Tracking simultaneous time-varying power harmonic distortions using filter banks

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Abstract—Although it is well known that the Fourier analysis is only accurately applicable to steady-state waveforms, it is a widely used tool to study and monitor time-varying signals, such as are commonplace in electrical power systems. The disadvantages of the Fourier analysis, such as frequency spillover or problems due to sampling (data window) truncation can often be minimized by various windowing techniques, but they nevertheless exist. This paper demonstrates that it is possible to track and visualize amplitude and time-varying power systems harmonics, without frequency spillover caused by classical time-frequency techniques. This new tool allows for a clear visualization of time-varying harmonics, which can lead to better ways to track harmonic distortion and understand time-dependent power quality parameters. It has been applied to extract the harmonic contents of a rolling mill. It also has the potential to assist with control and protection applications.

Index Terms—Harmonic distortion, harmonic decomposition, power quality, time-varying power harmonics, multi-resolution analysis, wavelet transform.

I. INTRODUCTION

While estimation techniques are concerned with the process of extracting useful information from a signal, such as amplitude, phase, and frequency, signal decomposition is concerned with the way that the original signal can be split in other components, such as harmonics, inter-harmonics, sub-harmonics, etc.

Harmonic decomposition of a power system signal (voltage or current) is an important issue in power quality analysis. There are at least two reasons to focus on harmonic decomposition instead of harmonic estimation: (i) If separation of the individual harmonic component from the input signal can be achieved, the estimation task becomes easier. (ii) The decomposition is carried out in the time-domain, such that the time-varying behaviors of each harmonic component are observable.

Some existing techniques are used to separate time-varying frequency components. For example, the Short Time Fourier Transform (STFT) and Wavelets Transforms have known decomposition techniques. Both can be seen as a particular case of filter bank theory [2]. In essence, the STFT uses a filter with purely imaginary coefficients, which generates a complex output signal whose magnitude corresponds to the amplitude of the harmonic component in the band. The main disadvantage of this method is the difficulty with the construction of an efficient band-pass filter with minimum frequency spillover. Though the wavelet transform uses filters with real coefficients, the common mother wavelets do not have good magnitude response in order to prevent frequency spillover. Additionally, the traditional binary tree structure is not able to divide the spectrum conveniently for harmonic decomposition, and the discrete wavelet packet must be chosen in order to obtain a good decomposition.

The adaptive notch filter and the Phase-Locked Loop (PLL) [3,4] have also been used for extracting time-varying harmonics components. However, the methods only work well in the case where a few harmonics components are present in the input signal. In other cases, the energies of adjacent harmonics spill over into each other and the decomposed signal becomes contaminated. A similar approach was proposed in [5], where an adaptive filter-bank, labeled the resonator-in-a-loop filter bank, tracks and estimates the voltage and current harmonic signals. The author only presents cases with odd harmonic tracking, and the non-stationary example deals just with the case of time-varying frequencies.

In [6] the author presents a technique, based on a multi-stage implementation of narrow low-pass digital filters, to extract stationary harmonic components. The technique uses a multi-rate approach for the filter implementation, but it must know the system frequency to implement the modulation stage. The order of the narrow low-pass filter is large, and consequently the transient response time is long, and the computational effort is substantial.

Attempts to analyze time-varying harmonics using the wavelet transform have been proposed in [7], [8] and [9]. In references [8] and [9] the analysis of non-stationary power system waveforms includes two steps. In the first step, the signal is decomposed into sub-bands using Discrete Wavelet Packet Transform (DWPT). In the second step, a Continuous Wavelet Transform (CWT) is applied to obtain the amplitude and phase of the harmonics. However, the methods are either

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computationally too complex or are not able to decouple the frequencies completely.

In [10] the authors present a methodology for separating the harmonic components, up through the 15th, using selected digital filters and down-sampling to obtain the equivalent band-pass filters centered at each harmonic. After the signal is decomposed by the analysis bank, each harmonic is reconstructed using a synthesis bank structure. This signal processing structure is composed of filters and up-sampling stages that reconstruct each harmonic back into its original sampling rate.

