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Long Distance Synchronization of Mobile Robots

Alejandro Alvarez-Aguirre, Henk Nijmeijer, and Toshiki Oguchi

Abstract—This paper considers the long distance master-slave and mutual synchronization of unicycle-type mobile robots. The issues that arise when the elements of a robotic network are placed in different locations are addressed, specifically the time-delay induced by the communication channel linking the robots. Experiments between wirelessly controlled mobile robots located in Eindhoven, The Netherlands and Tokyo, Japan demonstrate the applicability of the proposed approach.

I. INTRODUCTION

In several fields the use of robots can be beneficial in order to complete tasks that are complex, that have rigorous timing or precision requirements, or that are to be carried out in hostile environments. Specifically, the use of groups of mobile robots that work together allows for greater flexibility and coverage. Possible applications include logistics [1], payload transportation [2], simultaneous localization and mapping [3], automated highway systems [4], and reconnaissance and surveillance [5], [6]. Additional applications are presented in [7] while a recent overview is provided in [8].

In order to fully exploit the capabilities of these configurations several theoretical and technological issues remain, such as the development of appropriate group coordination and cooperative control strategies. In this sense, the leader-follower, the virtual structure and the behavioral approaches are the most recurrent.

In the leader-follower approach leader robots define the tasks for a group of followers [9]-[11]. Since there is no feedback from the followers to the leaders, synchronized behavior cannot be maintained if a follower is perturbed.

In the virtual structure approach there is no hierarchy and the elements that conform the group influence its overall performance, allowing for a tradeoff between synchronized behavior and individual performance [12]-[14].

The behavior-based approach focuses on decentralization, is suitable for large groups with multiple objectives, and defines a task for each robot [10], although its applicability is limited because of the resulting complex dynamics.

The definition of controlled synchronization of mechanical systems as presented in [15], [16] has a close relationship with the last two approaches. Considering master-slave (coordination) and mutual synchronization (cooperation) as defined in [16], a clear resemblance appears between these ideas and the leader-follower and the virtual structure approaches. Along these lines, previous definitions of synchronization have been translated to mobile robots in [9].

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The current paper focuses on exploiting this relationship in order to extend the synchronization strategies developed in [9] to the case when the couplings connecting the robots are affected by a time-delay. This requires a framework which allows the synchronization of the mobile robots independently of the distance between them, as long as they are able to exchange information. In this sense, synchronization based approaches have proven to be quite successful when dealing with time-delays, as shown in [15] for bilateral teleoperation and [17] for a boiler subsystem. In essence, the paper addresses the long distance master-slave and mutual synchronization of a group of mobile robots in which a constant time-delay affects the coupling between the elements of the group.

The reasons behind pursuing this objective may differ. For instance, in order to remove the robots' geographical constraints and provide greater flexibility when executing coordinated or cooperative tasks. Another reason is that even a small delay will affect the performance of a synchronized task if the task requires very high precision. Moreover, exchanging information using a communication channel such as the Internet induces a delay even when considering small distances separating the robots. Finally, it constitutes a first approach towards combining locally synchronized robotic networks cooperating or coordinating with similar networks in remote locations. The idea is to provide further insight on how a coordinated task carried out locally can also be carried out over a long distance, starting with a simple task such as tracking and moving on to more complex behaviors.

The paper is organized as follows. Section II elaborates on the idea of long distance master-slave synchronization of mobile robots. In Section III the concept of long distance mutual synchronization by means of the virtual structure approach is explained. Section IV provides an overview of the experimental platforms used for validation, while Section V presents the experimental results. Conclusions and ideas for future work are provided in Section VI.

II. LONG DISTANCE MASTER-SLAVE SYNCHRONIZATION

Master-slave synchronization or coordination of robotic manipulators was presented in [16]. The same work reviewed this concept applied to unicycles, and was studied more thoroughly in [14]. The current work considers a time-delay in the communication channel coupling the mobile robots.

In this case, a master robot tracks its own reference, while its real trajectory constitutes the base for the reference of the i th slave, for $i = 1, \dots, N$; while a constant time-delay τ_i affects the system's unidirectional coupling. This synchronization scheme for two robots is shown in Fig. 1.

Fig. 1. Long distance master-slave synchronization of two mobile robots.

