On the Optimal Radiation Bandwidth of Printed Slot Antennas Surrounded by EBGs

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Abstract—This paper describes a design strategy to achieve the maximum bandwidth and efficiency for a printed slot antenna surrounded by EBGs. First the dielectric constant and the thickness of the dielectric slab that guarantees an acceptable front to back radiation ratio is identified. Then electromagnetic bandgap (EBG) structures are designed to achieve the optimal bandwidth (BW) while obtaining high surface wave efficiency in the radiating half space. To achieve this goal the wave interaction between the slot and the EBG structures is investigated in depth and clearly described. For the case of Planar circularly symmetric (PCS) EBGs the maximum of radiation BW is shown to occur when the distance between the central antenna and the EBG is approximately half wavelength of the first surface wave, \( \lambda_{sw}/2 \).

Index Terms—Electromagnetic bandgap (EBG), planar circularly symmetric electromagnetic bandgap (EBG), planar printed antennas, slot antennas, surface waves, wideband antennas.

I. INTRODUCTION

THE design of antennas surrounded by electromagnetic bandgap (EBG) structures has been the object of many investigations (see [1] for an overview). In many cases the introduction of the EBG materials was aimed at enhancing the directivity [2] of the antenna, at reducing the mutual coupling [3] or at increasing the scan performances in arrays [4]. When the antenna and EBG are printed on the same slab, a problem that systematically arises is choice of the optimal distance between the antenna and the surrounding EBG. In [5] simulations for different distances have been presented showing a significant impact of this parameter on the input impedance. In [6], the fact that the distance parameter should be selected to avoid reactive coupling between the antenna and the EBG was emphasized.

This paper describes a design strategy to achieve the maximum bandwidth and efficiency, in terms of both front to back ratio and surface wave suppression, for a printed slot antenna surrounded by EBGs. When a slot antenna is printed on a surface-wave less substrate, the real part of the admittance of the slot becomes the key parameter, since it is essentially associated only to power radiated in space. In this situation, the input impedance BW corresponds to the radiation BW.

In Section II, assuming that surface waves are not launched, the front to back radiation ratio of the slot is traded off with radiation BW using the dielectric constant and the thickness of the slab as design parameters. Unfortunately substrates that do not support surface waves do not exist and, even assuming an ideal EBG, this condition would only be fulfilled at one single frequency.

Also ideal EBG structures do not exist, and thus the remaining part of the paper is devoted to the design of a realistic EBG structure that minimizes, over a certain frequency band, the surface waves excited by a central slot antenna. In particular, in Sections III and IV, it is described how the distance between the antenna and the EBG can be selected to control the impact of the EBG on the antenna admittance. A simplified analytical model of the interaction between the antenna and the EBG is introduced. The simplification consists of considering a two dimensional problem that is infinitely extended in one dimension and that represents the interaction between the antenna and the EBG via a single surface wave mode.

The applicability of the single mode representation of the field is discussed in Section V. In that section a real slot antenna excited via microstrip and surrounded by a planar circularly symmetric (PCS) EBG, Fig. 1, has been simulated via full wave tools, for different values of the distance \( p_1 \). The wave interaction described in this paper provides a clear physical explanation for the increment in BW that was already anticipated in [7].

II. SUBSTRATE SELECTION

The slot, in absence of the EBG, is shown in Fig. 2. It is etched on a ground plane that divides two dielectric slabs with the same dielectric constant \( \varepsilon_r \) and different heights \( h_1 \) and \( h_{m} \). The length and width of the slot are \( l_s \) and \( w_s \), respectively. The slot is excited via a microstrip whose extension beyond the slot crossing is \( l_{stub} \), while its width is \( w_{m} \). The efficiency of printed antennas using elementary dipoles was already discussed in [8]. In this section, this discussion is extended to introduce a BW versus front to back ratio trade off in absence of surface waves. This condition will be then realized via EBGs.
The optimal substrate thickness and dielectric constant, from the point of view of radiation bandwidth, can be determined by just considering the properties of radiation of an elementary slot in the upper half space alone, characterized by $\varepsilon_r, h$. The power radiated in the lower half space by an elementary magnetic dipole located on the ground plane can be approximated as the one radiated in a free half space. This is due to the fact that the thickness of the lower dielectric slab is taken in the order of $\lambda_d/100$, where $\lambda_d$ is the free space wavelength. The power radiated in the upper half space, divided by the power radiated in the lower half space is plotted in Fig. 3, as a function of the thickness of the slab in terms of the wavelength in the dielectric $\lambda_d$, for $\varepsilon_r = 9.8$ (continuous line). This normalization of the power allows to directly obtain a measure of the front to back radiation of the slot. The graph also shows the power launched in the first surface waves, normalized to the downward radiated power. Similar graphs can be obtained for different values of the dielectric constant.

