On the performance of static mixers : a quantitative comparison

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On the performance of static mixers: A quantitative comparison

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ABSTRACT

The performance of industrially relevant static mixers that work via chaotic advection in the Stokes regime for highly viscous fluids, flowing at low Reynolds numbers, like the Kenics, the Ross Low-Pressure Drop (LPD) and Low-Low-Pressure Drop (LLPD), the standard Sulzer SMX, and the recently developed new design series of the SMX, denoted as SMX(n) (n , N_p , N_x) = (n , $2n - 1$, $3n$), is compared using as criteria both energy consumption, measured in terms of the dimensionless pressure drop, and compactness, measured as the dimensionless length. Results are generally according to expectations: open mixers are most energy efficient, giving the lowest pressure drop, but this goes at the cost of length, while the most compact mixers require large pressure gradients to drive the flow. In compactness, the new series SMX(n), like the SMX($n = 3$) (3, 5, 9) design, outperform all other devices with at least a factor 2. An interesting result is that in terms of energy efficiency the simple SMX (1, 1, 4, $\theta = 135^\circ$) outperforms the Kenics RL 180°, which was the standard in low pressure drop mixing, and gives results identical to the optimized Kenics RL 140°. This makes the versatile “X”-designs, based on crossing bars, superior in all respects.

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Contents

1. Introduction	1334
1.1. Kenics type of mixers	1336
1.2. SMX type of mixers	1336
1.3. Perfect (stratifying) and small mixers (microfluidics).....	1336
2. Qualitative results	1337
2.1. Interfacial stretch	1337
2.1.1. Kenics RL 180°	1337
2.1.2. Kenics RL 140°	1338
2.2. Flow and mixing in different designs	1338
2.3. Characteristic stretch in different designs	1340
2.3.1. Ross LPD, LLPD	1340
2.3.2. Sulzer SMX (1, 1, 3).....	1340
2.3.3. Sulzer SMX (n , N_p , N_x) = (n , $2n - 1$, $3n$)	1340
2.4. Conclusions on interface stretch in the different designs	1341
2.5. A new mixer design, the SMX (1, 1, 4, 135°).....	1341
3. Quantitative results	1343
3.1. Choice and motivation of an appropriate mixing measure.....	1344

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3.2.	Performance for compactness	1344
3.3.	The SMX-plus	1344
3.4.	Performance for energy efficiency	1346
4.	Conclusions	1346
	Acknowledgement	1347
	References	1347

1. Introduction

Numerous static mixers are extensively used in various homogenization processes in industrial operations like, e.g. polymer blending, chemical reactions, food processing, heat transfer, and in cosmetics and pharmaceuticals, but also in waste-water treatment. They are moreover frequently applied in disposable applications, like in situ mixing of two-component epoxy adhesives and sealants. The question posed in this paper is which static mixer is the best, in particular for systems where efficient mixing via turbulence is absent and mixing can only be achieved by chaotic advection, which is the repeated stretching and folding of material. Static mixers, also known as motionless mixers, are typically devices that contain static mixer elements in a cylindrical, or squared, housing. Elements and housing are made from metals or polymers, depending on particular applications like sustainable versus disposable.

The actual first patent on a static mixer goes back to 1874 where Sutherland describes a single element, multilayer, motionless mixer, used to mix air with a gaseous fuel [1]. Since the first application of static mixers in industry in the 1970s, a wealth of papers have been published that deal with various scientific and more technological questions. One of the first papers on static mixers is that of Thomas Bor discussing the application of static mixers as a chemical reactor [2]. Static mixers in process industry are initially developed for blending of fluids in laminar flow [3] and applications in heat transfer, turbulence and multiphase systems appear much later, although Nauman presents, also already in 1970s, a study on the enhancement of heat transfer and thermal homogeneity using static mixers [4]. Later he extends this work to studies on reactions taking place based on residence time distributions [5,6]. Other simulations are conducted in the 1980s by Arimond and Erwin [7]. Since those seminal papers the number of publications have grown exponentially dealing with the applications, including liquid–liquid systems (e.g. liquid–liquid extraction), gas–liquid systems (e.g. absorption), solid–liquid systems (e.g. pulp slurries) and solid–solid systems (e.g. solids blending). The review paper of Thakur et al. [8] provides a nice summary that discusses in what applications static mixers are beneficial and to be preferred above stirred vessels and other conventionally agitated vessels. Over the years various groups work on mixing in those stirred and agitated vessels using both new experimental and computational methods, like for example Barrue, who focuses on particle trajectories in stirred vessels [9] and also studies mixing in the turbulent regime [10]. Important handbooks and textbooks that discuss fluid mixing and applications in process technology from a much

wider perspective are those of Oldshue [11], Nienow et al. [12] and Kresta [13].

In general, in the evaluation of mixers for highly viscous liquids, two different criteria can be used to judge their efficiency: the first is energy consumption (measured, e.g. in terms of the dimensionless pressure drop) and the second is compactness (measured in terms of dimensionless length). Until now only a few studies have been reported that compare the performance of static mixers [14–16]. During the last decade, we developed in our group an efficient and accurate evaluation tool to analyze mixing in prototype flows, and later also in static and dynamic mixers, known as the mapping method. The method provides time or spatial resolved qualitative mixing profiles as well as a quantitative measure: the flux-weighted cross-sectional area-averaged intensity of segregation [17], see e.g. [18–21]. One of the drawbacks of the original mapping formulation was the computational difficulty to compute the coefficients of the matrix. Later, our group and the group of Philippe Tanguy implemented alternative, simplified formulations [22,23] both based on regular particle tracking. Using this new mapping method as a research tool, we evaluated and optimized various industrially relevant static mixers, like the Kenics mixer [24], the Ross Low-Pressure Drop (LPD) and Low-Low-Pressure Drop (LLPD) mixer [25], and the Sulzer SMX mixer including a recently developed new series of SMX mixers, the SMX(n) [26].

