Blind extraction algorithm with direct desired signal selection.

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BLIND EXTRACTION ALGORITHM WITH DIRECT DESIRED SIGNAL SELECTION

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Abstract—In many practical applications we are interested in the extraction of only one desired signal out of a mixture of signals. A disadvantage of most blind extraction approaches proposed in the literature is that they are inefficient in the sense that they also separate or extract undesired signals. To deal with this inefficiency we exploit an a priori guess of direction of arrival related parameters of the desired signal, which serves as a mold. Based on this mold we create linear combinations of noise-free correlation matrices that are used to construct a single matrix with a specific eigenstructure. The eigenvector that corresponds to the smallest eigenvalue of this matrix is the desired extraction filter. Finally it is shown that this approach paves the way to make the algorithm flexible in the utilization of additional a priori information.

I. INTRODUCTION

The extraction of only one desired signal from a linear mixture of signals is the objective in a large variety of signal processing problems, e.g., the cocktail party problem. Many approaches to solve this problem use Blind Signal Processing (BSP) techniques. In BSP problems the source signals as well as the mixing system are unknown. Therefore, BSP techniques generally use no training data and no a priori knowledge about the mixing system. However, an essential problem in BSP is the permutation problem, which implies that signals can be separated or extracted in an arbitrary order only.

In the literature several Blind Signal Extraction (BSE) approaches are proposed that deal with this permutation problem. In [1] and [2] sequential BSE algorithms are discussed. In these algorithms the first step is to extract one likely interesting signal. The second step is to classify the extracted signal. If the extracted signal is not the desired signal, a deflation method is used and a new, potentially interesting signal is extracted. The order in which the signals are extracted is depending on properties like sparseness, non-Gaussianity, smoothness, and linear predictability. These properties are assumed to lead to the extraction of only interesting signals because noise signals typically carry properties like Gaussianity and whiteness. An alternative BSE approach is based on Blind Source Separation (BSS) [1]–[3]. Such a method first separates all signals simultaneously with a BSS algorithm. Second, a classifier selects the desired signal from the set of separated signals. A disadvantage of both these approaches is that they are inefficient in the sense that they also separate or extract undesired signals.

In [4] a novel BSE approach is introduced that randomly extracts a signal. The extraction filters are the eigenvectors from the Generalized Eigenvalue Decomposition (GEVD) of correlation matrices from which the structure is composed in a very specific way. It is observed from there that each eigenvalue depends only on the mixing parameters of the signal that is extracted by the corresponding eigenvector.

We believe that the approach in [4] can be extended towards convolutive mixtures, which are applicable in many acoustic applications. In these applications a priori knowledge about the mixing system is often available in terms of a rough guess of the Direction Of Arrival (DOA) of the desired signal, which is a parameterization of elements in the mixing system. Therefore, we assume that we have a mold available, which is an a priori estimation of the mixing column that belongs to the desired signal. In the current work, this mold is incorporated in the work in [4] in order to extract directly the desired signal and this rationale is validated by means of simulations.

The outline of this paper is as follows. In Section II, the BSE problem scenario is given and in Section III assumptions on the Second Order Statistics (SOS) are given. In Section IV the BSE algorithm is derived and in Section V simulation results are discussed. Finally, Section VI contains conclusions and future research.

II. BSE PROBLEM SCENARIO

A model of the extraction scenario is depicted in the upper branch of Figure 1. Here, $S$ unknown source signals $s_1[n], \ldots, s_S[n]$ are mixed by an unknown instantaneous mixing system $A$. We observe $D$ sensors, which generate sensor signals $x_1[n], \ldots, x_D[n]$. These sensor signals consist of mixtures of the source signals combined with additive noise signals $\nu_1[n], \ldots, \nu_D[n]$. The index $n \in \mathbb{Z}$ is the discrete time index and we assume that the sampling rate is sufficiently high to prevent aliasing. If we model the mixing system as a matrix