From Fig. 3 one can observe that the curve of the power radiated by a slot has a peak for thickness $h \approx (\lambda_d)/4$. In this condition the front to back radiation ratio of the antenna is optimum but the efficiency of the antenna is very low since the surface waves are strongly excited. If one defines the surface wave efficiency of the slot $\eta_{SW}$ as

$$\eta_{SW} = \frac{P_{rad}}{P_{rad} + P_{SW}}$$  \hspace{1cm} (1)

the efficiency of a slot, printed on a $\varepsilon_r = 9.8$ slab, has acceptable values (larger than 80%) only when the dielectric slab is $h/\lambda_d < 0.1$, and consequently the front to back radiation ratio is small. For these reasons slot antenna designs are typically done for substrate parameters that provide a high surface wave efficiency but at the cost of a poor front to back ratio.

A. Surface-Wave Less Substrate

The front to back ratio of slot antennas could be greatly enhanced if one was able to suppress both TE and TM surface waves. Let us assume for the moment that this situation can be realized. The dispersion diagram in Fig. 4 shows that, besides the first surface waves, $TM_0$ and $TE_1$, also a TM leaky wave is present before the $TE_1$ enters in propagation. The leaky pole seems to become less relevant for slabs thicker than $0.26\lambda_d$, and
our search algorithms could not track it any more after it merges with the branch in \( k_0 \). In a certain frequency range, typically just before the cutoff of the \( \text{TE}_3 \) mode, the leaky wave radiation cannot be neglected and can give rise to significant pattern deformations. This pattern deformation is related to a low value of the attenuation constant of the leaky mode, \( |\text{IM}[k_{LW}]| < 1.5k_0 \). Accordingly, both a deformation of the radiation pattern, and an increase of the mutual coupling between elements in array environment would be unavoidable if one operated the slot at a frequency at which the slab thickness is around \( \lambda_d/4 \). In the example of Fig. 4, the thicknesses from \( h/\lambda_d = 0.239 \) to \( h/\lambda_d = 0.26 \) are associated with strong leaky wave effects.

Such a leaky wave is difficult to suppress due to the fact that its wave velocity is significantly different from the one of the \( \text{TM}_0 \) surface wave. For this reason, slab thicknesses that would support leaky waves must be avoided. In the example of Fig. 4, the slab can be chosen with normalized thickness \( h/\lambda_d \) either smaller than 0.239 or larger than 0.26. The former case is convenient since it supports only the existence of the \( \text{TM}_0 \) wave, while the latter would support both \( \text{TM}_0 \) and \( \text{TE}_3 \). Considering that suppressing only one mode is easier, the slab’s thickness in this paper is selected to be \( h/\lambda_d < 0.239 \).

This choice does not imply any sacrifice in terms of radiation BW in comparison with the thicker slab alternative, \( h/\lambda_d > 0.26 \). In fact, the maximum radiation BW that can be achieved is proportional only to the derivative of the radiation power curve in Fig. 3. This is approximately the same for the two alternative ranges of the dielectric slab thickness.

Let us now quantify the maximum radiation BW associated with the optimal substrate for a slot antenna in a surface-wave less substrate considering only the radiation contribution associated to the space wave emanating from the slot directly. For frequencies where the leaky wave is negligible, \( |\text{IM}[k_{LW}]| > 1.5k_0 \), the space wave power is proportional to the radiated power curve of Fig. 3. Consequently, the space wave slot conductance associated to the upper media, normalized to the downward conductance, can be seen as the normalized power

\[
P_{\text{rad}} = G_{\text{rad}}.\tag{2}
\]

One can then assume \(-10\) dB as acceptable reflection coefficient \( \Gamma \) at the point of connection between the slot and a transmission line \( |\Gamma| < 0.316 \).