In this paper, we further explore the analytical basis and present new visualization forms for time-varying harmonics. The new methodology may be used for monitoring individual time-varying power systems harmonics. Future applications may include enhanced power quality monitoring, resilient control and protection functions, as well as inter-harmonic measurements. The paper is divided into sections covering the method description, odd/even harmonic extraction, and simulation results.

II. SIGNAL PROCESSING METHOD DESCRIPTION

A multi-rate system employs a bank of filters with either a common input or a summed output. The first process is known as an analysis filter bank [11,12]. This bank divides the input signal into different sub-bands in order to facilitate the analysis or processing of the signal. The second process is known as a synthesis filter bank and is used, if needed, to reconstruct the signal. In addition, a multi-rate system must include operations for altering the sampling rate (up and down-sampling). Figure 1 shows two basic processing structures used in a multi-rate system. Process (a) is a decimator, composed of a filter, $H(z)$, followed by a down-sampler with a down sampling factor of $M$. The interpolator process (b) is composed of an up-sampler with an up sampling factor of $L$, followed by a filter, $F(z)$. The decimator process is responsible for reducing the sampling rate while the interpolator process increases it. The right side of Fig. 1 shows the symbolic representation for the cases of $M = 2$ and $L = 2$.

The direct way to build an analysis filter bank, for dividing the input signal in its odd harmonic component, is represented in Fig. 2. In this structure the filter $H_k(z)$ is a band-pass filter centered at the $k$th harmonic and must have a 3 dB bandwidth less than $2f_0$, where $f_0$ is the fundamental frequency. If only odd harmonics are expected to be present in the input signal, the 3 dB bandwidth can be relaxed to be less than $5f_0$. Note that Fig. 2 is not a multi-rate system, because the process structure does not include any sampling rate alternations, which means that there is only one sampling rate throughout the whole signal processing system.

The practical problem concerning the structure shown in Fig. 2 is the difficulty to design each individual band-pass filter. This problem becomes more challenging when a high sampling rate must be used to handle the signal and the consequential abrupt-transition band.

In this situation the best way to construct an equivalent filter bank is to use the multi-rate technique. Figure 3 shows an equivalent structure, obtained using the multi-rate approach, in contrast to the one in Fig. 2. The filters $H_0(z)$ and $H_1(z)$ are quadrature mirror filters (QMF), designed using the power-symmetric approach [11]. $H_0(z)$ is a low-pass filter, and $H_1(z)$ is a high-pass filter.

Figure 4 shows the amplitude response of a filter bank for the first 15 harmonics. This filter bank was obtained for a signal sampling rate of 64 samples per cycle using QMF FIR filters of the 69th order, (which contains 70 filter coefficients). The central frequency of each filter corresponds to one of the odd harmonics. The band-pass filters have poor rejection for the even harmonics, so if they are present in the input signal they could leak signal energy into the adjacent odd harmonics. In a later section, this paper focuses on a novel way to eliminate this problem.

The main difference between the filter bank shown in Fig. 2, and the filter structure obtained using the multi-rate technique, is the fact that the equivalent band-pass filter is obtained using
the QMF filter and decimating operator. In fact, Fig. 3 can be represented by the equivalent filter bank shown in Fig. 5. The decimated signal at the output of each filter has a sampling rate 16 times lower than the input signal.

Fig. 4–Amplitude response of the single-sampling-rate filter bank for the first 15 harmonics.

![Image](image_url)

Fig. 5–Equivalent multirate analysis filter bank

The equivalent filters, $H_i(z)$, with $i = 1, 2, \ldots, k$, are obtained using the noble identities for multi-rate systems [12]. These identities are shown in Figure 6. The first identity shows that $H(z)$ must be changed to $H(z^M)$ when the down-sampling operator is moved from left to right. The second identity shows the reverse case.