A. Master robot controller

Consider the posture kinematic model of a unicycle-type mobile robot,

$$\dot{x}(t) = v(t) \cos \theta(t), \quad (1a)$$

$$\dot{y}(t) = v(t) \sin \theta(t), \quad (1b)$$

$$\dot{\theta}(t) = \omega(t), \quad (1c)$$

in which $x(t)$ and $y(t)$ denote the robot's position in the global coordinate frame X - Y , $\theta(t)$ corresponds to its orientation w.r.t the X axis, and $v(t)$ and $\omega(t)$ represent its translational and rotational velocities respectively, and constitute the system's control inputs. The robot is subject to a nonholonomic constraint due to the no-slip condition on its wheels and its state is defined as $q(t) = [x(t) \ y(t) \ \theta(t)]^T$.

The control objective of the master, with state $q_m(t)$, is to track the reference trajectory generated by an exosystem, with state $q_r(t)$, which satisfies (1) and whose reference velocities are $v_r(t)$ and $\omega_r(t)$ defined by,

$$v_r(t) = \sqrt{\dot{x}_r^2(t) + \dot{y}_r^2(t)}, \quad (2a)$$

$$\omega_r(t) = \frac{\dot{x}_r(t)\ddot{y}_r(t) - \ddot{x}_r(t)\dot{y}_r(t)}{\dot{x}_r^2(t) + \dot{y}_r^2(t)}. \quad (2b)$$

The difference between $q_r(t)$ and $q_m(t)$ expressed in terms of their local coordinate frames defines the error coordinates $q_{e_m}(t)$, given by the clockwise rotation of the position differences between the reference exosystem and the master mobile robot, i.e.,

$$\begin{bmatrix} x_{e_m}(t) \\ y_{e_m}(t) \\ \theta_{e_m}(t) \end{bmatrix} = \begin{bmatrix} \cos \theta_m(t) & \sin \theta_m(t) & 0 \\ -\sin \theta_m(t) & \cos \theta_m(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r(t) - x_m(t) \\ y_r(t) - y_m(t) \\ \theta_r(t) - \theta_m(t) \end{bmatrix}. \quad (3)$$

Differentiating (3) w.r.t. time yields the error dynamics,

$$\dot{x}_{e_m}(t) = \omega_m(t)y_{e_m}(t) + v_r(t)\cos \theta_{e_m}(t) - v_m(t), \quad (4a)$$

$$\dot{y}_{e_m}(t) = -\omega_m(t)x_{e_m}(t) + v_r(t)\sin \theta_{e_m}(t), \quad (4b)$$

$$\dot{\theta}_{e_m}(t) = \omega_r(t) - \omega_m(t). \quad (4c)$$

A tracking controller designed by expressing the closed-loop error dynamics (4) in a cascaded form is proposed in

[18], [19], and is given by,

$$v_m(t) = v_r(t) + c_{2_m}x_{e_m}(t) - c_{3_m}w_r(t)y_{e_m}(t), \quad (5a)$$

$$\omega_m(t) = \omega_r(t) + c_{1_m}\sin \theta_{e_m}(t), \quad (5b)$$

with $c_{1_m}, c_{2_m} > 0$ and $c_{3_m} > -1$ ensuring stability.

B. Slave trajectory generation

The master's real path constitutes the base for the reference trajectory of the i th slave, with state $q_{s_i}(t)$. For simplification purposes, subindex i denoting the i th slave will be obviated hereinafter. The slave's reference may be defined in terms of possibly time varying displacements l_x and l_y w.r.t the master's inertial (local) coordinate frame $X_m - Y_m$. In this case, the slave's reference position is given by the counterclockwise rotation of l_x and l_y by θ_m , i.e.,

$$\begin{aligned} x_{r_s}(t) &= x_m(t - \tau) + l_x(t - \tau) \cos \theta_m(t - \tau) \\ &\quad - l_y(t - \tau) \sin \theta_m(t - \tau), \end{aligned} \quad (6a)$$

$$\begin{aligned} y_{r_s}(t) &= y_m(t - \tau) + l_x(t - \tau) \sin \theta_m(t - \tau) \\ &\quad + l_y(t - \tau) \cos \theta_m(t - \tau), \end{aligned} \quad (6b)$$

where the robots' coupling induces a constant time-delay τ due to the communication channel linking them.

The slave's reference velocities have the same form of (2), whereas its reference orientation is given by,

$$\theta_{r_s}(t) = \arctan \left(\frac{\dot{y}_{r_s}(t)}{\dot{x}_{r_s}(t)} \right). \quad (7)$$

Note that computing these signals will require the velocity and acceleration of (6).

