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A two-dimensional nonequilibrium model of cascaded arc plasma flows

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A nonequilibrium model is developed for the prediction of two-dimensional flow, electron and heavy particle temperatures, and number density distributions in cascaded arcs of monatomic gases. The system of strongly coupled elliptic partial differential equations describing plasma flow is solved by a numerical method based on a control volume with a nonstaggered numerical grid. The model is applied for the computation of both stagnation and flowing argon arc plasmas. The results show that the plasma in stagnation arcs is nearly in local thermal equilibrium (LTE), except very close to the wall, whereas fast flowing arc plasmas exhibit a significant degree of nonequilibrium, both close to the wall and in the inlet region. The results of the calculations are in satisfactory agreement with experimental data, both for the cases of stagnation and flowing argon cascaded arc plasmas.

I. INTRODUCTION

Thermal plasmas, as plasma torches and inductively coupled plasmas (ICP), have become very important for many industrial applications such as plasma spraying, cutting, welding, powder processing, high-pressure light sources, and spectrochemistry. Also, more recently, for plasma deposition thermal plasmas and cascaded arcs, in particular, have proven to offer great advantages in terms of effectiveness of the process and hence of the deposition rate.

For the latter application, a good control of the plasma properties such as ionization degree and flow velocity is crucial. Therefore it is essential to have a two-dimensional model of the plasma flow, which takes into account correctly phenomena such as high nonisothermal flow, deviations from local thermal equilibrium (LTE), as well as diffusion and heat conduction.

In accordance with the applications, extensive experimental work has been conducted to obtain insight into the extent to which nonequilibrium exists in cascaded arc thermal plasmas. Most of these investigations concern stagnant, and thus homogeneous, cylindrical plasmas. This permitted, besides side-on, end-on (axial) observation and, in particular, for argon, the small deviations from LTE in stagnant arcs, have been measured with great precision.

The electron density has been measured very accurately with interferometry; electron temperature and density obtained by the source function method yields the effect of non-LTE, expressed in an overpopulation factor of the ground state b, which is the ratio of the actual neutral ground state density and the Saha-equilibrium value. It also appears that above a critical current density and thus ionization degree in stagnant arcs, the heavy particle temperature equals the electron temperature. This is no more guaranteed in flowing cascaded arc plasmas. Here, in particular, experimental work has been done concerning the flow velocity. This experimental work has been supported by one-dimensional modeling, which gave reasonable agreement with experiments. In this work, it was felt that there exists a need to develop a concise two-dimensional model. This was particularly true with regard to a correct description of the neutral flow besides electron density, temperatures, and flow velocity, and their radial dependencies. Therefore an attempt has been made in the present work to develop a basic two-dimensional two-temperature nonequilibrium model and a prediction procedure for cascaded arc monatomic plasma flows. After this basic model is verified by experiments, it can be extended to other plasma flow situations, such as the expansion of plasmas into vacuum, complex gas mixture plasmas, strongly recombining plasma jets, and impinging jets at solid surfaces for the purpose of plasma deposition processes.

In the approach adopted in this work, the plasma is treated as a two-phase medium that consists of heavy particles (neutral atoms and ions) and light particles (electrons). For the full description of this two-phase flow one needs to solve the mass, momentum, and energy equations for both phases. However, because of the small electron mass, their moments are neglected in comparison with the heavy particle momentum, and both phases are assumed to have the same convective velocity field, except for the electron velocity associated with the divergence-free current density. So the two-phase flow character is kept only with respect to the heavy particle and electron temperatures, which are assumed to be different and described by separate energy equations. The cascaded arc plasma is created by Ohmic heating of the gas due to the current flowing between the cathode and the anode. Ohmic heat input is consumed by electrons whose temperature increases. Frequent elastic and inelastic collisions with the heavy particles result in an increasing gas temperature and a higher ionization degree. The mass of the electrons is small and if the flow velocity is large, the elastic collisions are not always efficient enough to equalize the temperatures of the two-phases, especially at inlet regions of the flow, where the cold gas is introduced and in the vicinity of cooled walls.

The model is first applied for the case of a stagnation
argon plasma at very low gas flow rates at different arc currents, thus simulating the experiments of Rosado\(^1\) and Timmermans et al.,\(^2\) which makes it possible to assess the modeling of near-equilibrium processes. In the next step the model is applied for the prediction of strongly flowing arcs, experimentally investigated by de Haas,\(^3\) Kroesen\(^5\) and Beulens,\(^6\) which allows the verification of the nonequilibrium plasma modeling.

