Almost decentralized model predictive control of power networks

Citation for published version (APA):

Hermans, R. M., Lazar, M., Jokic, A., & Bosch, van den, P. P. J. (2010). Almost decentralized model predictive control of power networks. In *Proceedings 15th IEEE Mediterranean Electrotechnical Conference, MELECON 2010, 26-28 April 2010, Valletta, Malta* (pp. 1551-1556). Institute of Electrical and Electronics Engineers. <https://doi.org/10.1109/MELCON.2010.5476277>

DOI:

[10.1109/MELCON.2010.5476277](https://doi.org/10.1109/MELCON.2010.5476277)

Document status and date:

Published: 01/01/2010

Document Version:

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

[Link to publication](#)

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license above, please follow below link for the End User Agreement:

www.tue.nl/taverne

Take down policy

If you believe that this document breaches copyright please contact us at:

openaccess@tue.nl

providing details and we will investigate your claim.

Almost Decentralized Model Predictive Control of Power Networks

R. M. Hermans ^{#1}, M. Lazar [#], A. Jokić [#], P. P. J. van den Bosch [#]

[#] *Department of Electrical Engineering, Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven, The Netherlands*

¹ *r.m.hermans@tue.nl*

Abstract—Stable operation of the electrical power grid in the future will require novel, advanced control techniques for supply and demand matching, as a consequence of the liberalization and decentralization of electrical power generation. Currently, there is an increasing interest for using model predictive control (MPC) for power balancing. However, a centralized implementation of MPC is hampered by the large scale and complexity of power networks. Non-centralized, scalable control schemes are more suited for future application. In this paper we therefore propose a novel almost-decentralized Lyapunov-based predictive control algorithm for power balancing, i.e., for asymptotic stabilization of the network frequency. The algorithm is particularly suited for large-scale power networks, as it requires only local information and limited communication between directly-neighboring control areas to provide a stabilizing control action. We assess the suitability of this scheme and compare it with state-of-the-art non-centralized MPC in a benchmark case study.

I. INTRODUCTION

Reliable supply of electrical energy, and consequently, stable operation of the power grid have become of paramount importance to society. Traditionally, a large portion of the power production could be efficiently scheduled in an open-loop manner, whereas simple and relatively slow control methods sufficed for real-time balancing of supply and demand. However, today electrical power networks are going through a number of fundamental restructuring processes. Firstly, power networks are subject to an increasing integration of small-scale distributed generators (DG) (see e.g., [1] and the references therein), often based on renewable energy sources, leading to large and unpredictable fluctuations on the supply side of future power systems. Secondly, from a regulated, one-utility controlled operation, the system is restructured to include many parties that compete for power production and consumption, while pushing the system towards its stability boundaries (see e.g., [2] and the references therein). These observations point out that preservation of the robust and stable supply of electrical power that was attained in the past will become a major challenge for future power system control.

Recently, it was observed that model predictive control (MPC) has a potential for solving the problems that will appear in future electrical power networks (see for instance [3]–[5]). MPC can explicitly take constraints on states and inputs into account when computing the control action, and it can employ disturbance models to counteract the fluctuations in supplied power introduced by renewable energy sources. Nonetheless, the fact that MPC is a centralized control method is a major issue if it is to be used in power systems. Centralized MPC requires a single controller to measure all the system outputs,

compute and apply the control input to all actuators in the network, all within one sampling period. As power grids are large-scale systems, both computationally and geographically, it is practically impossible to implement a predictive controller in a centralized fashion. This motivates the search for non-centralized formulations of MPC for power systems, in which the overall control scheme equals the ensemble of a set of local control laws, each assigned to a separate control area.

The non-centralized implementations of MPC that have been proposed by the literature, see e.g., [3], [4], [6]–[10], can roughly be divided into two categories: *decentralized* techniques, in which local controllers operate without communication, and *distributed* techniques that exploit mutual exchange of information over a usually predefined structured communication network to compute the control action. Distributed methods that employ iterations or global information can generally outperform decentralized MPC in terms of optimality with respect to a global objective, at the cost of higher computational and communication requirements (see [11]). However, the sampling periods required in power system control (in the order of seconds) are too short for MPC algorithms to perform iterations or to exchange global information in a reliable fashion. Consequently, iterative or globally communicating distributed MPC approaches are not suited for control of large-scale power networks. A demand for globally optimal performance of MPC-controlled power systems is currently not realistic. Given these observations, we focus in this article on asymptotic stabilization of the grid frequency, i.e., power balancing, using a new non-iterative MPC scheme that was recently presented in [12]. An attractive feature of the proposed method is that it needs no global coordination and can be implemented in an almost decentralized fashion. By this we mean that the controller only requires one run of information exchange between directly neighboring control areas per sampling instant. The limited (inter-area) communication exploited by the proposed MPC scheme is more realistic than requiring a global exchange of information, and it is easy to implement, as present transmission lines are usually equipped with communication links.

The effectiveness of the MPC algorithm presented in this paper is illustrated in a simulation study, by comparing its performance and complexity with a similar state-of-the-art non-centralized MPC method. Given the results of this benchmark test, we discuss the usefulness of the proposed control method for frequency control in power networks.

