Low-complexity model predictive control of electromagnetic actuators

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LOW-COMPLEXITY MODEL PREDICTIVE CONTROL OF ELECTROMAGNETIC ACTUATORS

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Abstract: Electromagnetically driven mechanical systems are characterized by fast non-linear dynamics that are subject to physical and control constraints. This paper describes a Model Predictive Controller (MPC) for a general ElectroMagnetic (EM) actuator that satisfies both the performance constraints and the strict requirements on the computation time. Novel aspects of the MPC design are a one-step-ahead prediction horizon and an infinity-norm artificial Lyapunov function that is employed to drive the system to a desired reference. An additional optimization variable is introduced to relax the conditions on the Lyapunov function, which is not forced to decrease monotonically. This feature improves feasibility considerably. The resulting MPC problem is transformed into a low-complexity linear program that can be solved by modern microprocessors within tenths of milliseconds. An even simpler piecewise affine explicit controller is obtained via multiparametric programming. Simulation results are reported and compared with the results achieved by existing state-of-the-art explicit MPC.

Index Terms: Mechatronics, predictive control for non-linear systems, stability of nonlinear systems.

I. INTRODUCTION

Over the last few years, increasing operating demands for mechatronic applications in fields as diverse as precision, power, and automotive engineering [1] have intensified the need for fast and accurate stabilizing control schemes. Regardless of their application area, mechatronic systems are characterized by strict operating requirements (low power consumption, fast transition times, accurate reference tracking etc.), severe nonlinearities, and hard input and state constraints that need to be enforced. In addition, these operating requirements must be met robustly, considering component variability due to part-to-part differences and aging.

Due to these characteristics, the controller design task is challenging. Traditional methods such as Proportional-Integral-Derivative (PID) or Linear-Quadratic Regulator (LQR) control cannot explicitly enforce hard constraints. This is in fact one of the main reasons why Model Predictive Control (MPC) has become successful [2]–[4]. In MPC, the actual control action is computed by solving a finite horizon open-loop optimization problem at each control sample instant, using the measured current state as starting condition, while satisfying input and state constraints.

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Fig. 1. A magnetically actuated mass-spring-damper system.

Although until recently only “slow” systems found in chemical or process industry permitted implementation of MPC, the field of application of MPC is growing with advances in computing power. Recently, an explicit MPC approach was used to tackle the electromagnetic actuator control problem in [5], with promising results. However, some improvements are still needed to make it suitable for practical implementation, both regarding the complexity of the control law and closed-loop stability. Existing MPC schemes with a stability guarantee [3] are either too complex or too conservative for real-time implementation. This paper proposes a new low-complexity nonlinear MPC strategy that is more suitable for real-time control. This is achieved by using a one-step-ahead prediction horizon in the MPC optimization problem and relaxed stabilization constraints formulated using infinity-norm based Lyapunov functions. This particular setup yields a single Linear Program (LP) to be solved on-line, despite the fact that a full nonlinear model is still used for predictions. Compared to [5], which requires a PieceWise Affine (PWA) approximation of the nonlinear actuator model, this ultimately results in improved performance and lower complexity. Even tighter timing and control hardware requirements can be handled by making use of an explicit version of the developed MPC controller [6].

This article is organized as follows. Section II describes the electromagnetic actuator to be controlled. An existing MPC approach for this actuator is given in Section III. Section IV presents the novel MPC scheme, including stability results and implementation issues. After that, simulation results are discussed in Section V and conclusions are summarized in Section VI.