![Fig. 1: Diagram depicting the proposed BSE algorithm](image-url)
then the mutual relationship of the signals is given as

\[ x[n] = \sum_{i=1}^{S} a^i s_i[n] + \nu[n] = As[n] + \nu[n] \quad \forall n \in \mathbb{Z} \tag{1} \]

with the vectors defined by \( x[n] \triangleq (x_1[n], \ldots, x_D[n])^T \), \( s_i[n] \triangleq (s_1[n], \ldots, s_S[n])^T \), \( \nu[n] \triangleq (\nu_1[n], \ldots, \nu_D[n])^T \), and the mixing matrix defined by: \( A \triangleq [a^1 \quad \cdots \quad a^S] \), with \( a^i \) the mixing vector of the \( i \)’th signal for \( i \in \{1, \ldots, S\} \). The output \( y[n] \) of the extraction system is obtained by applying a linear filter \( w \) to the sensor signals. Ideally, this output signal is exactly the desired signal, which we indicate by \( s_d \); however, by allowing for only linear filters, the best extraction filter produces a noisy observation of this desired signal. The task of our BSE algorithm is to identify this best filter, which is the \( d \)’th row vector of the (pseudo-) inverse of the mixing system. This (pseudo-)inverse only exists if there are at least as many sensors as sources; therefore, in our analysis we assume to have the same amount of sensors as sources.

Our filter identification strategy is depicted in the lower branch of Figure 1. As discussed in Section I, we use the approach from [4] where correlation matrices \( C_\Gamma \) were constructed from the sensor signals. These matrices have a very specific structure and are taken from an a priori available Noise-Free Region Of Support (NF-ROS) as discussed in Section III. From these noise-free correlations matrices linear combinations \( \Gamma_l \) are created based on a mold, which is a guess of the mixing column corresponding to the desired signal. From the linear combinations one new matrix \( M \) is constructed with a specific eigen-structure. The eigenvector that corresponds to the smallest eigenvalue of \( M \) is the desired extraction filter, as derived in Section IV.

### III. Second Order Statistics

The approach in this work exploits the structure in correlation matrices from the observed sensor signals. First we introduce our assumptions on the auto- and crosscorrelation functions of the source, noise and sensor signals. Then we indicate the structure in the correlation matrices.

**Definition III.1.** The correlation function value of a source signal pair \( (s_{i_1}, s_{i_2}) \) for \( 1 \leq i_1, i_2 \leq S \) at a given time \( n \in \mathbb{Z} \) with a certain lag \( k \) in \( \mathbb{Z} \) is defined by

\[ r_{i_1i_2}[n,k] \triangleq \mathbb{E}\{s_{i_1}[n]s_{i_2}[n-k]\} \tag{2} \]

where \( \mathbb{E}\{\cdot\} \) is the mathematical expectation operator.

By using time-lag pairs \( (n,k) \) we are able to cope with both non-stationary and non-white signals. Similar to the source signal correlation functions, the noise, sensor, and source-noise correlation functions are defined by

\[ r_{\nu i_2}[n,k] \triangleq \mathbb{E}\{\nu_i[n]\nu_{i_2}[n-k]\} \quad \forall 1 \leq i, i_2 \leq S = D \]

\[ r_{xi_2}[n,k] \triangleq \mathbb{E}\{x_i[n]\nu_{i_2}[n-k]\} \quad \forall 1 \leq i, i_2 \leq S = D \]

\[ r_{xi}[n,k] \triangleq \mathbb{E}\{s_i[n]\nu_{i}[n-k]\} \quad \forall 1 \leq i, k \leq S \]

By using these definitions we are able to define a Noise-Free Region of Support (NF-ROS), or shortly \( \Omega \).

**Definition III.2.** The Noise-Free Region Of Support (NF-ROS), also denoted by \( \Omega \), is a set of time lag-pairs \((n,k)\) for which the noise correlation, source-noise crosscorrelation, and source crosscorrelation functions equal zero. The total number of time-lag pairs in the NF-ROS is denoted by \( N \), thus: \( \Omega \triangleq \{\Omega_1, \ldots, \Omega_N\} \) and \( |\Omega| = N \).

We assume from now on that we only take time-lag pairs from the NF-ROS. This implies a noise-free relation between the sensor correlations and the source autocorrelations, i.e.,

\[ r_{i_1i_2}^\nu[\Omega] = \sum_{j=1}^{S} a^j_{i_1} a^j_{i_2} r_{jj}[\Omega] \quad \forall 1 \leq i_1, i_2 \leq D \tag{3} \]

An additional assumption is that all source autocorrelation functions are linearly independent in the NF-ROS. Finally, we assume that we have the same amount of time-lag pairs in the NF-ROS as we have sources, \( N = S \).