In the reference trajectory previously derived the relative distance between the robots depends on how l_x and l_y are defined, resulting in a *formation oriented* behavior of the slave, as depicted in Fig. 2. Nevertheless, the slave's reference may also be defined w.r.t the global coordinate frame $X - Y$. In this case, l_x and l_y will determine the absolute distance between the robots, resulting in a *location oriented* behavior, also shown in Fig. 2. In this case the slave's reference position is given by,

$$x_{r_s}(t) = x_m(t - \tau) + l_x(t - \tau), \quad (8a)$$

$$y_{r_s}(t) = y_m(t - \tau) + l_y(t - \tau), \quad (8b)$$

with the reference orientation and velocities defined as in the formation oriented case.

C. Slave robot controller

The error coordinates between the slave and its reference are defined by the state $q_{e_s}(t)$, and have the same form of (3). Differentiating these coordinates w.r.t time produces the slave error dynamics $\dot{q}_{e_s}(t)$, which have the same form of (4). Considering this, a tracking controller based on a cascaded structure for the slave is proposed as in (5),

$$v_s(t) = v_{r_s}(t) + c_{2_s}x_{e_s}(t) - c_{3_s}w_{r_s}(t)y_{e_s}(t), \quad (9a)$$

$$\omega_s(t) = \omega_{r_s}(t) + c_{1_s}\sin \theta_{e_s}(t), \quad (9b)$$

with $c_{1_s}, c_{2_s} > 0$, and $c_{3_s} > -1$ ensuring stability.

Fig. 2. Master robot trajectory, location oriented reference, and formation oriented reference.

Since the slave's reference is formed with the master's delayed output, it follows that the time-delay induced by the communication channel only affects this reference. In consequence, the control problem remains one of a tracking nature, where the master and slave robots are fitted with equivalent tracking controllers which have already been proven to achieve local exponential stability in [18] and [19].

III. LONG DISTANCE MUTUAL SYNCHRONIZATION

Mutual synchronization or cooperation of rigid or flexible joint robotic manipulators is presented in [16]. The concept has been applied to unicycle-type mobile robots in [14] by using the virtual structure approach. In this case, all robots receive a common signal known as the virtual center, which is used to define their individual reference trajectories. Additionally, all robots communicate their position and orientation errors to each other, forming a bidirectional coupling.

This work considers a time-delay in the robots' coupling induced by the communication channel linking them. This may imply that the robots are in different locations, so the possibility of a reference trajectory generated elsewhere is considered, inducing an additional delay. The proposed synchronization scheme for two robots is depicted in Fig. 3.

A. Trajectory generation

The reference trajectory of the i th robot, defined in terms of the virtual center, will be subject to a constant time-delay τ_{r_i} if generated at a different location. Such reference may be defined in terms of possibly time-varying displacements l_{x_i} and l_{y_i} w.r.t. the virtual center's inertial (local) coordinate frame, resulting in the following reference position,

$$\begin{aligned} x_{r_i}(t) &= x_{vc}(t - \tau_{r_i}) + l_{x_i}(t - \tau_{r_i}) \cos \theta_{vc}(t - \tau_{r_i}) \\ &\quad - l_{y_i}(t - \tau_{r_i}) \sin \theta_{vc}(t - \tau_{r_i}), \end{aligned} \quad (10a)$$

$$\begin{aligned} y_{r_i}(t) &= y_{vc}(t - \tau_{r_i}) + l_{x_i}(t - \tau_{r_i}) \sin \theta_{vc}(t - \tau_{r_i}) \\ &\quad + l_{y_i}(t - \tau_{r_i}) \cos \theta_{vc}(t - \tau_{r_i}). \end{aligned} \quad (10b)$$

The reference trajectory derived above implies a *formation oriented* behavior of the i th robot w.r.t the virtual center (cf. Fig. 2), whereas a *location oriented* behavior may be

Fig. 3. Long distance mutual synchronization of two mobile robots.

achieved by defining the reference position w.r.t the global coordinate frame $X - Y$, i.e.,

$$x_{r_i}(t) = x_{vc}(t - \tau_{r_i}) + l_{x_i}(t - \tau_{r_i}), \quad (11a)$$

$$y_{r_i}(t) = y_{vc}(t - \tau_{r_i}) + l_{y_i}(t - \tau_{r_i}). \quad (11b)$$

In both cases the reference orientation and velocities are defined as in the master-slave case.