II. MODEL AND PHYSICAL LIMITS

The system to be controlled, shown in Fig. 1, is a typical magnetically actuated Mass-Spring-Damper System (MSDS) that is common in mechatronic applications. The mechanical MSDS subsystem can be modeled by the well-known second-order linear dif-

ferential equation

$$\ddot{x} = \frac{F}{m} - \frac{c}{m}\dot{x} - \frac{k}{m}x, \quad (1)$$

with mass m [kg], position x [m], damping coefficient c [N·s·m⁻¹] and spring constant k [N·m⁻¹]. The mechanical subsystem is coupled to a nonlinear electromagnetic driving circuit via the magnetic force F [N], induced by applying an input voltage V [V] to the terminals of a coil with resistance R [Ω]. Controlling the nonlinear EM-actuator dynamics is complex and might therefore be inappropriate for implementation in standard microcontrollers. The complexity is decreased by adopting an inner-outer control loop strategy, in which the electromagnetic subsystem is driven by a feedback-linearizing control law [5]. In this way, the inner-loop closed-loop dynamics can be made much faster than the dynamics of the mechanical subsystem, so that the predictive outer-loop controller can be designed based on a time-discretization of the mechanical subsystem (1), with the magnetic force F as the control input.

Like most mechatronic actuator applications, the controlled system is subject to several constraints originating from physical limits or performance requirements. The mass position should not exceed the physical limits

$$-d \leq x \leq d. \quad (2)$$

For safety and performance reasons, a soft-landing constraint is imposed on the mass velocity with respect to its position,

$$-\varepsilon - \beta(d - x) \leq \dot{x} \leq \varepsilon + \beta(d - x), \quad (3)$$

where ε and β are chosen such that the constraint is essentially inactive for $x = 0$ mm, while the constraint is tight for $x = d$. Furthermore, the magnetic force is only able to attract the mass, and the outer-loop controller needs to include a saturation constraint on the control input F that is a direct effect of the maximum coil current $i_{\max}$ [A] allowed:

$$0 \leq F \leq \frac{k_a i_{\max}^2}{(d + k_b - x)^2}, \quad (4)$$

with constants k_a , k_b [m] originating from the EM architecture. Note that this constraint is nonlinear and non-convex in the state variable x .

After defining the dynamics and the limitations of the actuator, the following sections will focus on controller design.

III. EXISTING OUTER-LOOP MPC APPROACH

Although the model (obtained by time-discretization of (1)) used by the outer-loop MPC controller is linear, the controller needs to find an optimizing input u from a non-convex feasible input set, defined by linear and nonlinear constraints (2)–(4). The nonlinear constraint poses a computational problem: including the saturation

bound in the MPC controller requires solving a nonlinear optimization problem each sampling instant. Nonlinear optimization solvers are not guaranteed to find the global optimum with respect to the criterion to be minimized, and they are slow compared to Linear or Quadratic Program (LP/QP) solvers.

In [5], an outer-loop MPC controller is proposed with a prediction horizon of 3 sampling instants and a quadratic minimization criterion. The state and input restrictions are also enforced within a constraint horizon of 3 instants. The complexity problem is tackled by formulating the MPC optimization as a Mixed-Integer Quadratic Program (MIQP), in which the nonlinear saturation constraint (4) is lower bounded by a PWA approximation. In general, this approximated constraint can result in loss of performance. Furthermore, the controller developed in [5] does not include stabilization as part of its design. Although stability can be checked a posteriori for the explicit form of the controller, if the check fails it is not clear how to modify the control scheme such that stability is guaranteed.

As such, in the remainder of this paper we propose an MPC algorithm that offers low computational complexity, and explicitly includes stabilization constraints as part of its design. These desirable properties are attained for discrete-time nonlinear systems that are affine in the control input, hence, without using a PWA approximation of nonlinear terms, which can degrade performance.

IV. LOW-COMPLEXITY NONLINEAR MPC

Before the new outer-loop MPC controller is described, it is necessary to recall some preliminary notions concerning Lyapunov stability and Lyapunov functions, which will be instrumental in the MPC setup.

A. Preliminaries

1) *Basic notions:* Let $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{Z}$, $\mathbb{Z}_+$ denote the set of real numbers, the set of non-negative reals, the set of integers and the set of non-negative integers, respectively. Also recall that a polyhedron (or polyhedral set) in $\mathbb{R}^n$ is a convex set obtained as the intersection of a finite number of open and/or closed half-spaces. For a set $S \subseteq \mathbb{R}^n$, let $\text{int}(S)$ denote the interior of S .

