Tales of a So(u)rcerer: Optimal Sourcing Decisions Under Alternative Capacitated Suppliers and General Cost Structures

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April 1, 2010

Abstract: Most companies must procure items necessary for their businesses from outside sources, where there are typically a number of competing suppliers with varying cost structures, price schemes, and capacities. This situation poses some interesting research questions from the outlook of different parties in the supply chain. We consider this problem from the perspective of (i) the party that needs to outsource, (ii) the party that is willing to serve as the source, and (iii) the party that has in-house capability to spare. We allow for stochastic demand, capacitated facilities (in-house and suppliers’), and general structures for all relevant cost components. Some simpler versions of this problem are shown to be NP-hard in the literature. We make use of a novel dynamic programming model with pseudo-polynomial complexity to address all three perspectives by solving the corresponding problems to optimality. Our modeling approach also lets us analyze different aspects of the problem environment such as pricing schemes and channel coordination issues. We derive several managerial insights, some of which are counter to collective intuition.

Keywords: Sourcing; Supplier Selection; Inventory; Production; General Costs; Capacity; Supply Chain; Channel Coordination

1. Introduction and Related Literature

Consider a manufacturer that produces and sells a certain product that is subject to stochastic demand. The manufacturer procures the main component used in production. This component (or, the ‘item’) can be produced or supplied by a finite number of capacitated external suppliers (or, ‘sources’), and the manufacturer must decide which of these sources to utilize and to what extent. This decision is made by using a request for quotes reverse

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auction where each of the existing sources quotes a quantity-price list. These sources may have a different capacity, cost, and price structure but they are identical in terms of their function, i.e. the item’s characteristics do not depend on the supplier. We do not restrict our analysis to a particular cost function for sourcing, hence we allow, e.g. for a separate fixed cost for initiating the usage of each source, logistics costs that might depend on the geographical location of the suppliers, and non-linear unit variable costs. Progressive or all-units quantity discounts are special cases. Such a generality, together with the capacitated nature makes decision making a difficult task in this environment, so careful analysis is required such that the sourcer does not need to be a sorcerer for optimal decision making.

We consider three problems (or sorcerer’s tales) in such an environment:

1. Manufacturer’s Sourcing Problem: “How to do the trick?” That is, which sources should be utilized to what extent?

2. Supplier’s Problem: “How to cast a counter-spell?” That is, what is the best price a supplier should quote and what is the capacity he should dedicate?

3. Manufacturer’s In-House Production Capacity Problem: “Cast your own spell.” That is, if the manufacturer can allocate some of her resources for in-house manufacturing of the item, how much capacity should she dedicate for this?

While we refer to the case of ordering from alternative suppliers as our problem environment, there are many other environments where our work would apply. To name some possible applications, suppose that the materials ordered by a manufacturer or a retailer are shipped by vehicles with certain capacities. For each vehicle utilized there may exist a fixed cost as well as a unit variable cost, and the total order may be satisfied with a number of vehicles with varying characteristics. Similarly, consider a production environment where the process is heating to be performed in industrial ovens. Each oven may have a different capacity and a fixed cost of operation. As another possible application, consider a production environment with flexible and dedicated machines, where each machine incurs different set-up and production costs.

To analyze this problem, we consider a single-item, single-period make-to-stock setting. We note that the sourcing problem in general is NP-hard even in the single period setting due its combinatorial nature. Some deterministic versions of this problem have been considered.
in the literature to a certain extent. In this study, we extend those studies by considering
stochastic demand and by including general cost structures.

Sourcing decisions must consider the cost of materials procured, delivery punctuality, the
quality of items procured, creating effective strategic partnerships, and the like. Therefore,
one of the key processes of effective supply chain management is the supplier selection process,
which process consists of determining a supplier base (a set of potential suppliers to operate
with), the supplier(s) to procure from, and the procurement quantities. We refer the reader
to Elmaghraby (2000) for an overview of research on single and multiple sourcing strategies.
Aissaoui et al. (2007) present a comprehensive review of literature related to several aspects
of the procurement function including the supplier selection process and in-house versus
outsourcing decisions. Firms sometimes employ multiple criteria in selecting their suppliers.
A recent survey of multi-criteria approaches for supplier evaluation and selection processes
is presented by Ho et al. (2010). In our work, we do not include the multi-criteria supplier
evaluation phase and assume that the supplier base has already been determined and that
the immediate supplier selection decisions are based on the cost criterion.

The supplier selection problem has received attention mostly under deterministic demand
assumption. When the demand is deterministic, the problem becomes either (i) to determine
the set of suppliers to purchase a given quantity, or (ii) to determine the suppliers and the
purchasing frequency for a given demand rate. Chauhan and Proth (2003) consider a version
of the problem when there is a lower and an upper bound for the quantity supplied from
each supplier and the supply costs are concave. The authors propose heuristic algorithms.
Chauhan et al. (2005) show that the problem considered by Chauhan and Proth (2003)
is NP-hard. Burke et al. (2008b) study a similar problem where suppliers offer quantity
discounts. The suppliers are capacitated and each supplier’s cost function is concave. The
authors first discuss that this particular problem is a version of the ‘continuous knapsack
problem’ where the objective is to minimize the sum of separable concave functions and then
show that this problem is NP-hard. Romeijn et al. (2007) analyze the continuous knapsack
problem with nonseparable concave functions and propose a polynomial time algorithm. We
note that the supplier selection problem with stochastic demand results in a nonseparable
cost function, and is actually not a knapsack problem because the size of the knapsack (the
amount allocated to the suppliers) is itself a decision variable. Some other examples of the
supplier selection problem under deterministic demand when the quantity to be purchased
is known are by Qi (2007), Burke et al. (2008a), and Kawtummachai and Hop (2005).