B. Controller structure

The error coordinates $q_{e_i}(t)$ between the i th robot and its reference have the same form of (3), while the resulting error dynamics $\dot{q}_{e_i}(t)$ have the same form of (4). A synchronizing controller based on (5) is proposed for the i th robot. The controller accounts for the interaction between robot i and the j remaining robots thorough couplings subject to a time-delay τ_{ij} , and is defined as follows,

$$\begin{aligned} v_i(t) &= v_{r_i}(t) + c_{2i} x_{e_i}(t) - c_{3i} \omega_{r_i}(t) y_{e_i}(t) \\ &\quad + \sum_{j=1, i \neq j}^N k_{2ij} (x_{e_i}(t - \tilde{\tau}_{ij}) - x_{e_j}(t - \tau_{ij})) \\ &\quad - \sum_{j=1, i \neq j}^N k_{3ij} \omega_{r_i}(t - \tilde{\tau}_{ij}) (y_{e_i}(t - \tilde{\tau}_{ij}) - y_{e_j}(t - \tau_{ij})), \end{aligned} \quad (12a)$$

$$\begin{aligned} \omega_i(t) &= \omega_{r_i}(t) + c_{1i} \sin \theta_{e_i}(t) \\ &\quad + \sum_{j=1, i \neq j}^N k_{1ij} \sin(\theta_{e_i}(t - \tilde{\tau}_{ij}) - \theta_{e_j}(t - \tau_{ij})), \end{aligned} \quad (12b)$$

with $c_{1i}, c_{2i} > 0, c_{3i} > -1$ and $k_{1ij}, k_{2ij}, k_{3ij}$ determining the tracking performance and coupling strength. To establish a proper comparison within the coupling terms, time-delay $\tilde{\tau}_{ij}$ is induced on purpose. Given the i th and j th robots and assuming their coupling delay τ_{ij} to be equal to or greater than the difference between their reference trajectory delays, i.e. $\tau_{ij} \geq |\tau_{r_i} - \tau_{r_j}|$, the induced delays $\tilde{\tau}_{ij}$ and $\tilde{\tau}_{ji}$ yield,

$$\tilde{\tau}_{ij} = \tau_{ij} - (\tau_{r_i} - \tau_{r_j}), \quad (13a)$$

$$\tilde{\tau}_{ji} = \tau_{ij} + (\tau_{r_i} - \tau_{r_j}). \quad (13b)$$

C. A stability result

Considering $\xi_1 = [x_{1e} \ y_{1e} \ x_{2e} \ y_{2e}]^T$ and $\xi_2 = [\theta_{1e} \ \theta_{2e}]^T$, the closed-loop error dynamics of two mutually synchronized mobile robots are written in the following cascaded form,

$$\begin{aligned} \dot{\xi}_1(t) = & A_0(t)\xi_1(t) + A_1(t)\xi_1(t - \tilde{\tau}_{12}) + A_2(t)\xi_1(t - \tilde{\tau}_{21}) \\ & + A_3(t)\xi_1(t - \tau_{12}) + g_0(t, \xi_1, \xi_2) \\ & + g_1(t, t - \tau_{12}, t - \tilde{\tau}_{12}, t - \tilde{\tau}_{21}, \xi_1, \xi_2), \end{aligned} \quad (14a)$$

$$\dot{\xi}_2(t) = f_{20}(t, \xi_2) + f_{21}(t - \tau_{12}, t - \tilde{\tau}_{12}, t - \tilde{\tau}_{21}, \xi_2), \quad (14b)$$

where,

$$A_0(t) = \begin{bmatrix} -c_{21} & (1 + c_{31})\omega_{r1}(t) & 0 & 0 \\ -\omega_{r1}(t) & 0 & 0 & 0 \\ 0 & 0 & -c_{22} & (1 + c_{32})\omega_{r2}(t) \\ 0 & 0 & -\omega_{r2}(t) & 0 \end{bmatrix},$$