The Hölder p -norm of a vector $\xi \in \mathbb{R}^n$ is defined as

$$\|\xi\|_p := \begin{cases} (|\xi|_1^p + \dots + |\xi|_n^p)^{\frac{1}{p}}, & p \in \mathbb{Z}_{[1,\infty)} \\ \max_{i=1,\dots,n} |\xi|_i, & p = \infty, \end{cases}$$

where $[\xi]_i$, $i = 1, \dots, n$, is the i -th component of ξ and $|\cdot|$ is the absolute value. For brevity, let $\|\cdot\|$ denote an arbitrary p -norm. For a matrix $Z \in \mathbb{R}^{m \times n}$ let $\|Z\| := \sup_{\xi \neq 0} \frac{\|Z\xi\|}{\|\xi\|}$ denote its corresponding induced matrix norm. Let $\mathbf{z} := \{z[l]\}_{l \in \mathbb{Z}_+}$ with $z[l] \in \mathbb{R}$ for all $l \in \mathbb{Z}_+$ denote an arbitrary sequence and

define $\|z\| := \sup\{\|z[l]\| \mid l \in \mathbb{Z}_+\}$. Let $z_{[m,n]}$ with $m \leq n$ and $m, n \in \mathbb{Z}_+$ denote the truncated sequence $\{z[l]\}_{l=m, m+1, \dots, n}$.

A function $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a class $\mathcal{K}$ function if it is continuous, strictly increasing and $\varphi(0) = 0$. A function $\varphi \in \mathcal{K}$ is a class $\mathcal{K}_\infty$ function if it is radially unbounded (i.e. $\lim_{s \rightarrow \infty} \varphi(s) = \infty$). A function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ belongs to class $\mathcal{KL}$ if for each fixed $k \in \mathbb{R}_+$, $\beta(\cdot, k) \in \mathcal{K}$ and for each fixed $s \in \mathbb{R}_+$, $\beta(s, \cdot)$ is non-increasing and $\lim_{k \rightarrow \infty} \beta(s, k) = 0$.

2) *Lyapunov asymptotic stability*: Consider the discrete-time, autonomous nonlinear system described by the difference inclusion

$$\xi[k+1] \in \Phi(\xi[k]), \quad k \in \mathbb{Z}_+, \quad (5)$$

where $\xi[k] \in \mathbb{R}^n$ is the state at the discrete-time instant k . The function $\Phi : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is an arbitrary set-valued mapping. The origin is assumed to be an equilibrium for (5), implying that $\Phi(0) = \{0\}$.

Definition IV.1 A set $\mathcal{P} \subseteq \mathbb{R}^n$ is *Positively Invariant (PI)* for system (5) if $\forall \xi \in \mathcal{P}$ it holds that $\Phi(\xi) \subseteq \mathcal{P}$.

Definition IV.2 Let $\mathbb{X}$ with $0 \in \text{int}(\mathbb{X})$ be a subset of $\mathbb{R}^n$. System (5) is *asymptotically stable in the Lyapunov sense in $\mathbb{X}$* if there is a $\mathcal{KL}$ -function $\beta(\cdot, \cdot)$ such that, for all $\xi[0] \in \mathbb{X}$ it holds that all corresponding state trajectories of (5) satisfy

$$\|\xi[k]\| \leq \beta(\|\xi[0]\|, k), \quad \forall k \in \mathbb{Z}_+. \quad (6)$$

The next theorem lists conditions that are sufficient for regional asymptotic Lyapunov stability.

Theorem IV.3 Let $\mathbb{X}$ be a PI set for system (5) and let $0 \in \text{int}(\mathbb{X})$. Furthermore, let $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$ and let $V : \mathbb{R}^n \rightarrow \mathbb{R}_+$ be a function such that

$$\alpha_1(\|\xi\|) \leq V(\xi) \leq \alpha_2(\|\xi\|), \quad (7a)$$

$$V(\xi^+) - V(\xi) \leq -\alpha_3(\|\xi\|), \quad (7b)$$

for all $\xi \in \mathbb{X}$ and all $\xi^+ \in \Phi(\xi)$. Then system (5) is *asymptotically Lyapunov stable in $\mathbb{X}$* .