The stochastic demand version of the sourcing problem under capacitated suppliers has also received attention to a certain extent in the literature. Alp and Tan (2008) and Tan and Alp (2009) analyze the problem with two supply options in a multi-period setting under fixed costs of procurement. The authors characterize the optimal ordering policy for a single-period problem and propose a dynamic programming model to solve the multi-period problem. Awashti et al. (2009) consider multiple suppliers where they have minimum order quantity requirements and/or a maximum supply capacity, but no fixed cost is associated with procurement. The authors show that this problem is NP-hard even when the suppliers quote the same unit price to the manufacturer, and they propose a heuristic algorithm for the general version. Hazra and Mahadevan (2009) analyze an environment where the buyer reserves capacity from a set of suppliers through a contracting mechanism. The capacity is reserved before the random demand is observed and allocated uniformly to the selected suppliers. If the capacity turns out to be short upon demand realization, then the shortage is fulfilled from a spot market at a higher unit price. The authors show that the supplier’s cost function in this setting is concave. Our work is different from these articles because we consider multiple suppliers, general cost functions, and we do not impose a particular structure on the allocation of purchased quantity to the suppliers.

Zhang and Ma (2009) consider a relatively similar environment to ours, where the manufacturer produces multiple products observing stochastic demand and there are multiple suppliers that can supply the raw materials used for these products. The manufacturer and the suppliers are all assumed to be capacitated. The suppliers offer quantity discounts resulting in a concave cost function. There is also a fixed cost of doing business with each supplier. The authors propose a mixed integer nonlinear programming formulation that determines the optimal production quantities of each product, purchasing quantities of the raw materials, and the corresponding suppliers to make the purchases. The main differences of our model and this one are that we consider general cost functions and we adopt a dynamic programming approach with pseudo-polynomial complexity. Moreover, our modeling approach lets us analyze different aspects of the problem environment such as the decisions of an entrant supplier, channel coordination issues, and in-house versus outsourcing decisions. Another work that is related to our problem environment, particularly considering the problem from the suppliers’ point of view, is by Li and Debo (2009). The authors consider an
existing and an entrant supplier that compete for the business of the manufacturer. Using a unit variable cost structure and considering a two-period setting where the demand in the second period is stochastic, the authors derive several managerial insights regarding the capacity investment and price quotation decisions of both suppliers.

There are also a number of studies that consider the supplier selection problem with unreliable suppliers. For example, Yang et al. (2007) and Dada et al. (2007) consider this problem under unit variable costs. Federgruen and Yang (2008) also consider fixed costs at the supplier level but the variable supply costs are assumed to be identical for all suppliers. In their model, the quantity to be purchased is determined according to a service level target. The authors propose approximate expressions to determine the purchasing quantity and heuristic algorithms for allocating this quantity to the suppliers.

Our study also elaborates on the value of coordinating the business channel between a supplier and the manufacturer. Several mechanisms such as contracting, quantity discounts, return options, etc. have been proposed in the literature in order to coordinate the channel and create a win-win situation. Li and Wang (2007) present a comprehensive review of the channel coordination literature. Toptal and Cetinkaya (2008) quantify the value of channel coordination between a supplier and a buyer under a certain cost structure. Kheljani et al. (2009) consider a buyer’s sourcing decisions by focusing on optimizing the channel’s profit. Both of these studies consider deterministic demand. Xia et al. (2008) consider the channel coordination problem for a multiple supplier and multiple buyer setting. The order quantity and frequency of the buyers are exogenous parameters. The authors present models that can be used to coordinate the channel by matching the suppliers’ cost functions and the buyers’ purchasing behaviors.

The major contributions of our paper can be summarized as follows:

- We build a novel dynamic programming model that we use for finding the optimal solution to the NP-hard sourcing problem under a fairly general setting consisting of stochastic demand, general cost structures, and capacitated suppliers in one shot. The computational complexity of the solution that we propose is pseudo-polynomial.

- We evaluate the performance of separately making sourcing and production decisions.

- We develop a methodology to find the optimal pricing decision of a supplier who competes with other suppliers.
We develop a methodology to find the optimal capacity allocation decision of the manufacturer for in-house manufacturing under the existence of alternative production sources.

Finally, we build several managerial insights, some of which are contrary to the collective intuition that traditional inventory/production models generate.

The rest of the paper is organized as follows: We present the manufacturer’s sourcing problem in Section 2. The supplier’s problem is analyzed in Section 3 and the manufacturer’s in-house production capacity problem is analyzed in Section 4. We conclude the paper in Section 5.

2. Manufacturer’s Sourcing Problem: How to do the trick?

In this section, we analyze the sourcing problem for the main component used in production, under a given set of alternative capacitated suppliers. There are two decisions that must be made in such an environment: Which sources should be utilized, and in what quantities? The total quantity to be procured also determines the optimal production quantity to be materialized. It is common practice in industry that sourcing and production decisions are handled separately. The production department (or the department that needs the item) determines required quantities based on available inventories and related production costs, and relays this information to the purchasing department who makes the purchase. The purchasing department determines the sources from which to purchase this quantity with the least cost. We refer to this approach as the ‘decoupled approach’. We first discuss how the problem could be solved in the decoupled approach. Then, we present a dynamic programming model to formulate the problem under consideration and show how the optimal solution can be found. Finally, we present the results of the numerical study we conducted to investigate (i) the effect of problem parameters on the optimal solution, and (ii) the performance of the decoupled approach.