The maximum height of the dielectric slab that allows to neglect the leaky wave radiation is \( h = 0.239\lambda_d \) (see Fig. 4). One then obtains the corresponding maximum normalized conductance, \( G_{\text{rad}}^{\text{max}} \), from Fig. 3. From this value and the requirement on the reflection coefficient amplitude, it is possible to derive the normalized characteristic admittance of the best matching transmission line

\[
G_{\text{line}} = G_{\text{rad}}^{\text{max}} \left\{ \frac{1 - |\Gamma|}{1 + |\Gamma|} \right\}.\tag{3}
\]

The minimal value of the normalized conductance, \( G_{\text{rad}}^{\text{min}} \), that gives the same reflection coefficient can be expressed as

\[
G_{\text{rad}}^{\text{min}} = G_{\text{line}} \left\{ \frac{1 - |\Gamma|}{1 + |\Gamma|} \right\}.\tag{4}
\]

Accordingly, the relative radiation BW can be expressed in terms of the relative variation of the normalized slab’s thickness

\[
\frac{2f_{\text{max}} - f_{\text{min}}}{f_{\text{max}} + f_{\text{min}}} = \frac{2h/\lambda_d-\text{min} - h/\lambda_d-\text{max}}{h/\lambda_d-\text{min} + h/\lambda_d-\text{max}}\tag{5}
\]

where \( \lambda_d-\text{min} \) corresponds to \( f_{\text{max}} \), the higher operational frequency defined by \( G_{\text{rad}}^{\text{max}} \), and \( \lambda_d-\text{max} \) corresponds to the minimal operational frequency associated to \( G_{\text{rad}}^{\text{min}} \).

Following this procedure, the maximum theoretical bandwidths for a surface-wave less substrate with \( \epsilon_r = 9.8 \) is 43.9%.

Table I reports the maximum values of normalized conductances, \( G_{\text{rad}}^{\text{max}} \), for several dielectric constants. Their actual values are important since they can be read as front to back ratios at the band extreme. For lower dielectric constants, the BW are larger, [9], however the front to back ratios are smaller, thus a preliminary trade off between the front to back ratio and the BW must be performed, even before considering the surface wave efficiency.

### B. Ideal EBG

It is not possible to completely avoid exciting surface waves in dielectric slabs since they are intrinsically needed to satisfy the boundary conditions. However, a source will not launch real power into surface waves when it is surrounded by a cavity that guarantees the cancellation of the outgoing wave with a reflected wave of the same amplitude. In the present study, rather than cavities [10], we will consider an ideal EBG as a periodic structure operating in the bandgap to reproduce the cavity behavior.

The surface wave component of the slot admittance in presence of such ideal EBG presents real part equal to zero since no power is lost in surface waves. However at nearly all frequencies, except at resonance, the ideal EBG would induce an imaginary part to the admittance associated to surface wave reflections. The resonant point is defined by tuning the cavity dimension corresponding to the distance between the antenna and the EBG. The presence of the imaginary part of the admittance induced by the EBG significantly reduces the bandwidth with respect to the theoretically optimal case of the previous subsection.

The scope of this paper is the analysis of PCS structures as in Fig. 1 that are realized in planar technology and aim at suppressing \( \text{TM}_0 \) waves only. In next section, the different components of the admittance of a slot surrounded by a planar EBG in a two-dimensional (2-D) case will be described. The deviations from the ideal behavior will be explicitly quantified.

### III. 2-D Model

In order to design a realistic EBG that achieves the best TM surface wave suppression it is important to establish the distance between the antenna and the EBG that best approaches the ideal EBG behavior. A simplified analytical model of the interaction between the antenna and the EBG is represented by

<table>
<thead>
<tr>
<th>( \epsilon_r )</th>
<th>4</th>
<th>9.8</th>
<th>16</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_{\text{rad}}^{\text{max}} )</td>
<td>2.1</td>
<td>6.46</td>
<td>11.9</td>
<td>23.8</td>
</tr>
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| TABLE I
| Maximum Conductances for Slots in the Presence of Surface Wave Less Slabs for Several Dielectric Constants |
the 2-D, \( y \)-independent, structure of Fig. 5(a), with a TM field configuration (magnetic field along \( y \)). The 2-D structure is obtained as the cut for \( \phi = 0 \) of the three-dimensional (3-D) structure of Fig. 1. The source is an aperture of width \( w_s \) etched on the ground plane with current distribution \( \bar{m}_s \). The same finite number of strip gratings is printed on top of the slab at the left and the right of the aperture. The first gratings are located at a distance \( x_1 \) from the central source, which corresponds to \( \rho_1 \) of the 3-D structure (Fig. 1).