II. PRELIMINARIES

A. Basic Notions and Definitions

Let $\mathbb{R}$, $\mathbb{R}_+$, $\mathbb{Z}$ and $\mathbb{Z}_+$ denote the field of real numbers, the set of non-negative reals, the set of integer numbers and the set of non-negative integers, respectively. We use the notation $\mathbb{Z}_{\geq c_1}$ and $\mathbb{Z}_{(c_1, c_2]}$ to denote the sets $\{k \in \mathbb{Z}_+ \mid k \geq c_1\}$ and $\{k \in \mathbb{Z}_+ \mid c_1 < k \leq c_2\}$, respectively, for some $c_1, c_2 \in \mathbb{Z}_+$. For a finite set of vectors $\{x_i\}_{i \in \mathbb{Z}_{[1, N]}}$, $x_i \in \mathbb{R}^{n_i}$, $N \in \mathbb{Z}_+$, we use $\text{col}(\{x_i\}_{i \in \mathbb{Z}_{[1, N]}})$, and equivalently $\text{col}(x_1, \dots, x_N)$, to denote the column vector $(x_1^\top, \dots, x_N^\top)^\top$. Let $\mathbf{0}_n$ denote the zero vector in $\mathbb{R}^n$. For a set $\mathcal{S} \subseteq \mathbb{R}^n$, we denote by $\text{int}(\mathcal{S})$ the interior of $\mathcal{S}$. For a vector $x \in \mathbb{R}^n$, let $\|x\|$ denote an arbitrary p -norm and let $[x]_i$, $i \in \mathbb{Z}_{[1, n]}$ be the i -th component of x . The ∞ -norm of a vector $x \in \mathbb{R}^n$ is defined as $\|x\|_\infty := \max_{i=1, \dots, n} |[x]_i|$, where $|\cdot|$ denotes the absolute value. For a matrix $M \in \mathbb{R}^{m \times n}$, let $\|M\| := \max_{x \neq \mathbf{0}_n} \frac{\|Mx\|}{\|x\|}$ denote its corresponding induced matrix norm. Moreover, by $M \succ 0$ and $M \succeq 0$ we mean that M is positive definite or positive semi-definite, respectively. A function $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ belongs to class $\mathcal{K}$ if it is continuous, strictly increasing and $\varphi(0) = 0$. A function $\varphi : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ belongs to class $\mathcal{K}_\infty$ if $\varphi \in \mathcal{K}$ and it is radially unbounded, i.e., $\lim_{s \rightarrow \infty} \varphi(s) = \infty$.

B. Lyapunov Stability

Consider the discrete-time, autonomous nonlinear system

$$x(k+1) \in \Phi(x(k)), \quad k \in \mathbb{Z}_+, \quad (1)$$

where $x(k) \in \mathbb{X} \subseteq \mathbb{R}^n$ is the state at the discrete-time instant $k \in \mathbb{Z}_+$. The (possibly nonlinear) set-valued mapping $\Phi : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$ is such that $\Phi(x)$ is compact and nonempty for any $x \in \mathbb{X}$. We assume that the origin is an equilibrium of (1), i.e., $\Phi(\mathbf{0}_n) = \{\mathbf{0}_n\}$.

Definition II.1 A set $\mathcal{P} \subseteq \mathbb{R}^n$ is *Positively Invariant (PI)* for system (1) if $\forall x \in \mathcal{P}$ it holds that $\Phi(x) \subseteq \mathcal{P}$.

Definition II.2 (i) System (1) is *Lyapunov stable* if $\forall \varepsilon > 0$, $\exists \delta(\varepsilon) > 0$ such that for all state trajectories of (1) it holds that $\|x(0)\| \leq \delta(\varepsilon) \Rightarrow \|x(k)\| \leq \varepsilon$ for all $k \in \mathbb{Z}_+$. (ii) Let $\mathbb{X} \subseteq \mathbb{R}^n$ and $\mathbf{0}_n \in \text{int}(\mathbb{X})$. The origin of (1) is *attractive in $\mathbb{X}$* if for any $x(0) \in \mathbb{X}$ it holds that all corresponding trajectories of (1) satisfy $\lim_{k \rightarrow \infty} \|x(k)\| = 0$. (iii) System (1) is *asymptotically stable in $\mathbb{X}$* if it is Lyapunov stable and attractive in $\mathbb{X}$.

Theorem II.3 Let $\mathbb{X}$ be a PI set for system (1) and let $\mathbf{0}_n \in \text{int}(\mathbb{X})$. Furthermore, let $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$, $\rho \in \mathbb{R}_{[0,1]}$ and let $V : \mathbb{R}^n \rightarrow \mathbb{R}_+$ be a function such that

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|) \quad (2a)$$

$$V(x^+) \leq \rho V(x) \quad (2b)$$

for all $x \in \mathbb{X}$ and all $x^+ \in \Phi(x)$. Then system (1) is asymptotically stable in $\mathbb{X}$.

A function V that satisfies the conditions of Theorem II.3 is called a *Lyapunov function*. The proof of Theorem II.3 is

given in [13], Theorem 2.8. Note that in [13] continuity of the function V is required only to show certain robustness properties. See also [14] for results on stability of discrete-time systems via discontinuous Lyapunov functions.

C. CLFs for discrete-time systems

Consider the discrete-time constrained nonlinear system

$$x(k+1) = \phi(x(k), u(k)), \quad k \in \mathbb{Z}_+, \quad (3)$$

where $x(k) \in \mathbb{X} \subseteq \mathbb{R}^n$ is the state and $u(k) \in \mathbb{U} \subseteq \mathbb{R}^m$ is the control input at the discrete-time instant $k \in \mathbb{Z}_+$. The function $\phi : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ is nonlinear with $\phi(\mathbf{0}_n, \mathbf{0}_m) = \mathbf{0}_n$. We assume that $\mathbb{X}$ and $\mathbb{U}$ are bounded sets with $\mathbf{0}_n \in \text{int}(\mathbb{X})$ and $\mathbf{0}_m \in \text{int}(\mathbb{U})$. Next, let $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ and let $\rho \in \mathbb{R}_{[0,1]}$.

Definition II.4 A function $V : \mathbb{R}^n \rightarrow \mathbb{R}_+$ that satisfies

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|), \quad \forall x \in \mathbb{R}^n, \quad (4)$$

and for which there exists a control law, possibly set valued, $\pi : \mathbb{R}^n \rightrightarrows \mathbb{U}$ such that

$$V(\phi(x, u)) \leq \rho V(x), \quad \forall x \in \mathbb{X}, \forall u \in \pi(x),$$

is called a control Lyapunov function (CLF) in $\mathbb{X}$ for (3).