The relevant costs in our environment are the costs of procuring from suppliers, and underage and overage costs, all of which are exogenously determined and non-negative. We do not impose any conditions on the costs of procuring from suppliers, and hence these costs
might assume any form, possibly including fixed costs for procurement, stepwise costs for
shipments, different forms of quantity discounts, and the like. Our approach allows for the
underage and overage costs of the remaining inventory position after demand materialization
to assume any form as well, via the corresponding loss function. Nevertheless, linear underage
and overage costs are presented in the remainder of the text for ease of exposition. We also
allow for capacitated suppliers with fixed and known capacities. We assume full availability
of the ordered quantities, and we also assume that the differences between procurement
lead times from alternative suppliers can be neglected. In case the latter assumption is
significantly violated, different lead times can be approximately incorporated into the model
by considering appropriate costs associated with purchasing from each supplier, reflecting
the cost effect of corresponding procurement lead times. Similarly, other non-biddable price
factors such as delivery punctuality, the quality of items procured, strategic partnership
concerns, and the like are also valued by the manufacturer and reflected in the procurement
costs. For a discussion on the valuation of non-biddable price factors, see Kostamis et al.
(2009). We summarize our major notation in Table 1 for ease of reference.

<table>
<thead>
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<th>Table 1: Summary of notation.</th>
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<tr>
<td>( N ) : Number of alternative suppliers</td>
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<td>( Q ) : Total procurement quantity</td>
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<tr>
<td>( U_n ) : Capacity of supplier ( n ), ( n = 1, 2, ..., N )</td>
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<tr>
<td>( q_n ) : Quantity procured from supplier ( n )</td>
</tr>
<tr>
<td>( C_n(q_n) ) : Cost of procuring ( q_n ) units from supplier ( n ), ( n = 1, 2, ..., N )</td>
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<td>( h ) : Overage cost per unit unsold</td>
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<td>( b ) : Underage cost per unit of unmet demand</td>
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<tr>
<td>( W ) : Random variable denoting the demand</td>
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<td>( G(w) ) : Distribution function of ( W )</td>
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If \( Q \) units are procured with a total purchasing cost of \( PC(Q) \), the resulting average
unit procurement cost is \( c = PC(Q)/Q \). The problem is to minimize expected total costs
\( ETC(Q) = PC(Q) + L(Q) \), where \( L(Q) = h \int_0^Q (Q - w)dG(w) + b \int_Q^\infty (w - Q)dG(w) \) is the
regular loss function.

In the decoupled approach, the production department decides on the ‘optimal’ required
quantity \( Q \) mainly based on inventory-related costs, as the exact value of \( PC(Q) \) is unknown
to that department. The purchasing department procures this quantity based on sales prices
and capacities quoted by various suppliers, by solving the following problem (P):

\[
\begin{align*}
\text{Min.} & \quad \sum_n C_n(q_n) \\
\text{st} & \quad \sum_n q_n = Q \\
& \quad q_n \leq U_n \text{ for all } n.
\end{align*}
\]

If the production department considers inventory-related costs only, then the optimal procurement quantity is \( Q = G^{-1}(\frac{b}{b+h}) \). But this approach results in over-estimation of the required quantity since it neglects procurement costs. If one incorporates a unit procurement cost of \( c \), the resulting optimal procurement quantity is \( Q = G^{-1}(\frac{b-c}{b+h}) \). However, there is no way of knowing what the procurement cost would be, until the required quantity is known. One could prepare a list of all possible quantities, but each entry in the list requires solving problem P, which is a knapsack problem with a general objective function. A special case is the fixed-charge continuous knapsack problem (see Haberl, 1999), which is NP-hard with some known pseudo-polynomial algorithms.

A simple approach is to incorporate an estimate of the purchasing cost, \( \tilde{c} = \frac{\sum_n C_n(U_n)}{\sum_n U_n} \), and decide on \( Q \) accordingly, after which the purchasing department procures \( Q \) units by solving problem P. Nevertheless, this approach can be improved: Once the purchasing department knows the optimal \( PC(Q) \) and \( c \) corresponding to procuring \( Q \) units, they could communicate this information to the production department, upon which the production department can come up with the corresponding ‘new’ optimal value of \( Q \), and so forth. Exploiting this idea, one can come up with the following solution approach:

**Step 0.** Set \( i = 1 \), decide on the order quantity \( Q^i = G^{-1}(\frac{b-\tilde{c}}{b+h}) \).

**Step 1.** Compute the average unit cost associated with purchasing \( Q^i \) units, \( c^i \), by solving problem P.

**Step 2.** Find \( Q^{i+1} = G^{-1}(\frac{b-c^i}{b+h}) \).

**Step 3.** If the solution converges or the algorithm is run for a sufficiently long time, quit with \( Q = Q^{i+1} \). Otherwise, set \( i = i + 1 \), and go to Step 1.

A major disadvantage of this approach is that it tackles the problem in an iterative manner rather than an integrated manner, and hence the solution it generates will not
necessarily be optimal. Clearly, $Q$ must be decided by taking the sourcing alternatives into account. Any approach such as dynamic programming (DP) that considers the allocation of an additional unit will not guarantee optimality either, since the solution may change drastically by this additional unit. Furthermore, the problem cannot be seen as a special case of a knapsack problem with a non-separable objective function, because the ‘knapsack size’ (i.e. the total amount to be purchased and allocated to the suppliers) is also a decision variable.

Nevertheless, the following DP formulation can be used to solve the integrated problem of finding optimal sourcing decisions including the procurement quantity, with $f_n(x)$ defined as the minimum cost of purchasing from suppliers $n, n+1, \ldots, N$, if $x$ units are procured from suppliers $1, 2, \ldots, n-1$.

The Manufacturer’s Problem (MP):

$$f_n(x) = \min_{y: x \leq y \leq x + U_n} \{C_n(y-x) + f_{n+1}(y)\}$$

where

$$f_{N+1}(x) = \mathcal{L}(x).$$

The stages in this model are the suppliers (in an arbitrary order), and the state variable $x$ (at stage $n$) is the quantity that is already purchased using the first $n-1$ suppliers, with the existing inventory (if any) prior to the sourcing decision being $x$ at stage 1. At each stage, MP searches for how many units should be ordered from supplier $n$, i.e. $y-x$, to minimize $f_n(x)$, where inventory-related costs are taken into account at the terminal stage. Hence, all possible sourcing combinations are considered in one shot. We also note that the ‘sequence’ of suppliers while defining them as stages is irrelevant in this model. The computational complexity of this DP is $O(N \sum_n U_n \max_n(U_n))$.