Theorem IV.3 can be proven analogously to the method given in [7], replacing the difference equation by the difference inclusion in (5). A function $V(\cdot)$ that satisfies the conditions of Theorem IV.3 is called a *Lyapunov function*.

Next, consider the discrete-time constrained nonlinear system described by the difference equation

$$\xi[k+1] = \phi(\xi[k], u[k]), \quad \forall k \in \mathbb{Z}_+, \quad (8)$$

where $\xi[k] \in \mathbb{X} \subseteq \mathbb{R}^n$ is the state and $u[k] \in \mathbb{U} \subseteq \mathbb{R}^m$ is the control input at the discrete-time instant k . The function $\phi : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ is arbitrary nonlinear with $\phi(0, 0) = 0$ and we assume that $\mathbb{X}$ and $\mathbb{U}$ are bounded sets with $0 \in \text{int}(\mathbb{X})$ and $0 \in \text{int}(\mathbb{U})$. Let $\alpha_1, \alpha_2, \alpha_3 \in \mathcal{K}_\infty$ and let $\tilde{\mathbb{X}}$ be a subset of $\mathbb{X}$ with the origin in its interior.

Definition IV.4 A function $V : \mathbb{R}^n \rightarrow \mathbb{R}_+$ that satisfies (7a) for all $\xi \in \mathbb{R}^n$ and for which there exists a control law $u : \mathbb{R}^n \rightarrow \mathbb{U}$ such that

$$V(\phi(\xi, u(\xi))) - V(\xi) \leq -\alpha_3(\|\xi\|), \quad \forall \xi \in \tilde{\mathbb{X}}, \quad (9)$$

is called a *Control Lyapunov Function (CLF)* in $\tilde{\mathbb{X}}$ for system (8).

Theorem IV.3 and Definition IV.4 will be used throughout the rest of the paper to assess the stability of the resulting closed-loop system.

B. Control and Stabilization via Optimization

Given the above notions, the new outer-loop MPC scheme can be formally defined. The main idea is to use a one-step-ahead prediction scheme to reduce the overall controller complexity, while closed-loop stability is achieved in a non-conservative way by relaxing the conditions on the CLF. By using stabilizing constraints independent from the MPC cost, this approach decouples performance from stability and no longer requires the global optimum to be attained at each sampling instant, as typically required in MPC [3].

Consider the relaxed CLF constraint for system (8)

$$V(\phi(\xi[k], u[k])) - V(\xi[k]) + \alpha_3(\|\xi[k]\|) \leq \tau[k], \quad (10)$$

where $\tau[k] \in \mathbb{R}_+$ is an additional manipulated variable that improves feasibility. Next, we define the following optimization problem:

Problem IV.5 Let $J(\cdot)$ denote a cost function, such that $J(\tau) \rightarrow \infty$ for $\tau \rightarrow \infty$ and $J(\tau) \rightarrow 0$ for $\tau \rightarrow 0$. Furthermore, let $\mathbb{X}$ and $\mathbb{U}$ be the sets defined by the state and input constraints, respectively, which contain the origin in their interiors, let $V(\cdot)$ be a CLF in $\tilde{\mathbb{X}} \subseteq \mathbb{X}$ for system (8) and let $\alpha_3 \in \mathcal{K}_\infty$ be given. At time $k \in \mathbb{Z}_+$, measure the state $\xi[k]$ and minimize the cost $J(\cdot)$ over $u[k]$ and $\tau[k]$, subject to the constraints

$$u[k] \in \mathbb{U}, \quad \phi(\xi[k], u[k]) \in \mathbb{X}, \quad \tau[k] \geq 0, \quad (11a)$$

$$V(\phi(\xi[k], u[k])) - V(\xi[k]) + \alpha_3(\|\xi[k]\|) \leq \tau[k]. \quad (11b)$$

Let $\Pi(\xi[k]) := \{u[k] \in \mathbb{R}^m \mid \exists \tau[k] \text{ s.t. (11) holds}\}$ and let the corresponding closed-loop map be $\phi_{CL}(\xi[k], \Pi(\xi[k])) := \{\phi(\xi[k], u[k]) \mid u[k] \in \Pi(\xi[k])\}$.

