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Capacity of weakly (d, k) -constrained sequences

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Abstract — In the presentation we find an analytic expression for the maximum of the normalized entropy $-\sum_{i \in T} p_i \ln p_i / \sum_{i \in T} i p_i$ where the set T is the disjoint union of sets S_n of positive integers that are assigned probabilities P_n , $\sum_n P_n = 1$. This result is applied to the computation of the capacity of weakly (d, k) -constrained sequences that are allowed to violate the (d, k) -constraint with small probability.

I. PROBLEM DESCRIPTION AND RESULTS

Let T be a set of positive integers, and assume that T is the disjoint union of a (finite or infinite) number of non-empty sets S_n , $n \in M$. Also assume that there are given numbers $P_n \geq 0$, $n \in M$, with $\sum_n P_n = 1$. We show the following result.

Theorem: The maximum of

$$H := \frac{-\sum_{i \in T} p_i \ln p_i}{\sum_{i \in T} i p_i} \quad (1)$$

($\ln$: natural logarithm) under the constraints that $p_i \geq 0$, $\sum_{i \in S_n} p_i = P_n$, $n \in M$, equals z_0 , where $z_0 > 0$ is the unique solution z of the equation

$$-\sum_{n \in M} P_n \ln Q_n(z) = -\sum_{n \in M} P_n \ln P_n \quad (2)$$

with for $z > 0$

$$Q_n(z) := \sum_{i \in S_n} e^{-iz}, \quad n \in M. \quad (3)$$

Moreover, the optimal p_i are given by

$$p_i = \frac{P_n}{Q_n(z_0)} e^{-iz_0}, \quad i \in S_n, n \in M, \quad (4)$$

and for these p_i we have that

$$\sum_{i \in T} i p_i = \frac{d}{dz} \left[-\sum_{n \in M} P_n \ln Q_n(z) \right] (z_0). \quad (5)$$

As an application of this result we consider *weakly constrained* (d, k) sequences [1]. A binary (d, k) -constrained sequence has by definition at least d and at most k 'zeros' between consecutive 'ones'. Weakly constrained codes produce sequences that violate the specified constraints with a small probability. It is argued that if the channel is not free of errors, it is pointless to feed the channel with

perfectly constrained sequences. A (d, k) -constrained sequence can be thought to be composed of 'phrases' 10^i , $d \leq i \leq k$, where 0^i means a series of i 'zeros'. In order to compute the channel capacity, i.e. the maximum $z_0/\ln 2$ of the entropy $H/\ln 2$, we define

$$T = \{1, \dots, d\} \cup \{d+1, \dots, k+1\} \cup \{k+2, k+3, \dots\} =: S_1 \cup S_2 \cup S_3, \quad (6)$$

where $d = 0, 1, \dots$, and $k = d+1, d+2, \dots$ are given, and we compute the capacity for the case that the probabilities P_1, P_3 assigned to the sets S_1, S_3 are both small. Clearly, the quantities P_1 and P_3 denote the probabilities that phrases are transmitted that are either too short or too long, respectively. We find that the familiar capacities of (d, k) -constrained sequences [2] are approached from above as $P_1, P_3 \rightarrow 0$ with an error $A(P_1 \ln P_1 + P_3 \ln P_3)$, where we can evaluate the A explicitly. We obtain a similar result for the case that T is as in (6) with S_1, S_3 merged into a single set $S_1 \cup S_3$. Further results are published in [3].

Conclusions

We have presented an analytic expression for the maximum of the normalized entropy $-\sum_{i \in T} p_i \ln p_i / \sum_{i \in T} i p_i$ under the condition that T is the disjoint union of sets S_n of positive integers that are assigned probabilities P_n , $\sum_n P_n = 1$. We computed the capacity of weakly (d, k) -constrained sequences that are allowed to violate the (d, k) -constraint with given probability.

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