The periodicity \( d \) and length of the strips \( l_g \) are designed, as explained in [7], in order to obtain a bandgap of the TM\(_0\) mode in the frequencies from 4.8 to 6.4 GHz. The procedure is based on the solution of an integral equation derived enforcing the vanishing of the electric field (EFIE) on the metallic loadings, which is considered to be periodic with the structure infinitely extended also in \( x \). The real and imaginary parts of the normalized solutions, \( k_f/k_0 \), of the dispersion equations pertinent to the grounded dielectric slab (\( \varepsilon_r = 9.8, h = 3.81 \) mm) loaded by a strip grating are shown, with continuous line, in Fig. 6. Note that the specific dielectric thickness has been chosen since it is commercially available.

Once the design of the EBG is fixed, the finite structure in Fig. 5(a), can be analyzed using a standard element by element method of moments (MoM). Given a lumped electric current along \( I_x \) as excitation of the slot, the current on all strips, the voltage on the slot, the impedance and the efficiency of the system can all be calculated as a function of the frequency. The surface wave efficiency, (1), is plotted in Fig. 7 for the slot alone, and when one, two and four strips are located on each side of the slot. Here \( x_1 \) is chosen arbitrarily to be equal to the periodicity \( x_1 = d = 13.7 \) mm. With the infinite model, the attenuation constant of the mode inside the bandgap can be derived, see Fig. 6, and then used as indication of the number of EBG elements necessary to attenuate the mode. A significant improvement of the efficiency is obtained in the bandgap region when the number of elements goes from 1 to 2. A smaller improvement is obtained when other two EBG elements are added. With further inclusions of elements there are no significant improvements.

A. Single Mode Field Representation

In parallel to the full wave analysis and in order to derive the physical insight necessary to determine the influence of the parameter \( x_1 \), we can set up a simplified model to characterize the wave interaction. The \( x \)-component of the electric field can be represented, resorting to the equivalence theorem, as the sum of the field radiated by the slot on an infinite substrate in absence of the EBG plus the field scattered by the EBG

\[
E_{\text{tot}}(\mathbf{r}) = E_{\text{sl}}(\mathbf{r}) + E_{\text{elbg}}(\mathbf{r})
\]  

(6)

where \( \mathbf{r} = (x, z) \). In general all the spectral components are necessary to represent the \( E_{\text{sl}} \) in the vicinity of the antenna or \( E_{\text{elbg}} \) in the vicinity of the EBG

\[
E_{\text{sl}}(\mathbf{r}) = E_{\text{sl}}^{\text{SW}}(\mathbf{r}) + E_{\text{sl}}^{\text{rad}}(\mathbf{r})
\]

(7)

\[
E_{\text{elbg}}(\mathbf{r}) = E_{\text{elbg}}^{\text{SW}}(\mathbf{r}) + E_{\text{elbg}}^{\text{rad}}(\mathbf{r}) + E_{\text{elbg}}^{\text{elbg}}(\mathbf{r})
\]

(8)
where the apex rad stands for radiated field which is defined as the difference between the complete fields and their TM$_0$ surface wave components. Note that in other contexts [11], these difference fields are indicated as fringe fields rather than radiated fields. The surface wave components are indicated via the superscript $+$ and $-$ for outgoing or ingoing waves respectively. When the EBG field, $E_{\text{EBG}}$, is observed in the vicinity of the antenna, or conversely the slot field, $E_{\text{sl}}$, is observed in the vicinity of the EBG, the surface waves contributions are dominant. Thus, in the following, we will concentrate only on the surface wave contribution. The projection on the surface wave components of the total field, can be represented as a standing wave pattern [see Fig. 5(b)], for instance for the electric field.

$$E_{\text{tot}}^\text{sw}(\mathbf{r}) = E_{\text{sl}}^\text{sw}(\mathbf{r}) + E_{\text{EBG}}^\text{sw}(\mathbf{r}) + P_{\text{EBG}}^\text{sw}(\mathbf{r})$$

(9)

The surface wave electric and magnetic fields radiated inside the slab can all be derived by capturing the residue at the surface wave pole $k_{\text{sw}}$ that appears in the spectral representation of the pertinent Green’s function of the substrate. All the residue expressions used in this paper are given in the Appendix. $E_{\text{sl}}^\text{sw}(\mathbf{r})$, radiated by a slot of current distribution $m_s$, can be expressed as

$$E_{\text{sl}}^\text{sw}(\mathbf{r}) = jM_s(k_{\text{sw}})\text{Res}(V_{TM}^j(k_{\text{sw}}, z))e^{-jk_{\text{sw}}z}\chi.\quad (10)$$

For narrow slots one can approximate the magnetic current distribution of the source, $m_s$, via a constant function so that $M_s = FT(m_s)$ is also known.