We conducted a numerical study to investigate (i) the effect of problem parameters on the optimal solution (Sections 2.1 and 2.2) and (ii) the performance of the decoupled approach (Section 2.3). While it is not possible to present all of the results here, in what follows we present the most interesting managerial insights. We considered the following setting: The demand has a Gamma distribution with coefficient of variation (CV) values of 0.5, 1, 1.5, and an expected value of $E[W] = 40$. We also considered a Poisson-distributed demand with the same mean. The cost parameters are $h = 1, b = 5, 10, 50$. There are $N = 5$ alternative
suppliers \((n = 1, 2, \ldots, 5)\) with capacities \(U_n = 40, 20, 20, 10,\) and 10, respectively. There exists a fixed-cost component of ordering from supplier \(n\), with \(K_n = 40, 20, 20, 10,\) and 10, respectively, and a linear unit variable cost component of \(c_n\), where \(c_1 \in \{1.5, 2, 2.5\}, c_2\) and \(c_3 \in \{2, 2.5, 3\}, c_4\) and \(c_5 \in \{2.5, 3, 3.5\}\).

### 2.1 Effects of Demand Variability and Cost Parameters

Contrary to the collective intuition that traditional inventory/production models generate, we first show in this section that the optimal procurement quantity does not necessarily increase as the variability of demand increases. Table 2 illustrates this phenomenon, where the syntax is as follows: The procurement quantity from a supplier is listed as long as it is nonzero, and the full capacity of the supplier is indicated in parentheses in case the capacity is under-utilized. For example, when \(b = 50\) and \(CV = 0.5\), the optimal sourcing decision is to order the full capacity of 40 and 20 units from suppliers 1 and 2, respectively, and to order 17 units from supplier 3, whose capacity is 20. We have \(c_n = 1.5, 2, 2, 3,\) and 3, for \(n = 1, \ldots, 5\), respectively, in this experiment.

If the cost of backordering is high, which refers to an environment with a high service level, then the total amount procured increases as the variability of the demand increases, in line with collective intuition. Nevertheless, if the cost of backordering is low, then the total amount procured decreases as the variability of the demand increases. This is because the risk of being left with unsold goods (as in obsolescence) outweighs the risk of goodwill loss, due to relatively high procurement and holding costs.

Table 2: Optimal sourcing decision at different coefficients of demand variation and backorder costs.

<table>
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<tr>
<th>CV</th>
<th>b= 5</th>
<th>b=10</th>
<th>b=50</th>
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<tr>
<td>(0.5)</td>
<td>40</td>
<td>40</td>
<td>40, 20, 17(20)</td>
</tr>
<tr>
<td>(1.0)</td>
<td>20</td>
<td>40</td>
<td>40, 20, 20, 10</td>
</tr>
<tr>
<td>(1.5)</td>
<td>0</td>
<td>40</td>
<td>40, 20, 20, 10</td>
</tr>
</tbody>
</table>

We also observe that the optimal solution might be extremely sensitive to cost parameters. For example, when \(CV = 1.0, b = 5, N = 3, U_n = 40, 20, 10,\) \(K_n = 40, 20, 10,\) and \(c_n = 1.5, 2.5, 2.5,\) for \(n = 1, 2, 3,\) respectively, the optimal solution is \(37(40)\). When we keep all the parameters the same, except for \(c_1 = 2\) instead of \(1.5,\) the optimal solution becomes \(10,\) which represents not only a \(73\%\) decrease in total procurement quantity, but also a
completely different supplier selection. This example shows that the managerial insights as to the optimal solution of a particular situation would not necessarily hold even when a single parameter gets changed, indicating a lack of robustness, which emphasizes the importance of having a methodology to find the optimal solution.

2.2 Effects of Flexibility

In this section, we analyze the impact of flexibility on optimal sourcing decisions. We call a problem environment ‘more flexible’ than another when there is at least one more sourcing option to choose from in addition to the existing ones. An interesting observation we make is that a more flexible environment may lead to lower procurement quantities. We illustrate this situation with the following parameter setting: \( CV = 0.5, b = 5, c_n = 2.5, 3, 3, 2.5, \) and 2.5, for \( n = 1, \ldots, 5, \) respectively. Let us first set \( U_3 = U_5 = 0, \) i.e., only suppliers 1, 2, and 4 are available. In that case, the optimal solution is 34(40), following the same syntax as in Section 2.1. When we make this system more flexible by letting \( U_3 = 20, \) and \( U_5 = 10, \) the optimal solution becomes 10, 10, decreasing the total procurement by 41%. This result is due to interactions within the cost parameters, particularly dictated by fixed-cost components. For many different parameter settings, we did observe the total procurement quantity increasing as the system became more flexible, as might be expected.

2.3 Value of the Integrated Approach

Finally, we compare the optimal solution of the integrated approach with the solution found by the decoupled approach. The problem setting is the same as the one originally introduced. The average cost deviation percentages relative to the optimal solution are presented in Figure 1.

The decoupled approach performs quite well as the backorder cost gets higher (average cost deviation = 0.67% for \( b = 50 \)), because sourcing takes place in large quantities, sometimes consuming the full available capacity (which is 2.5 times the average demand). When we considered more flexible systems and the same high backorder cost, the performance of the decoupled approach deteriorated, while still being small enough for practical purposes. Nevertheless, when the backorder cost is not as high, the value of the integrated approach over the decoupled approach appears to be very significant. The average cost deviation...
Figure 1: Percent of cost deviation due to decoupled approach.

over all cases considered is 7.3%, and the maximum is 88.9%, which also demonstrates the importance and non-triviality of finding the optimal solution.