Given these assumptions we collect all sensor correlation data in the following correlation matrices, for \( i \in \{1, \ldots, D\} \):

\[ C^\nu_i \triangleq \begin{bmatrix} r_{11}[\Omega] & r_{12}[\Omega] & \cdots & r_{1D}[\Omega] \\ r_{21}[\Omega] & r_{22}[\Omega] & \cdots & r_{2D}[\Omega] \\ \vdots & \vdots & \ddots & \vdots \\ r_{D1}[\Omega] & r_{D2}[\Omega] & \cdots & r_{DD}[\Omega] \end{bmatrix} \tag{4} \]

Using (3), the structure of these matrices is described in terms of the mixing system and a source autocorrelation matrix

\[ C^s_i \equiv A \text{ diag}(a_i) C^s \quad \forall i \in \{1, \ldots, D\} \tag{5} \]

where the source autocorrelation matrix is defined by

\[ C^s \triangleq \begin{bmatrix} r_{11}^s[\Omega] & r_{12}^s[\Omega] & \cdots & r_{1D}^s[\Omega] \\ r_{21}^s[\Omega] & r_{22}^s[\Omega] & \cdots & r_{2D}^s[\Omega] \\ \vdots & \vdots & \ddots & \vdots \\ r_{D1}^s[\Omega] & r_{D2}^s[\Omega] & \cdots & r_{DD}^s[\Omega] \end{bmatrix} \tag{6} \]

and \( a_i \triangleq [a^1_i, \cdots, a^S_i] \) is the \( i \)’th row vector of the mixing matrix. This source autocorrelation matrix has full rank because we assumed that the source autocorrelation functions are linearly independent in the NF-ROS. Later in this work we use the following linear combinations of correlation matrices:

\[ \Gamma_l \triangleq \sum_{i=1}^{D} \xi_i^l C^\nu_i \equiv A \text{ diag}(\alpha_i) C^s \tag{7} \]

where \( \xi^l \triangleq ([\xi_1^l, \cdots, \xi_D^l])^T \) and \( \alpha_i^l \triangleq \langle \xi_i^l, a_i \rangle \). Here \( \langle \cdot, \cdot \rangle \) is the standard Euclidean inner product. A vector \( \xi^l \) can be any arbitrarily chosen vector from a set of \( L \) unequal vectors \( \xi^l, \ldots, \xi^L \). These vectors are used further on to incorporate the mold such that the desired extraction filter is selected.

### IV. Performing BSE

In [4] it is shown how to identify a random extraction filter from the GEVD of linear combinations of correlation matrices as in (7). Here we generalize these results and introduce an algorithm that directly identifies the desired extraction filter.
\textbf{Definition IV.1.} The GEVD of two square, equal size, full rank matrices $\Gamma_1$ and $\Gamma_2$ of size $S \times S$ is denoted by

$$\{w, \lambda\} = \text{gevd}(\Gamma_1, \Gamma_2)$$

where $\{w, \lambda\}$ is the set of all eigenvectors and eigenvalues that solve the system $\lambda w \Gamma_1 = w \Gamma_2$.

\textbf{Theorem IV.1.} Suppose that we have two random, full rank linear combinations of correlation matrices $\Gamma_1$ and $\Gamma_2$ as in (7). Then the extraction filters for all source signals are the eigenvectors of $\text{gevd}(\Gamma_1, \Gamma_2)$.

Sketch of proof: In (7) we give the structure of linear combinations of correlation matrices. As long as $\alpha_i^2 \neq 0$, these matrices remain square, full rank matrices. If we substitute (7) into Definition IV.1, it follows that the $S$ eigenvectors are exactly the $S$ row vectors from the inverse of the mixing system, up to an unknown scaling. Therefore we can use the symbol $w$ for eigenvectors as well as extraction filter. The unknown scaling is an inevitable problem of BSP, which we deal with by normalizing the eigenvectors; furthermore, no specific ordering is chosen in the set of eigenvectors and eigenvalues, which is related to the permutation problem.

We observe that using different linear combinations of correlation matrices leads to the same eigenvectors for each GEVD, as long as both linear combinations are different. On the other hand, the eigenvalues have the form

$$\lambda_{i1,i2}^l = \frac{\alpha_{i1}}{\alpha_{i2}} \langle \xi^l_{i1}, \mathbf{a}_i \rangle \langle \xi^l_{i2}, \mathbf{a}_i \rangle$$

which depends on the choice of linear combinations $\xi^l_{i1}$ and $\xi^l_{i2}$. A physical interpretation of these eigenvalues is that we project all mixing vectors $\mathbf{a}_1, \cdots, \mathbf{a}_S$ onto the two dimensional plane that is spanned by the vectors $\xi^l_{i1}$ and $\xi^l_{i2}$. The eigenvalues are related to the angles between the projected mixing vectors and the vectors $\xi^l_{i1}$ and $\xi^l_{i2}$, respectively. Furthermore it follows that each eigenvector and eigenvalue pair is depending only on one source. The mixing information $\mathbf{a}_i$ in the eigenvalue $\lambda_{i1,i2}^l$ belongs to the signal $s_i[n]$ that is extracted by the corresponding eigenvector, without knowing the label $i$. This property leads us to our new BSE approach. Given the mold and two vectors $\xi^1$ and $\xi^2$ we are able to calculate an estimation of the generalized eigenvalue that belongs to the desired source, which is extracted by its corresponding eigenvector.