The surface wave electric field scattered by the EBG, $E_{\text{EBG}}^\text{sw}(\mathbf{r})$, can be described, in the region $|x| < x_1$, resorting to two equal equivalent elementary electric currents located at $x_1$ and $-x_1$, and oriented along $x$ [see Fig. 5(b)]

$$I_{\text{EBG}}^2 = I_{\text{EBG}}^2(x = \pm x_1)\chi.\quad (11)$$

The amplitude of these currents is unknown at this stage. The surface wave electric fields radiated by these currents can be expressed, highlighting the outgoing and ingoing waves, as

$$E_{\text{EBG}}^\text{sw}(\mathbf{r}) = jI_{\text{EBG}}\text{Res}(V_{TM}^j(k_{\text{sw}}, z))e^{-jk_{\text{sw}}z}e^{j\pi k_{\text{sw}}x}\chi.\quad (12)$$

Using these analytical, residue based contributions, the actual value of the equivalent current amplitude $I_{\text{EBG}}$ can be obtained characterizing the EBG structure in terms of an equivalent reflection coefficient. Observing the field at $(x = x_1, z = h)$, one can write the reflection coefficient as the ratio between the ingoing and outgoing waves fields

$$R_{\text{EBG}} = \frac{E_{\text{EBG}}^\text{sw}}{E_{\text{tot}}^\text{sw}} = \frac{E_{\text{EBG}}^\text{sw}(\mathbf{r})}{E_{\text{sl}}^\text{sw}(\mathbf{r}) + E_{\text{EBG}}^\text{sw}(\mathbf{r})}^{\chi = \pm h}.\quad (13)$$

Substituting (10) and (12) into (13), one can obtain the expression for the amplitude of the equivalent current as a function of the reflection coefficient of the EBG

$$I_{\text{EBG}} = M_s(k_{\text{sw}})\frac{\text{Res}(V_{TM}^j(k_{\text{sw}}, h))}{\text{Res}(V_{TM}^j(k_{\text{sw}}, h))} \frac{R_{\text{EBG}}e^{-j\pi k_{\text{sw}}x_1}}{jR_{\text{EBG}}e^{-j\pi k_{\text{sw}}x_1}}.\quad (14)$$

### B. 2-D-EBG Reflection Coefficient

The important aspect of the single mode representation of the fields is that all quantities are expressed analytically once the reflection coefficient of the surface wave impinging on the EBG is known. We define the reflection coefficient with respect to the component of the electric field parallel to the direction of propagation of the wave, $i_x$. Since the boundary condition that the EBG enforces on the electric field is $E_i + E_s = E_{\text{tot}} = 0$ and assuming an ideal EBG where $E_s = \Gamma E_i$ the reflection coefficient must be $\Gamma = -1$. In real EBGs, this reflection coefficient is instead a frequency dependent complex value.

The reflection coefficient can be obtained from the analysis of the canonical problem in Fig. 5(c). Only a few elements of the EBG are retained in the analysis since the electric current distribution decreases rapidly for the elements that are further away from the source when the frequency of operation is well inside the bandgap of the structure [7]. In this canonical configuration the reflection coefficient for 2-D-EBG is defined as follows:

$$R_{\text{EBG}} = \frac{E_{\text{EBG}+}(\mathbf{r})}{E_{\text{EBG}+}(\mathbf{r})}^{\chi = \pm h}$$

(15)

where $E_{\text{EBG}+}$ and $E_{\text{EBG}-}$ are the amplitudes of the direct and reflected TM$_0$ surface waves, respectively.

The analysis can be performed solving the same MoM code that was used to obtain the results of Fig. 7. In this case it solves the (EFIE) that imposes the vanishing of the tangent electric field on the metallic gratings under surface wave incidence

$$E_{\text{EBG}+}(\mathbf{r}) = E_0e^{-j\pi k_{\text{sw}}x}i_x.$$

(16)

Once the MoM has been solved, the electric current on the entire grating is known, $j_{\text{gr}}(\mathbf{r})$. One can then obtain the field $E_{\text{EBG}}$ by first expressing the scattered field in the spectral domain and by then applying again the residue theorem to retain only the surface wave contribution at $k_x = -k_{\text{sw}}$. The actual expression is

$$E_{\text{EBG}}^- = jI_{\text{gr}}(-k_{\text{sw}})\text{Res}(V_{TM}^j(k_{\text{sw}}, z))e^{j\pi k_{\text{sw}}x}$$

(17)

where $j_{\text{gr}}(k_x)$ is the Fourier transform of $j_{\text{gr}}(\mathbf{r})$.

