Improved design equations for asymmetric coplanar strip folded dipoles on a dielectric slab
Visser, H.J.

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IMPROVED DESIGN EQUATIONS FOR ASYMMETRIC COPLANAR STRIP FOLDED DIPOLES ON A DIELECTRIC SLAB

H.J. Visser*

* Holst Centre – TNO
P.O. Box 8550
5605 KN Eindhoven, The Netherlands
E-mail: huib.j.visser@tno.nl

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Abstract

Design equations for the input impedance of the asymmetric strip folded dipole, developed by Lampe, [2,3], depend, amongst others, on the characteristic impedance of an asymmetric coplanar strip (CPS) transmission line. Lampe assumes a homogeneous surrounding medium. In practice, the antenna will be realised on a dielectric slab. Employing the Lampe equations for this inhomogeneous case may lead to relative errors in the CPS characteristic impedance as much as 32% and therefore to large errors in the input impedance. Improved design equations for the input impedance are discussed for the inhomogeneous case. These improved design equations rely on an accurate calculation of the characteristic impedance of an asymmetric CPS on a dielectric slab and further employ correction factors applied to the homogeneous case dipole length and equivalent radius.

1 Introduction

Although the resonant folded dipole antenna is known for its improved frequency bandwidth over that of the ordinary dipole antenna, [1], its main attraction at the moment lies in its ability to adjust the input impedance over a wide range of values. This is especially true for the antenna realised as a planar folded dipole using PCB technology, see figure 1, which may be employed e.g. for RF power scavenging or RFID.

1.1 Lampe model

Design equations for the input impedance of the asymmetric coplanar strip folded dipole antenna were developed by Lampe, [2,3]. These equations give three means of controlling the input impedance: the impedance of a dipole of equivalent radius, the step-up impedance ratio that depends on the widths of the two arms of the planar folded dipole antenna and the impedance of the CoPlanar Strip (CPS) transmission line formed by these two arms.

The input impedance of the antenna is given by, [2,3]

$$Z_{in} = \frac{2(1+a)^2 Z_d Z_x}{(1+a)^2 Z_d + 2 Z_x}.$$  \hspace{0.5cm} (1)

Figure 1: Coplanar strip folded dipole antenna.

Where $Z_d$ is the impedance of an equivalent dipole, i.e. a cylindrical dipole of equivalent radius $\rho_d$, $Z_x$ is the impedance of the transmission line mode and $(1+a)^2$ is the step-up impedance ratio.

The impedance of the transmission line mode is the impedance of a shorted CPS of length $L/2$, see figure 1,

$$Z_x = j \left[ \frac{120 \pi K(k)}{\sqrt{\varepsilon_r} K'(k)} \right] \tan \left( \frac{\beta L}{2} \right).$$ \hspace{0.5cm} (2)

The expression between the brackets is the characteristic impedance of the CPS in a homogeneous medium of relative permittivity $\varepsilon_r$. $K(k)$ is the complete elliptic function of the
first kind, $K'(k)=K(k')$, where $k'^2=1-k^2$. $\beta$ is the wave number in the medium. The complete elliptic function of the first kind $K(k)$ is approximated by, \[ K(k) \approx \frac{1}{2\pi} \ln \left[ \frac{2\sqrt{1+k^2}}{\sqrt{1+k^2-\frac{1}{2}k^2}} \right] \left[ \frac{2\pi}{2\sqrt{1+k^2-\frac{1}{2}k^2}} \right] \left[ \frac{2\pi}{2\sqrt{1+k^2-\frac{1}{2}k^2}} \right] ; 0 \leq \frac{k}{\sqrt{c}} \leq 1; 0 \leq k \leq \frac{1}{\sqrt{c}} \]

and $k$ is given by, \[ k = \frac{b}{\sqrt{c+b(W_1+W_2) - \sqrt{W_1W_2(b+W_1)(b+W_2)}}} \]

where

\[ e = \frac{W_1W_2 + \frac{b}{\sqrt{c+b(W_1+W_2) - \sqrt{W_1W_2(b+W_1)(b+W_2)}}}}{\left(\frac{b}{\sqrt{c+b(W_1+W_2) - \sqrt{W_1W_2(b+W_1)(b+W_2)}}}\right)^2(W_1-W_2)} \]