For results on CLFs for discrete-time systems we refer the interested reader to [15] and the references therein.

III. MAIN RESULTS

In order to set-up the control algorithm, we first introduce a framework for defining a network of systems (e.g., a set of interconnected control areas in power networks). Consider a directed connected graph $\mathcal{G} = (\mathcal{S}, \mathcal{E})$ with a finite number of vertices $\mathcal{S} = \{\varsigma_1, \dots, \varsigma_N\}$ and a set of directed edges $\mathcal{E} \subseteq \{(\varsigma_i, \varsigma_j) \in \mathcal{S} \times \mathcal{S} \mid i \neq j\}$. In a network of dynamically coupled systems, a dynamical system is assigned to each vertex $\varsigma_i \in \mathcal{S}$, with the dynamics governed by the following difference equation:

$$x_i(k+1) = \phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k))), \quad k \in \mathbb{Z}_+, \quad (5)$$

for vertex indices $i \in \mathcal{I} := \mathbb{Z}_{[1, N]}$. In (5), $x_i \in \mathbb{X}_i \subseteq \mathbb{R}^{n_i}$ denotes the state and $u_i \in \mathbb{U}_i \subseteq \mathbb{R}^{m_i}$ represents the control input of the i -th system, i.e., the system assigned to vertex ς_i . With each directed edge $(\varsigma_j, \varsigma_i) \in \mathcal{E}$ we associate a function $v_{ij} : \mathbb{R}^{n_j} \rightarrow \mathbb{R}^{n_{v_{ij}}}$ that defines the interconnection signal $v_{ij}(x_j(k)) \in \mathbb{R}^{n_{v_{ij}}}$, $k \in \mathbb{Z}_+$, between system j and system i , i.e., $v_{ij}(x_j(k))$ characterizes how the states of system j influence the dynamics of system i . We use $\mathcal{N}_i := \{j \mid (\varsigma_j, \varsigma_i) \in \mathcal{E}\}$ to denote the set of indices corresponding to the *direct neighbors* of system i . A direct neighbor of system i is any system in the network whose dynamics (e.g., states or outputs) appear explicitly (via the function $v_{ij}(\cdot)$) in the state equations that govern the dynamics of system i . Clearly, if system j is a direct neighbor of system i , this does not necessarily imply the reverse. Let $\mathcal{N}_i := \mathcal{N}_i \cup \{i\}$. We define $x_{\mathcal{N}_i}(k) := \text{col}(\{x_j(k)\}_{j \in \mathcal{N}_i})$ as the vector that collects all the state vectors of the direct neighbors

of system i and $v_i(x_{\mathcal{N}_i}(k)) := \text{col}(\{v_{ij}(x_j(k))\}_{j \in \mathcal{N}_i}) \in \mathbb{R}^{n_{v_i}}$ as the vector that collects all the vector valued interconnection signals that “enter” system i . The functions $\phi_i(\cdot, \cdot, \cdot)$ and $v_{ij}(\cdot)$ are arbitrary nonlinear and satisfy $\phi_i(\mathbf{0}_{n_i}, \mathbf{0}_{m_i}, \mathbf{0}_{v_i}) = \mathbf{0}_{n_i}$ for all $i \in \mathcal{I}$ and $v_{ij}(\mathbf{0}_{n_j}) = \mathbf{0}_{n_{v_{ij}}}$ for all $(i, j) \in \mathcal{I} \times \mathcal{N}_i$. For all $i \in \mathcal{I}$ we assume that $\mathbf{0}_{n_i} \in \text{int}(\mathbb{X}_i)$ and $\mathbf{0}_{m_i} \in \text{int}(\mathbb{U}_i)$.

Finally, let

$$x(k+1) = \phi(x(k), u(k)), \quad k \in \mathbb{Z}_+, \quad (6)$$

denote the dynamics of the overall network of interconnected systems (5), written in a compact form. In (6), $x = \text{col}(\{x_i\}_{i \in \mathcal{I}}) \in \mathbb{R}^n$, $n = \sum_{i \in \mathcal{I}} n_i$, and $u = \text{col}(\{u_i\}_{i \in \mathcal{I}}) \in \mathbb{R}^m$, $m = \sum_{i \in \mathcal{I}} m_i$, are vectors that collect all local states and inputs, respectively.

A. Structured max-CLFs

Next, we introduce the notion of a set of “structured max-CLFs”, which provides a novel alternative to the structured CLFs defined recently in [16].

Definition III.1 Let $\alpha_1^i, \alpha_2^i \in \mathcal{K}_\infty$ for $i \in \mathcal{I}$ and let $\{V_i\}_{i \in \mathcal{I}}$ be a set of functions $V_i : \mathbb{R}^{n_i} \rightarrow \mathbb{R}_+$ that satisfy

$$\alpha_1^i(\|x_i\|) \leq V_i(x_i) \leq \alpha_2^i(\|x_i\|), \quad \forall x_i \in \mathbb{R}^{n_i}, \quad \forall i \in \mathcal{I}. \quad (7a)$$

Then, given $\rho_i \in \mathbb{R}_{[0,1]}$ for $i \in \mathcal{I}$, if there exists a set of control laws, possibly set-valued, $\pi_i : \mathbb{R}^{n_i} \times \mathbb{R}^{n_{v_i}} \rightarrow \mathbb{U}_i$ such that

$$\begin{aligned} V_i(\phi_i(x_i, u_i, v_i(x_{\mathcal{N}_i}))) &\leq \rho_i \max_{j \in \mathcal{N}_i} V_j(x_j), \\ \forall x_i \in \mathbb{X}_i, \forall u_i \in \pi_i(x_i, v_i(x_{\mathcal{N}_i})), \end{aligned} \quad (7b)$$

the set of functions $\{V_i\}_{i \in \mathcal{I}}$ is called a set of “structured max control Lyapunov functions” in $\mathbb{X} := \{\text{col}(\{x_i\}_{i \in \mathcal{I}}) \mid x_i \in \mathbb{X}_i\}$ for system (6).