3. Supplier’s Problem: How to cast a counterspell?

In this section, we take the suppliers’ point of view into consideration. Consider a particular supplier (referred to as ‘the supplier’ from now on) who intends to earn the manufacturer’s business by forming a channel between himself and the manufacturer. The supplier could be a new entrant to the market or one of the existing suppliers, who either needs to install new capacity or spare a portion of his existing capacity for the manufacture of the item. What the supplier must determine are the optimal capacity to dedicate and the optimal price to quote to the manufacturer, under the existence of other suppliers (referred to as the ‘alternative suppliers’ in the rest of the text) who are kept at the peripherals of the business channel. After receiving all quotations, the manufacturer will determine her optimal course of action by using the methodology explained in Section 2.

The supplier would benefit from the price and capacity information of the alternative suppliers in order to make a better decision about the capacity to dedicate and the price to quote to the manufacturer. Such information might be available to the supplier if (i) the supplier has enough experience in the market, (ii) the supplier has the power or sources
to collect this information, (iii) there exists a business-to-business establishment where this information is available, (iv) the manufacturer shares the information with the supplier for reasons such as the intention to form a strategic partnership, and the like. If the supplier has information on the manufacturer’s demand distribution or he can anticipate it, he could use the methodology presented in Section 2 to predict how the manufacturer would operate. The question that we address in this section is how the supplier can make use of this information to form a list of price quotations at various quantities that will result in the manufacturer procuring the quantity that maximizes the supplier’s profit. If the supplier could find such a price and capacity pair then he would eliminate the uncertainty as to the capacity he should dedicate to this manufacturer.

Prior to the quotation of the supplier, the manufacturer has a certain course of action. However, any capacity and price quotation offered by the supplier might change the manufacturer’s decision considerably. As noted in Section 2.1, this problem is very sensitive to the problem parameters; the effect of a change in even one of the parameters or an increase in the number of available suppliers cannot be easily anticipated without solving the problem under the new settings to optimality. Therefore, even if the supplier has all the necessary information, it is not straightforward to derive insights and to set a price and capacity pair without a methodology to find the optimal solution. The supplier needs to solve the following optimization problem.

The Supplier’s Problem (SP):

\[
\begin{align*}
\max_{p(Q) \geq 0, 0 < Q \leq U_1} & \quad Z(p(Q), Q) = C_1(Q) - K_1(Q) - A(Q) \\
\text{s.t.} & \quad C_1(Q) + f_2(Q) \leq C_1(y) + f_2(y) \quad \forall y \leq U_1
\end{align*}
\]

(1)

where

- \( Q \): quantity quoted
- \( p(Q) \): average price per unit corresponding to selling \( Q \) units
- \( K_1(Q) \): costs associated with purchasing \( Q \) units from the supplier that are not accrued by the supplier
- \( Z(p(Q), Q) \): total profit of the supplier
- \( A(Q) \): total cost of dedicating \( Q \) units of capacity to the manufacturer
- \( C_1(Q) = Qp(Q) + K_1(Q) \)

We index the supplier as ‘Supplier 1’ in SP without loss of generality. The objective
function of SP is to maximize the profit generated by the supplier when the manufacturer purchases \( Q \) units with a cost of \( C_1(Q) \), resulting in an average price of \( p(Q) \) per unit, accrued by the supplier (possibly as a result of a non-linear cost scheme quoted by the supplier). The cost \( C_1(Q) \) also includes all costs associated with purchasing \( Q \) units from the supplier that are not accrued by the supplier, such as the shipping costs charged by a logistics service provider. In the constraint set, the expression on the left hand side is the total cost of the manufacturer’s optimal purchasing strategy when the supplier quotes \( Q \) units at an average price of \( p(Q) \) per unit, whereas the right hand side is the manufacturer’s cost of procuring any quantity less than \( U_1 \) from the supplier. This constraint set ensures that the price quoted for each \( Q \) value makes it economical for the manufacturer to procure \( Q \) units in full from the supplier with a cost of \( C_1(Q) \). Note that SP is a nonlinear programming model as the functions \( A(Q) \), \( C_1(Q) \), and \( f_2(Q) \) can have any functional form. Nevertheless, we devise an algorithm to find the optimal solution by inspection.

For a given value of \( Q \), \( p(Q) \) attains the largest possible value, since we have a maximization problem. We first note that the constraint (1) at \( y = 0 \) provides an upper bound on \( p(Q) \) because the manufacturer would procure only from the alternative suppliers for any price quotation above \( p(Q) \). Since \( C_1(0) = 0 \), this upper bound turns out to be \( p(Q) \leq \frac{f_2(0) - f_2(Q)}{Q} \). Repeating this for all \( 0 < Q \leq U_1 \) generates a list of price quotations at each possible \( Q \) such that the manufacturer is indifferent between procuring \( Q \) units at a price of \( p(Q) \) from the supplier and procuring \( Q \) units elsewhere. This means that the constraint set (1) is equivalent to \( C_1(Q) + f_2(Q) \leq f_2(0) \quad \forall 0 < Q \leq U_1 \), which decreases the complexity of the problem.

While SP generates a list of price quotations for all \( 0 < Q \leq U_1 \), the supplier would not be interested in \((Q, p(Q))\) pairs with \( Z(p(Q), Q) \leq 0 \). Hence, the list consists of the \((Q, p(Q))\) pairs with positive profit. The supplier needs to give an incentive to the manufacturer to make sure that \( Q^* = Q \) that maximizes \( Z(p(Q), Q) \) is procured by quoting a price of \( p^*(Q^*) = p(Q^*) - \varepsilon \) for \( Q^* \), with \( \varepsilon > 0 \). We note that \( Z(p^*(Q^*), Q^*) \) is the maximum benefit that can be generated by the business channel between the supplier and the manufacturer. The supplier enjoys all of this benefit but the incentive, where the incentive ensures that the manufacturer is also better off compared to the situation without this business channel, resulting in channel coordination.