Theorem IV.6 Let $\tau^*[k]$ denote the optimal value of $\tau[k]$ with respect to Problem IV.5. Suppose that Problem IV.5 is feasible for all $\xi[k] \in \mathbb{X}$ and assume that $\tau^*[k] \rightarrow 0$ as $k \rightarrow \infty$. Then the closed-loop system

$$\xi[k+1] \in \phi_{CL}(\xi[k], \Pi(\xi[k])), \quad k \in \mathbb{Z}_+, \quad (12)$$

is *asymptotically Lyapunov stable in $\mathbb{X}$* .

Proof: The proof is quite straightforward, following the steps indicated in for instance [7, Ch. 2] and [8]. Due to constraint (11a), Problem IV.5 remains

feasible for $\xi[k+1]$. Then, exploiting recursively (11b) and the upper bound in (7a) yields

$$V(\xi[k+1]) \leq \eta^{k+1} \alpha_2(\|\xi[0]\|) + \sigma(\|\tau_{[0,k]}^*\|),$$

for some $\sigma(\cdot) \in \mathcal{K}$ and $\eta \in \mathbb{R}_{[0,1)}$. Using the lower bound (7a) and recalling that $\alpha_1^{-1} \in \mathcal{K}_\infty$ gives

$$\|\xi[k]\| \leq \beta(\|\xi[0]\|, k) + \gamma(\|\tau_{[0,k-1]}^*\|), \quad (13)$$

for some functions $\beta(\cdot) \in \mathcal{KL}$ and $\gamma(\cdot) \in \mathcal{K}$. By the hypothesis the optimal solution of Problem IV.5 (in terms of the optimization variable τ) will satisfy $\tau^*[k] \rightarrow 0$ for $k \rightarrow \infty$. Together with (13) this implies that the closed-loop system is “converging-input converging-state” (with $\tau[k]$ as input) [8], and hence, asymptotic Lyapunov stability can be concluded (see [7] and [8] for further details). ■

Remark IV.7 Asymptotic convergence of $\tau[k]$ to 0 can be enforced by augmenting Problem IV.5 with the constraint $\tau[k] \leq \rho \tau^*[k-1]$, for some $\rho \in \mathbb{R}_{[0,1)}$. However, requiring monotonic decrease of τ can be too conservative. As such, it is of interest to develop other, less conservative solutions for achieving $\tau^* \rightarrow 0$ as $k \rightarrow \infty$. This will be the aim of further research.

The next section describes how Problem IV.5 can be implemented for the nonlinearly constrained MSDS, defined by (1)–(4), as a single linear program.

C. Implementation as a Linear Program

First, we define the prediction model. The outer-loop controller uses a zero-order-hold time-discretized version of the MSDS model, with sampling period $T_s = 0.5$ ms. This gives the state-space representation

$$\xi[k+1] = A_d \xi[k] + B_d u[k], \quad (14)$$

with state $\xi[k] = \begin{bmatrix} x[k] \\ \dot{x}[k] \end{bmatrix}$ and $u[k] = F[k]$, where $x[k]$ is the position and $\dot{x}[k]$ is the velocity. Additionally, let $u_r[k]$ and $\xi_r[k] = \begin{bmatrix} r_x[k] \\ r_{\dot{x}}[k] \end{bmatrix}$ be the corresponding input and state reference values respectively. The system's output is defined by $y[k] = C\xi[k]$, where $y = \begin{bmatrix} x + \beta x \\ \dot{x} - \beta x \end{bmatrix}$. Next, consider the cost function to be minimized by the MPC controller