Two problems arise with this basic approach. First, if more than two sensors are used, the projection of the mixing column vectors on the two dimensional subspace reduces the information in the eigenvalues. This could lead to the selection of an undesired signal. Second, by choosing the vectors $\xi^1$ and $\xi^2$ randomly, the estimated eigenvalue can take any value. In practical algorithms it is more common to search for a typical eigenvalue, such as a zero, the largest or the smallest eigenvalue. These problems lead to the following generalization such that the desired signal is selected directly and that we can use an efficient algorithm such as the power method [5] to search for the eigenvalue that corresponds to the smallest eigenvalue.

\textbf{Theorem IV.2.} We denote the mold by the vector $\mathbf{a}^0$. If we choose the set of $S$ vectors $\xi^1, \cdots, \xi^S$ as an orthonormal basis, with $\xi^1 = \mathbf{a}^0 / ||\mathbf{a}^0||$ and $||\cdot||$ the Euclidean norm, then

$$m_i \triangleq \sqrt{(\lambda_{D1}^2)^2 + \cdots + (\lambda_{D1}^2)^2} \quad \forall \ i \in \{1, \cdots, S\}$$

is minimal for $i = d$ if

$$\frac{||\mathbf{a}^0, \mathbf{a}^0||}{||\mathbf{a}||} > \frac{||\mathbf{a}^0, \mathbf{a}^i||}{||\mathbf{a}^i||} \quad \forall \ i \neq d \in \{1, \cdots, S\}$$

Notice that $m_i$ collects the generalized eigenvalues for several GEVD problems that correspond to source $i$.

Theorem IV.2 means that $m_i$ has the smallest value for the mixing column with the smallest angle towards the mold.

\textbf{Proof of Theorem IV.2:} We decompose the measure $m_i$ in terms of the basis vectors $\xi^1, \cdots, \xi^S$ by using (9)

$$m_i = \sqrt{\sum_{l=2}^S \left(\langle \xi^l, \mathbf{a}^i \rangle / ||\mathbf{a}^i||\right)^2} = \frac{1}{||\mathbf{a}^i||} \sqrt{\sum_{l=2}^S (\alpha_i^l)^2}$$

where $\alpha_i^l \triangleq \langle \xi^l, \mathbf{a}^i \rangle$. We may assume that all mixing vectors are normalized because these vectors appear in the nominator as well as in the denominator. From (11) it follows that $\alpha_i^1$ has the largest value for the desired mixing vector $\mathbf{a}^d$. Because the vectors $\xi^1, \cdots, \xi^S$ are orthonormal it holds that

$$\sqrt{\sum_{l=2}^S (\alpha_i^l)^2} = \sqrt{||\mathbf{a}^i|| - (\alpha_i^1)^2} \quad \forall \ i \in \{1, \cdots, S\}$$

Equation (13) has always the smallest value for the mixing vector that has the smallest angle with respect to the mold, thus for $i = d$. Next, combining (12) and (13) leads to

$$m_i = \frac{\sqrt{1 - (\alpha_i^1)^2}}{||\mathbf{a}^i||} \quad \forall \ i \in \{1, \cdots, S\}$$

which has the smallest value for $i = d$.

From Theorem IV.2 it follows that we are able identify the desired extraction filter as the eigenvector that corresponds to the smallest value of $m_i$ if (11) holds. The measure $m_i$ can be calculated from the solutions of $S - 1$ GEVD problems. However, this is computationally expensive. Therefore we give a new, less expensive algorithm.

Notice that $\text{gevd}(\Gamma_1, \Gamma_1)$ results in the same eigenvector and eigenvalue pairs as the eigenvalue decomposition $\text{eig}(\Gamma_1(\Gamma_1)^{-1})$ that solves the system $\lambda w \mathbf{I} = w \Gamma_1(\Gamma_1)^{-1}$, if $\Gamma_1$ is invertible [5].