Property 1. Any list that includes \((Q^*, p^*(Q^*))\) and has \((Q, p(Q))\) such that \( p(Q) \geq p(Q) \)
for all $Q \neq Q^*$ is an optimal list.

In what follows, we provide an algorithm that can be used to generate a profitable quotation list, based on SP.

**Step 0.** Number the alternative suppliers starting from 2 and find $f_2(Q)$ by solving MP for all $0 \leq Q \leq U_1$.

**Step 1.** For each value of $Q$ such that $0 \leq Q \leq U_1$, $p(Q) = (f_2(0) - f_2(Q))/Q$.

**Step 2.** Let $Q^* = \arg \max_Q Z(p(Q), Q)$ and $p^*(Q^*) = p(Q^*) - \varepsilon$, such that $p^*(Q^*) > 0$ and $Z(p^*(Q^*), Q^*) > 0$. If no such $(p^*(Q^*), Q^*)$ exists, quit the algorithm as there is no profitable quotation list.

**Step 3.** An optimal quotation list consists of $(Q^*, p^*(Q^*))$ and $(Q, p(Q))$ for all $0 < Q \leq U_1$ such that $Q \neq Q^*$, $p(Q) > 0$ and $Z(p(Q), Q) > 0$.

Step 0 and Step 1 take $O\left(N \left(\sum_n U_n\right) \max_n (U_n)\right)$ and $O(U_1)$ computational time, respectively, and Step 2 can already be computed within the effort required in Step 1. Therefore, the computational complexity of this algorithm is $O\left(N \left(\sum_n U_n\right) \max_n (U_n)\right)$, i.e. it does not add to the complexity of MP.

Although for any non-speculative cost structure $f_2(Q)$ is non-increasing in $Q$, we note that $p(Q)$ is not necessarily monotonic in $Q$. Figure 2 depicts an example under the parameter setting introduced in Section 2 with $b = 10$, $CV = 1.5$, and $c_n = 1.5, 2, 2, 2.5, 2.5$ for $n = 1, \ldots, 5$, respectively (the dotted line on the figure). The optimal $(Q^*, p^*(Q^*))$ is also encircled in Figure 2, which is not even a local optima of the $Q$ versus $p(Q)$ graph. While the $Q$ versus $p(Q)$ graph may yield any form, it would be unconventional and complicated to quote such a non-monotone pricing scheme. To that end, the supplier can adopt a more practical scheme, such as quantity discounts, as long as it is in line with Property 1. Note that such a scheme would require the supplier to apply artificial mark-ups to optimal prices. We also depict an example pricing scheme with quantity discounts after the artificial mark-up in Figure 2 (the solid line). A possible disadvantage of quoting elevated prices in practice is the prospective loss of goodwill of the manufacturer. Therefore, a remedy would be to apply a constant unit-price scheme (or, ‘linear’ scheme), which is observed frequently in practice. Furthermore, the manufacturer might specifically require a linear scheme. Nevertheless,
which constant unit price must be quoted is not a trivial decision and requires further analysis. Quoting $p^*(Q^*)$ is not necessarily optimal, and moreover, it violates Property 1 unless $p^*(Q^*) = \max_Q p(Q)$. Therefore, in the remainder of this section, we consider the ‘special case’ of linear price quotations between the supplier and the manufacturer.

Figure 2: An optimal pricing scheme example.

If the manufacturer requires a linear pricing scheme from the supplier, then the supplier’s problem becomes the following:

$$(SP^L) : \max_{p \geq 0, 0 < Q \leq U_1} Z(p, Q) = pQ - A(Q)$$

s.t. $pQ + K_1(Q) + f_2(Q) \leq py + K_1(y) + f_2(y)$ \quad \forall y \leq U_1

$Q, y : \text{integer}$

For any $0 < Q \leq U_1$, we have the following relations from the constraint set:

$$p \leq \frac{(f_2(y) - f_2(Q) + K_1(y) - K_1(Q))/(Q - y)}{Q - y} \quad \forall y \leq U_1 \quad (2)$$

Let the optimal price to quote that would result in ordering $Q$ units from the supplier be $\overline{p}(Q)$. The maximum unit price that will not violate (2) is given by $\overline{p}(Q) = \min_y (f_2(y) - f_2(Q) + K_1(y) - K_1(Q))/(Q - y)$. In what follows we provide an algorithm that can be used to generate a profitable quotation list, based on $SP^L$:
Step 0. Number the alternative suppliers starting from 2 and find \( f_2(Q) \) by solving MP for all \( 0 \leq Q \leq U_1 \).

Step 1. For each value of \( Q \) such that \( 0 \leq Q \leq U_1 \), \( p(Q) = \min_{y:0 \leq y \leq U_1} (f_2(y) - f_2(Q) + K_1(y) - K_1(Q))/(Q - y) \).

Step 2. Let \( Q^* = \arg \max_Q Z(p(Q), Q) \) and \( p^* = p(Q^*) \) such that \( p^* > 0 \) and \( Z(p^*, Q^*) > 0 \).

If no such \((p^*, Q^*)\) exists, quit the algorithm as there is no profitable quotation.

Step 3. The optimal unit price is to quote available capacity \( U_1 \) at a unit price of \( p^* \).