$$J_{\text{MPC}}(\xi, u, \tau) := \|Q_0(\xi - \xi_r)\|_\infty + \|Q_1(A_d \xi + B_d u - \xi_r)\|_\infty + \|R_u(u - u_r)\|_\infty + J(\tau), \quad (15)$$

where the dependency of time for all variables is removed, for brevity. The cost on the optimization variable τ is defined as $J(\tau) := \|\Gamma \tau\|_\infty$. Here, Q_0, Q_1, R_u and Γ are known full-column rank matrices of appropriate dimensions. Notice that the cost $J(\cdot)$ is chosen as required in Problem IV.5.

A single sample prediction scheme is not only beneficial for decreasing the controller complexity, but also because all constraints depending nonlinearly on the measured state appear now linearly with respect

to the variables in the optimization problem. The one-step-ahead saturation constraint (4) is linear in u for instance, since the right-hand-side is just a constant determined by the current, known position $x[k]$, i.e.

$$0 \leq u[k] \leq \frac{k_d i_{\text{max}}^2}{(d + k_b - x[k])^2}. \quad (16)$$

The other performance or control constraints are linear in u and specified as

$$y_{\min} \leq C(A_d \xi + B_d u) \leq y_{\max}, \quad (17)$$

with $y_{\min} = \begin{bmatrix} -d \\ -\infty \\ -\varepsilon - \beta d \end{bmatrix}$ and $y_{\max} = \begin{bmatrix} d \\ \varepsilon + \beta d \end{bmatrix}$.

Only the stabilizing constraints remain. Therefore, consider the following infinity-norm based CLF

$$V(\xi) = \|P\xi\|_\infty, \quad (18)$$

where $P \in \mathbb{R}^{p \times n}$ is a full column-rank matrix to be determined. Note that this function satisfies all Lyapunov requirements (7a), with $\alpha_1(s) = \frac{\sigma}{\sqrt{p}}s$ (σ is the smallest singular value of P) and with $\alpha_2(s) = \|P\|_\infty s$. Substituting (14) and (18) in (10) yields

$$\begin{aligned} \|P(A_d \xi + B_d u - \xi_r)\|_\infty - \|P(\xi - \xi_r)\|_\infty \\ \leq -\alpha_3 \|\xi - \xi_r\|_\infty + \tau. \end{aligned} \quad (19)$$

Although the cost function and the CLF constraints appear to be nonlinear in the optimization variables, the corresponding optimization problem can be recast as a linear program via a particular set of linear inequalities, without introducing conservatism, as follows. Since $\|\xi\|_\infty = \max_{j \in \{1, 2, \dots, n\}} |\xi_j|$ by definition, for $\|\xi\|_\infty \leq c$ to be satisfied, it is necessary and sufficient to require that $\pm \xi_j \leq c$ for all $j \in \{1, 2, \dots, n\}$. So, for (19) to be satisfied it is necessary and sufficient to require that

$$\begin{aligned} \pm [P(A_d \xi + B_d u - \xi_r)]_j - \|P(\xi - \xi_r)\|_\infty \\ \leq -\alpha_3 \|\xi - \xi_r\|_\infty + \tau, \end{aligned} \quad (20)$$

for $j = 1, 2$. This yields a total of $2p$ linear inequalities in the optimization variables u and τ . Furthermore, instead of optimizing over the cost (15), auxiliary optimization variables $\varepsilon_1, \dots, \varepsilon_3$ can be introduced to rewrite the MPC optimization problem as

$$\min \sum_{i=1}^3 \varepsilon_i \quad (21)$$

subject to (16), (17), (20) and

$$\begin{aligned} \pm [Q_1(A_d \xi + B_d u - \xi_r)]_j + \|Q_0(\xi - \xi_r)\|_\infty &\leq \varepsilon_1 \\ \pm R_u(u - u_r) &\leq \varepsilon_2 \\ \pm \Gamma \tau &\leq \varepsilon_3, \end{aligned}$$

for $j = 1, 2$.