In the above definition the term *structured* emphasizes the fact that each V_i is a function of x_i only, i.e., the structural decomposition of the dynamics of the overall interconnected system (5) is reflected in the functions $\{V_i\}_{i \in \mathcal{I}}$. Moreover, the term *max* originates from the corresponding convergence condition, i.e., (7b). Next, based on Definition III.1, we formulate the following feasibility problem.

Problem III.2 Let $\rho_i \in \mathbb{R}_{[0,1]}$, $i \in \mathcal{I}$ and a set of candidate “structured max-CLFs” $\{V_i\}_{i \in \mathcal{I}}$ be given. At time $k \in \mathbb{Z}_+$, let the state vector $\{x_i(k)\}_{i \in \mathcal{I}}$, the set of interconnection signals $\{v_i(x_{\mathcal{N}_i}(k))\}_{i \in \mathcal{I}}$ and the values $\{V_i(x_i(k))\}_{i \in \mathcal{I}}$ be known, and calculate a set of control actions $\{u_i(k)\}_{i \in \mathcal{I}}$, such that:

$$u_i(k) \in \mathbb{U}_i, \quad \phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k))) \in \mathbb{X}_i, \quad (8a)$$

$$\begin{aligned} V_i(\phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)))) \\ \leq \rho_i \max_{j \in \mathcal{N}_i} V_j(x_j(k)), \end{aligned} \quad (8b)$$

for all $i \in \mathcal{I}$. $\square$

It can be proven that the control law $\pi(x(k)) := \{\text{col}(\{u_i(k)\}_{i \in \mathcal{I}}) \mid (8) \text{ holds}\}$ asymptotically stabilizes the

difference inclusion $x(k+1) \in \{\phi(x(k), u(k)) \mid u(k) \in \pi(x(k))\}$ in $\mathbb{X}$. This proof, given in [12], exploits the fact that the function $V(x) := \max_{i \in \mathcal{I}} V_i(x_i)$ is a CLF for the overall network if (8) is recursively feasible. The result then directly follows from Theorem II.3.

Notice that in Problem III.2, the functions V_i do not need to be CLFs (in conformity with Definition II.4) in $\mathbb{X}_i$ for each system $i \in \mathcal{I}$, respectively. More precisely, (8b) allows V_i to increase, as long as for each system the value of its function V_i at the next time instant is less than ρ_i times the maximum over the current values of its own function and those of its direct neighbors. Still, constraint (8b) could be restrictive in practice, as it can be difficult to find functions $\{V_i\}_{i \in \mathcal{I}}$ that satisfy (7) for all $x_i \in \mathbb{X}_i$. Therefore, we formulate the following feasibility problem, which permits a non-strict decrease of both local functions $V_i(x_i)$ and the full-network candidate CLF $V(x)$.

Problem III.3 Let $N_\tau \in \mathbb{Z}_{\geq 1}$ be given. Consider Problem III.2 for a set of “structured max-CLFs” $\{V_i\}_{i \in \mathcal{I}}$ in $\tilde{\mathbb{X}} \subset \mathbb{X}$, with (8b) replaced by

$$\begin{aligned} V_i(\phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)))) \\ \leq \rho_i \max_{\tau \in \mathbb{Z}_{[0, N_\tau-1]}} \max_{j \in \mathcal{N}_i} V_j(x_j(k-\tau)), \end{aligned} \quad (9)$$

for all $k \in \mathbb{Z}_{\geq N_\tau-1}$ and $i \in \mathcal{I}$. $\square$

In [12] it is proven that the control law $\bar{\pi}(x(k)) := \{\text{col}(\{u_i(k)\}_{i \in \mathcal{I}}) \mid (8a) \text{ and } (9) \text{ hold}\}$ renders the closed-loop system $x(k+1) \in \{\phi(x(k), u(k)) \mid u(k) \in \bar{\pi}(x(k))\}$ asymptotically stable in $\mathbb{X}$. The proof demonstrates that the function $V(x) := \max_{i \in \mathcal{I}} V_i(x_i)$ asymptotically converges to 0 when k goes to infinity, under the assumption that (8a) and (9) are recursively feasible. This and (7a) imply attractivity and Lyapunov stability of the closed-loop network.

Next, note that Problem III.2 and Problem III.3 are separable in $\{u_i\}_{i \in \mathcal{I}}$. Therefore, it is possible to compute the control action $u(k) := \text{col}(\{u_i(k)\}_{i \in \mathcal{I}})$ by solving N feasibility problems independently, with each subproblem in $u_i(k)$ assigned to one local controller, corresponding to one system $i \in \mathcal{I}$. In order to compute $u_i(k)$, each controller needs to measure or estimate the current state $x_i(k)$ of its system, and have knowledge of the interconnection signals $v_{ij}(x_j(k))$, $j \in \mathcal{N}_i$, and the values $V_j(x_j(k))$, $j \in \mathcal{N}_i$. In practice, it is possible to measure many interconnection signals directly at node i , whereas a single run of exchanging information among direct neighbors per sampling instant is sufficient to acquire the non-locally measurable signals. For example, in electrical power systems, where each control area represents a dynamical system, the interconnection signal can be the frequency of adjacent control areas and the power flows in the tie lines that connect these neighbors. The power flow $\Delta P_{\text{tie}}^{ij}(k)$ is directly measurable at node i , whereas the frequency $\Delta \omega_j(k)$ can only be determined in the corresponding control area and needs to be transmitted to node i . The value of $V_j(x_j)$, $j \in \mathcal{N}_i$, can be computed both at node j and i , although the latter option

requires j to send its full state measurement x_j to i , instead of only $V_j(x_j)$. Note that the above described exchange of information between possibly different market players does not carry competitive risks, as specific system parameters cannot be deduced from state information and $V_j(x_j)$ alone. This makes Problem III.2 and Problem III.3 well suited for use in a liberalized market environment.