In this paper we mutually compare the global performance of all these mixers in the regime of laminar flow to find the optimum design using both criteria, i.e. pressure drop and length. We start with an overview on earlier approaches to quantify mixing in motionless mixers and most studied are the Kenics RL-180 mixer, with a right–left twist and an angle of blade twist of 180°, and the Sulzer SMX. The first is characterized by its relatively simple geometry, see Fig. 1a, which gives excellent mixing at low pressure gradients at the cost of long lengths. The second mixer, see Fig. 1c, represents the complete opposite and combines compactness with a complex geometry that needs large pressure gradients to sustain the flow. Apart from compactness and energy use, a third issue is whether mixer geometries can be (injection) molded which is of utmost importance for disposable applications, which represent a huge market. Since basically only simple geometries can be (de)molded, disposables generally suffer from too long lengths, which implies a large wasted volume. Also this aspect is part of this study and via a thorough analysis of the elementary working principles of the various designs a new geometry is proposed that combines compactness with simplicity of shape.

Fig. 1. Motionless mixers: (a) Kenics (right twist–left twist; angle of blade twist 180°), (b) Ross LPD (right rotation–left rotation; crossing angle $\theta = 90^\circ$), (c) standard Sulzer SMX (n, N_p, N_x) = (number of crosses over the height, number of parallel bars over the length, number of crossing bars over the width) = (2, 3, 8), and two examples of the new design series of the most efficient SMX(n) (n, N_p, N_x) = ($n, 2n - 1, 3n$), here in rectangular version of (d) the “working horse” ($n = 1$), and of (e) the compact ($n = 3$); see Ref. [26] for nomenclature and explanation.

1.1. Kenics type of mixers

In the eighties it proved that even the simple geometry of the Kenics mixer was too complex for a detailed analysis and the so-called PPM, partitioned pipe mixer, geometry was proposed instead [27]. It consists of straight plates with length $1.5D$, crossing each other under an angle of 90° , placed inside a rotating tube with diameter D . The transverse flow in the PPM is comparable to that in the Kenics mixer, allowing relevant mixing analyses, but this only holds for Newtonian fluid flows. When shear thinning is present, the difference in driving mechanisms, pressure flow in the Kenics versus drag flow in the PPM, yields significantly different velocity fields, and thus different mixing. In particular pressure-driven mixers are far less sensitive to changes in the rheology. From a theoretical point of view the rather simple PPM has several advantages and simplifying assumptions can be made. One of the main assumptions is that axial and radial flow can be decoupled and expressed in closed-form analytical relations; a second simplification is that transition zones from one blade to another are infinitely thin. With these assumptions, the theory and dynamical tools developed for prototype flows based on concepts of chaotic advection as introduced by Aref [28,29] could be applied. Poincaré maps, three-dimensional islands separated from the main flow by KAM (Kolmogorov, Arnold and Moser) boundaries, were detected and minimalization of these KAM tubes formed the basis for mixing analyses and optimization by different groups, i.e. Khakhar et al. [27], Meleshko et al. [30], Muzzio [31], and Ottino [32]. Also the standard Kenics geometry allowed for numerical analyses, starting in 1995, see [33–36]. Dynamical system techniques were applied to understand and optimize mixing, see [31–38], and applied to the Kenics geometry [39], in the low [40], and high [41,42], Reynolds number regime, all the way to turbulent flows [43–46], and the use of the Kenics as reactor [47–49]. The group of Muzzio developed analytical and numerical methods to quantify stretching distributions in these types of mixers, which is far from trivial since interfaces grow exponentially in time. A somewhat related study was conducted by Galaktionov et al. [50], who extended the original mapping method to also map a microstructural variable, the area tensor, in time. One of the striking conclusions of that study was the importance in choice of the mixing measure while comparing different designs of a mixer. In particular, it was found that in pressure driven mixers, like static mixers, a mixing measure should be flux weighted. The distribution of material close to walls is far less interesting and important compared to the material in a zone of maximum velocity. In almost all studies the rheology was chosen to be Newtonian, where some groups also included the influence of shear thinning [51], both theoretically [52], and experimentally [53], as well as the influence of two phase flows [54–56]. Interestingly, the Kenics mixer is often used to promote heat exchange [57], and the optimum geometry for mixing a 50–50% equal viscosity fluid differs from that for exchange of wall fluid [21], which basically is a warning against too rapid conclusions. Finally, the availability of proper experimental data concerning quantification of mixing quality are scarce; the best data result from the PhD work of Jaffer [58].