II. MATHEMATICAL MODEL

In the experiments of Timmermans\(^1,2\) and Kroesen,\(^5\) the argon plasma is produced in cascaded arcs, as sketched in Fig. 1. The arc channel has a circular cross section with diameters of 4 mm (Kroesen) and 5 mm (Timmermans). The channel is 60–90 mm long and formed by the central bore of a series of water cooled copper plates insulated electrically from each other. Three cathodes are fixed just at the argon inlet to the arc channel, while a ring-shaped anode is fixed at the outlet.

The argon flow in the arc channel is a laminar circular pipe flow, which can be described by parabolic partial differential equations, but since the mathematical model is aimed to be applied for the predictions of more complex recirculating flows, the system of elliptic flow equations is solved, together with the energy and concentration conservation equations. The flow is assumed to be axisymmetric, two-dimensional, compressible, and strongly nonisothermal. The argon plasma is considered as singly ionized, locally quasineutral \((n_i = n_e)\) with local temperature nonequilibrium \((T_e \neq T_h)\). The electric field is assumed to be one-dimensional, uniform over the arc cross section \((E_x = 0, \partial E_x/\partial r = 0)\). The total current is specified as an input parameter and is kept constant along the arc channel.

A. Flow equations

The plasma flow is described by the following continuity and momentum equations:

\[
\frac{\partial}{\partial r} \left( \rho u \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \rho rv \right) = 0, \tag{1}
\]

\[
\frac{\partial}{\partial x} \left( \rho u^2 \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \rho rv^2 \right) = 2 \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \mu \frac{\partial v}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \rho u \frac{\partial u}{\partial r} \right) - \frac{\partial p}{\partial x}, \tag{2}
\]

\[
\frac{\partial}{\partial x} \left( \rho wv \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \rho wv^2 \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( \mu \frac{\partial u}{\partial r} \right) + \frac{2}{r} \frac{\partial}{\partial r} \left( \mu \frac{\partial v}{\partial r} \right) + \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) - \frac{2\mu v}{r} = -\frac{\partial p}{\partial r}, \tag{3}
\]

in which \(u\) and \(v\) denote the axial and radial velocities, \(\rho\) is the plasma mass density, \(p\) is the pressure, \(\mu\) is the dynamic viscosity, and \(x\) and \(r\) are the axial and radial coordinates. As mentioned before, the contribution of the electrons to the plasma continuity and momentum equations is neglected, because of the small electron mass. However, the influence of electrons is held in the definition of the plasma mass density \(\rho = (n_i + n_e)\rho_n\) and the plasma pressure \(p\), which is based on Dalton’s law of partial pressures,

\[
p = (n_i + n_e)kT_h + n_e kT_e, \tag{4}
\]

where \(n_i, n_n,\) and \(n_e\) denote the number densities of ions, neutral atoms, and electrons, respectively; \(k\) is the Boltzmann constant and \(T_h\) and \(T_e\) are the heavy particle and electron temperatures. With the local quasineutrality assumption \(n_i \approx n_n\), and introducing the heavy particle number density \(n_n = n_i + n_n\), the equation of state (4) becomes

\[
p = n_h k \left[ T_h + (n_e/n_h)T_e \right]. \tag{5}
\]

Substituting \(R_e \rho = n_h k\) and the ionization degree \(\alpha = n_e/n_h\), the equation of state is finally written as

\[
p = p/R_e (T_h + \alpha T_e), \tag{6}
\]

where \(R_e\) denotes the universal gas constant divided by the atom mass number. From Eq. (6), it can be seen that the plasma mass density is influenced by the presence of electrons. It has to be noted that all possible body forces that may originate from electrical and magnetic fields are neglected in the momentum equations (2) and (3). This is certainly justified for the present values of the current density in the absence of externally applied magnetic fields \(J \times B \in \nabla p\).
B. Plasma composition equations

The model is formulated for a monatomic gas undergoing single ionization at locally quasineutral conditions. Therefore the plasma composition is defined by a single conservation equation; the electron continuity equation

$$\frac{\partial}{\partial x} (n_e u_e) + \frac{1}{r} \frac{\partial}{\partial r} (n_e v_e)$$

$$= \frac{\partial}{\partial x} \left( D_a \frac{\partial n_e}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( D_a \frac{\partial n_e}{\partial r} \right) + R_{ne},$$  (7)

where $D_a$ denotes the ambipolar diffusion coefficient and $R_{ne}$ is the electron source term, which coincides with the volumetric ionization-recombination rate.