The MPC algorithm can now be summarized as

Algorithm IV.8 At each sampling instant k :

Step 1: Measure or estimate the state $\xi_0 = \xi[k]$;

Step 2: Solve LP (21) and pick any feasible control action $\bar{u}_0$;

Step 3: Set $F[k] = \bar{u}_0$ as inner-control-loop reference.

D. Explicit form of the proposed MPC scheme

Although the optimization problem solved by the novel predictive controller is a simple linear program, implementation might still be hampered if the time required to find a feasible input exceeds the sampling period or if the memory available for the execution code is too limited to contain an efficient LP solver. However, [6] shows that the solution to an LP can be obtained as a function of parameters θ that appear linearly in the program, via multiparametric Linear Programming (mp-LP). The optimal solution is calculated off-line using the MultiParametric Toolbox (MPT) [9] and has the form of a piecewise affine feedback law $u(\theta)$. The standard choice of $\theta = \begin{bmatrix} \xi_0 \\ \xi_r \end{bmatrix}$, cannot be applied here however, because the saturation bound (16) and stabilizing constraints (20) are not linear in θ . Defining the parameter vector as

$$\theta = \begin{bmatrix} \xi_0 \\ \xi_r \\ -\alpha_3 \|\xi_0 - \xi_r\|_\infty + \|P(\xi_0 - \xi_r)\|_\infty \\ k_a \frac{i_{\max}^2}{(d+k_b-x)^2} \end{bmatrix}$$

solves this problem, so that the explicit controller can be computed.

It should be stressed that the optimal solution of the on-line MPC problem and its explicit counterpart return identical results, but there is a significant difference in the computational burden required in each sampling interval. This difference is consistent with the solution of an on-line optimization problem versus the evaluation of a set of linear equalities and the calculation of an affine state feedback term.

The explicit MPC algorithm can now be summarized as follows:

Algorithm IV.9 At each sampling instant k :

Step 1: Measure or estimate the state $\xi_0 = \xi[k]$;

Step 2: Detect in which region of the explicit-control parameter space the corresponding θ lies and calculate the optimal input u_0^* using the corresponding affine control law;

Step 3: Set $F[k] = u_0^*$ as inner-control-loop reference.

In short, the explicit MPC controller acts as a simple PWA state feedback law, without introducing conservatism, whereas it preserves the properties of the on-line optimization based MPC scheme.

V. SIMULATION RESULTS

In this section we present the performance results of the one-step-ahead predictive controller developed in this article and compare them with those of the MPC scheme of [5]. All simulations discussed in this section are obtained using the predictive controller in closed-loop with the discretized MSDS plant (14).

Fig. 2. Comparison of controller performance (top: Position trajectory tracking, bottom: Velocity trajectory tracking and soft-landing bounds).

This subsystem is shaped to resemble a second-order under-damped system with damped frequency peak at $\omega_r = 950$ [rad·s⁻¹] and 3 dB-bandwidth $BW_3 = 3 \cdot 10^3$ [rad·s⁻¹].

Fig. 2 shows the closed-loop state trajectories obtained with the one-step-ahead MPC controller (IMPC-1 for short) when tracking a certain reference profile. The weight matrices of the cost (15) used by the IMPC-1 setup are $Q_1 = Q_0 = \begin{bmatrix} 10^4 & 0 \\ 0 & 2 \end{bmatrix}$ and $R_u = 0$. The CLF optimization variable penalty was chosen as $\Gamma = 10^3$. The technique of [10] was used to compute off-line the weight $P \in \mathbb{R}^{2 \times 2}$ of the CLF $V(\xi) = \|P\xi\|_\infty$ for $\alpha_3(s) := 0.001s$ and the linear model of the MSDS in closed-loop with $u[k] := K\xi[k]$, $K \in \mathbb{R}^{1 \times 2}$, yielding

$$P = \begin{bmatrix} 7.0793 & 0.0005 \\ 2.9330 & 0.0028 \end{bmatrix}, \quad K = \begin{bmatrix} -1.1582 \\ -0.0033 \end{bmatrix}^\top \times 10^5.$$

