<tr>
<td>--------------------------------------------------------</td>
</tr>
<tr>
<td>Martin (1976)</td>
</tr>
<tr>
<td>( L_0 ) (current)</td>
</tr>
<tr>
<td>( L(t), \alpha = 0 )</td>
</tr>
<tr>
<td>( L(t), \alpha = 0.1 )</td>
</tr>
</tbody>
</table>
Fluctuating reservoir-head and closed end

The reservoir pressure head is fluctuating in the test case $A_{or} = 0$, $n = 1.2$ and $\alpha = 0.1$. Figure 5 shows that the analytical expression (7) and the numerical solution give the same result. The dimensionless parameter $\alpha (L_{\text{max}} - L_0) / H_{R,0} = 3.4\%$. The fluctuation results in a 4\% lower maximum pressure head, as listed in Table 1.

Constant reservoir-head and orifice end

The open system of Martin (1976) [1a] is simulated for a maximum of 32 s to verify the numerical solutions for the case $n = k = 1.4$, $\alpha = 0$, $H_{R,0}/H_{\text{gas},0} = 4$, 3 or 2, and with varying $A_{or}$. The calculations stop either after 32 s or when 5\% of the initial air volume is left (when $L_{\text{gas}} = 0.64$ m = 3.2D). For small air pockets the mathematical model becomes invalid thereby leading to very high pressure heads. Figure 6 shows the calculated column motions in terms of velocities and positions for $H_{R,0}/H_{\text{gas},0} = 4$. The closed-system behavior is shown as a reference and an extended Fig. 6a is shown in Appendix A. The water column bounces back at least once for small orifices up to $D_{or}/D = 0.07$. For large orifices there is a smooth liquid movement towards the pipe’s end, although the build-up of air pressure finally causes a deceleration. For a fully open end the liquid column slows down mainly due to skin friction thereby satisfying the analytical solution [4b, Eq. (35) for $\theta = 0$]:

$$v(L) = e^{-C_2 L} \sqrt{2 g H_{R,0}} \left[ \frac{L}{L_0} e^{2 C_2 L} \right] \frac{dL}{L}$$  \hspace{1cm} (9)

By taking $L^* = L_0$ in the denominator this function can be approximated by [4c, Eq. (5)]:

$$\tilde{v}(L) = \frac{g H_{R,0}}{C_2 L_0} \left( 1 - e^{2 C_2 (L_0 - L)} \right)$$  \hspace{1cm} (9a)
Figure 6e shows solution (9) with the maximum attainable velocity \( v(x_L) = 7.4 \text{ m/s} < \bar{v}(x_L) = 7.7 \text{ m/s} \). Figure 7 shows air pressure heads as function of the monotonously decreasing air mass.

Martin’s results for \( f \frac{H_\text{gas,0}}{H_\text{air,0}} = D^3 \) in Fig. 8 and the current results in Fig. 9 are consistent, keeping in mind that the variable \( L(t) \) (instead of constant \( L_0 \)) herein leads to higher air pressures \( (H_{\text{air,max}}) \) than in Ref. [1a], see Table 1. Other sources of discrepancy include numerical error, and the possibly different values of \( T_\text{gas,0} \) and hence \( \rho_\text{gas,0} \) and to a smaller extent \( \rho \). The arbitrary air-volume threshold of 5% cuts off the maximum pressures for the cases without reverse flow \( (v > 0 \text{ all the time}) \). When the water column hits the closed end, there is the potential risk of a Joukowsky pressure rise of magnitude \( \rho c v_{\text{end}} \), although the vent will relieve the water hammer.

Fig. 6 Water velocity versus column length for different orifice sizes and \( H_{R,0} / H_{\text{gas,0}} = 4 \).
Fig. 7  Air pressure head versus air mass for different orifice sizes and $H_{R,0} / H_{gas,0} = 4$.

Fig. 8  Martin (1976): Effect of orifice on maximum air pressure.

Fig. 9  Current paper: Effect of orifice on maximum air pressure.
CONCLUSION

Previous work on entrapped air pockets has been extended with a vent. The obtained results are consistent with C.S. Martin’s (1976) diagrams. One new element herein is that the reservoir pressure head fluctuates in phase with the motion of the liquid column. For this situation, with a closed end, an analytical expression is found for the liquid velocity as function of column length. The work is applicable and will be extended to situations with multiple gas pockets and vents.