In Fig. 8 the amplitude and phase of the reflection coefficient is shown, for the structure of Fig. 5(c) including 1, 2, or 4 elements in the x-direction. The curve outlines the bandgap region shown in Fig. 7 via a significant change of derivative of the reflection coefficient as a function of the frequency. The reflection coefficient of real EBGs have amplitudes lower than one. However, since the surface wave efficiencies observed in Fig. 7 for the same configurations correspond to almost 100%, it can be concluded that the remaining portion of the power is radiated. This radiation is responsible for an increase of the directivity of an antenna surrounded by the EBG, as was experimentally observed in [7].

### IV. Input Admittance

The single mode representation allows to rigorously define the surface wave admittances briefly introduced in Section II-B. Generally, these are considered more useful design parameters
than the field themselves. The admittance can be expressed in the space domain as

\[ Y = \int_{-w_s/2}^{w_s/2} m_b(x)H(x) \, dx \]  

(18)

where \( m_b \) is the magnetic current distribution on the slot and \( H \) is the tangent magnetic field (along \( y \)). Then, the total input admittance of the slot can be expressed as the superposition of two reaction integrals. The first involves the magnetic field in absence of the EBG for an infinite substrate, while the second one involves the field radiated by the equivalent EBG currents. Accordingly, an appropriate representation is

\[ Y_{\text{tot}} = Y_{\text{sl}} + Y_{\text{ebg}}. \]  

(19)

This expression is the circuit equivalent of (6). In the self reaction \( Y_{\text{sl}} \) it is possible to distinguish a surface wave term and a radiated term

\[ Y_{\text{sl}} = Y_{\text{sl}}^{\text{sw}+} + Y_{\text{sl}}^{\text{rad}}. \]  

(20)

The term \( Y_{\text{sl}}^{\text{sw}+} \) can be calculated analytically resorting to standard slab’s GF theory and applying the residues theorem

\[ Y_{\text{sl}}^{\text{sw}+} = G_{\text{sl}}^{\text{sw}+} = -j M_8^2(k_{\text{sw}}) \text{Res}(I_{\text{TM}}^N(k_{\text{sw}}, 0)). \]  

(21)

Notice that the surface wave contribution is completely real since all the power launched by the slot in surface waves is lost in the substrate which is assumed infinitely extended.

In Fig. 9 the real and imaginary part of the total slot admittance in absence of an EBG is shown together with its two components: the surface wave \( G_{\text{sl}}^{\text{sw}+} \) and the radiated \( Y_{\text{sl}}^{\text{rad}} \) one. One can note that \( G_{\text{sl}}^{\text{sw}+} > Y_{\text{sl}}^{\text{rad}} \) is an explicit manifestation of the low efficiency of the slot alone.

The reaction, \( Y_{\text{ebg}} \), can be expressed via a surface wave contribution only, for \( \lambda_1 \) sufficiently large, i.e., \( Y_{\text{ebg}} \approx Y_{\text{ebg}}^{\text{sw}} \)

\[ Y_{\text{ebg}}^{\text{sw}} = -j 2 R_{\text{ebg}} M_b(k_{\text{sw}}) \text{Res}(I_{\text{TM}}^N(k_{\text{sw}}, 0)) e^{-j k_{\text{sw}} x_1} \]  

(22)

where the factor 2 is due to the presence of the two equivalent EBG currents.

Substituting in (22) the expressions of the residues (see Appendix) and the equivalent current amplitude (14), then using (21) one can derive the following relation:

\[ Y_{\text{ebg}}^{\text{sw}} = 2 G_{\text{sl}}^{\text{sw}+} \frac{R_{\text{ebg}}}{e^{j 2 k_{\text{sw}} x_1} - R_{\text{ebg}}}. \]  

(23)

This analytical relation explicitly shows how the impact of \( \lambda_1 \) on the admittance of the slot is modulated by \( R_{\text{ebg}} \).