1.2. SMX type of mixers

Similar analyses of mixing in the SMX designs seriously suffered from its geometrical 3D complexity where meshing becomes non-trivial and details of the boundary conditions are important. Only few groups were capable of performing simulations, like Muzio [59,60] and Tanguy [61]. Application of methods from dynamical systems theory is complicated due to the inherent numerical nature of the velocity field and any attempt of optimization remained a problem given the limited number of computations that could be performed in reasonable time [62]. Interestingly it was believed, even by the manufacturers of the Sulzer SMX, that the standard geometry, which we called the SMX (2, 3, 8) based on the number and structure of the crossing bars inside the mixer, performed optimal, since neither experimentally nor via modeling improvements were found [63]. The first proper analysis of mixing in the SMX geometry had to wait until the new way of computing the components of the mapping matrix was developed and incorporated as a post-processing operation added to standard CFD software [23]. Understanding mixing, even in these complex geometries, is based on investigating the optimum interface stretch in the cross section of the device, which is extensively dealt with in Section 2 below, and resulted not only in improvements in the design of the SMX but moreover in a complete new SMX series, the SMX(n) (n , N_p , N_x) = (n , $2n - 1$, $3n$), see [26].

1.3. Perfect (stratifying) and small mixers (microfluidics)

Two more, important application areas of motionless mixers exist. The first where the aim is producing stratified systems with uniform layer distributions and the prime example here is the beautiful Multiflux mixer, developed by Sluijters while working at Akzo, and originally meant to improve the melt temperature homogeneity in spin lines [64,65]. The working principle of the Multiflux most closely realizes the perfect bakers transformation of stretching, cutting and stacking, see [66] and the overview *Mixing of immiscible liquids* in [67]. Modifications of the original design were the addition of so-called I and H elements to even improve the homogeneity in layer distribution [68], and splitting not in the middle, to realize a hierarchical layer thickness distribution [69]. The second area is that of microfluidics where, since the Reynolds numbers are small and the Péclet numbers are large, chaotic advection is again the only way to mix fluids in reasonable time, see the overview *Scale-down of mixing equipment: microfluidics* in [70] and the review paper by Hessel et al. [71]. Basically people working in microfluidics reinvented what was already rather well known in polymer processing, where not the small channel dimensions but the high viscosities require efficient mixing in the laminar flow regime. All designs realized on the micro-level have in common that they possess a simple geometry such that they can be realized, e.g. on the interface between bottom and top part of the device. Optimized mixers create crossing streamlines, the prerequisite for periodic points around which chaotic advection is realized, by generating two counter rotating vortices of unequal size in the cross section of the

channel, that change position during axial flow. The prime example is the staggered herringbone mixer, see Stroock et al. [72] for precise and spectacular experiments, and later [73,74] for computations and optimization. A second example is the barrier-embedded mixer [75], which uses slanted grooves at the bottom of the channel with a barrier placed at the top, see [76] for explanation and optimization. The third example is the serpentine channel. Originally, this one only worked at Reynolds numbers higher than a critical value, $Re > 50$, see [77] for design and [78] for calculations. Later, it was recognized that the original serpentine mixer relied on inertia to induce folding, and the splitting serpentine channel was developed, greatly improving its performance in the low Re number regime by completely changing the working principle [79–81]. Further optimizing the geometry resulted in even better layer distributions by bringing fluid from outwards inwards and vice versa. This design is in our lab also macroscopically realized notably on the two halves on a mold for injection molding, making the fabrication of multiple layers during the injection process possible. Finally based on these principles an interesting multiple flow splitting and recombining design was developed, also for use on the macroscopic scale [82].

After these excursions, partly to mixing on the microscale, we are back on the macroscale and start the analysis of the flow inside the most commonly used motionless mixers, in order to understand how they stretch interfaces, which is the basis of making better new designs. This is the topic of Section 2 presenting the qualitative results. Section 3 reports the quantitative results where the mapping method is applied, and the flux-weighted cross-sectional area-averaged discrete intensity of segregation is computed, that allows optimizing the mixers using the two different criteria, compactness and energy use. Given the relevance of disposable mixers with small volumes, also (de)molding issues are sometimes touched when appropriate.

2. Qualitative results

2.1. Interfacial stretch

Fig. 1 shows the mixing devices that are compared in this paper, and clarifies in the legend the notation used, and Table 1 reveals their characteristic dimensionless length,

Fig. 2. The original Kenics RL 180°: the frames show the evolution of concentration patterns within the first firblade: (a) start, (b) one quarter, (c) one half, (d) three quarters, and (e) exit, and in the subsequent blades: (f) two, (g) three, and (h) five [24].

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L/D , and pressure drop ΔP^* ($=\Delta P/\Delta P_0$ where ΔP is the pressure drop per element and ΔP_0 is the pressure drop in an empty pipe with the same diameter D), and their mixing efficiency expressed in the characteristic interfacial stretch generated. N_{elem} reflects the number of elements of the device. These characteristics form the basis in understanding the working principle of all motionless mixers and allow the comparison of their mutual performance as summarized in Fig. 8.

2.1.1. Kenics RL 180°

The standard Kenics mixer, RL 180°, is used to illustrate the approach and Fig. 2 shows, in a Lagrangian way (moving with the fluid through the mixer), how black and white fluids mix. The interface between black and white is first cut into two halves, each with a length $0.5D$, by the edge of the twisted blade and is, during the rotation caused by the transversal flow working in the cross section of the mixer, subsequently stretched inside the two halves of the mixer to result in two interfaces, each with length D .