$$A_1(t) = \begin{bmatrix} -k_{212} & k_{312}\omega_{r1}(t - \tilde{\tau}_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_2(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -k_{221} & k_{321}\omega_{r2}(t - \tilde{\tau}_{21}) \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_3(t) = \begin{bmatrix} 0 & 0 & k_{212} & -k_{312}\omega_{r1}(t - \tilde{\tau}_{12}) \\ 0 & 0 & 0 & 0 \\ k_{221} & -k_{321}\omega_{r2}(t - \tilde{\tau}_{21}) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$g_0(t, \xi_1, \xi_2) = \begin{bmatrix} c_{11}y_{e1}(t)\sin\theta_{e1}(t) + v_{r1}(t)(\cos\theta_{e1}(t) - 1) \\ -c_{11}x_{e1}(t)\sin\theta_{e1}(t) + v_{r1}(t)\sin\theta_{e1}(t) \\ c_{12}y_{e2}(t)\sin\theta_{e2}(t) + v_{r2}(t)(\cos\theta_{e2}(t) - 1) \\ -c_{12}x_{e2}(t)\sin\theta_{e2}(t) + v_{r2}(t)\sin\theta_{e2}(t) \end{bmatrix}, \quad (15a)$$

$$g_1(t, t - \tau_{12}, t - \tilde{\tau}_{12}, t - \tilde{\tau}_{21}, \xi_1, \xi_2) = \begin{bmatrix} k_{112}y_{e1}(t)\sin(\theta_{e1}(t - \tilde{\tau}_{12}) - \theta_{e2}(t - \tau_{12})) \\ -k_{112}x_{e1}(t)\sin(\theta_{e1}(t - \tilde{\tau}_{12}) - \theta_{e2}(t - \tau_{12})) \\ k_{121}y_{e2}(t)\sin(\theta_{e2}(t - \tilde{\tau}_{21}) - \theta_{e1}(t - \tau_{12})) \\ -k_{121}x_{e2}(t)\sin(\theta_{e2}(t - \tilde{\tau}_{21}) - \theta_{e1}(t - \tau_{12})) \end{bmatrix}, \quad (15b)$$

$$f_{20}(t, \xi_2) = \begin{bmatrix} -c_{11}\sin\theta_{e1}(t) \\ -c_{12}\sin\theta_{e2}(t) \end{bmatrix}, \quad (15c)$$

$$f_{21}(t - \tau_{12}, t - \tilde{\tau}_{12}, t - \tilde{\tau}_{21}, \xi_2) = \begin{bmatrix} -k_{112}\sin(\theta_{e1}(t - \tilde{\tau}_{12}) - \theta_{e2}(t - \tau_{12})) \\ -k_{121}\sin(\theta_{e2}(t - \tilde{\tau}_{21}) - \theta_{e1}(t - \tau_{12})) \end{bmatrix}. \quad (15d)$$

Assumption 1: The desired rotational velocities ω_{ri} , $i = 1, 2$ are not only persistently exciting (P.E.) [18], but actually nonzero everywhere.

Remark 1: The delayed time varying terms $\omega_{r1}(t - \tilde{\tau}_{12})$ and $\omega_{r2}(t - \tilde{\tau}_{21})$ in matrix functions A_1 , A_2 and A_3 may be renamed as $\bar{\omega}_{ri}(t)$ since due to Assumption 1 the matrices, although time-varying, will always have entries in the same locations, i.e., from a practical viewpoint, their entry values

will change with time but not their structure. This is the reason why the matrix functions in (14a) were denoted as $A_1(t)$, $A_2(t)$ and $A_3(t)$.

The following theorem provides further insight into the local stability of the equilibrium point $(x_{ei}, y_{ei}, \theta_{ei}) = 0$, $i = 1, 2$ of the closed-loop error dynamics.

Theorem 1: Consider two unicycle-type mobile robots, placed at two different locations, whose kinematics are described by (1). Suppose that the reference position of each robot $q_{ri}(t)$, $i = 1, 2$ is given as in (10) or (11) and is derived from a common virtual center, whereas the reference orientation is derived from the reference velocities and is subject to a nonholonomic constraint. Moreover, this virtual center is generated locally for each robot, resulting in $\tau_{ri} = 0$, $i = 1, 2$. The mobile robots are coupled through their error coordinates $q_{ei}(t)$ and the coupling is subject to a constant time-delay τ_{12} caused by the communication channel linking them. Given the synchronizing controller (12), there exist gains c_{1i} , c_{2i} , c_{3i} , k_{1ij} , k_{2ij} , and k_{3ij} , $i = 1, 2$, $j = 1, 2$, $i \neq j$, such that:

- system (14b) is locally exponentially stable (LES);
- functions g_0 and g_1 in (14a) are bounded;
- the following subsystem in (14a) is LES,

$$\begin{aligned} \dot{\xi}_1(t) = & A_0(t)\xi_1(t) + A_1(t)\xi_1(t - \tilde{\tau}_{12}) + A_2(t)\xi_1(t - \tilde{\tau}_{21}) \\ & + A_3(t)\xi_1(t - \tau_{12}); \end{aligned} \quad (16)$$

meaning the equilibrium point $(x_{ei}, y_{ei}, \theta_{ei}) = 0$, $i = 1, 2$, of the closed-loop error dynamics (14) is LES.