Now we can clarify the ideal EBG concept that was mentioned in Section II-B. In fact for \( \lambda_1 = \lambda_{\text{sw}}/2 \) and assuming an EBG that realizes a short circuit for the impinging surface wave, \( R_{\text{ebg}} = -1 \), the EBG contribution to the surface wave admittance, (23), becomes

\[ Y_{\text{ebg-ideal}}^{\text{sw}} = -G_{\text{sl}}^{\text{sw}+}. \]  

(24)

This expression describes the situation in which all the impinging surface wave power is reflected from the EBG toward the slot with such a phase to cancel the outgoing surface wave. The real part of \( Y_{\text{ebg-ideal}}^{\text{sw}} \) is equal to \( -G_{\text{sl}}^{\text{sw}+} \) for all frequencies, while its imaginary part is equal to zero at least for one frequency point. In that frequency point, only \( Y_{\text{sl}}^{\text{rad}} \), which is essentially associated to the radiated space wave, would remain to be accounted for. Fig. 10 shows the admittances of the slot for the cases of ideal EBG and real EBG using the same value of \( \lambda_1 = \lambda_{\text{sw}}/2 \approx 21 \text{ mm} \). In the real EBG case, the real part
of surface wave admittance associated to an infinitely extended slab is not completely cancelled by the reflected waves.

Fig. 11 plots the EBG surface wave admittance, (22), the slot admittance, (20), and sum of the two, $Y_{\text{tot}}$, as a function of $x_1$. The frequency selected is $f = 5.5$ GHz at the center of the band. The total wave admittance when the EBG is present oscillates around the value of the admittance in absence of the EBG. It can be observed that the minimum value of the real part of the total admittance is obtained when $x_1 = \lambda_{sw}/2 = 20.7$ mm (i.e., half of the surface wave length).

Using (19), in which $Y_{\text{sl}}$ is evaluated full wave and $Y_{\text{ebg}}$ with the analytical approximation in (23), one can evaluate the input impedance of the antenna in the presence of the EBG, Fig. 12. To validate the formulation, the same structure has also been analyzed with the standard 2-D el. by el. MoM. The comparison is very good highlighting that the impact of the EBG on the input impedance is very well described by the surface wave interactions only. In the same figure, also the impedance considering an ideal EBG is plotted. In this specific case, even if the actual value of the total impedance is derived using the ideal reflection coefficient, the resonant frequency point is nevertheless well approximated.

V. APPLICABILITY OF THE SINGLE MODE REPRESENTATION

In the previous section, a 2-D model has been developed to analyze the effect of the EBG on the admittance of a slot. More-

over the model was based on a single mode interaction that is assumed valid for large values of $x_1$ in the 2-D case. Both these aspects would seem to limit the applicability of the results presented in realistic (3-D) configurations where the EBG is so close to the antenna that the $TM_0$ surface wave is not sufficient to describe the complete interaction.

However the important aspect of the previous field representation is not the quantitative one, there are full wave simulators for that. The key issue is that the physical insight of the single mode interaction guides the initial slot-EBG design already very close to the final one. Even without knowing the actual value of the reflection coefficient, but just thinking in terms of an ideal EBG, one knows that the radius $\rho_1$, in configurations as those in Fig. 1, has to be close to half of the surface wave wavelength for maximum radiation bandwidth. The actual phase of the reflection coefficient will only moderately affect the value of $\rho_1$. Moreover, the amplitude of the reflection coefficient being lower than unity does not really play a major role besides giving some indication on how much radiation emerges from the EBG.

To emphasize this design aspect, three different slots in the presence of PCS-EBG configurations (see Fig. 1) have been investigated using Ansoft Designer. The slabs are the same in all the cases. The radii were $\rho_1 = 10$ mm, $\rho_1 = 18$ mm.
and $r_1 = 27$ mm, roughly corresponding to $\lambda_{sw}/4, \lambda_{sw}/2$ and $3/4\lambda_{sw} \approx \lambda_0/2$. In order to perform a fair comparison between the different configurations the central slots were tuned in the three cases to achieve the best match at the central frequency of 5.5 GHz when fed by a 50 $\Omega$ microstrip line extended beyond the slot in an open circuit stub, see Fig. 2. The reflection coefficients pertinent to these three configurations are shown in Fig. 13 together with the reflection of the antenna without EBG. It is apparent that the largest bandwidth, around 13$\%$ at -10 dB, is obtained with $r_1 \approx \lambda_{sw}/2$. In this case the BW is even larger than the one of the slot in absence of the EBG (a case with surface wave efficiency less than 40$, \%$, see Fig. 7). Much lower BWs are obtained for the other radii. In the case of $r_1 = \lambda_0/2$ case, the BW is reduced because the reflections from the EBG return in phase with the outgoing surface waves. For the smallest radius case, the BW is even smaller as the reactive coupling between the slot and the EBG has also a significant effect on the frequency behavior.

One may note that 18 mm is not exactly half of the surface wave wavelength at the center of the bandwidth. It was the value selected for the antenna prototype in [7]. In a realistic case, the difference between 18 mm and 20.7 mm is the range of approximation of the simplified design rule, $r_1 \approx \lambda_{sw}/2$.