C. Energy equations

The energy conservation equations are written both for the heavy particles and the electrons. The heavy particle energy equation is given in the form of conservation of the heavy particle stagnation enthalpy

$$h = C_{ph} T_h + \frac{1}{2} v^2, \quad v^2 = u^2 + v^2,$$

$$\frac{\partial}{\partial x} \left( \rho h c_{ph} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \rho u c_{ph} v \right)$$

$$= \frac{\partial}{\partial x} \left[ \lambda_h \frac{\partial h}{\partial x} \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[ \lambda_h \frac{\partial h}{\partial r} \right]$$

$$+ \frac{1}{r} \frac{\partial}{\partial r} \left[ \mu \left( 1 - \frac{1}{P_s} \right) \left( \frac{\partial u^2}{\partial r} \right) \right]$$

$$+ \frac{1}{r} \frac{\partial}{\partial r} \left[ \mu \left( 1 - \frac{1}{P_s} \right) \left( \frac{\partial v^2}{\partial r} \right) \right] + Q_{eh},$$  (9)

where $C_{ph} = 5R_h/2$ denotes the specific heat of heavy particles, $\lambda_h$ is the heavy particle heat conductivity, and $P_s = \mu C_{ph}/\lambda_h$ is the heavy particle Prandtl number. The viscosity term $\nabla \cdot \mathbf{u}$ is approximated by $\nabla \cdot \mathbf{u} = \nabla \nu \cdot \mathbf{u}$. The energy gain and loss because of collisions induced by the electric field and the current density $\mathbf{E}$ and $\mathbf{J}$, respectively, vanish when for $E = v$ and $R = - \mathbf{v} \cdot \mathbf{j} - \mathbf{R}$ in which $\mathbf{R}$ is the thermal force. The thermal force $(1 / n_e) j \cdot \mathbf{R}$ can be neglected because $\mathbf{R}$ depends on the gradient of the electron temperature, which is very small in the direction of the current density, which is still valid near the walls of the arc. The last term on the right-hand side of Eq. (9) $Q_{eh}$ is the net volumetric energy exchange rate between electrons and heavy particles due to elastic collisions.

The intrinsic electron energy equation has the following form:

$$\frac{\partial}{\partial x} \left( \frac{3n_e u_e k T_e}{2} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{3n_e v_e k T_e}{2} \right)$$

$$= \frac{\partial}{\partial x} \left( \lambda_{ce} \frac{\partial T_e}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \lambda_{ce} \frac{\partial T_e}{\partial r} \right)$$

$$+ u_e \frac{\partial}{\partial x} (n_e k T_e) + v_e \frac{\partial}{\partial r} (n_e k T_e)$$

$$+ Q_{Ohm} + Q_{ion} - Q_{eh} + Q_{rad},$$  (10)

where $\lambda_{ce}$ is the electron heat conductivity, $Q_{Ohm}$ is the energy source due to the Ohmic heating, $Q_{ion}$ is the energy sink for ionization of neutral atoms or source if ion recombination takes place, and $Q_{rad}$ is the net volumetric radiative energy exchange rate. In this equation, the velocities $u_e$ and $v_e$ are the electron velocities. These can be rewritten using the current density $j = n_e q (v_e - v_0)$, in which $v_0$ is the ion velocity and equal to the gas velocity, and $v_e$ is the electron velocity.

D. Electric field equations

The electric field in the arc channel is described by a very simple algebraic model that assumes a one-dimensional field such that the radial component $E_r$ is zero and the axial component $E_z$ is uniform over the arc cross section:

$$E_z = 0, \quad \frac{\partial E_z}{\partial r} = 0.$$  (11)

The axial electric field component $E = E_z$ is calculated from the specific, axially preserved total discharge current $I$ and the electrical conductivity $\sigma$, averaged over the cross section:

$$E = \frac{I}{(2\pi R^5 \sigma \, dr)},$$  (12)

where $R$ is the radius of the arc channel. The local axial current density $j$ is determined from the known local electrical conductivity and electric field

$$j = \sigma E_z, \quad j_z = 0.$$  (13)

E. Source terms

The system of equations given above is very near to the exact mathematical description of a locally neutral, singly ionized monatomic plasma flow. However, modeling is needed for the physical phenomena that enter in the source terms, transport coefficients, and boundary conditions of the differential equations. In many situations, the source terms are dominant and some differential terms in the equations can be neglected. That is why the source terms bring the most important physical information in the plasma model.

The volumetric ionization recombination rate $R_{ne}$ can be written as

$$R_{ne} = n_e (n_n - n_{ns}) K_{1+},$$  (14)

where $n_{ns}$ is defined by the Saha ionization equilibrium coefficient $S_{1+}$.

$$n_{ns} = \frac{n_e}{S_{1+}}, \quad S_{1+} = \frac{2g_0 + \left( \frac{2\pi mn_e k T_e}{h^2} \right)^{3/2} \exp \left( \frac{H_{le}}{k T_e} \right)}{g_0},$$

where $m_e$ is the electron mass. The statistical weights $g_0$ and $g_{1+}$, according to Drawin and Felenbok, are

$$g_0 = 1, \quad g_{1+} = 4 + 2 \exp ( - 2059 / T_e).$$

The ionization energy $H_{le} (J)$ is defined by $H_{le} = \epsilon (E_{01} - \Delta E)$, where $E_{01}$ (eV) is 15.76 eV and the lowering of the ionization energy is given by $\Delta E = 2.086 \times 10^{-11} \sqrt{2n_e / T_e}$ in eV, where $\epsilon$ is the elementary charge.