This interfacial stretching results in a $2^{N_{elem}}$ increase in interface length, see Fig. 3, top row, for a schematic presentation, and Scheme 1 in Table 1. But there is more. In each element one extra interface is generated at the trailing edge of the twisted blade where the two fluids that flow on both sides of the blade recombine while having a different color, see Fig. 2e. Including those extra interfaces, that

Table 1

Geometrical features of different static mixers and their interfacial stretch; for details see text.

Mixer type	L/D	$\Delta P^*/\text{element}$	Interfacial stretch	
			Scheme 1: expected	Scheme 2: actual
Kenics (RL 180°)	1.5	5.5	$2^{N_{elem}}$	$4^{N_{elem}} - 1$
LPD (RL 90°)	1	8.5	$3^{N_{elem}}$	$2^{N_{elem}}$
LLPD (RL 120°)	$\sqrt{3}$	7.4	$3^{N_{elem}}$	$2^{N_{elem}}$
Standard SMX (2, 3, 8)	1	38	$8^{N_{elem}} - 1$	$5^{N_{elem}}$
SMX(n) (3, 5, 9)	1	83	$9^{N_{elem}}$	$9^{N_{elem}}$
SMX(n) (4, 7, 12)	1	171	$12^{N_{elem}}$	$12^{N_{elem}}$
SMX(n) (1, 1, 3)	1	9.2	$2^{N_{elem}}$	$4^{N_{elem}}$

Scheme 1: as one would obtain from a first examination of interface stretching, e.g. from textbooks. Scheme 2: based on actual interface stretching and cutting, using column C¹ of Fig. 4.

Fig. 3. Schematic interfacial stretch in the Kenics mixer: (a) the interface multiplication and (b) including the extra interfaces.

are also cut by the next element, and rotated and stretched by the secondary flow, results in a much higher efficiency of $4^{N_{\text{elem}}} - 1$; see the bottom row of Fig. 3, and Scheme 2 in Table 1.

2.1.2. Kenics RL 140°

This straightforward and well-known result teaches us a lot. For example, cutting an interface with length D in two halves, $0.5D$, to subsequently stretch them each to length D is the best we can do in a device with internal diameter D . That is why this is the optimum stretch. The fact that the optimum blade twist angle in a Kenics is 140°, rather than the standard 180°, as demonstrated in [24], can be directly deduced from Fig. 2d and e. Clearly a 180° twist, Fig. 2e, overstretches the interface, while at $3/4$ of the length of the first blade, the interface length D is already reached, see Fig. 2d. The disadvantage of overstretching is best demonstrated with the Kenics RL 270°, see Figs. 8e and 9e in Galaktionov et al. [24], where overrotation brings the interface length, prior to be cut by the next element, from $0.5D$ via D back to $0.5D$. No mixing results.

Based on this concept of the characteristic interfacial stretch, we compare in the next section different mixer layouts, starting with a detailed analysis of the flow inside the complex geometries of the mixers, and subsequently analyzing the resulting mixing patterns.

2.2. Flow and mixing in different designs

Mathematical tools are available to numerically model 3D flows, even in complex geometries, and the mapping method has been developed to analyze distributive mixing based on the computed velocity fields. In its original implementation, first the boundaries of a large number of N cells, forming a fine grid within the fluid, are accurately tracked during a characteristic flow distance Δz . Subsequently, the deformed grid is superposed on the original undeformed one, and the mutual intersections are computed and stored as elements in the mapping matrix, φ of the order $N \times N$. The elements have values between zero and one and they reflect the fraction of fluid that is advected from each cell to every other cell during the flow distance Δz . Only the

non-zero numbers (zero means that no fluid was exchanged between the cells over this flow distance) are stored in the matrix φ that becomes huge if Δz is chosen large, the grid is fine and the problem is 3D. Using the mapping matrix it is straightforward to compute from the original concentration C^0 defined as a column vector of length N , the concentration distribution C^1 after Δz , since it follows from the (mapping matrix φ – concentration vector C) multiplication $C^1 = \varphi C^0$, which is a fast operation. The procedure is repeated during the total flow distance $z = N \cdot \Delta z$ going from step $j^* \Delta z$ to step $(j+1)^* \Delta z$ with $j = 0, \dots, N-1$, computing $C^{j+1} = \varphi C^j$. After each mapping step, thus at each distance $j^* \Delta z$, the local concentration distribution is averaged over each cell, which leads to some numerical diffusion. Later, an improved implementation of the method was proposed by Singh et al. [23] based on tracking a large number of particles in each cell, rather than tracking the cell's boundaries. The definition of the elements of the mapping matrix simplify accordingly and only needs to count the number of particles from each cell as they end up in all other cells after a flow distance Δz . This new implementation is easily applicable to 3D flows and represents therefore a major improvement, allowing the use of the method even in complex geometries like those of the Sulzer mixer, but also in dynamic mixers like co-rotating twin screw extruders, see e.g. [83–85].

In conclusion it can be stated that the mapping method is elegant and fast, and allows, once the necessary matrices are determined via parallel computing, for hundreds or thousands of computations in a reasonable time on a single processor PC, indeed making optimization of distributive mixing possible by investigating the influence of different geometries, protocols and processing parameters. Fig. 4 summarizes the results of mixing in the Kenics, Ross and Sulzer mixers of different designs. The concentration distribution of black and white fluid, entering the devices, the first column C^0 , is plotted after 1–4, and 8 elements as C^1 – C^4 , and C^8 , respectively. Basically already the concentration distribution after one element, the column C^1 , reveals the characteristic interfacial stretch of every design. Before starting to quantify the concentration distributions, we will first qualitatively analyze this column of Table 1 in some more detail.