If we combine Problem III.2 or Problem III.3 with the optimization of a set of local cost functions, the feasibility-based stability guarantee and the possibility of an almost decentralized implementation still hold. This enables the formulation of a one-step-ahead predictive control algorithm in which stabilization is decoupled from performance, and in which the controllers do not need to attain the global optimum at each sampling instant, as typically required for stability in classical MPC. For the remainder of the article we therefore consider the following almost-decentralized MPC algorithm, supposing that a set of local objective functions $\{J_i(x_i(k), u_i(k))\}_{i \in \mathcal{I}}$ is known.

Algorithm III.4 At each instant $k \in \mathbb{Z}_+$ and node $i \in \mathcal{I}$:

Step 1: Measure or estimate the current local state $x_i(k)$ and transmit $v_{ji}(x_i(k))$ and $V_i(x_i(k))$ to nodes $\{j \in \mathcal{I} \mid i \in \mathcal{N}_j\}$.

Step 2: Specify the set of feasible local control actions $\bar{\pi}_i(x_i(k), v_i(x_{\mathcal{N}_i}(k))) := \{u_i(k) \mid (8a) \text{ and } (9) \text{ hold}\}$. Minimize the cost $J_i(x_i(k), u_i(k))$ over $\bar{\pi}_i(x_i(k), v_i(x_{\mathcal{N}_i}(k)))$ and denote the optimizer by $u_i^*(k)$;

Step 3: Use $u_i(k) = u_i^*(k)$ as control action.

The interested reader is referred to [12] for more information on the algorithms and results presented in this section.

B. Implementation Issues

For infinity-norm based CLFs (i.e., $V_i(x_i) = \|P_i x_i\|_\infty$, where $P_i \in \mathbb{R}^{p_i \times n_i}$ is a full-column rank matrix) and input-affine prediction models $x_i(k+1) = f_i(x_i(k), v_i(x_{\mathcal{N}_i}(k))) + g_i(x_i(k), v_i(x_{\mathcal{N}_i}(k)))u_i(k)$, it is possible to formulate (9) as a set of linear inequalities, without introducing conservatism. By definition of the infinity norm, for $\|x\|_\infty \leq c$ to be satisfied for some vector $x \in \mathbb{R}^n$ and constant $c \in \mathbb{R}$, it is necessary and sufficient to require that $\pm[x]_j \leq c$ for all $j \in \mathbb{Z}_{[1,n]}$. So, for (9) to be satisfied, it is necessary and sufficient to require that

$$\begin{aligned} & \pm[P_i\{g_i(x_i(k), v_i(x_{\mathcal{N}_i}(k)))u_i(k)\}]_j \\ & \leq \zeta_i(k) \mp [P_i\{f_i(x_i(k), v_i(x_{\mathcal{N}_i}(k)))\}]_j, \end{aligned} \quad (10)$$

for $j \in \mathbb{Z}_{[1,p_i]}$ and $k \in \mathbb{Z}_{\geq N_\tau-1}$, and where

$$\zeta_i(k) := \rho_i \max_{\tau \in \mathbb{Z}_{[0, N_\tau-1]}} \max_{j \in \mathcal{N}_i} V_j(x_j(k-\tau)) \in \mathbb{R}_+$$

is constant at any $k \in \mathbb{Z}_{\geq N_\tau-1}$. This yields a total of $2p_i$ linear inequalities in the optimization variables u_i . Therefore, in combination with polytopic state/input constraints and an infinity-norm or quadratic cost function, it is possible to implement step 2 of Alg. III.4 as a linear or quadratic program, respectively.

Fig. 1. Schematic representation of a 4 control area power network.

IV. BENCHMARK TEST

The suitability of Alg. III.4 for frequency control is assessed via comparison with the non-centralized *Stability-constrained distributed MPC* (SC-DMPC) scheme for linear time-invariant systems, proposed in [4]. SC-DMPC is similar to Alg. III.4 in the sense that it employs an identical prediction model, non-iterative computations and communication among direct neighbors only. SC-DMPC imposes an alternative stability constraint on the state-prediction, by optimizing over $u(k) := \text{col}(\{u_i(k)\}_{i \in \mathcal{I}})$ such that

$$\begin{aligned} & \|\phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)))\|_2^2 \\ & \leq \|x_i(k)\|_2^2 - \beta_i \|x_i^1(k)\|_2^2, \end{aligned} \quad (11)$$

for some parameter $\beta_i \in \mathbb{R}_{[0,1]}$ and all $i \in \mathcal{I}$. Here, $x_i^1(k)$ denotes the decentralized controllable companion form of $x_i(k)$, which is obtained via a similarity transformation [4]. Inputs that satisfy (11) stabilize the global closed-loop system, which is proven in [4]. However, existence of these control actions is only guaranteed in the absence of state/input constraints.

The control schemes are assessed by simulating them in closed-loop with the power network setup given in [3], which is schematically depicted in Fig. 1. The system consists of 4 control areas (or lumped generators), interconnected via tie lines. The linearized continuous-time dynamics of each subsystem are given by the following standard model [17]:

$$\frac{d\Delta\omega_i}{dt} = \frac{1}{J_i} (\Delta P_{M_i} - D_i \Delta\omega_i - \sum_{j \in \mathcal{N}_i} \Delta P_{\text{tie}}^{ij} - \Delta P_{L_i}), \quad (12a)$$

$$\frac{d\Delta P_{M_i}}{dt} = \frac{1}{\tau T_i} (\Delta P_{V_i} - \Delta P_{M_i}), \quad (12b)$$

$$\frac{d\Delta P_{V_i}}{dt} = \frac{1}{\tau G_i} (\Delta P_{\text{ref}_i} - \Delta P_{V_i} - \frac{1}{r_i} \Delta\omega_i), \quad (12c)$$

$$\frac{d\Delta P_{\text{tie}}^{ij}}{dt} = b_{ij} (\Delta\omega_i - \Delta\omega_j), \quad (12d)$$

$$\Delta P_{\text{tie}}^{ji} = -\Delta P_{\text{tie}}^{ij}, \quad i \in \mathcal{I} := \mathbb{Z}_{[1,4]}, \quad j \in \mathcal{N}_i. \quad (12e)$$