The combined rate coefficient for excitation and ioniza-
tion $K_{1+}$ in the source term (14) is approximated by the following semiempirical formula, obtained by fitting the numerical data of Willems,\textsuperscript{9} based on the model of Van der Sijde,\textsuperscript{10}

$$K_{1+} = \exp (-2.69983 \times 10^{-15} T_e^6 + 1.2137 \times 10^{-19} T_e^7 + 2.1524 \times 10^{-6} T_e^3 + 1.89074 \times 10^{-7} T_e^2 - 114.838) \text{ in (m}^3\text{/sec}).$$

The energy source terms $Q_{\text{Ohm}}$, $Q_{\text{con}}$, $Q_{\text{e}}$, and $Q_{\text{rad}}$ are very important for the correct prediction of the plasma flow. Ohmic heating is the only source of thermal energy in the plasma:

$$Q_{\text{Ohm}} = j \cdot E. \quad (15)$$

The ionization-recombination energy source term is calculated from the ionization-recombination rate (14)

$$Q_{\text{con}} = -H_{\text{con}} R_{\text{con}}. \quad (16)$$

The net volumetric energy exchange rate between electrons and heavy particles due to elastic collisions is given by

$$Q_{\text{eh}} = 3n_e \frac{m_e}{m_h} v_{\text{eh}} k(T_e - T_h), \quad (17)$$

where $m_h$ is the heavy particle mass and $v_{\text{eh}} = v_{\text{el}} + v_{\text{en}}$ denotes the average elastic collision frequency between electrons and heavy particles, which is determined from the characteristic electron–ion ($\nu_{\text{el}}$) and electron–neutral ($\nu_{\text{en}}$) collision frequencies:

$$\nu_{\text{el}} = (e^2 n_e \Lambda / 12 \pi \epsilon_0^2) \sqrt{2 \pi m_e k^2 T_e}, \quad (18)$$

$$\nu_{\text{en}} = n_e Q_{\text{en}} \sqrt{8 k T_e / \pi m_n}. \quad (19)$$

The Coulomb logarithm $\Lambda$ is defined by

$$\Lambda = -\lambda_D / b_0, \quad (20)$$

where $\lambda_D$ and $b_0$ are the Debye length and impact parameter for a 90° scattering of the particles:

$$\lambda_D = [e^2 k T_e / (n_e + n_i)]^{1/2}, \quad (21)$$

$$b_0 = e^2 / 12 \pi \epsilon_0 k T_e. \quad (22)$$

Finally, $\Lambda$ can be written as

$$\Lambda = 6 \pi \sqrt{2} [(e^2 k / e^4)^{1/2} \sqrt{T_e^2 / n_e}], \quad (23)$$

which, after substitution of all physical constants, gives

$$\Lambda = 8.76 \times 10^{10} [(\text{m/K})^{1.5}] \sqrt{T_e^2 / n_e}. \quad (24)$$

This value is $\sqrt{2}$ times lower than the value obtained from a definition of the Debye length without taking into account the shielding of the ions,

$$\lambda_D' = [(e^2 k T_e / n_e) e^2]^{1/2}, \quad (25)$$

$$\Lambda' = 1.24 \times 10^{10} [(\text{m/K})^{1.5}] \sqrt{T_e^2 / n_e}. \quad (26)$$

The choice of the definition of the Debye length may have an important influence on the results of model predictions at lower temperatures, especially for high density low-temperature plasmas where also nonideality effects should be taken into account. These effects change the electrical conductivity and electric field calculations and to much less extent the thermal energy transfer. In the present work, a definition of the Debye length, which takes into account both electrons and ions suffices [Eq. (21)], in accordance with Gunther et al.,\textsuperscript{11} which results in the Coulomb logarithm defined by Eqs. (23) and (24). However, the calculations are performed also with Eqs. (25) and (26) for comparison purposes.