The gain calculated by Ansoft Designer can be interpreted as a measure of the radiation efficiency of the antenna. Fig. 14 shows the gains for the four configurations at broadside as a function of the frequency. The antenna without EBG has a very low gain due to the fact that more than 60$\%$ of the power is lost into the $TM_0$ surface wave (Ansoft Designer considers an infinitely extended substrate). The case of $r_1 \approx \lambda_{sw}/2$ provides the highest gain overall the frequency band. The case of $r_1 = \lambda_0/2$ provides essentially the same gain of $r_1 \approx \lambda_{sw}/2$ at the central frequency, but lower gain in average over the complete band. The fact that a distance between the antenna and the EBG of $r_1 \approx \lambda_0/2$ results in a higher gain was emphasized also in [12] for a single soft-surface structure.

VI. CONCLUSION

The role of the distance between a central resonant antenna and a surrounding EBG structure printed on the same slab has been addressed in this paper. The optimal radiation bandwidth for a printed slot surrounded by an PCS-EBG is achieved when the inner radius, $r_1$, of the equivalent EBG cavity is approximately half of the first surface wave wavelength $\lambda_{sw}/2$. In this situation the wave reflected from the EBG cancels out the outgoing waves emanated by the source. At least in first approximation all the power delivered to the slot is then radiated. Any other configuration reduces the BW. If $r_1$ is too small the coupling between the slot and the EBG is reactive and the field around the slot does not have sufficient room to develop itself into radiation. If $r_1$ is too large the SWs reflected from the EBG add in phase with the ones launched by the slot. Thus the optimal value of $r_1$ can be tuned using a single mode model that characterizes the interaction of the SW with a planar EBG by means of a reflection coefficient. The procedure was demonstrated to be accurate and close to analytical in a simple 2-D case. It is possible to extend the approach to more realistic 3-D structures. However, for simple planar antenna+EBG structures full wave commercial tools are fast and accurate enough to be used in the
final refinement phase, if the initial design, based on the 2-D analysis, is already close to the final one.

APPENDIX

The surface wave electric and magnetic fields radiated inside the slab can be derived by applying the residues theorem at the surface wave pole \( k_{sw} \) to the spectral representation of the field using the Green’s function of the substrate. If a function \( F(k_p) = (N(k_p))/(D(k_p)) \) has a pole in \( k_p = k_{sw} \), the residues can be calculated as 

\[
\text{Res}(F(k_p)) = (F(k_{sw}))/(D(k_p)|_{k_p=k_{sw}}).
\]

The residue of the voltage solution inside the slab, \( z < h \), of the TM transmission line associated to the grounded slab with a series voltage source at the ground plane, used in (10), has the following expression:

\[
\text{Res}(V_{TM}^m(k_{sw}, z)) = \frac{\sin(k_z z) - \sin(k_z h)}{\sin(k_z h)} D_{TM}^m(k_{sw}) \tag{25}
\]

where, see the equation at the top of the page.

The residue of the voltage solution at \( z < h \) of the TM transmission line associated to the grounded slab with a parallel current source at \( z = h \), used in (12) and (17), has the following expression:

\[
\text{Res}(\tilde{V}_{TM}^m(k_{sw}, z)) = j \frac{\cos(k_z z)}{k_0} \frac{\sin(k_z h)}{D_{TM}^m(k_{sw})} \tag{26}
\]

where \( D_{TM}^m(k_{sw}), k_z \) and \( k_{zo} \) are the same as in (25).

The residue of the current solution at \( z = 0 \) of the TM transmission line associated to the grounded slab with a series voltage source at the ground plane, appearing in (21), assumes the form

\[
\text{Res}(P_{TM}^m(k_{sw}, 0)) = j \frac{\cos(k_z z)}{k_0 \sin(k_z h)} \frac{\sin(k_z h)}{D_{TM}^m(k_{sw})} \tag{27}
\]

where, see the equation at the top of the page.

The residue of the current solution at \( z = 0 \) of the TM transmission line associated to the grounded slab with a parallel current source at \( z = h \), appearing in (22), can be expressed as

\[
\text{Res}(\tilde{I}_{TM}^m(k_{sw},0)) = \frac{\cos(k_z z)}{k_0} \frac{\sin(k_z h)}{D_{TM}^m(k_{sw})} \tag{28}
\]

where \( D_{TM}^m(k_{sw}), k_z \) and \( k_{zo} \) are the same as in (25).

ACKNOWLEDGMENT

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