Here, (12a)–(12c) describe the dynamics of the generator (or the equivalent of multiple generators) in control area i , with $\Delta\omega_i$ denoting the local grid frequency, and ΔP_{M_i} , ΔP_{V_i} being the turbine and governor states, respectively, all measured with respect to their nominal values. The dynamics of the tie lines that connect two areas are modeled by (12d) and (12e), where $\Delta P_{\text{tie}}^{ij}$ denotes the deviation in the power flow from area i to j compared to its scheduled value. The control input to system i is ΔP_{ref_i} , which represents the change in the reference value for the power production in that area with respect to the planned value. The exogenous disturbance input ΔP_{L_i} represents the accumulated change of power demand in control area i . The parameters used in model (12) and the values used in our simulation are listed in Table I.

TABLE I
SIMULATION PARAMETERS

Local states	$x_1 = \text{col}(\Delta P_{V1}, \Delta P_{M1}, \Delta \omega_1)$ $x_2 = \text{col}(\Delta \delta_{12}, \Delta P_{V2}, \Delta P_{M2}, \Delta \omega_2)$ $x_3 = \text{col}(\Delta \delta_{23}, \Delta P_{V3}, \Delta P_{M3}, \Delta \omega_3)$ $x_4 = \text{col}(\Delta \delta_{34}, \Delta P_{V4}, \Delta P_{M4}, \Delta \omega_4)$
Generator damping	$D_1 = 3, D_2 = 0.275, D_3 = 2, D_4 = 2.75$
Generator inertia	$J_1 = 4, J_2 = 40, J_3 = 35, J_4 = 10$
Speed regulation	$r_1 = 0.12, r_2 = 0.28, r_3 = 0.16, r_4 = 0.12$
Governor time constant	$\tau_{G1} = 4, \tau_{G2} = 25, \tau_{G3} = 15, \tau_{G4} = 5$
Turbine time constant	$\tau_{T1} = 5, \tau_{T2} = 10, \tau_{T3} = 20, \tau_{T4} = 10$
Transmission line gain	$b_{12} = 2.54, b_{23} = 1.5, b_{34} = 2.5$
One-step-ahead penalty	$F_1 = \begin{bmatrix} 1.222 & 3.847 & 1.312 \\ 3.847 & 20.29 & 20.57 \\ 1.312 & 20.57 & 542.7 \end{bmatrix}$ $F_2 = \begin{bmatrix} 1092 & 33 & 101 & -233 \\ 33 & 19 & 38 & 159 \\ 101 & 38 & 130 & 967 \\ -233 & 159 & 967 & 13266 \end{bmatrix}$ $F_3 = \begin{bmatrix} 2287 & 8 & -145 & -4833 \\ 8 & 10 & 59 & 270 \\ -145 & 59 & 505 & 3511 \\ -4833 & 270 & 3511 & 37607 \end{bmatrix}$ $F_4 = \begin{bmatrix} 1245 & 6 & -2 & -792 \\ 6 & 2 & 13 & 28 \\ -2 & 13 & 122 & 456 \\ -792 & 28 & 456 & 3555 \end{bmatrix}$
Current-state penalty	$Q_1 = 100 \cdot \text{diag}(0, 0, 5)$ $Q_2 = Q_3 = Q_4 = 100 \cdot \text{diag}(5, 0, 0, 5)$
Input penalty	$R_1 = R_2 = R_3 = R_4 = 0.1$

In our simulation, we compare the performance of the closed-loop network when recovering from a large state perturbation (or imbalance), given by

$$\begin{aligned}
 x_1(0) &= 0.01 \cdot \begin{bmatrix} -1.164 \\ -27.996 \\ 2.352 \end{bmatrix}, & x_2(0) &= 0.01 \cdot \begin{bmatrix} 0.096 \\ -2.064 \\ 0.072 \\ -0.336 \end{bmatrix}, \\
 x_3(0) &= 0.01 \cdot \begin{bmatrix} -0.168 \\ 0.132 \\ 26.184 \\ 13.404 \end{bmatrix}, & x_4(0) &= 0.01 \cdot \begin{bmatrix} 0.852 \\ -10.152 \\ -5.736 \\ 1.008 \end{bmatrix}.
 \end{aligned}$$

Moreover, we set $\Delta P_{L_i}(k) := 0$ for $k \in \mathbb{Z}_+$. The prediction model, i.e., (5), employed by all algorithms is obtained via time-discretization of (12), using the sampling period $T_s = 1$ s. This yields the discrete-time linear state-space representation

$$\begin{aligned}
 x_i(k+1) &= \phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k))) \\
 &:= A_{ii}x_i(k) + B_{ii}u_i(k) + v_i(x_{\mathcal{N}_i}(k)) \\
 v_i(x_{\mathcal{N}_i}(k)) &:= \sum_{j \in \mathcal{N}_i} A_{ij}x_j(k),
 \end{aligned} \tag{13}$$

for $i \in \mathcal{I}$, where $A_{ii} \in \mathbb{R}^{n_i \times n_i}$, $B_{ii} \in \mathbb{R}^{n_i \times m_i}$ and $A_{ij} \in \mathbb{R}^{n_i \times n_j}$. All algorithms employ the same set of quadratic cost functions, i.e.,

$$\begin{aligned}
 J_i(x_i(k), u_i(k)) &:= x_i(1|k)^\top F_i x_i(1|k) \\
 &\quad + x_i(k)^\top Q_i x_i(k) + u_i(k)^\top R_i u_i(k),
 \end{aligned}$$

with one-step-ahead state prediction $x_i(1|k) := \phi_i(x_i(k), u_i(k), v_i(x_{\mathcal{N}_i}(k)))$. The values for $F_i \succ 0$, $Q_i \succeq 0$ and $R_i \succ 0$ are listed in Table I. Note that F_i satisfies the discrete-time Riccati equation $F_i = A_i^\top F_i A_i + A_i^\top F_i B_i L_i + Q_i$, with linear quadratic regulator feedback gain $L_i = (R_i + B_i^\top F_i B_i)^{-1} B_i^\top F_i A_i$. This specific value was chosen to optimize performance, but it does neither in SC-DMPC nor in Alg. III.4 play a role in guaranteeing closed-loop stability, in contrast to classical MPC.