The average electron–neutral collision cross section $\langle Q_{\text{en}} \rangle$ in Eq. (19) is evaluated by the semiempirical expression, which is obtained by fitting the data of Milloy:\textsuperscript{12}

$$\langle Q_{\text{en}} \rangle = 2.8 \times 10^{-24} T_e - 4.1 \times 10^{-34} T_e^3 - 3 \times 10^{-21} \text{ in (m}^2). \quad (27)$$

For evaluation of the radiative energy exchange rate $Q_{\text{rad}}$, a modeling of the radiative energy transfer is needed, which should take into account emission, absorption, and scattering of radiation. In strongly flowing arcs, the radiative terms are small compared to the convection terms.\textsuperscript{13} For stagnant arcs, however, radiation must be taken into account because then it makes up a significant part of the energy source term. According to Kroesen,\textsuperscript{14} the radiation to the ground state is mathematically taken into account in the term containing $K_1$, as in Eq. 14. Therefore only free–free radiation, line radiation, and free–bound radiation, except to the ground state, are needed. The electron–neutral free–free term can be represented by

$$Q_{\text{rad}}^{ee} = 910 \bar{n}_e, \quad (28)$$

in which the dimensionless electron density $\bar{n}_e = n_e / 10^{20}$, and the dimensionless neutral density $\bar{n}_0 = n_0 / 10^{24}$. In the same way, the electron–ion free–free radiation loss is approximated by

$$Q_{\text{rad}}^{ei} = 2.572 \times 10^4 \bar{n}_e. \quad (29)$$

For the line radiation, the 4p–4s radiation is dominant and represented by

$$Q_{\text{rad}}^{\text{line}} = 2.572 \times 10^6 \bar{n}_e^{1.57}. \quad (30)$$

In this simple model, no absorption is taken into account, especially because radiation to the ground state is strongly absorbed.

### F. Transport properties

A two-temperature treatment of the plasma requires the determination of nonequilibrium transport properties. Specific heat and Prandtl number of the heavy particles are assumed to be constant.

$$c_{ph} = 5 \bar{R}_h / 2 \quad \text{and} \quad P_r = \delta. \quad (31)$$

The heat conductivity of the heavy particles is assumed to have the near-equilibrium values calculated by Devoto,\textsuperscript{14} which at lower temperatures can be approximated by

$$\lambda_h = 2.43 \times 10^{-4} T_e^{7/4} \text{ for } T_e \leq 7000 \text{ K}, \quad (32)$$

and at higher temperatures by the expression obtained by Spitzer and Harm,\textsuperscript{15}

$$\lambda_h = 1.84 \times 10^{-10} (T_e^{3/2} / \Lambda) \text{ for } T_e > 7000 \text{ K}. \quad (33)$$
The plasma viscosity is calculated from the definition of the Prandtl number,
\[ \mu = \nu_0 (\lambda_\nu / C_{ph}), \]  
which is in agreement with the data tabulated by Vargaf-tik.\(^{16}\)

The ambipolar diffusion coefficient is determined by the expression of Devoto,\(^{17}\)
\[ D_{amb} = 3kT_e/4\pi\Omega_m, \]  
in which \( \Omega_m \) is a first approximation to the ion–atom collision integral\(^{17}\)
\[ \Omega_m = 2.84 \times 10^{-17}T_e^{0.36} (\text{m}^2/\text{sec}). \]  

In order to follow the local quasineutrality assumption, i.e., the correspondence between the electron and ion diffusion fluxes, we have replaced \( T_e \) by \( T_h \) in Eq. (34), which has negligible influence on the results of the calculations. The replacement results in a better stability of the model, without significant expedience. For strong nonequilibrium \((T_e/T_h < 1)\), e.g., near walls, this replacement leads to an underestimated ambipolar diffusion. For the nonflowing cascaded arc the influence is negligible because \( T_e = T_h \), except very close to the walls, while for the strongly flowing arc the diffusion term is negligible with respect to the convection flow. Note that, e.g., for inductively coupled plasmas, the influence is significant because of both a strong nonequilibrium and a low flow velocity near the edge of the plasma and the skin regions.

The electron heat conductivity and the plasma electrical conductivity are determined by using Frost mixture rules given by Mitchner and Kruger,\(^{18}\) in which the Coulomb logarithm is evaluated by expressions (21)-(24), taking into account the ion shielding.

G. Boundary conditions

The system of partial differential equations (1)-(10) needs the boundary conditions for all variables to be specified at all boundaries of the flow domain. The cascaded arc, as shown in Fig. 1(a), is treated as axisymmetrical and the computational flow domain is taken to be bounded by the inlet and outlet plains, the arc channel wall and the symmetry axis [Fig. 1(b)]. For this flow, the following boundary conditions are specified.

(a) Inlet cross section \((x = 0)\). The average axial flow velocity \( v_{in} \) is calculated from a given mass flow rate \( G_{in} \), the radial flow velocity \( v_{in} = 0 \). The inlet pressure \( p_{in} \) is specified, \( T_{in} = T_h = 300 \text{ K} \), and the electron number density, which corresponds to a very low ionization degree of \( \alpha_{in} = 10^{-6} \), is also given.

(b) Arc channel wall \((r = R)\). Here \( u = v = 0 \), \( T_{sw} = 500 \text{ K} \), \( T_{ew} = 1000 \text{ K} \), and \( \alpha_e \) corresponds to the ionization degree of \( \alpha_e = 10^{-6} \).