The parameter $a$ in the step-up impedance ratio is given by

\[ a = \frac{\ln \left\{ 4c + 2\left(2c^2 - \left(\frac{W_1}{\sqrt{c}}\right)^2\right) \right\} - \ln(W_1)}{\ln \left\{ 4c + 2\left(2c^2 - \left(\frac{W_1}{\sqrt{c}}\right)^2\right) \right\} - \ln(W_2)} \]

and the dipole equivalent radius $\rho_e$ is given by

\[ \rho_e = \left(\frac{W_1}{4}\right)^{\frac{1}{\pi}} \left( c + \sqrt{c^2 - \left(\frac{W_1}{\sqrt{c}}\right)^2} \right)^{\frac{1}{\pi}} \]

where $c$ is defined in figure 1.

As an example, in figure 2 the real and imaginary part of the input impedance of an asymmetric coplanar strip folded dipole are shown as function of frequency as calculated with a full wave method (Finite Integration, CST Microwave Studio®) and as calculated with the above equations (Transmission Line method). The dimensions of the antenna are, with reference to figure 1: $W_1=3\text{mm}$, $W_2=1\text{mm}$, $b=1\text{mm}$, $L=62.5\text{mm}$, $\varepsilon_r=1$. The dipole impedance has been calculated by applying the empirical double polyfit equations for the King-Middleton second-order solution as given in [5].

Figure 2: Calculated real and imaginary part of the input impedance vs. frequency for an asymmetric coplanar strip folded dipole antenna in free space.

The agreement between the two simulation results is very good around resonance thus demonstrating the usefulness of the Lampe model.

In figure 3, we show the results as full-wave simulated for the same antenna on a dielectric slab of thickness $t=1.6\text{mm}$ and having a relative permittivity $\varepsilon_r=4.28$. In the same figure the results of the Lampe transmission line (TL) model for the free space antenna are shown.

Figure 3: Calculated real and imaginary part of the input impedance vs. frequency for an asymmetric coplanar strip folded dipole antenna on a dielectric slab and the same antenna in free space.

The figure shows that the input impedance of the antenna on a dielectric slab as function of frequency is very different from that of the same antenna in free space. Therefore it is necessary to adapt the TL model for the dielectric slab effects. The dielectric slab affects both the transmission line mode and the antenna mode.

2 Asymmetric coplanar strip transmission line

Closed-form equations for the characteristic impedance of asymmetric coplanar strip transmission lines on a dielectric slab of finite thickness are not readily available. For
symmetric CPS transmission lines, analytic formulas may be found in [6] and [7].

In a first attempt to derive the required equations, we could try to modify the equation for the characteristic impedance \( Z_0 \) of an asymmetric CPS that is used in equation (2)

\[
Z_0 = \frac{120\pi}{\sqrt{\varepsilon_r}} K(k) K'(k)
\]

(8)

2.1 Uniform medium

First, in a very crude approximation, we could substitute for \( \varepsilon_r \) in equation (8) the relative permittivity of the dielectric slab. This means that we assume the coplanar strips to be present in a uniform medium with relative permittivity equal to that of the dielectric slab. The characteristic impedance for different values of dielectric slab permittivities, heights, strip separations and widths have thus been calculated. Results for symmetric CPS transmission lines have been compared with full wave simulation results as reported in [7], see table 1. In this table, \( t \) is the height of the dielectric and \( b \) is the separation of the identical strips of width \( W \).