The method of [18] was used to compute the weights $P_i \in \mathbb{R}^{n_i \times n_i}$, $i \in \mathcal{I}$, of the local infinity-norm based candidate CLFs for Alg. III.4, i.e., $V_i(x_i) = \|P_i x_i\|_\infty$ with $\rho_i = 0.8$,

$\forall i \in \mathcal{I}$, and system (13), in closed-loop with the local feedback laws $u_i(k) := K_i x_i(k)$, $K_i \in \mathbb{R}^{1 \times n_i}$, yielding

$$\begin{aligned}
 P_1 &= \begin{bmatrix} 5.063 & 8.939 & -1.58 \\ 7.346 & 5.591 & 2.907 \\ -0.0994 & 1.688 & 11.13 \end{bmatrix}, & K_1 &= \begin{bmatrix} -0.852 \\ -0.099 \\ 6.819 \end{bmatrix}^\top, \\
 P_2 &= \begin{bmatrix} 4.897 & -3.156 & 0.992 & 10.033 \\ -1.104 & 6.513 & 22.107 & -8.855 \\ 1.53 & 3.026 & -1.283 & 21.787 \\ 4.545 & 6.887 & 2.357 & -4.246 \end{bmatrix}, & K_2 &= \begin{bmatrix} -0.415 \\ -10.292 \\ -0.772 \\ 2.524 \end{bmatrix}^\top, \\
 P_3 &= \begin{bmatrix} -9.145 & -2.669 & -4.32 & 41.963 \\ 4.489 & -0.446 & 3.866 & 27.328 \\ 3.742 & 0.669 & 4.2 & 1.067 \\ 12.231 & 1.892 & 7.73 & -41.379 \end{bmatrix}, & K_3 &= \begin{bmatrix} -1.129 \\ -3.106 \\ 0.311 \\ 74.01 \end{bmatrix}^\top, \\
 P_4 &= \begin{bmatrix} 3.194 & -4.292 & 5.041 & 20.535 \\ 3.281 & 0.866 & 13.903 & -4.559 \\ -3.756 & -4.284 & 2.553 & 7.299 \\ 7.662 & -4.822 & 7.531 & -1.119 \end{bmatrix}, & K_4 &= \begin{bmatrix} 5.821 \\ -3.575 \\ 6.52 \\ 11.11 \end{bmatrix}^\top.
 \end{aligned}$$

It is important to stress that the control laws $u_i(k) = K_i x_i(k)$ are only employed off-line, to calculate the weight matrices P_i and they are never used for controlling the system. Moreover, we set $N_\tau = 12$ in Alg. III.4. Note that by choosing infinity-norm CLFs, we are able to formulate Alg. III.4 as a quadratic program (QP), as explained in Section III-B. The implementation of SC-DMPC is based on a (more complex) quadratically constrained quadratic program (QCQP), as a result of its state-contraction constraint, i.e., (11).

We first consider an unconstrained state/input scenario, i.e., with $\mathbb{X}_i := \mathbb{R}^{n_i}$ and $\mathbb{U}_i := \mathbb{R}^{m_i}$, to compare the considered algorithms in terms of performance. The performance attained by each control scheme is measured as $\sum_{k \in \mathbb{Z}_{[0, 149]}} \sum_{i \in \mathcal{I}} x_i(k)^\top Q_i x_i(k) + u_i(k)^\top R_i u_i(k)$ and the settling time, calculated as $k_s := \arg \min \{l \in \mathbb{Z}_+ \mid \|x(k)\| \leq 10^{-4}, \forall k \in \mathbb{Z}_{\geq l}\}$. These values are listed in Table II, along with the worst-case time to compute the control action (using Matlab's quadprog and fmincon¹ solvers for Alg. III.4 and SC-DMPC, respectively, on a 3.48 GB RAM, 2.66 GHz Pentium-E PC) to assess the computational complexity of both schemes. Both schemes stabilize the state in this simulation. Moreover, the performance attained by Alg. III.4 matches that of SC-DMPC, whereas the QP problem corresponding to Alg. III.4 is of much smaller complexity than the QCQP implementation of SC-DMPC.

TABLE II
PERFORMANCE COMPARISON

Unconstrained scenario			
	Performance	Settling time	CPU time
SC-DMPC	218.1468	125	41 ms
Alg. III.4	197.1744	86	1.5 ms
Constrained scenario			
SC-DMPC (infeasible)	—	—	42 ms
Alg. III.4	216.0599	98	7.5 ms

In practice, power networks will always be subject to constraints, for physical, performance or safety reasons. Hence, in a second scenario we constrain the control inputs as

$$-0.25 \leq \Delta P_{\text{ref}_i} \leq 0.25, \quad i \in \mathcal{I}.$$

Table II summarizes the corresponding performance figures for Alg. III.4 and the SC-DMPC simulation. In contrast to the method proposed in this paper, the SC-DMPC scheme is

¹More efficient algorithms for solving QCQPs exist, but they generally require more computational effort than any QP solver.

Fig. 2. Frequency, flow and input trajectories under constrained structured max-CLF control.