Proof: For the sake of brevity only a sketch of the proof is provided here.

The theorem is derived from the results for cascaded systems presented in [18] and [19], which have been successfully applied for the mutual synchronization of mobile robots in [14].

- The condition that subsystem (14b) is LES can be proven by linearizing and decomposing (14b) into two one-dimensional systems with delay. Given symmetric gains c_{1i} and k_{1ij} , $i, j = 1, 2$, $i \neq j$, all the eigenvalues of the linearized coupling matrix are real. Therefore, proving the local stability of (14b) can be reduced to analyzing the stability of two independent one-dimensional systems with a state delay.
- Functions g_0 and g_1 as given in (15a) and (15b) respectively have been shown to be bounded in [9] for the delay free case. For (15b) a similar statement as in Remark 1 applies, so the previous analysis holds. Moreover, the fact that the equilibrium point $\theta_{ei} = 0$, $i = 1, 2$ is LES means that these terms will vanish.

Regarding (16), the following conditions are taken from [21] and establish the stability of delayed linear time-varying (LTV) systems,

- the matrix functions $A_1(t)$, $A_2(t)$, and $A_3(t)$ are bounded on $\mathbb{R}^+$;
- a matrix $P(t) > 0$, bounded on $\mathbb{R}^+$, exists and satisfies the Lyapunov inequation,

$$\dot{P}(t) + A_0^T(t)P(t) + P(t)A_0(t) + mI \leq 0, \quad (17)$$

Fig. 4. Experimental setup at TU/e.

and the conditions,

$$\eta(A_0) := \sup_{t \in \mathbb{R}^+} \eta(A_0(t)) < +\infty, \quad (18a)$$

$$\eta(A_0) + \alpha \|P_I\| + m e^{2\alpha h} \|P_I\|^2 \|A\|^2 < 0, \quad (18b)$$

where,

$$P_I = \sup_{t \in \mathbb{R}^+} \|P(t) + I\|, \quad \|A\|^2 = \sup_{t \in \mathbb{R}^+} \|A(t)\|^2, \quad (19)$$

h denotes the maximum delay, $\alpha > 0$ the convergence rate, m the number of delayed matrix functions (i.e. 3), and $\eta(A)$ the matrix measure of A as defined in [21].

Note that due to Assumption 1 the matrix functions $A_1(t)$, $A_2(t)$ and $A_3(t)$ are bounded. Moreover, the existence of matrix $P(t) > 0$ can be guaranteed provided the tracking and coupling gains c_{2i} , c_{3i} , k_{2ij} , and k_{3ij} , for $i, j = 1, 2$, $i \neq j$ are chosen adequately. Although the nature of the reference angular velocity $\omega_{r_i}(t)$ will affect this choice, because of practical limitations in real robots and since the posture kinematic model of the mobile robot is being used, the value of the reference velocity can be assumed to be small. Moreover, because of the structure of $A_0(t)$ it is known beforehand that matrix $P(t)$ is block diagonal. Considering these remarks, the computation of matrix $P(t)$ can be significantly simplified.

Remark 2: Simulations show that the error dynamics remain stable in a more general setting, e.g. for a remotely generated virtual center, i.e. $\tau_{r_i} \neq 0$, or for more than two robots. Since the corresponding stability analysis becomes more complex, the current paper will rather focus on the experimental validation of the approaches proposed so far.

IV. EXPERIMENTAL PLATFORM DESCRIPTION

Two multi-robot setups are available for experimental validation; one at the Eindhoven University of Technology (TU/e) and the other at Tokyo Metropolitan University (TMU). Further details regarding their design and implementation can be found in [9] (cf. Fig. 4)

In order to implement the long distance synchronization strategies proposed in Section II and Section III, data exchange between the setups has been added. Because of its

Fig. 5. Cartesian workspace behavior and error coordinates evolution for master-slave remote synchronization.

widespread availability and low cost, the Internet has been chosen as the communication channel.