Moving all down-sampling operators in Fig. 3 to the right side, and using the noble identity 1, one can rewrite the transfer function. For example, the fundamental and seventh harmonic filters are transformed into the following set of cascaded $H_0$ and $H_1$ periodic filters.

$$H_i(z) = H_0(z)H_0(z^2)H_0(z^4)H_0(z^8)$$ \hspace{1cm} (1)

or,

$$H_i(z) = H_0(z)H_0(z^2)H_1(z^4)H_0(z^8)$$ \hspace{1cm} (2)

Figure 7 shows how the $H_i(z)$ response is built and obtained using ideal QMF filters for $H_0(z)$ and $H_1(z)$. The frequency axis is normalized where 1 corresponds to $f_s/2 = 32f_0$. $H_0$ and $H_1$ denote periodic filters, whose periodicity is equal to $2\pi/k$ where $k = 1, 2, 4$ and 8 [10]. The intersection between all cascaded filters results in the response of the equivalent multi-rate filter as illustrated in Fig. 5. In this example (see Fig. 7) the intersection area ranges from $[3/16$ and $4/16]$ or in actual frequency between the $6^{th}$ and $8^{th}$ harmonic, and consequently has the central frequency at the $7^{th}$ harmonic. This analysis explains why the third output (from the bottom up) in Fig. 3 corresponds to $7^{th}$ harmonic instead of the $5^{th}$.

The order of each harmonic in the bank corresponds to the well known digital Gray code. The Gray code is the binary numeral system where two successive values differ in only one digit. Table 1 provides the Gray code for four-bit sequence. In applying the Gray code, let 0 correspond to $H_0(z)$ and let 1 correspond to $H_1(z)$. The structural pattern can be identified in Fig. 3 for each harmonic output. Despite the fact that the example of Fig. 3 shows the output of the first 15 odd harmonics, the extension for the first 30 odd harmonics is straightforward.

![Image](image_url)

Fig. 6–Noble identities of multi-rate systems: (a) Identity 1; (b) Identity 2.

![Image](image_url)

Fig. 7–Obtaining the equivalent magnitude response for $H_i(z)$. (a) $H_0(z)$ ideal magnitude response; (b) $H_1(z)$ ideal magnitude response and (c) the equivalent Magnitude response for $H_i(z)$.

<table>
<thead>
<tr>
<th>GRAY CODE</th>
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<th>GRAY CODE</th>
<th>HARMONIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>$1^{st}$</td>
<td>1100</td>
<td>$17^{th}$</td>
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<tr>
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<td>$3^{rd}$</td>
<td>1101</td>
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<tr>
<td>0100</td>
<td>$15^{th}$</td>
<td>1000</td>
<td>$31^{st}$</td>
</tr>
</tbody>
</table>

TABLE 1–THE GRAY CODE AND CORRESPONDING HARMONIC OUTPUT
A. The Sampling Frequency

In order to obtain the frequency response shown in Fig. 4 the sampling frequency used was 64 samples per cycle. This rate is related to the number of decomposition levels used in the filter bank (4 in this case) and is expressed through the following equation:
\[ f_s = 2^{N+2} \cdot f_0 \]  
(3)
where \( f_0 \) is the power frequency and \( N \) is the number of decomposition levels. The highest odd harmonics that can be extracted by the filter bank is given by the Nyquist Theorem and is equal to \( \left( 2^{N+1} - 1 \right) f_0 \).

B. The Down Sampling Effect

It is very important to understand what happens after the down sampling in Fig. 5. Depending on the harmonic component present at the output \( H(z) \) it can become undersampled. This is the case of the 7th harmonic in \( H(z) \), originally with 64 samples per cycle, and after a down-sampling operation, by a factor of 16, it will contain just 4 samples. In this case, this signal will appear at the output as a new sinusoidal signal with a different frequency. This frequency is defined as an apparent frequency, \( f_a \), and can be found through the equation given by [13],
\[ f_a = \frac{\theta}{2\pi M} f_s \]  
(4)
where \( M \) is the down sampler factor and \( \theta \) is the angle, in radians, obtained by,
\[ \theta = \text{mod}(2\pi M f_s / f_s, 2\pi) \]  
(5)
and
\[ \theta = \begin{cases} \theta, & \theta \leq \pi \\ 2\pi - \theta, & \text{otherwise.} \end{cases} \]  
(6)
where \( f_s \) is the \( i \)th harmonic frequency in Hz, and \( \text{mod}(a,b) \) returns the module of the rest of the division \( a / b \). Note that the 7th harmonic after down sampling will have the apparent frequency of 60 Hz, considering \( f_s = 64 f_0 \) and \( M=16 \). All other harmonics will result in the same apparent frequency of 60 Hz.