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NOMENCLATURE

\[ A = \text{cross-sectional pipe area (m}^2) \]
\[ A_{or} = \text{orifice area (m}^2) \]
\[ A_R = \text{reservoir area (m}^2) \]
\[ C_d = \text{orifice discharge coefficient} \]
\[ C_1, C_2 = \text{constants} \]
\[ c = \text{acoustic wave speed (m/s)} \]
\[ D = \text{pipe diameter (m)} \]
\[ D_{or} = \text{orifice diameter (m)} \]
\[ e = \text{exponential function} \]
\[ f = \text{Darcy-Weisbach friction coefficient} \]
\[ f = \text{vector function} \]
\[ g = \text{acceleration due to gravity (m/s}^2) \]
\[ H_{atm} = \text{atmospheric absolute pressure head (m)} \]
\[ H_{gas} = \text{gas absolute pressure head (m)} \]
\[ H_R = \text{reservoir absolute pressure head (m)} \]
\[ H = \text{Heaviside step function} \]
\[ K = \text{total loss coefficient} \]
\[ K_e = \text{entrance loss coefficient} \]
\[ K_v = \text{valve loss coefficient} \]
\[ k = \text{adiabatic constant, ratio of specific heats} \]
\[ L_e = \text{length of liquid column (m)} \]
\[ L^* = \text{dummy length in integral (m)} \]
\[ L_{gas} = \text{length of gas pocket (m)} \]
\[ m = \text{mass of liquid column (kg)} \]
\[ m_{gas} = \text{mass of gas pocket (kg)} \]
\[ n = \text{constant polytropic exponent} \]
\[ R = \text{specific gas constant (m}^2/(s^2 \text{K}) \]
\[ T = \text{absolute temperature (K)} \]
\[ t = \text{time (s)} \]
\[ V_{gas} = \text{gas volume (m}^3) \]
\[ v = \text{velocity of liquid column (m/s)} \]
\[ v_{end} = \text{velocity of liquid column when hitting pipe end (m/s)} \]
\[ v_R = \text{vertical velocity of reservoir free surface (m/s)} \]
\[ x = \text{axial position (m)} \]
\[ x_L = \text{pipe length (m)} \]
\[ Y = \text{adiabatic gas expansion factor} \]
\[ y = \text{vector of unknowns} \]
\[ \alpha = \text{ratio } A/A_R \]
\[ \Delta P = \text{pressure difference across orifice (Pa)} \]
\[ \theta = \text{angle of downward inclination of pipe (rad)} \]
\[ \rho = \text{mass density of liquid (kg/m}^3) \]
\[ \rho_{gas} = \text{mass density of gas (kg/m}^3) \]

Subscripts

max = maximum value
0 = constant initial value

REFERENCES

APPENDIX A

The solution space of Eq. (8) is four-dimensional. A three-dimensional trajectory is shown in Fig. 6a, to illustrate the nice dynamic behavior of the studied system.

Fig. 6a Water velocity, column length and air pressure head, for $D_{or}/D = 0.03$ and $H_{L,0} / H_{gas,0} = 4$. 


APPENDIX A

The solution space of Eq. (8) is four-dimensional. A three-dimensional trajectory is shown in Fig. 6a, to illustrate the nice dynamic behavior of the studied system.

Fig. 6a Water velocity, column length and air pressure head, for $D_{or}/D = 0.03$ and $H_{L,0} / H_{gas,0} = 4$. 

Four-dimensional trajectories are shown (three different views) in Fig. 6aa, where color provides the fourth dimension.

The corresponding projections on the x-z, y-z and x-z planes are shown in Fig. 6aaa.

**Fig. 6aa** Water velocity (x), column length (y), air pressure head (z) and air mass (color), for $D_{\text{col}}/D = 0.03$ and $H_{\text{R,0}} / H_{\text{gas,0}} = 4$ (by courtesy of Martijn Anthonissen).

**Fig. 6aaa** Projections of trajectory shown in Fig. 6aa.
APPENDIX B: Pipe filling with venting gas pocket

Input data: (C.S. Martin, 1976, PS2, Paper F2, 16-28) 

Orifice system - with fluctuating reservoir pressure and choking flow

\[ \beta = 1 \]
\[ \alpha = 1 \]
\[ \gamma = 1.4 \]

\[ V_{inlet} = 60 \text{ m/s} \]
\[ V_{exit} = 40 \text{ m/s} \]
\[ L_{gas} = 2 \text{ m} \]

\[ L_{liquid} = 100 \text{ m} \]