By solving \( SP_L \), the supplier finds the optimal quantity \( Q^* \) that will be ordered by the manufacturer from the quoted capacity of \( U_1 \), and the corresponding unit price \( p^* \) that will maximize his profit. If the algorithm generates a non-empty quotation list, then the supplier will be in business. In this case, the manufacturer is also better off and benefits due to the presence of the supplier.

In the following discussion, we examine the impact of problem parameters on operating characteristics. As a numerical test bed, we use the parameter set introduced above, and in addition we let \( A(Q) = 1.5Q \) and include \( b = 100 \). For this discussion, let \( \Pi^s \) denote the benefit (i.e. the profit) of the supplier, \( \Pi^m \) the benefit of the manufacturer, and \( \Pi = \Pi^s + \Pi^m \) the total benefit of the system due to the presence of the supplier. If the supplier decides not to engage in business due to a non-positive profit, then the benefits are zero. We first investigate the impact of the demand variability on the supplier’s and manufacturer’s benefits under different backordering costs (see Figure 3).

For low values of the backordering cost \((b = 5 \text{ or } 10 \text{ in Figure 3})\), we observe that the benefit to the supplier decreases as the demand variability increases. This is because the manufacturer prefers to decrease\(^1\) the total procurement amount from the market (see Table 3), cf. Section 2.1. For larger values of \( b \), the manufacturer’s total procurement quantity increases in demand variability, which leads to an increase in the benefit to the supplier.

The optimal prices quoted by the supplier under different demand variations and backordering costs are also shown in Table 3. The behavior of the optimal price strongly depends on the problem parameters and, in general, there is no monotonicity. When \( b = 100 \),

\(^1\)In this discussion, we use the term ‘decreasing’ (‘increasing’) in the weak sense, to mean ‘non-increasing’ (‘non-decreasing’).
Figure 3: Impact of demand variability on the benefits to the supplier and the manufacturer.

Table 3: Optimal procurement quantities and the unit price.

<table>
<thead>
<tr>
<th>CV</th>
<th>( Q^* )</th>
<th>( Q_m )</th>
<th>( p^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>33</td>
<td>33</td>
<td>2.59</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>17</td>
<td>3.02</td>
</tr>
<tr>
<td>1.5</td>
<td>10</td>
<td>10</td>
<td>2.64</td>
</tr>
</tbody>
</table>

\( Q_m \): Capacity procured by the manufacturer from all suppliers.

we observe that \( p^* \) increases as CV increases, even though \( Q^* \) remains about the same. Recall that as CV increases, the manufacturer is willing to procure more capacity under high backordering costs. This makes the supplier’s capacity more valuable and gives him an opportunity to elevate his prices. To be more specific, we explain the rationale behind this opportunistic behavior as follows: In this problem instance, a total of 100 units of capacity are available from alternative suppliers, and the maximum capacity that the supplier can quote is also 100 units. When CV=0.5, the supplier competes with all alternative suppliers for the existing range of capacity that would be procured by the manufacturer and quotes a price of 2.49, which beats all alternative suppliers. When CV=1, the manufacturer is willing to procure more than 100 units in total. As 40 units are procured from the ‘cheapest’ alternative supplier, the supplier now competes with the remaining ‘relatively more expensive’ ones that have a total capacity of 60, and hence is able to increase his quoted price while achieving higher sales (90 units rather than 84). When CV=1.5, the supplier competes with even more expensive suppliers, and hence increases the quoted price.

A similar but reverse effect is observed for \( b = 5 \). When CV is increased from 1 to 1.5,
the manufacturer prefers to procure less this time, since the backordering cost is relatively low. Hence the supplier needs to compete for a smaller market size with ‘relatively cheaper’ suppliers, which forces him to decrease the quoted prices.

When \( b = 10 \), the manufacturer normally prefers to procure less as CV increases from 0.5 to 1 (all other parameters are kept constant). However in this case, the supplier slightly decreases his price (from 2.5 to 2.48) by forcing the manufacturer to procure the same quantity (47). Had the manufacturer procured 46 units at a price of 2.5, the supplier’s profit would have been 115, which is less than the profit he makes (116.56) by selling 47 units at a price of 2.48.

As illustrated above, the optimal prices are determined according to the particular interactions of the problem parameters and there is no monotonic behavior. Figure 4 (Figure 5) depicts the optimal quantity procured from the supplier versus the unit price quoted, for different backordering costs when CV=0.5 (coefficient of variation values when \( b = 5 \)). The optimal procurement quantity and unit price pair is shown with a circle. In both figures, we plot the graphs for all unit prices in the feasible range, irrespective of profitability. As the unit cost for the supplier is 1.5, quoting any price less than 1.5 would not be rational in the short run; nevertheless, the supplier might prefer to operate with negative profits in return for capturing a large portion of the market and garnering a strategic benefit in the long run.

![Figure 4: Quantity procured from the supplier's quoted capacity vs. unit price, when CV=0.5.](image)

Finally, we elaborate on the benefit of channel coordination under the linear pricing scheme as modeled by \( \text{SP}^L \), making use of a numerical example with a Poisson demand and \( b = 100 \). In this case, the optimal course of action for the supplier is to quote a unit price
of 2.5, which results in a procurement quantity of 52 units with $\Pi^s = 52$, $\Pi^m = 20.84$, and $\Pi = 72.84$. This is the only operational point that would be materialized without any further coordination effort. However, the manufacturer’s benefit would have been maximized had she requested 39 units from the supplier, which dictates the supplier to quote a unit price of 2 according to $\text{SP}^L$. In this case, $\Pi^s = 19.50$, $\Pi^m = 68.15$, and $\Pi = 87.65$. Nevertheless, neither of these two operating points coordinate the channel. The maximum benefit of the channel is attained when the supplier quotes a unit price of 2.14, resulting in a procurement quantity of 41 units, with $\Pi^s = 26.28$, $\Pi^m = 62.24$, and $\Pi = 88.52$. A particular mechanism in the form of a tailored contract is necessary to ensure that both parties are better off and this channel-coordinating point is attained. Hence, the maximum channel profit of 88.52, which stands for an additional benefit of 15.68 compared to the situation without coordination, could be shared between the parties in such a way that the supplier’s profit exceeds 52 and the manufacturer’s benefit exceeds 20.84.