Fig. 4. Mixing profiles in most common static mixers, Kenics, Ross and Sulzer and their modifications computed using the mapping method, see Galaktionov and coworkers [20,21,24] and Singh and coworkers, [23,25,26,74].

Fig. 5. Schematic interfacial stretch in the Ross mixer: (a) no interface multiplication, (b) optimal and (c) realistic interface folding.

2.3. Characteristic stretch in different designs

2.3.1. Ross LPD, LLPD

After the short analysis of the Kenics mixer in Section 2.1, we continue with the working principle of the Ross mixers. The presence of the two crossing blades causes a rotation in the flow, causing a rotation of the interface. From horizontal to vertical in Fig. 5a, while subsequent elements keep on rotating the interface further. This gives no mixing.

However, as can be seen from Fig. 4, rows 2 and 3, the interface does not remain straight during rotation. Instead it starts folding meanwhile stretching. In the schematics of Fig. 5b the interface length increases by this folding from D to $3D$. An efficiency of $3^{N_{elem}}$ results for this—optimum—stretch; Scheme 1 in Table 1. However, since folding and stretching are clearly not complete, see Fig. 4 row 2 for the RL 90° which is the LPD, and row 3 for the RL 120° , which is the LLPD (that is slightly better in terms of stretch) the schematic of Fig. 5c gives a better estimate, and the interface length increases to something more close to $2D$, rather than $3D$. A more realistic factor of $2^{N_{elem}}$ results; Scheme 2 in Table 1.

2.3.2. Sulzer SMX (1, 1, 3)

Next we investigate the Sulzer SMX designs, starting with the most simple ($n=1$) configuration of the new $(n, N_p, N_x)=(n, 2n-1, 3n)$ series, with n the number of units over the height of the channel, N_p the number of parallel bars along the length of one element, and N_x the number of cross bars over the width of the channel. In Singh et al. [26] we called this (1, 1, 3) design the “working horse” of the proper SMX series.

The two mirrored LPD-type crossing blades over the width of the channel of the (1, 1, 3) cause two counter

rotating vortices that split the fluid in the middle creating two interfaces each of length D , resulting in a factor $2^{N_{elem}}$, see Fig. 6a, which is Scheme 1 in Table 1. Also in this case, during rotation, folding of the interfaces occurs, like in the LPD, compare Figs. 5b and 6b, which increases the efficiency of the (1, 1, 3) to $6^{N_{elem}}$. However, since also here interface stretching by interface folding during rotation is not optimal, a schematic as depicted in Fig. 6c is more realistic and the efficiency becomes $4^{N_{elem}}$; Scheme 2 in Table 1.

2.3.3. Sulzer SMX $(n, N_p, N_x)=(n, 2n-1, 3n)$

Finally we turn to the compact SMX mixers. For simplicity we will here temporary neglect the extra interface stretch due to folding during rotation, which was the basis of the working principle of the Ross mixers and which gives the extra length in interfaces reflected in the typical “hairy” structures (interfaces with extra folds) in all SMX’s, see Fig. 4, rows 4–10. Even without taken the extra interfaces into account, the arguments on the point we want to make remain the same.

The $(n, N_p, N_x)=(n, 2n-1, 3n)$ design series is expected to simply result in an interface multiplication factor of $(3n)^{N_{elem}} - 1$, see Fig. 7a (that shows the $n=2$ configuration (2, 3, 6)). Because this series is based on optimum interfacial stretching per element [26], Schemes 1 and 2 in Table 1 are identical. The standard SMX is defined as a (2, 3, 8) mixer, that is somewhere in between the (2, 3, 6) and (3, 5, 9) configurations of the optimum interfacial stretch series, and clearly does not have this advantage. Where ideally it would result in $8^{N_{elem}} - 1$, see Fig. 7b, the more realistic interface stretch given in Fig. 7c results in $5^{N_{elem}} - 1$. This disappointing factor should be compared to the $6^{N_{elem}} - 1$ of the (2, 3, 6) design and the $9^{N_{elem}} - 1$ of the (3, 5, 9) version.

Fig. 6. Schematic interfacial stretch in the Sulzer SMX(n), $n = 1$ mixer: (a) interface multiplication, (b) optimal and (c) realistic interface folding.

Fig. 7. Schematic interfacial stretch in the Sulzer SMX(n), $n = 2$ mixer (a), compared to the standard SMX (2, 3, 8) mixer with ideal (b) and more realistic (c) interface stretching.

2.4. Conclusions on interface stretch in the different designs

Fig. 8 gives a graphical interpretation of the results of this section as summarized in Table 1 and plots on a semi-logarithmic scale the number of layers generated along the length of the mixer (left, criterion compactness) and as a function of pressure drop needed (right, criterion energy efficiency). Results are as expected: simple geometries need large lengths, compact mixers need large pressure

gradients. The most energy efficient mixer is the Kenics, shortly followed by the SMX (1, 1, 3) while the most compact mixer is the SMX (4, 7, 12) shortly followed by the SMX (3, 5, 9).

Before turning to the quantitative mutual comparison of the performance of all mixers in Section 3, using the (flux-weighted, cross-sectional area-averaged, discrete) intensity of segregation as an objective mixing measure, we now first use our growing understanding of interface stretching in the different geometries to arrive at a new mixer design that tries to meet the two opposite requirements: simple geometry and, nevertheless, compact mixing.