Figure 8 shows the position of all harmonics after down sampling. Note that 60 Hz corresponds to the digital frequency of \( \pi / 2 \) radians.

C. Filtering Out the Even Harmonic

Figure 8 shows the ideal situation when only one sinusoid is present inside of each filter band. However the frequency response of the filter bank (Fig. 4) shows that it is unable to filter the even harmonic component, so if an even harmonic is present at the input signal, the filter output will be polluted by this component. It is possible to show that all even harmonics, after down sampling, have apparent frequency equal to zero or \( \pi \) radian. In this way, a single second order band pass filter can be used to filter the even harmonics. Equation (7) shows the transfer function of a typical second order parametric bandpass filter,
\[ H_{BP}(z) = \frac{1-\alpha}{2} \frac{1-z^{-2}}{1-\beta(1+\alpha)z^{-1} + \alpha z^{-2}} \]  
(7)

In this equation, \( \beta \) defines the frequency where the filter has unitary gain and \( \alpha \) defines the 3 dB bandwidth. Figure 9 shows the magnitude response when \( \alpha = 0.6 \) and \( \beta = \cos(\pi / 2) = 0 \). Note that this filter rejects the zero and \( \pi \) radian frequency components, and it has been used at the output of each stage at Fig. 3.

D. The Synthesis Filter Banks

As described in Section B, all harmonics at the output of the filter bank have the apparent frequency of \( f_0 \) sampled at a rate of 4 samples per cycle. One could use a single interpolator at this output to increase the sampling rate and improve the graphical resolution. However, this approach could not change the frequency to the original harmonic frequency. To reconstruct each harmonic into its original frequency, it is necessary to use the synthesis filter bank structure.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{magnitude_response.png}
\caption{Magnitude response of the second-order IIR band-pass filter.}
\end{figure}

The synthesis filter bank, which has been used in this work, is not a conventional one. As the objective is to reconstruct each harmonic instead of the original signal, the filter bank must be implemented in cascade, in order to obtain the corresponding harmonic, according to Fig. 10.

A practical way to obtain the cascade of reconstruction filters is to just invert the order of the Gray code in Table 1 and consider 0 as \( F_0(z) \) and 1 as \( F_1(z) \).
E. Extracting the Even Harmonics

In order to extract the even harmonics the same filter bank can be used together with preprocessing of the input signal. The Hilbert transform is then used to implement a single sideband (SSB) modulation [11]. The SSB modulation moves all the frequencies in the input signal up the spectrum by \( f_0 \). In this way, even harmonics are changed to odd harmonics and vice-versa. Figure 11 shows the whole system for extracting odd and even harmonics.

III. SIMULATION RESULTS

The filter bank structure, shown in Fig. 11, is verified for tracking time-varying harmonics using two example signals: (i) a synthetic signal, which has been generated in MatLab\textsuperscript{TM} using a mathematical model, and (ii) a signal obtained from the PSCAD-EMTDC\textsuperscript{TM} program. The filter bank is used to decompose the signal into sixteen different harmonic orders, including the fundamental (60 Hz) and the DC component.

A. Testing with a Synthetic Signal

The synthetic signal utilized is represented by:

\[
x(t) = \sum_{h=1}^{N} A_h \sin(h \omega_0 f(t) + g(t))
\]

(8)

where \( h \) is the harmonic order (1\textsuperscript{st} up to 15\textsuperscript{th}) and \( A \) is the magnitude of the component; \( \omega_0 \) is the fundamental frequency; and finally, \( f(t) \) and \( g(t) \) are exponential functions (crescent, de-crescent or alternated one) or simply a constant value \( \in \mathbb{R} \). \( x(t) \) is portioned in four different segments in such a way that the generated signal is distorted with some harmonics in steady-state and others varying in time, including abrupt and modulated changes of both magnitude and phase, as well as a DC component. Figure 12 illustrates the synthetic signal.