**Orifice**

\[ Q_w = \sqrt{\frac{2}{\gamma}} \frac{V_{inlet}^2}{\gamma} \]

\[ V_{exit} = \frac{Q_w}{A_{exit}} \]

\[ A_{exit} = \frac{V_{exit}^2}{\frac{2}{\gamma}} \]

\[ L_{gas} = \frac{V_{exit}^2}{\gamma} \]

\[ L_{liquid} = \frac{V_{exit}^2}{\gamma} \]

**Fluctuating reservoir pressure**

\[ H_{gas} = 4.6 \text{ m} \]
\[ L_{gas} = 100 \text{ m} \]

\[ H_{liquid} = 10 \text{ m} \]

\[ L_{liquid} = 100 \text{ m} \]

\[ H_{gas} = 4.6 \text{ m} \]

**Euler forward**

\[ \frac{dy}{dt} = \frac{y_{exit} - y_{inlet}}{t_{exit} - t_{inlet}} \]

\[ H_{gas} = 4.6 \text{ m} \]

**Closed end**

\[ H_{gas} = 4.6 \text{ m} \]

**Orifice**

\[ Q_w = \sqrt{\frac{2}{\gamma}} \frac{V_{inlet}^2}{\gamma} \]

\[ V_{exit} = \frac{Q_w}{A_{exit}} \]

\[ A_{exit} = \frac{V_{exit}^2}{\gamma} \]

**Fluctuating reservoir pressure**

\[ H_{gas} = 4.6 \text{ m} \]

**Euler forward**

\[ \frac{dy}{dt} = \frac{y_{exit} - y_{inlet}}{t_{exit} - t_{inlet}} \]

\[ H_{gas} = 4.6 \text{ m} \]
Euler forward Numerical solution
Orifice with choking flow for \(H_{o,\text{sys}} / H_{\text{inlet}} > 1.89\)

\[
Y_{\text{sys}} = \begin{bmatrix} Y_0 \\ \Delta I \end{bmatrix} = 0.8 \quad N = 12000
\]

\(k = 1.89, N_{\text{sys}} = 10.4\)

\[
Y_{\text{sys}} = \begin{bmatrix} Y_0 \\ \Delta I \end{bmatrix} = 0.8 \quad N = 12000
\]

\(k = 1.89, N_{\text{sys}} = 10.4\)

***Acceleration***

\[
\begin{align*}
\text{accel}_{t} &= \frac{1}{2} \left( \frac{\Delta Y_{\text{sys}}}{\Delta t} \right) \\
\text{accel}_{k} &= \frac{1}{2} \left( \frac{\Delta Y_{\text{sys}}}{\Delta t} \right)
\end{align*}
\]

\[
\begin{align*}
\text{accel}_{t} &= 1000 \\
\text{accel}_{k} &= 1000
\end{align*}
\]

Slug velocity as function of \(x\)

\[
\begin{align*}
\text{vel}_{t} &= 1000 \\
\text{vel}_{k} &= 1000
\end{align*}
\]

Slug velocity as function of \(L\)

\[
\begin{align*}
\text{vel}_{t} &= 1000 \\
\text{vel}_{k} &= 1000
\end{align*}
\]

Air pressure head as function of air mass

\[
\begin{align*}
\text{H}_{\text{air}} &= 1.89 \times \text{H}_{\text{inlet}} \\
\text{H}_{\text{air}} &= 1.89 \times \text{H}_{\text{inlet}}
\end{align*}
\]

Air mass as function of \(t\)

\[
\begin{align*}
\text{mass}_{t} &= 0 \\
\text{mass}_{k} &= 0
\end{align*}
\]

Air density as function of \(t\)

\[
\begin{align*}
\text{density}_{t} &= 0 \\
\text{density}_{k} &= 0
\end{align*}
\]
Mathcad 15 CASA-16-14.xmcd B9

**Results with venting orifice** (ř, ě, L, Hgas, mair, Vair)

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
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<td>ř</td>
<td>3.5690</td>
<td>3.5930</td>
<td>3.6170</td>
<td>3.6410</td>
<td>3.6650</td>
<td>3.6890</td>
</tr>
<tr>
<td>ě</td>
<td>0.0096</td>
<td>0.0176</td>
<td>0.0256</td>
<td>0.0336</td>
<td>0.0416</td>
<td>0.0516</td>
</tr>
<tr>
<td>L</td>
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<td>11.0270</td>
<td>11.0271</td>
<td>11.0272</td>
<td>11.0273</td>
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</tr>
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<tr>
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**Air pressure head as function of t**

**Air temperature as function of t**

**Air volume as function of t**

**3D phase space**
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