4. Manufacturer’s In-House Production Capacity Problem: Cast your own spell

In this section, we switch back to the manufacturer’s point of view, with the consideration that she might allocate some in-house production capacity to produce the item if she has (the ability to acquire) the technology to do so. This might be desirable for the manufacturer not only because of cost advantages, but also due to the strategic decision of being less dependent on suppliers. Moreover, the solution to MP is highly sensitive to relatively small
changes in problem parameters, as discussed in Section 2.1, and the manufacturer might need to build or allocate some in-house capacity as a remedy. Assuming that such concerns can be translated into financial terms (i.e. updating the quotations accordingly to incorporate them), we take the cost perspective into account in what follows.

As the quotations of prospective suppliers are available to the manufacturer, she may be better off manufacturing (part of) the items in-house, depending on the quotations and the cost of allocating her own manufacturing capability or acquiring this capability. Therefore, the manufacturer makes the in-house production-versus-outsourcing decision, where combining the two is also an option. The methodology we introduced in Section 2 can be used as the key facilitator to that end. We note that it does not suffice to simply use ‘in-house production option’ as an alternative supplier in that methodology, because the capacity to allocate is also a decision variable now. Nevertheless, the manufacturer can determine her optimal course of action in terms of best in-house capacity allocation versus outsourcing strategy as follows:

Let the cost of acquiring/allocating in-house production capability for manufacturing $Q$ items be $A(Q)$, which may assume any form. Then, the manufacturer’s in-house production capacity problem (MCP) can be modeled as:

$$\min_{0 \leq Q \leq U_0} A(Q) + f_2(Q)$$

where $U_0$ is the maximum in-house capacity that can be allocated. Note that this model considers all possible outsourcing options in combination with in-house production in one shot. The solution complexity is the same as that of MP, i.e. $O(N(\sum_n U_n) \max_n(U_n))$. That is, once the cost structure of allocating in-house capability is known, there is no additional complexity required for solving MCP.

MCP shows a similarity to SP, as the capacity to be quoted is also a decision variable in SP. Nevertheless, the objective of the supplier is to maximize his profit, whereas that of the manufacturer is to minimize her costs. The maximum benefit that can be generated by introducing a ‘new source’ of capacity (i.e. the supplier’s capacity in Section 3 and the in-house option here) to the system is the same in both models. Hence, if $A(Q)$ is the same in those two models and the quotation list of the supplier is determined by the solution of SP as proposed with the algorithm provided in Section 3 with $\varepsilon = 0$, the difference between those two cases rests on who collects the benefit, and the total production remains the same.
Nevertheless, the total benefit generated with different quotation structures (as in SP\textsuperscript{L}) might be less than that with MCP, which might encourage the manufacturer to produce in-house and eventually avoid double marginalization. Similarly, the total production quantities with MCP and with different quotation structures (as in SP\textsuperscript{L}) are also not necessarily the same.

5. Conclusions

In this paper, we consider a problem where a manufacturer must procure a critical component of her main product, from three perspectives: (i) Supplier selection problem of the manufacturer where she determines which supplier(s) to utilize and to what extent, (ii) Capacity and price quotation problem of a supplier, (iii) In-house versus outsourcing decision of the manufacturer. We allow for stochastic demand, capacitated production facilities (in-house and suppliers), and general structures for all relevant cost components. We make use of a dynamic programming model with pseudo-polynomial complexity to address all three perspectives by solving the corresponding problems to optimality. Accordingly, we derive the following managerial insights, some of which might be counter to collective intuition:

- The total quantity procured by the manufacturer does not necessarily increase as variability of demand increases. For relatively low service level requirements, the total quantity procured decreases as the variability of the demand increases, whereas a reverse effect is observed otherwise.

- An increase in the availability of sourcing options (a more flexible system) may lead to a decrease in the total quantity procured.

- There is significant value in integrating the decisions as to the supplier selection and the production quantity.

- A change in even a mere cost parameter might completely change the optimal courses of action of all parties involved. The manufacturer might need to build or allocate some in-house capacity to ensure a more robust situation.

- The entrance of a new supplier to the market can form a business channel between the supplier and the manufacturer, which brings a non-negative benefit to both parties (in terms of decreased sourcing costs for the manufacturer and profit for the supplier). The
party that reaps the maximum benefit that can be generated is the supplier, as long as he has the liberty of setting a quotation list in any form, such as non-monotonically quoted prices. As such a quotation list might be impractical, the supplier may be forced to adopt a particular pricing scheme such as a constant unit-price. However, in that case, the generated channel benefit might be limited and is shared by the supplier and the manufacturer. Consequently, the supplier and the manufacturer need to collaborate and tailor a contract in order to ensure that the channel is coordinated and both parties are better off. Traditional policies proposed for channel coordination such as quantity discounts, buy back policies, etc. do not necessarily “do the trick” for coordinating the channel.

As discussed in Section 1, the problem under consideration also applies to other environments such as retailers using capacitated vehicles to replenish their inventory, or production environments with alternative in-house capacitated production facilities. One needs to interpret accordingly the problem environment we depicted, such as ‘the supplier’ being translated into ‘alternative in-house capacitated production facilities’, or ‘in-house manufacturing capability’ being translated into ‘outsourcing possibility’.

References


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Li, C., L.G. Debo. 2009. Second Sourcing vs. Sole Sourcing with Capacity Investment
and Asymmetric Information. Manufacturing & Service Operations Management. 11 448-470.