Note that the control law $u[k] = K\xi[k]$ is only employed off-line, to calculate the weight matrix P of the CLF $V(\cdot)$, and it is never used for controlling the system. Also, although the system model is linear, $V(\xi) = \|P\xi\|_\infty$ is still only a “local” CLF, i.e. in a subset of the state-space where the auxiliary feedback law $u[k] = K\xi[k]$ satisfies the input constraints. This justifies the usage of a relaxed stabilization constraint, where the variable $\tau[k]$ is optimized at each sampling instant, as a function of the measured state $\xi[k]$.

Table I lists the cumulated squared position errors $\sum_k (x[k] - r_x[k])^2$ and the input effort $\sum_k u[k]^2$ for the quadratic-cost, 3 sample prediction controller (QMPC-3 for short) of [5] and the IMPC-1 scheme developed in this paper. Table I shows that the tracking performance of the IMPC-1 controller implementation matches the performance achieved by the existing QMPC-3 setup; recall that the performance reported in Table I is attained with a simpler LP-solving predictive controller, that does not approximate nonlinear constraints or dynamics with PWA functions.

Furthermore, Fig. 3 shows the effectiveness of the

TABLE I
REFERENCE TRACKING ERROR AND CONTROL EFFORT

MPC controller	Cumulated position error [mm ²]	Input effort [kN ²]
QMPC-3	83.1005	26.6588
IMPC-1	71.1129	27.0000

Fig. 3. Input-position trajectory and force-saturation bound.

IMPC-1 scheme in terms of enforcing the nonlinear saturation constraint (16). The MPC controller developed in this paper is able to exploit the full feasible input range, whereas the performance of the QMPC-3 scheme is restricted by a slightly conservative PWA lower bound on the saturation characteristic. Finally, the IMPC-1 setup does not violate the soft-landing constraint, represented by the dashed lines in the bottom plot of Fig. 2.

A. Explicit Controller

As stated before, the explicit controller and its corresponding on-line optimization problem deliver identical performance, although their computational complexity differs significantly. Whereas the explicit QMPC-3 controller, based on a PWA model and a mp-MIQP problem, has a polyhedral complexity of 671 regions, the explicit IMPC-1 parametric space only consists of 42 polyhedrons, which is a significant complexity reduction.

Finally, Fig. 4 shows the worst-case input calculation times for both the on-line and the explicit IMPC-1 setup, determined from multiple simulation runs using the reference of Fig. 2. The results are acquired using the GNU Linear Programming Kit (GLPK) solver in Matlab running on a 1.0 GB RAM, 3.2 GHz Pentium 4 PC. As expected, the explicit controller is on average faster than the on-line version, although both implementations are capable of calculating the optimal input in less than the sampling period of 0.5 ms. Optimized C-code implemented versions of the controllers running on dedicated computing hardware, for instance an FPGA device as proposed in [11], [12], are expected to require even less computation time.

VI. CONCLUSIONS

This article proposed a novel, low-complexity nonlinear model predictive control scheme for controlling general electromagnetic actuators. The MPC controller optimizes the behavior of the mass-spring-damper system, decoupled from the electromagnetic subsystem,

Fig. 4. Comparison of worst-case controller computation times.

subject to hard performance and control constraints, while taking into account the nonlinear constraint arising from the magnetic driving circuit. Previous MPC approaches used PWA approximations of this constraint, see e.g. [5].

By adopting a one-step-ahead prediction strategy and an infinity-norm based optimization objective, the MPC optimization problem reduces to a single linear program, which makes the developed MPC scheme particularly attractive for systems with fast dynamics that require sampling periods below 1 millisecond. Even tighter chronometric requirements can be handled by using an explicit version of proposed MPC controller. The decoupled controller has been tested in simulations, in which it performed as good as existing MPC solutions, although it is computationally simpler.

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