(c) Symmetry axis \((r = 0)\). Here \( v = 0 \), and the gradients of all other variables are equal to zero.

(d) Outlet cross section. Here \( \partial^2 p/\partial x^2 = 0 \), i.e., a constant pressure gradient, and zero gradients of all other variables.

It has to be noted that the solutions of the electron continuity and energy equations depend only weakly on the boundary conditions at the inlet and at the wall, because of the small values of the diffusion coefficient and electron heat conductivity at low temperatures, and because of the large local source terms. Hence zero gradients of the electron number density and temperature can be specified at the inlet and at the wall, without any noticeable difference in the results. For strongly flowing arcs, the outlet conditions are not correct because the Mach number, which is coupled with the heavy particle temperature and indirectly with the inlet pressure and the mass flow rate, should equal unity. Because of the elliptic differential equations, the outlet conditions influence the upstream calculations only slightly.

III. NUMERICAL SOLUTIONS

The system of partial differential equations (1)-(10) is solved by a control volume numerical method of Patankar and Spalding,\(^{19,20}\) However, their basis numerical method and iterative “SIMPLE” solution procedure for the elliptic flow equations are considerably extended in this work by incorporating a nonstaggered grid approach of Rhie\(^{21}\) with momentum interpolation at control volume faces, as suggested and discussed in detail by Majumdar.\(^{22}\) For the treatment of strongly compressible flow in the cascaded arc, density and temperature corrections from the equation of state are introduced into the pressure correction equation, as proposed by Jensen \textit{et al}.\(^{23}\) and Karki and Patankar,\(^{24}\) but with an additional upwindlike scheme, representing the influence of density on the pressure, which ensures always positive coefficients and convergence of the pressure correction equation. The system of algebraic discretization equations for each variable are solved by the strongly implicit (SIP) procedure of Stone.\(^{25}\)

The solution procedure of the cascaded arc plasma flow equations has the following basic steps.

(a) Guess the initial field of variables and transport coefficients.

(b) The solution of the laminar gas flow equations by the extended version of the “SIMPLE” algorithm, which includes the solution of the axial and radial momentum equations, interpolation of control volume face velocities, and \textit{face mass fluxes}, solution of the pressure correction equation, including density corrections from the equation of state (6). The calculated pressure correction field is used for the corrections of the pressure, velocities, and control volume face mass fluxes.

(c) The solution of the plasma composition equations, in this case, the electron number density conservation equation.

(d) The solution of the energy equations for the electrons and for the heavy particles.

(e) Updating the plasma properties and the transport coefficients, to be used in the next iteration.

The numerical solution is assumed to be converged as soon as the maximum relative residual sum for all equations is smaller than 0.5%. Because of the strong nonlinearity of the coupled set of equations, especially because of the exponential dependence of some source terms on the temperature, more than 400 and in some cases even up to 2000 iterations


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are needed to obtain a converged solution. A numerical grid with 23 × 22 nonuniformly distributed points is used for all calculations in this work, which were checked to give grid-independent results by changing the grid expansion rate.

IV. RESULTS AND DISCUSSION

The cascaded arc (CARC) plasma flow model described in the previous sections is applied for the calculations of both stagnation and flowing arcs, in order to check and verify the model assumptions, especially the modeling of the source terms and transport coefficients, by means of comparison with the existing experimental data of Rosado, Timmermans, and Kroesen. At the same time, the calculations are used to investigate some of the plasma properties that are not measured, such as the current density distribution over the cross section of the arc channel, plasma velocity, and the evolution of the heavy particle temperature profile along the arc, as well as the local temperature nonequilibrium in the arc.

A. Stagnation arcs

Stagnation arcs are cascaded arcs operated at low gas flow rates, so that the convection phenomena can be neglected, the plasma properties are one dimensional, i.e., they vary only in the radial direction and the variations of all properties in the axial direction are negligible. It can be recognized that our model is too complex for the treatment of such a one-dimensional diffusion-source-dominated plasma. It should be noted here that, unless the low flow rates, a change in rates by a factor 2, results in a change in, e.g., electron temperature by 2%-5%. However, the application of the model to stagnation arcs enhances a separate verification of the model assumptions for diffusion processes, ionization rate, electron-heavy particle collisional energy transfer, radiative losses, modeling of the electric field, the current density distribution, and the Ohmic heating. In the figures, the end effects (first two and last two grid points), where the deviation from a one-dimensional character of the model is the strongest, are not shown. The results for axial points in between the end-effect regions show negligible difference, so results for only one axial position (in the middle of the arc) are shown.