By squaring this matrix: $\Gamma_1(\Gamma_1)^{-1} \Gamma_1(\Gamma_1)^{-1}$, the eigenvalues are squared while the eigenvectors remain the same. Therefore, the following matrix $\mathbf{M}$ has $S$ eigenvalues that are the squared values of $m_i$:

$$\mathbf{M} \triangleq \sum_{l=2}^S \{\Gamma_1(\Gamma_1)^{-1} \Gamma_1(\Gamma_1)^{-1}\}$$
From Theorem IV.2 it follows that we have to select the eigenvector that corresponds to the smallest eigenvalue of $M$.

The method is summarized in the following algorithm.

1) Calculate or estimate the sensor correlation matrices $C^i_d$ for $1 \leq i \leq D$, in the NF-ROS.

2) Find a set of orthonormal basis vectors $\xi^1, \cdots, \xi^2$, where $\xi^i = a^0 / ||a^0||$, where $a^0$ is the mold.

3) Calculate $S$ linear combinations $\Gamma_i$ of the correlation matrices as is defined in (7).

4) Combine the matrices $\Gamma_i$ as in (15) such that the eigenvalues of $M$ are $(m^i)^2$ and the eigenvectors are the extraction filters.

5) Use an efficient algorithm to find the eigenvector that corresponds to the smallest eigenvalue of $M$.

V. SIMULATION RESULTS AND DISCUSSION

In [4] the use of generalized eigenvectors to extract sources is introduced. The main focus of the current work is to select the desired source based on the mold; therefore, we validate the selection procedure by evaluating the eigenvalues of the matrix $M$.

In our simulations we use a BSE scenario where two sensors measured 50000 samples of two stationary sources. The sources have unit variance and an autoregressive temporal structure with a pole at $z = 0.5$ and $z = 0.9$ for the desired and undesired sources respectively. Furthermore, the sensor noise was spatially and temporally white with variances of 0.1. All signals were created with a Gaussian distribution. Based on the properties of the stationary signals the NF-ROS was chosen as the lags $k = 1$ and $k = 2$.

The mixing system was constructed as follows. The mixing columns are parameterized by a Direction Of Arrival (DOA) parameter $\theta^i = \arctan(a^0_2/a^0_1)$. The mixing column elements are found as: $a^1 = \cos(\theta^i)$ and $a^2 = \sin(\theta^i)$. The real DOA was +30 degrees, while the guess for the mold uses a DOA of 0 degrees, thus $a^0 = (\begin{bmatrix} 1 & 0 \end{bmatrix})^T$. The DOA of the undesired source increased linearly from -90 degrees to +90 degrees.

The algorithm from Section IV was used and the eigenvalues $(m^i)^2$ of the matrix $M$ were transformed with the monotonically increasing function $\phi^i = \arctan(m^i)$; selecting the smallest value of $(m^i)^2$ still corresponds to selecting the smallest value of $\phi^i$. These transformed eigenvalues $\phi^i$ are depicted in Figure 2. In the upper graph the basis vectors $\xi^1$ and $\xi^2$ were chosen orthonormal with respect to each other and $\xi^2$ was equal to the mold. We know from constructing the simulations that the horizontal line of eigenvalues belongs to the desired source at 30 degrees, while the ‘V’-shaped eigenvalues correspond to the undesired source. We observe that if the DOA of the undesired source lies in between -30 and +30 degrees, then the undesired source is extracted. This corresponds with our analysis and implies that the source with the DOA closest to the mold is extracted by selecting the smallest eigenvalue.

In the lower graph of Figure 2 we made an extension to our work. We chose $\xi^1$ with a DOA of 60 degrees and $\xi^2$ orthogonal to the mold, with a DOA of 90 degrees. Now the undesired source is extracted when it’s DOA is in between -10 and +30 degrees. Furthermore, it follows that the algorithm prefers a source with a DOA equal to the mold; therefore, we conclude that the source selection is not symmetrical anymore with respect to the mold. If the desired source is not located at the DOA of the mold, then in this case a positive DOA is preferred over a negative DOA. This means that besides an available guess of the DOA we are also able to incorporate additional global DOA information via the weighting parameters $\xi^i$. 

VI. CONCLUSIONS AND FUTURE RESEARCH

We introduced a new blind extraction algorithm that directly selects and extracts the desired signal from a mixture of signals. If we have available a rough guess of the mixing parameters of this desired signal, then based on the GEVD of noise-free correlation matrices we have shown that the desired signal is selected directly. We validated our method with simulations and showed that additional a priori information can be used to obtain a more flexible selection procedure.

Future research topics are to investigate if extra sensors can be used for noise reduction. Furthermore, we will develop a BSE algorithm for convolutive mixtures based on a similar approach where we use a rough guess of the direction of arrival of the desired source as a priori information.

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