Figure 13 shows the decomposed signal from \( 4\text{th} \) up to \( 7\text{th} \) harmonic components. The left column represents the original synthesized components and the right column corresponds to the signal components obtained through the filter bank. For simplicity and space limitation the other components are not shown, but the result are similar.

It is important to remark that all waveforms of the time-varying harmonics are efficiently extracted with respect to time. Naturally, intrinsic delays are present at the transitions points. Furthermore, the filter banks will introduce a linear phase shift that can be found using the following [11].

\[
T(z) = \frac{1}{K} \sum_{k=0}^{K} \mathcal{H}_k(z) \mathcal{G}_k(z)
\]

(9)

where \( \mathcal{H}(z) \) and \( \mathcal{G}(z) \) are, respectively the equivalent filters for the analysis and synthesis bank, obtained from (1) and (2). As the phase of \( T(z) \) is linear the phase compensation for each harmonic can be obtained easily. Figure 14 shows both the original and the estimated components of the DC to \( 3\text{rd} \) harmonic over a short time scale interval after phase
compensation has been applied.

Table 2 shows the comparative results using different filters types inside the filter bank structure. Two of these filters are well know wavelet filters. The comparative parameters are the Mean Square Error (MSE), Mean Square Percentage Error (MSE%) and Mean Absolute Percentage Error (MAE%) obtained by taking the average of each harmonic. The first two types are the 69th and 33rd order QMF filters. The other types are the Meyer and Daubechies (db 30) wavelet filters with filter order of 101 and 59, respectively. Note that the QMF filters have the lower errors. The QMF 33rd order has better results than the Meyer and Daubechies filters with the advantage of lower computational effort over the QMF 69th order filter. Figure 15 shows the magnitude of the H0 filter’s frequency response for the db30, Meyer and QMF-33 filter types. Note that QMF-33 has the sharpest magnitude response.

![Figure 15 - Magnitude frequency response of the H0 filter for QMF-33, db60 and Meyer filter types.](image)

**TABLE 3 - TYPICAL HARMONIC CONTENT OF THE INRUSH CURRENT**

<table>
<thead>
<tr>
<th>Order</th>
<th>Content %</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>63</td>
</tr>
<tr>
<td>3</td>
<td>26.8</td>
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<tr>
<td>4</td>
<td>5.1</td>
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<td>3.7</td>
</tr>
<tr>
<td>7</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Independent of these improvements to transformer inrush harmonics, it is enlightening to examine the time-varying nature of the inrush currents. A transformer energization case was simulated using PSCAD-EMTDC. The resulting inrush current is shown in Fig. 16. The inrush signal has been analyzed with the filter bank structure.

![Figure 16 - Transformer inrush current seen in phase A of the EMTDC simulation.](image)

**B. Testing with a EMTDC Simulated Current Signal**

It is well known that the energization of a transformer draws a large inrush current from the supply system with a large even-harmonic content. Table 3 shows the typical harmonic components present in transformer inrush currents [14]. Although today’s power transformers have lower harmonic content, these values are normally used as a reference for network protection analysis. The inrush model does not take into account the time-varying nature of this phenomenon. Improvements in materials and transformer design have lead to inrush currents with lower distortion content [15]. The magnitude of the second harmonic, for example, has dropped to approximately 7% for some designs [16].
Another way to visualize the dynamic behavior of each component is through a polar plot. To obtain the polar plot, the phasor of each component must be computed. With the phasor estimate, the real versus the imaginary part are plotted. Figure 18 shows the three dimensional (3D) polar plot of the 2nd, 4th, 5th, and 7th harmonics with the time scale on the vertical axis. Figure 19 shows the two dimensional version of the polar plot curves. These plots highlight the dynamic behavior of the inrush current.