Calculations for stagnation arcs are performed for a 5 mm diam cascaded arc, operated at low (2 × 10⁻⁶ kg/sec) argon flow rate, at discharge currents of 40, 80, 140, and 200 A, in order to simulate the experimental conditions of Rosado and Timmermans et al.

Figure 2 shows the radial distributions of measured (symbols) and predicted electron temperatures (dashed curves), as well as predicted heavy particle temperatures obtained at different electrical currents. The agreement between the predictions and the experiments is quite satisfactory, but the measured electron temperature profiles are more uniform over the radius than predicted. At higher currents (100 and 200 A), the reason for this discrepancy can be the fact that the reabsorption of the radiation energy loss is not taken into account in the model, but the same discrepancy exists at lower currents, where the radiation loss presents only a minor loss. Another possible reason can be the fact that the spectroscopic measurements of the electron temperature and density were performed with a spatial resolution of 0.4 mm, which means an integration over almost 20% of the arc radius. This will result in more uniform profiles of the electron temperature. Figure 2 also shows that the plasma is nearly in equilibrium, except very close to the wall, where the electron and heavy particle temperatures differ significantly.

The comparisons of the measured and calculated electron density and ionization degree profiles are given in the Figs. 3 and 4, respectively. The agreement is very good and the differences are practically within experimental error, which is estimated to be 10% (Timmermans et al.). We emphasize that the comparison between experimental and model values is based on absolute values!

FIG. 2. Stagnation arc: electron and heavy particle temperature versus radial position in the arc for different arc currents; electron temperature measurements (Timmermans): □, I = 60 A; ▲, I = 80 A; +, I = 140 A; and ●, I = 200 A. Calculations (CARC) --, electron temperature; - - -, heavy particle temperature.

FIG. 3. Stagnation arc: electron density versus radial position in the arc for different arc currents; experiments (Timmermans): □, I = 60 A; ▲, I = 80 A; +, I = 140 A; and ●, I = 200 A. Calculations (CARC) are solid lines.
One of the most important characteristics of the plasma generator, for engineering purposes, is the relationship between the mean axial electric field and axial current. The comparison of measured and predicted electric field distributions for different total discharge currents is shown in Fig. 5. The calculation in which the ion shielding is taken into account in the definition of the Coulomb logarithm [see Eq. (24)], as shown by the solid line in Fig. 5, is in perfect agreement with the experiments. If the shielding of the ions is not taken into account [see Eq. (26)], the electric field is approximately 5% higher, as shown by the dashed curve in Fig. 5. The difference appears to be mainly due to the influence of the Coulomb logarithm on the electron-ion collision frequency and thus on the electrical conductivity.

The most critical part of the cascaded arc model is the calculation of the total radiation loss, as given by expressions (28)-(30). In order to illustrate the importance of the radiation loss at high arc currents, Fig. 6 shows the comparison of the calculated radial temperature profiles with the radiation loss taken into account (solid line) and neglected (dashed line) with the experimental values (symbols) for a discharge current of 200 A. The influence of the radiation loss is very pronounced and it is believed that a more elaborated modeling of the radiation transfer is needed for higher temperature plasmas.

Another simplified part of the cascaded arc model is the treatment of the electric field equations (11)-(13). It is assumed that the electric field is one dimensional and constant over the arc radius. That is why the radial distribution of the current density $j$ is influenced only by the distribution of the electrical conductivity [see Eq. (13)]. However, the calculated distribution of the current density appears to be different for different total currents, as shown in Fig. 7. The same figure also shows the rather high currents in the near-wall region, despite the fact that both the heavy particle temperature and the electron density are low in that region.

**B. Flowing arcs**

Cascaded arcs operated at high gas flow rates are named flowing arcs, due to their feature that the plasma properties

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**FIG. 4.** Stagnation arc: ionization degree versus radial position in the arc for different arc currents: ---, 60 A; - - - , 80 A; ---, 140 A; and ---, 200 A.

**FIG. 5.** Stagnation arc: electric field versus arc current; ■, experiments (Timmermans); calculations (CARC): —, with ion shielding in In A; - - - , without ion shielding in In A.

**FIG. 6.** Stagnation arc ($I = 200$ A): influence of radiation loss; ■, experiments (Timmermans); calculations (CARC): —, $T_e$; ---, $T_i$ with radiation loss; ---, $T_e$ without radiation loss (upper curve) and double radiation loss (lowest curves).

**FIG. 7.** Stagnation arc: calculated relative current density distributions versus the radial position in the arc for different total arc currents: ---, $I = 60$ A; ---, $I = 80$ A; ---, $I = 140$ A; and ---, $I = 200$ A.
are significantly influenced by flow phenomena. The arc becomes fully two dimensional and plasma nonequilibrium is more emphasized.