2.5. A new mixer design, the SMX (1, 1, 4, 135°)

Examination of Table 1 and especially comparing the first considerations on how motionless mixers increase interfaces, which is Scheme 1, with the more realistic view, as depicted with Scheme 2 in the same table, we learn that in all but two cases the realistic versions lead to either the same or (usually) lower values of the multiplication factor. These two exceptions are the Kenics RL 180°, where in every element an extra interface is created at the trailing edge of every twisted blade, and the SMX (1, 1, 3) where extra folding of interfaces during rotation and stretching create “hairy” structures, which are folded interfaces with larger length than expected based on applying the pure baker’s transformation. Of course the extra folds appear in all SMX(n) mixers, but were not taken into consideration in Table 1. Where the SMX (1, 1, 3), see Fig. 9a has two counter-rotating vortices that give symmetry by mirroring, the Kenics uses two non-symmetric co-rotating vortices that produce the extra interface, since white meets black fluid in the middle. But rotation of the flow in the cross section of the Kenics does not produce folding, while the

Fig. 8. Graphical summary of all results from the qualitative analyses thus far, Table 1. Plotted is the logarithm of the number of layers generated $N_{elem} \times \log(a)$ as a function of the length, defined as $N_{elem} \times L/D$ (left) and of the total pressure drop, defined as $N_{elem} \times \Delta P/elem$ (right) for all 7 mixers from Table 1. The red, bold line is for the standard Sulzer SMX (2, 3, 8), mixer number 4 in Table 1. It clearly only proves intermediate using both criteria.

rotation in the SMX does. The obvious question therefore is whether the two positive effects, an extra interface and folding can be combined, e.g. by avoiding symmetry in the SMX: the SMX (1,1,4), see Fig. 9b. It has four blades that give two co-rotating vortices.

Since the (1,1,4) is basically one quarter of a standard SMX (2, 3, 8), it has improper interface stretching, see Fig. 4, row 4, and Fig. 7c. The solution to this basic problem of the standard SMX mixer is an improved ratio between number of parallel N_p and cross N_x bars, and resulted in the

Fig. 9. Simple geometries in the SMX design: (a) (1, 1, 3), (b) (1, 1, 4), and (c) (1, 1, 6).

Fig. 10. Mixing in some ($n=1$) geometries of the SMX design: (a) entrance C^0 ; (b) after the first element C^1 for the (1, 1, 6, $\theta=90^\circ$); (c) C^1 for the (1, 1, 6, $\theta=135^\circ$); (d) C^1 for the (1, 1, 4, $\theta=135^\circ$); (e) C^1 – C^4 , and C^8 for the (1, 1, 4, $\theta=135^\circ$); and (f) C^1 – C^4 , and C^8 for the (1, 1, 4, $\theta=135^\circ$) mirrored.

optimized SMX(n) series: (n, N_p, N_x) = ($n, 2n-1, 3n$). For $n=1$ this gives the (1,1,3) that has the counter rotating mirrored symmetry and no extra interface and is, therefore, not the solution. In Singh et al. [26] we also investigated the influence of the crossing angle θ between bars, the last geometrical parameter available, to show that changing the angle has only influence on interface stretch near the walls. This is once more illustrated by investigating the influence of θ in an SMX (1, 1, 6) geometry, see Fig. 9c, and the results are plotted in Fig. 10. Fig. 10a shows the initial configuration and Fig. 10b the distribution and interface stretch after one element of the SMX (1, 1, 6, $\theta=90^\circ$). Interface stretching is insufficient as expected. Increasing the crossing angle to $\theta=135^\circ$ does not help in improving the layer distribution, instead interfaces curl up, see Fig. 10c. In contrast, for the SMX (1, 1, 4) configuration with 4 parallel bars, that basically only has near-wall interface stretching, the larger crossing angle helps in sufficiently far stretching the interfaces while indeed the extra folds remain, see Fig. 10d. Thus in the SMX (1, 1, 4, $\theta=135^\circ$) both beneficial effects are present: the extra interface created in the midplane and the extra folding of the stretched interfaces, the hairs, while the total interfacial stretching is OK.

Based on this positive result, we investigate how mixing proceeds in this SMX. In Fig. 10e every second mixing element is rotated over 90° , and indeed a uniform multiple interface stretching results over the cross section. In Fig. 10f every second mixing element is rotated over 90° and moreover mirrored to break symmetry. It gives very similar results, indicating that mirroring is for the SMX

geometry not necessary, while it is in the Kenics and Ross LPD and LLPD geometries.

3. Quantitative results

The qualitative results of the mutual comparison of the different motionless mixers of Fig. 1 in Section 2 gave understanding of their working principle which resulted in their mutual comparison using energy consumption and compactness as criteria, see Table 1 and Fig. 8, and in a new mixer design that combines all beneficial extra's: the SMX (1,1,4, $\theta=135^\circ$). Here we compare all designs of Fig. 1 in a quantitative way, by computing the flux-weighted cross sectional area-averaged discrete intensity of segregation. The mapping results of Fig. 4 are the basis of this computation. Two distinct design criteria are used: compactness as measured by the dimensionless length L/D , see Fig. 11 that can be compared to Fig. 8 (left), and energy efficiency based on the pressure drop per element normalized with that of an empty pipe with the same dimensions, $\Delta P^* = \Delta P_{elem} / \Delta P_0$, see Fig. 14 that can be compared to Fig. 8 (right). Velocity fields and pressure drops are computed using *Fluent V6.1.22*, applying Stokes flow of a Newtonian fluid with a viscosity of 1 Pa s and density of 846 kg m^{-3} . The same diameter is used in all devices, $D=0.052 \text{ m}$, while the thickness of all blades and bars equals $t=0.002 \text{ m}$. The same flux is applied, defined with the average entrance velocity of 0.01 m s^{-1} , yielding a Reynolds number of $Re=0.44$. Unstructured tetrahedral meshes are obtained with *Gambit V2.1.2*.

