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Minisymposium *"Multirate Time Integration for Multiscaled Systems"*

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Multiphysical¹ and network modeling usually lead to coupled systems that exhibit largely different timescales. In time domain it is called multirate, in space multiscale. Efficient algorithms need to take these phenomena into account. Such methods are specially requested by industry. As a practical example one can think of a cellular phone, which consists of coupled digital and analog subcircuits that operate at vastly different frequencies. The inclusion of memory cells extends the difference in dynamics even more. Other examples one can find in astrophysics; in multibody systems of vehicles; in robotics (where multiple time scales and hierarchy in the dynamics may exhibit); in coupling mechanical and electrical systems; in reaction and diffusion processes involving multiple chemical components; in combustion engines with chain drives. Similar effects have to be taken into account when designing Integrated Circuits (ICs) [19] and Power Systems [5].

To speed-up numerical integration of ordinary differential equations (ODEs), three research directions have been followed in the last decades, all based on exploiting multirate behaviour in time:

- Multi-method schemes (for systems containing both non-stiff and stiff parts): here a partitioning is done on the level of the discretization scheme, i.e., an explicit scheme is used for the non-stiff parts, and an implicit method for the stiff ones. First results for Runge-Kutta [13] and Rosenbrock-Wanner schemes [14] have been generalized to linearly-implicit RK methods [24]. Applications: chemical reaction kinetics and circuit simulation.
- Multi-order schemes (for non-stiff problems only): here the same explicit method, and the same step size is used for all parts, but the order of the method is chosen according to the activity level of the subsystem. For simulating accretion disks in astrophysics numerically, two approaches based on extrapolation and RK schemes have been developed [8]. Partitioning

¹ This minisymposium was an event of the ECMI Special Interest Group on Scientific Computing in Electronics Industry

can be done here with respect to solution components as well as to the right-hand side. A similar approach is known as multiple time stepping in molecular dynamics.

- Multirate schemes (for both stiff and non-stiff problems): here the same (implicit or explicit method) with the same order is applied to all sub-systems, but the step size is chosen according to the activity level. The first Multirate Time Integration algorithm traces back to [15] in 1960, where split explicit RK methods have first been analyzed. Nearly 25 years later, [9] applied the multirate idea to multi-step schemes (BDF methods). Stability investigations for these methods can be found in [1, 18]. Multirate Rosenbrock-Wanner schemes have been introduced in [11]. The drawback of all these schemes is the combination of numerical integration schemes with interpolation/extrapolation algorithms for coupling active to latent parts and vice versa, susceptible to stability problems. To overcome this flaw, so-called generalized multirate schemes have been developed since 1999. Here a macro time step on the latent part is combined with the first micro step on the active part (compound step). In this compound-strategy, coupling is done via internal stages, and not via interpolation/extrapolation. Runge-Kutta schemes [4] and Rosenbrock-Wanner schemes [3], resp., were developed for non-stiff and stiff problems, resp. For mixed non-stiff and stiff problems, multirate partitioned RK schemes [12] combine the idea of multi-method and (generalized) multirate schemes.

Though of much interest in academia, the applicability of these multirate schemes developed in the last part of the previous millennium was limited to ordinary differential equations and restricted to a low number of special industrial applications. In the last five years, however, a shift could be observed from purely ODE applications to Differential-Algebraic systems of Equations (DAE) and systems of Partial Differential Equations (PDE). In addition, industry became aware of these activities in academia.

Since 2003, European semiconductor industries (Philips Electronics, NXP Semiconductors, Infineon AG, Qimonda AG) initiated research for Multirate Time Integration for DAEs arising from circuit equations covering a dynamics spectrum varying from latent behaviour (memory applications) to highly time varying behaviour (RF applications) and by coupling these together into bigger systems. Additional coupling of circuit equations to heat transfer and to electromagnetics also has led to systems that show multirate time behaviour. This resulted in two PhD-Theses [20, 23]. In particular Multirate Time Integration using Backward Differential Formulae and using Rosenbrock-Wanner methods were studied. Automatic dynamical partitioning is essential to obtain successful methods that can deal with e.g. wake ups of parts of an electronic circuit [21]. Also a hierarchical multirate approach was developed to deal with various levels of dynamics behaviour [19, 20].

Parallel to this, research on reaction diffusion and on systems of hyperbolic

conservation laws gave rise to study Multirate Time Integration methods for the system of ODEs that arise after semidiscretization of these PDEs [6, 7, 17]. Here Rosenbrock methods were the starting point and also higher order methods were considered. The multirate time stepping caused deeper analysis in order to guarantee that stability, monotonicity, interface, stiff source term and conservation law constraints are met.

In vehicle system dynamics, multirate methods have recently been introduced in the context of co-simulation paradigms by [2].

This minisymposium addressed particular aspects of the various multirate time integration methods

- A ROW-based hierarchical mixed multirate method that can deal with an arbitrary amount of subcircuits for differential-algebraic equations of index 1 [19, 20] [Univ. of Wuppertal & Infineon AG & Qimonda AG].
- A self-adjusting multirate time stepping strategy, where the partitioning into different levels of slow to fast components is performed automatically during the time integration [16, 17] [Univ. of Amsterdam & CWI Amsterdam].
- A multi-step multirate implementation, where the interpolation error and the coarse discretisation error is controlled by the macro stepsize, while the micro stepsize controls the fine discretisation errors for the fast state part [21, 22, 23] [TU Eindhoven & Philips & NXP].
- Two families of explicit multirate time discretization methods based on Adams-Bashforth and partitioned Runge-Kutta schemes for hyperbolic conservation laws, which avoid the necessity to take small global timesteps restricted by the largest CFL number [6, 7] [Virginia Tech, Blackburg, VA].

In the application to systems of hyperbolic conservation laws the multirate time stepping caused deeper analysis in order to guarantee that stability, monotonicity, interface, stiff source term and conservation law constraints are met.

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