In order to simulate the experimental conditions of Kroesen,⁵ the flowing arc calculations are performed for a 4 mm diam, 60 mm long cascaded arc operated at a constant total discharge current of 50 A, with variable argon flow rates of 50, 100, and 200 scc/sec (standard cubic centimeters per second, which means at standard conditions of 273 K and 1 bar).

The measurements of Kroesen⁵ provided the variation of the average electron temperature and electron density along the arc, as well as the distributions of the pressure and electric field along the pipe wall.

Temperature distributions along the arc are shown in Fig. 8. The measured electron temperature (symbols) is at all argon flow rates somewhat lower than the calculated centerline values. This is considered to be a good validation for the model, because of the difficulty of the interpretation of the measuring volume in the experiments. All curves in Fig. 8 show qualitatively the same temperature distributions, in which temperature levels are decreasing with increasing argon flow rate. At the same time, the temperature nonequilibrium becomes stronger at the inlet region of the arc when the flow rate increases.

The electron density distributions along the arc are shown in Fig. 9. With increasing argon flow rate, the measured values of the electron density become closer to the predicted cross-section average than to the centerline values. Figure 9 clearly shows that there is an increasing difference between the centerline and the cross-section average values of the electron density with increasing argon flow, which implies that the radial distributions become steeper.

The variation of the pressure along the pipe wall is shown in Fig. 10 for three different argon flow rates, which
FIG. 12. Flowing arc: calculated radial profiles of the electron (upper curves) and heavy particle (lower curves) temperature at different axial positions and for argon flow rates of 200 scc/sec. The axial positions are denoted by +, $x/l = 0.045$; $\Delta$, $x/l = 0.21$; $\bullet$, $x/l = 0.52$; $\blacksquare$, $x/l = 0.72$; $\beta$, $x/l = 1$. The arc current is 50 A.

Figs. 12 for different axial positions. The differences between the electron and heavy particle temperatures are very large. At each axial location, the electron and heavy particle temperature profiles appear to be nearly equidistant over the radius. It can also be seen that all temperature profiles become practically fully developed at relative distances $x/l$ larger than approximately 0.5, especially at higher argon flow rates. This implies that the arc behaves similarly as a stagnation arc with much more emphasized temperature nonequilibrium.

The calculated radial electron density profiles at different axial positions are shown in Fig. 13. However, the electron density distributions do not show any fully developed character, probably because of the pressure and density variation along the flowing arc.

FIG. 13. Flowing arc: calculated radial profiles of the electron density at different axial positions and for argon flow rates of 200 scc/sec. The axial positions are denoted by +, $x/l = 0.045$; $\Delta$, $x/l = 0.21$; $\bullet$, $x/l = 0.52$; $\blacksquare$, $x/l = 0.72$; $\beta$, $x/l = 1$. The arc current is 50 A.

FIG. 15. Flowing arc: calculated centerline values of the axial plasma velocity along the arc, for different argon flow rates: $\cdots$, 50 scc/sec; $\cdots$- - -, 100 scc/sec; $\cdots$--$, 200 scc/sec. The arc current is 50 A.

The calculated radial temperature profiles are shown in Figure 11 shows the variation of the electric field along the arc. In this case, the present model overestimates the experimental values of Kroesen and Beulens. This is possibly due to the error in the calculations of the electrical conductivity at significant nonequilibrium conditions in the flowing arc. However, there is also a possibility of undeterminacy of the experimental results, due to measurement of voltage differences between neighboring cascaded arc plates, whose distance is comparable to their thickness.

The calculated radial temperature profiles are shown in
The ionization degree, which is caused both by the increase of the temperature and the decrease of the pressure along the arc. The sonic plasma velocity, which is approached at the arc exit, is proportional to the square root of the temperature. The exit temperature is nearly the same at all three argon flow rates (see Fig. 8). Consequently, the plasma velocities differ only slightly, as shown in Fig. 15, in which the calculated centerline velocities are compared.

One of the most important plasma parameters is the ionization degree. Figure 16 shows the calculated cross-section averaged ionization degree as a function of the axial position in the arc, for different argon flow rates. The ionization degree decreases with increasing flow rate. Because of the large variety of different regimes in which the plasma can be produced, it looks reasonable to take the ionization degree rather than the electron density as a parameter for the plasma production effectiveness.

V. CONCLUSIONS

The cascaded arc (CARC) plasma model is developed and applied for the description of both stagnation and flowing argon arcs. The results show that the stagnation arcs are close to local temperature equilibrium conditions, except in the regions near the wall. The calculated values of the electric field are practically identical to the measured values if both electron and ion shielding is taken into account in the definition of the Coulomb logarithm.

Flowing arcs show very strong temperature nonequilibrium effects, both in the regions near the wall and in the inlet region of the arc. The calculated values of the pressure distributions are in very good agreement with the experimental data, thus showing that the model is applicable to strongly compressible flows.

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