Fig. 3. Evolution of $V(x(k))$ and its upper bound over time.

unable to find feasible control actions at all simulated time instants $k \in \mathbb{Z}_{[0,149]}$. The structured max-CLF controlled state trajectories asymptotically converge to $\mathbf{0}_n$, and the relevant outputs ($\Delta\omega_i$, ΔP_{tie}^{ij}) and the corresponding control inputs ΔP_{ref_i} , $i \in \mathcal{I}$ are shown in Fig. 2. Note that the input constraint is not violated, although it is active for some time instants. Fig. 3 depicts the corresponding evolution of $V(x(k))$ and the upper bound for this function as generated by condition (9) in Alg. III.4. The simulation illustrates that $V(x(k))$ is allowed to vary arbitrarily within the asymptotically converging envelope defined by (9), resulting in closed-loop stability. The contraction constraint of SC-DMPC lacks this flexibility, which leads to infeasibility of the algorithm in this particular simulation.

V. CONCLUSIONS

Stable operation of the electrical power grid in the future will require advanced control techniques such as non-centralized MPC for supply and demand matching, as a consequence of the liberalization and decentralization of electrical power generation. Therefore, this paper proposed a novel almost-decentralized Lyapunov-based predictive control algorithm for power balancing, i.e., for asymptotic stabilization of the network frequency. The algorithm is particularly suited

for large-scale power networks, as it only requires local information and short-distance communication between directly-neighboring control areas to provide a stabilizing control action. We assessed the scheme in a non-trivial simulation example, in which its performance matched that of an existing state-of-the-art almost decentralized MPC scheme, whereas it is of much lower complexity and can provide a guarantee for closed-loop stability in the presence of state/input constraints.

ACKNOWLEDGEMENTS

This research is supported by the Veni grant “Flexible Lyapunov Functions for Real-time Control”, grant number 10230, awarded by STW (Dutch Science Foundation) and NWO (The Netherlands Organization for Scientific Research). R. M. Hermans is a researcher in the EOS-Regelduurzaam project that is funded by SenterNovem.

REFERENCES

- [1] A.-M. Borbely and J. F. Kreider, *Distributed generation: the power paradigm for the new millennium*. CRC Press, 2001.
- [2] S. Stoft, *Power System Economics: Designing Markets for Electricity*. Kluwer Academic Publishers, 2002.
- [3] A. N. Venkat, I. A. Hiskens, J. B. Rawlings, and S. J. Wright, “Distributed MPC strategies with application to power system automatic generation control,” *IEEE Transactions on Control Systems Technology*, vol. 16, no. 6, pp. 1192–1206, 2008.
- [4] E. Camponogara, D. Jia, B. H. Krogh, and S. Talukdar, “Distributed model predictive control,” *IEEE Control Systems Magazine*, vol. 22, no. 1, pp. 44–52, 2002.
- [5] A. Jokić, M. Lazar, and P. P. J. Van den Bosch, “Price-based optimal control of power flow in electrical energy transmission networks,” in *Hybrid Systems: Computation and Control*, ser. Lecture Notes in Computer Science, vol. 4416. Pisa, Italy: Springer, 2007, pp. 315–328.
- [6] A. Alessio and A. Bemporad, “Decentralized model predictive control of constrained linear systems,” *European Control Conference*, pp. 2813–2818, 2007.
- [7] T. Keviczky, F. Borrelli, and G. J. Balas, “Decentralized receding horizon control for large scale dynamically decoupled systems,” *Automatica*, vol. 42, no. 12, pp. 2105–2115, 2006.
- [8] L. Magni and R. Scattolini, “Stabilizing decentralized model predictive control of nonlinear systems,” *Automatica*, vol. 42, no. 7, pp. 1231–1236, 2006.
- [9] A. Richards and J. P. How, “Robust distributed model predictive control,” *International Journal of Control*, vol. 80, no. 9, pp. 1517–1531, 2007.
- [10] M. Lazar, W. P. M. H. Heemels, and A. Jokić, “Self-optimizing robust nonlinear model predictive control,” in *Nonlinear Model Predictive Control*, ser. Lecture Notes in Control and Information Sciences, vol. 348. Springer, 2009, pp. 27–40.
- [11] A. C. R. M. Damoiseaux, A. Jokić, M. Lazar, A. Alessio, P. P. J. van den Bosch, I. A. Hiskens, and A. Bemporad, “Assessment of decentralized model predictive control techniques for power networks,” *Power Systems Computation Conference*, 2008.
- [12] R. M. Hermans, M. Lazar, and A. Jokić, “Almost decentralized Lyapunov-based nonlinear model predictive control,” in *American Control Conference*, Baltimore, MD, USA, 2010.
- [13] C. M. Kellett and A. R. Teel, “On the robustness of $\mathcal{KL}$ -stability for difference inclusions: Smooth discrete-time Lyapunov functions,” *SIAM Journal on Control and Optimization*, vol. 44, no. 3, pp. 777–800, 2005.
- [14] M. Lazar, W. P. M. H. Heemels, and A. R. Teel, “Lyapunov functions, stability and input-to-state stability subtleties for discrete-time discontinuous systems,” *IEEE Transactions on Automatic Control*, vol. 54, no. 10, pp. 2421–2425, 2009.
- [15] M. Lazar, “Flexible control Lyapunov functions,” *American Control Conference*, pp. 102–107, 2009.
- [16] A. Jokić and M. Lazar, “On decentralized stabilisation of discrete-time nonlinear systems,” *American Control Conference*, pp. 5777–5782, 2009.
- [17] P. Kundur, *Power system stability and control*. McGraw-Hill, 1994.
- [18] M. Lazar, W. P. M. H. Heemels, S. Weiland, and A. Bemporad, “Stabilizing model predictive control of hybrid systems,” *IEEE Transactions on Automatic Control*, vol. 51, no. 11, pp. 1813–1818, 2006.