Fig. 11. Mixing performance of static mixers. Plotted is the flux-weighted cross-sectional averaged discrete intensity of segregation, computed using the mapping results of Fig. 4, versus the dimensionless length of the mixer, for the mixers of Figs. 1, 4 and 9.

3.1. Choice and motivation of an appropriate mixing measure

One of the main objectives of this paper is to present a quantitative comparison between most common static mixers applied in the limit of zero inertia, either based on compactness or pressure drop. Clearly static mixers have several different applications and an appropriate mixing measure is needed. In this work we choose to use a measure based on the intensity of segregation I . The intensity of segregation is a first-order statistic moment that quantifies the deviation of the composition of the mixture to the ideal case. Its value is scaled such that it ranks from 1 (poor mixing) $< I < 0$ (ideal mixing). Although the intensity of segregation provides a mean to compare different static mixers in a quantitative way, a more appropriate measure is the *flux-weighted* intensity of segregation I . Basically this means that in a cross section, after a characteristic unit length of a mixer, the intensity of segregation is weighted with the velocity distribution in the same cross section. The beauty of this mixing measure is that it automatically includes the residence time distribution of material. For clarity, the logarithm of I is plotted as vertical axis, $\log I$. As reference line in the comparison $\log I$ versus ΔP^* and $\log I$ versus L/D can serve the performance of the standard Sulzer SMX (2, 3, 8) since it proves to be in the middle in both comparison figures (basically demonstrating its poor, or only intermediate, performance using either of the two design criteria, see also Fig. 8).

3.2. Performance for compactness

In performance versus compactness, $\log I$ versus L/D see Fig. 11, we distinguish three groups of lines:

1. The middle group of lines are around the standard SMX (2,3,8, $\theta = 90^\circ$). Here we find the most simple version $n = 1$

of the new design series, SMX(n) (1,1,3) in its circular form, and the new square SMX (1, 1, 4, $\theta = 135^\circ$).

2. The poor ones have much longer lengths. They are the mixers with the simplest geometry: the Kenics RL 180° and the LPD RL 90° , and – as worst – the LLPD RL 120° . Rather interesting is the difference between the standard Kenics RL 180° and the optimized (see Section 2.1) Kenics RL 140° that uses 20% less length for the same quality of mixing, and starts approaching the standard Sulzer (2, 3, 8, $\theta = 90^\circ$) line.
3. The winners in compactness are – not that surprisingly – the mixers with increasing geometrical complexity that all are members of the optimized SMX(n) series: $(n, N_p, N_x) = (n, 2n - 1, 3n)$. We find the $n = 3$ (here in circular version) to be at least twice as good as the standard SMX (also circular) while the (here square) $n = 4$ can (of course) be laid out even more compact while achieving the same high mixing performance (a horizontal line in Fig. 11).

Not shown in the figure, for clarity, are results of slightly different variants, like the square (1, 1, 3) and (3, 5, 9) and the circular (4, 7, 12); they perform similar as their counterparts. The conclusion is that simple mixers are long and complex mixers short, with an interesting exception for the new SMX (1, 1, 3, $\theta = 90^\circ$) and SMX (1, 1, 4, $\theta = 135^\circ$) designs. They are both simple in geometry and have moderate lengths only, directly comparable to the standard Sulzer SMX (2, 3, 8, $\theta = 90^\circ$).

3.3. The SMX-plus

Recently, Sulzer introduced a new variant, the SMX-plus [86,87], see Fig. 12a. Its design is based on thorough advanced analyses, using 3D CFD as a design tool [63] and the basic idea is to reduce the width of the crossing bars to increase the volume available for the fluid, decreasing its resistance without losing mixing performance. We

Fig. 12. Geometry (a) and surface description (b) of the SMX-plus.

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analyzed mixing in the SMX-plus using the mapping method. In order to do that we computed the 3D velocity field under otherwise identical conditions as used in the computation of the velocity field in the other devices, based on the geometry provided by Sulzer [88], see Fig. 12b.

In contrast to just tracking particles in the 3D velocity field obtained and based thereupon compute the covariance [63], we compute the much more precise objective quantitative mixing measure, the flux-weighted area-averaged intensity of segregation $\log I$, that as explained in Section 3.1 allows direct comparison of performance with alternative mixers. This is done in Fig. 13. We first compare the SMX-plus with the standard SMX (2, 3, 8, $\theta = 90^\circ$) to conclude that it indeed performs much better, about twice as good, both in terms of length needed, Fig. 13a, as well as pressure consumption, Fig. 13b. Being especially designed for low pressures, by making thinner crossing bars that

are connected with complex bridges, the result in Fig. 13a seems somewhat surprising on first sight, and demonstrates that the improved performance is not only due to a lower pressure drop. The answer comes from counting the number of parallel and cross bars in the SMX-plus design, and we conclude that the mixer contains $N_p = 3$ and $N_x = 6$ bars and is, therefore, probably without Sulzer realizing this, the first fabricated member of the optimized series $(n, N_p, N_x) = (n, 2n - 1, 3n)$. This is confirmed by comparing its performance with a normal (2, 3, 6) circular design of the same optimized series. Results of both mixers are identical in performance versus length (Fig. 13a) while indeed the complex thinned bars in the SMX-plus give a slightly lower pressure drop for the same mixing performance, see Fig. 13b.