<table>
<thead>
<tr>
<th>( \varepsilon_r )</th>
<th>( t ) (mm)</th>
<th>( b ) (mm)</th>
<th>( W ) (mm)</th>
<th>( Z_0 ) full wave (Ohm)</th>
<th>( Z_0 ) analytic (Ohm)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>0.79</td>
<td>0.10</td>
<td>1.52</td>
<td>100.07</td>
<td>82.60</td>
<td>17.46</td>
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<td>0.30</td>
<td>0.76</td>
<td>149.79</td>
<td>125.56</td>
<td>16.18</td>
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<td>9.9</td>
<td>0.64</td>
<td>0.04</td>
<td>1.27</td>
<td>49.91</td>
<td>33.99</td>
<td>31.90</td>
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<td>0.64</td>
<td>0.37</td>
<td>0.51</td>
<td>99.98</td>
<td>69.84</td>
<td>30.15</td>
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<tr>
<td>12.9</td>
<td>0.25</td>
<td>0.026</td>
<td>0.38</td>
<td>50.00</td>
<td>34.23</td>
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<tr>
<td>12.9</td>
<td>0.25</td>
<td>0.15</td>
<td>0.13</td>
<td>100.05</td>
<td>70.51</td>
<td>29.53</td>
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<tr>
<td>50.0</td>
<td>0.25</td>
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<td>0.20</td>
<td>30.03</td>
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<td>50.0</td>
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<td>0.025</td>
<td>50.03</td>
<td>35.63</td>
<td>28.78</td>
</tr>
</tbody>
</table>

Table 1: Calculated characteristic impedances for different symmetric CPS transmission lines and relative differences.

The table reveals that the relative difference may be as high as 32%. The impact of this CPS characteristic impedance approximation used on the input impedance of an asymmetric coplanar strip folded dipole antenna on a dielectric slab is shown in figure 4 for \( W_1=5\text{mm} \), \( W_2=1\text{mm} \), \( b=1\text{mm} \), \( L=62.5\text{mm} \), \( t=1.6\text{mm} \) and \( \varepsilon_r=4.28 \).

It should be noted that the dielectric has only been taken into account for the CPS transmission line mode of the antenna. However, the effect on the antenna mode reveals itself merely as a shift in frequency of the impedance curve and a change of impedance levels, not a change in shape.

Although the impedance curves show a distinct improvement with respect to those shown in figure 3, there is still room for improvement, even when taking into account that the impedance has not yet been corrected for the antenna mode. Since the antenna will, most likely, be connected to a transceiver by a length of CPS transmission line, a need exists for calculating the CPS characteristic impedance with an accuracy higher than 68%.

2.2 Half spaces

A more realistic approximation than assuming the whole space being filled with slab dielectric is to assume the dielectric slab to fill up a half space. Then we may replace \( \varepsilon_r \) in equation (8) with an arithmetic average of the relative permittivities of two dielectric half spaces on both sides of the antenna, [8]

\[
\varepsilon_r = \frac{\varepsilon_{r_{\text{die}}} + 1}{2}.
\]

(9)

Characteristic impedance calculations for symmetric CPS transmission lines, having adapted this effective relative permittivity in equation (8), have been compared with full wave simulation results as reported in [7], see table 2.

<table>
<thead>
<tr>
<th>( \varepsilon_r )</th>
<th>( t ) (mm)</th>
<th>( b ) (mm)</th>
<th>( W ) (mm)</th>
<th>( Z_0 ) full wave (Ohm)</th>
<th>( Z_0 ) analytic (Ohm)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
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<td>1.52</td>
<td>100.07</td>
<td>96.86</td>
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<td>1.70</td>
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<td>99.98</td>
<td>94.13</td>
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<td>6.74</td>
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<tr>
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<td>0.15</td>
<td>0.13</td>
<td>100.05</td>
<td>96.06</td>
<td>3.99</td>
</tr>
<tr>
<td>50.0</td>
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<td>0.031</td>
<td>0.20</td>
<td>30.03</td>
<td>29.00</td>
<td>3.43</td>
</tr>
<tr>
<td>50.0</td>
<td>0.25</td>
<td>0.030</td>
<td>0.025</td>
<td>50.03</td>
<td>49.89</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 2: Calculated characteristic impedances for different symmetric CPS transmission lines and relative differences.

The table reveals that the relative difference is now less than 8.5%. The impact of this CPS characteristic impedance approximation used on the input impedance of an asymmetric...
A coplanar strip folded dipole antenna with the same dimensions as analysed in section 2.1 is shown in figure 5. Again it should be noted that only the transmission line mode of the antenna has been modified for the dielectric slab.