1) *Data exchange:* The setup at TU/e accesses TMU's network via a Virtual Private Network (VPN) in order to allow for a reliable and secure data exchange.

2) *Socket configuration:* Experimental data is exchanged between the setups as soon as it becomes available by means of non-blocking Transmission Control Protocol (TCP) sockets running over the Internet Protocol (IP). The system's low bandwidth allows the use of the TCP protocol, which guarantees reliable and orderly data delivery.

3) *Data payload:* The variables exchanged amount to the current time instant and desired position, orientation, and translational and rotational velocities in the master-slave case, and to the current time instant and position and orientation errors of each robot in the mutual case.

V. EXPERIMENTAL RESULTS

A. Master-slave synchronization (location oriented)

In this experiment the master is located at TU/e and the slave at TMU. The reference trajectory for the master is a sinusoid with origin at $[0.3\text{m}, 0.3\text{m}]$, an amplitude of 0.15m , an angular frequency of 0.3m and a translational velocity multiplier of 0.01m/s . The reference for the slave is location oriented and defined in terms of constant displacements $l_{x_s} = -0.2\text{m}$ and $l_{y_s} = -0.15\text{m}$. The initial conditions are $q_m(0) = [0.16\text{m} \ 0.16\text{m} \ 1.1\text{rad}]^T$ and $q_s(0) = [0.11\text{m} \ 0.24\text{m} \ 0.89\text{rad}]^T$.

The experimental results in Fig. 5 show in the upper plots the behavior of the master (TU/e) and the slave (TMU) in their respective Cartesian workspace, depicting their reference trajectory (solid line), actual trajectory (dashed line), and initial and final positions (a cross and a circle respectively). The lower plots show that the error coordinates $x_e(t)$, $y_e(t)$, and $\theta_e(t)$ for the master (solid line) and slave (dashed line) practically converge to zero, meaning that

Fig. 6. Cartesian workspace behavior and error coordinates evolution for mutual remote synchronization.

master-slave synchronization is achieved and that the slave's behavior is determined by the master.

B. Mutual synchronization (location oriented)

This experiment shows mutual synchronization between robot 1 at TMU and robot 2 at TU/e. The virtual center is a lemniscate with center at the origin, a length and width of 0.4m, and a velocity multiplier of 0.2m/s. The robots' displacements in the global coordinate frame of the virtual center are $l_{x1} = 0.5\text{m}$, $l_{y1} = 0.25\text{m}$ (TMU), and $l_{x2} = 0.85\text{m}$, $l_{y2} = 0.62\text{m}$ (TU/e). Their initial conditions are $q_1(0) = [0.47\text{m } 0.31\text{m } 0.74\text{rad}]^T$ and $q_2(0) = [0.75\text{m } 0.44\text{m } 0.81\text{rad}]^T$, and the controllers' gains are the same as in the master-slave case, while the coupling gains are $k_{1ij} = k_{2ij} = k_{3ij} = 1.0$.

The virtual center is generated locally, i.e. $\tau_{r1} = \tau_{r2} = 0$, and the coupling delay is measured at approximately $\tau_{12} = 0.150\text{s}$, resulting in $\tilde{\tau}_{12} = \tilde{\tau}_{21} = 0.150\text{s}$. The first robot (TMU) is perturbed by displacing it manually at approximately 14.6s.

In the experimental results shown in Fig. 6 the upper plots depict the behavior of robot 1 (TMU) and robot 2 (TU/e) in their respective Cartesian workspace, while the lower plots show that their error coordinates (1:solid, 2:dashed) practically converge to zero. The influence of the coupling terms in the controller can be noticed when robot 1 is perturbed, meaning the interaction between the robots is maintained even under a time-delay.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, control strategies developed for cooperation and coordination of unicycle-type mobile robots have been adapted in order to cope with delayed couplings, giving way to long distance synchronization. The necessary stability concepts for achieving mutual synchronization of mobile robots with delayed couplings have been extended using a cascaded approach. The proposed schemes do not restrict

the controller being used, meaning e.g. that the one with saturation constraints on the control signals in [1] can be readily implemented, although particular stability analyses should still be performed. Experiments between two remote setups successfully implement the strategies presented.

The theoretical framework and experimental setups are being extended in order to accommodate larger robotic networks, resulting in locally synchronized networks cooperating or coordinating with similar networks located elsewhere. Additionally, a better method for characterizing the system's performance is under development.

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