IV. INTER-HARMONIC

According to the definition given by the IEC-61000-2-2 standard [17], an inter-harmonic is "any frequency which is not an integer multiple of the fundamental frequency". Several articles have addressed the problem of inter-harmonic sources, the impacts, measurement and mitigation techniques [18-22]. However, the difficulty has remained in accurately finding the inter-harmonic frequencies and magnitudes. The presence of inter-harmonics produces a beat frequency in the adjacent harmonics when the filter bank structure is used. A simple example clarifies this principle. Consider the synthetic signal given by

$$v(n) = 10\sin(w_0 n) + 3\sin(2w_0 n) + \sin(3w_0 n) + 2\sin(w_1 n)$$

(7)

where $w_0 = 2\pi f_0 / f_s$ and $w_1 = 2\pi f_i / f_s$ is the digital frequency of the inter-harmonic. In this example let $f_i = 82$ Hz, and $f_s = 64 \times 60$. Performing the harmonic decomposition of the signal, $v(n)$, using the filter bank structure, the results are given in Fig. 20. It becomes clear that the inter-harmonic (82 Hz) causes a beating in the fundamental component and the second harmonic, while the third harmonic is not affected.
This is an interesting point, since this kind of beating may indicate the presence of an inter-harmonic between two consecutive harmonic components. Although the current methodology is not able to estimate the frequency and amplitude of the inter-harmonics, maybe the inclusion of other digital algorithms can be used for this propose.

Figure 21 show the polar plot curves for the fundamental and the second and third harmonic components of the signal along with the time plot of the fundamental amplitude estimation. As can be seen, the polar plot curves show that the fundamental and the second harmonic are modulated by the inter-harmonic frequency as a broad circular band. This characteristic is absent from the third harmonic polar plot.

A. Inter-harmonic Application to a Measured Signal

In this example, the current waveform, acquired at the 88-kV side of an aluminum sheet rolling mill facility, is analyzed. Figure 22 shows the odd harmonic decomposition. Even decomposition is not shown because these components are not significant. Note that the components are time varying and the output corresponding to the 5th harmonic is the most dominant after the fundamental. The 3rd harmonic seems to exhibit frequency beating, which identifies the presence of some inter-harmonic in this output. The same happens with the 7th, 9th, and 15th harmonics; only a further analysis is able to identify the existence of the inter-harmonics, but the current approach shows clearly the general behavior at each output. Signal reconstruction is shown in Fig. 22, considering both the odd and even components. Note that the reconstructed signal is close to the original signal, this means that the decomposition approach successfully captures the main information of the whole signal.
applications is related to the transient time response produced by the filter bank and the Hilbert Transformer implementation. Concerning the transient problem, the presence of abrupt change in the input signal, such as in inrush currents, the transient response can last more than 5 cycles, which is not appropriate for applications whose time delay must be as short as possible. Concerning the Hilbert Transformer implementation, some filters used in the literature (mainly in communication area) do not produces good results and further research should address this problem, however for off-line or block processing application, the FFT based implementation leads to excellent results.

The computational effort for real time implementation is another challenge that the authors are investigating. In fact, the high order filter (69th) used in the bank structure demand high computational effort. By using multi-rate techniques to implement the structure it is possible to show that the number of multiplications per second to implement one branch of the analysis filter bank of Fig. 3 is about 250,000. Some low price Digital Signal Processors (DSPs) available in the market are able to execute 300 million float point operations per second. This shows that, despite the higher computational effort of the structure, it is feasible to be implemented in hardware. However, new opportunities exist for overcoming the limitations and the development of improved and alternative algorithms. For example, the authors are developing a similar methodology based on Discrete Fourier Transform (DFT). The first results show similar visualization capabilities with the promise to provide a reduced transient time response and lower computational burdens. This process has the potential of addressing specific protective relaying needs such as detecting a high impedance fault during transformer energization and detection of ferro-resonance.

VI. CONCLUSIONS

This paper presents a new method for time-varying harmonic decomposition based on multi-rate filter banks theory. The technique is able to extract each harmonic in the time domain. The composed structure was developed to work with 256 samples per cycle and to track up to the 15th harmonic. The methodology can be adapted through convenient pre-processing for different sampling rates and higher harmonic orders.

VII. ACKNOWLEDGMENTS

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VIII. REFERENCES