![Figure 5](image)

Figure 5: Calculated real and imaginary part of the input impedance vs. frequency for an asymmetric coplanar strip folded dipole antenna on a dielectric slab and the same antenna on a half space with relative permittivity equal to that of the dielectric slab.

Upon a close inspection of figures 4 and 5 we see that the impedance curves in figure 5 for the transmission line model are – apart from a shift in frequency – in closer agreement with the full wave results as those shown in figure 4.

We aim at developing closed form equations for the input impedance of asymmetric coplanar strip folded dipole antennas on dielectric slabs with a reasonable accuracy. Therefore, having arrived at this point, we should continue with the calculation of the CPS characteristic impedance using equations (8) and (9).

However, to be able to improve upon the accuracy of the model if this will appear to be necessary, we take the calculation of the CPS characteristic impedance one step further and again improve upon accuracy.

### 2.3 Analogy with asymmetric coplanar waveguide

The characteristic impedance of a symmetric CPS on a dielectric slab of height \( t \) and relative permittivity \( \varepsilon_r \) is given by, [6]

\[
Z_0 = \frac{120\pi}{\sqrt{\varepsilon_{eff}}} \frac{K(k)}{K'(k)},
\]

(10)

where

\[
\varepsilon_{eff} = 1 + \frac{\varepsilon_r - 1}{2} \frac{K(k')}{K(k)} K(k_2'),
\]

(11)

and

\[
k = \frac{b}{b + 2W},
\]

(12)

\[
k_2 = \frac{\sinh\left(\frac{\pi}{4\ell}\right)}{\sinh\left(\frac{\pi}{4\ell}\left[\frac{\Delta}{2} + W\right]\right)}.
\]

(13)

The characteristic impedance of an asymmetric CoPlanar Waveguide (CPW), see figure 6, on a half space dielectric slab (\( t \to \infty \)) of relative permittivity \( \varepsilon_r \) is given by, [9]

![Figure 6](image)

Figure 6: Asymmetric coplanar waveguide.

\[
Z_0 = \frac{30\pi}{\sqrt{\varepsilon_{eff}}} \frac{K'(k)}{K(k)},
\]

(14)

where \( k \) is given by equation (4) and \( \varepsilon_{eff} \) is given by equation (9). For a CPW on a dielectric slab of height \( t \), the characteristic impedance is still given by equation (14), but \( \varepsilon_{eff} \) is now given by, [9]

\[
\varepsilon_{eff} = 1 + \frac{\varepsilon_r - 1}{2} \frac{K(k')}{K(k)} K(k_2'),
\]

(15)

where

\[
k_2 = \frac{W_A (1 + \alpha W_B)}{W_B + \alpha W_A},
\]

(16)

\[
W_A = \sinh\left(\frac{\pi}{4\ell}\right),
\]

(17)

\[
W_A = \sinh\left(\frac{\pi}{4\ell}\left[\frac{\Delta}{2} + W\right]\right),
\]

(18)
be noted that only the transmission line mode of the antenna has been modified for the dielectric slab. Section 2.1 is shown in figure 7. Also for this figure it should be noted that only the transmission line mode of the antenna has been modified for the dielectric slab.

The impact of the CPS characteristic impedance thus calculated on the input impedance of an asymmetric coplanar strip transmission line on a finite thickness dielectric is given by equation (10), where \( k \) is calculated with equations (4) and (5) and \( \varepsilon_{ep} \) is calculated with equations (15) till (20). The complete elliptic function of the first kind \( K(k) \) is calculated with equation (3). The relative differences with the full wave results for the impedances thus calculated for the symmetric CPS structures as stated in tables 1 and 2 are less than 1%.

The results are very similar to the previously shown results. The benefit of being able to calculate the CPS characteristic impedance with a high accuracy lies in the opportunity to calculate the input impedance of the folded dipole also with a high accuracy.

For the moment however, we will develop an approximate method to calculate this input impedance with a reasonable accuracy. To that purpose, also the approximate characteristic impedance calculation for a CPS based on two half spaces may be used. We will use the latest discussed equations however.

\[
W_E = -\sinh\left(\frac{\varepsilon}{2w_2 + W_1}\right), \tag{19}
\]

and

\[
\alpha = \frac{1}{W_B + W_E} \left[-1 - \frac{W_B W_E}{W_A^2} - \sqrt{\left(\frac{w_2^2}{w_1^2} - 1\right)^2 - 1}\right] \tag{20}
\]

Given the analogy between a coplanar waveguide and a coplanar strip transmission line, we may easily transform the characteristic impedance equations for an asymmetric CPW on a finite thickness dielectric slab to those of a CPS.

3 Dipole length and radius corrections

Although it is possible to accurately account for the strip dipole on the dielectric slab, we will develop an approximate method based on correction terms applied to the free space analysis of the folded dipole antenna. If a higher accuracy in the end-results is required, the accurate analysis of the transmission line mode of the antenna needs to be used together with an accurate analysis of the dipole mode. Such a dipole analysis method will be outlined in the next section.

3.1 Strip dipole analysis

An accurate way to account for the strip dipole antenna being situated on a dielectric slab is to start with a three-term model for a cylindrical dipole antenna that models a non-perfect conductor by means of a distributed impedance, [10,11]. By virtue of this distributed impedance, it will be possible to model a dielectric or magnetic coating of a cylindrical dipole antenna through substituting a distributed inductance for the distributed impedance, [14]. A strip dipole on a dielectric slab will now modeled as an equivalent, magnetically coated, cylindrical dipole antenna, [12]. In this analysis, [12], the static capacitance of a coupled strip transmission line is needed, where the strip widths are equal to the dipole strip width. This capacitance value is calculated by the method described in [13].

This analysis method, however, will not be applied to the problem at hand now. Instead we will attempt to correct the impedance curves resulting from accounting for the dielectric in the transmission line mode of the antenna by introducing correction terms applied to the free space dipole length and equivalent radius.

3.1 Strip dipole approximation

We have seen that accounting for the dielectric slab in the transmission line mode of the asymmetric coplanar strip folded dipole antenna has lead to improvement of the impedance vs. frequency curves. The curves resemble the ones obtained from full wave analyses apart from a frequency shift and a change in impedance level. We know that one of the main effects of a dielectric on a dipole antenna will be a lowering of the resonance frequency. Therefore, we could try by lengthening the dipole, in the dipole mode analysis part of the antenna, to make the resonance frequency coincide with that obtained by full wave analysis. Further, by increasing the equivalent radius we could try to make the impedance levels coincide. Thus

\[
L' = \alpha L
\]

\[
\rho'_c = \chi \rho_c
\]

For a large number of asymmetric coplanar strip folded dipole antennas, having different dimensions, being positioned on dielectric slabs of different heights having different relative permittivities, the correction factors \( \alpha \) and \( \chi \) have been determined.
Figure 8 shows a typical example of the thus calculated input impedance of a folded dipole antenna together with full wave analysis results.

![Figure 8: Calculated real and imaginary part of the input impedance vs. frequency for an asymmetric coplanar strip folded dipole antenna on a dielectric slab.](image)

The agreements between the thus calculated input impedances and the full wave simulation results tend to get better for smaller strip widths and separations. For the tested frequency range (1GHz-6GHz) the correction factors appear to be frequency independent and may be approximated by

\[
\alpha = \left(1 + \sqrt{r} \cdot 10^{\frac{\log(\epsilon_r)}{45}} \right)
\]

\[
\chi = 1.90
\]  

(22)

4 Conclusions

Improved design equations are derived for an asymmetric coplanar strip folded dipole antenna on a dielectric slab based on the analysis of the same antenna in a uniform medium. A fair to good agreement with full wave simulation results for the input impedance may be obtained by taking the dielectric slab into account in the transmission line mode of the antenna and by correcting for length and equivalent radius of the dipole antenna mode as calculated for free space.

References