These results contradict those of Sulzer, the inventor of the SMX plus. In their analysis Hirschberg et al.

Fig. 13. Performance of the SMX-plus: (a) in terms of length and (b) in terms of pressure drop.

Fig. 14. As Fig. 11, now plotting intensity of segregation versus the dimensionless pressure drop.

basically concluded the opposite, see their Figs. 5 and 8 in [63], by stating that changing the number of cross bars N_x (6 versus 8) while keeping the number of parallel bars $N_p = 3$ the same, does not influence mixing (their Fig. 8) but has, combined with slimmer bars, a tremendous effect of pressure consumption, which is halved (their Fig. 5). In the text (page 529 bottom) they claim that the 50% reduction was also experimentally verified, while their Fig. 13 seems to support their first statement. Arguments based on computed mixing qualities are somewhat less convincing when compared to ours, since in the mapping method the accuracy is superior, given the much larger number of particles tracked because the number of elements in the fine grid is huge, while we moreover computed the *flux-weighted* intensity of segregation, which is the better measure, see Section 3.1 and [50]. The differences in computed pressure drop remain a problem that is not just solved by pointing to the differences in flux used ($Re = 0.44$ versus $Re = 0.08$). Possibly slight differences in geometry (we used the one in Fig. 12b) are a cause, or other boundary conditions. Since in our analyses we used the same mapping method with the same grid and number of tracked points per element, based on velocity fields computed with the same software, boundary conditions, diameters, blade thicknesses, and the same fluxes, in all mixing devices dealt with, we think that, despite this yet unclarified issue, we still can trust the comparative results presented here.

Being roughly a factor 2 better than the standard SMX (2, 3, 8, $\theta = 90^\circ$) does not mean that the SMX-plus is the best mixer available since by comparing Figs. 11 and 13 we can conclude that the more complex ($n = 3$ and $n = 4$) mixers of the same series on their turn outperform the ($n = 2$) variant in terms of compactness, as expected.

3.4. Performance for energy efficiency

Next we compare the performance of the mixers using energy efficiency as the design criterion, $\log I$ versus ΔP^* . Intuitively, and based on the results of Fig. 8, we expect the

opposite order as in the case of compactness and indeed we find three groups of lines, see Fig. 14.

1. The middle one is again the standard SMX (2, 3, 8, $\theta = 90^\circ$).
2. The poor ones, which are now obviously the complex SMX(n) designs which were superior in compactness: the $n = 3$ (here circular) and $n = 4$ (here square) need large pressure gradients to sustain the flow.
3. And, finally, we observe a large group of lines representing low pressure drop mixers (with consequently long lengths). We find the LLPD indeed better than the LPD mixer, and the SMX (1, 1, 3) identical in pressure consumption to the standard Kenics RL 180° . The optimized Kenics RL 140° is again 20% better than its RL 180° brother, but more interesting is that the new square SMX (1, 1, 4, $\theta = 135^\circ$) outperforms the Kenics RL 180° mixer and behaves identical as the RL 140° .
4. Since the SMX (1, 1, 4, $\theta = 135^\circ$) can be injection molded, it can be made disposable, although the mold needs moveable inserts to allow to eject the part (whereas the SMX (1, 1, 3, $\theta = 90^\circ$) does not need those inserts since in that geometry a small axial off-set of the crossing bars is sufficient to allow demolding).

4. Conclusions

This study on the performance of static mixers is conclusive. Explained is why interface stretch can be optimal and, based thereupon, why some alternative versions in the same design family perform better. Based on the Mapping Method, the interface stretch in the different motionless mixers is visualized and quantified. Two surprises give better interface stretching than expected on first sight, based upon which a new mixer design is proposed that combines the two beneficial effects. The performance of all mixers is quantitatively compared, using compactness and energy efficiency as two independent criteria. In detail the conclusions read:

1. We can define “optimal interface stretching” as the best we can do given the limited internal diameter D of a mixing device. We used the Kenics mixer to explain the concept. After splitting an interface of length D into two halves, each of length $0.5D$, the maximum stretch we can give to those halves is back to maximum length D in the cross section of the mixer. And that sets the optimum blade twist, which is not 180° but 140° .
2. This optimal interface stretch is the basis of understanding and optimizing other motionless mixer devices. In the Ross design it explains why the LLPD (low, low pressure drop) performs better than the LPD (low pressure drop) (the nomenclature relates to the other Ross motionless mixer, the ISG, the interfacial surface generator that requires huge pressure gradients). This is clearly not because in the LLPD the pressure drop is less, but because the interface stretch via interface folding from length D approaches the length $3D$ closer, which is the optimum we can achieve in one fold. That is why the LLPD performs better, despite its longer length. Similarly is the SMX-plus not better than the standard SMX because of its narrowed bar-widths that lower the pressure drop, but because, being part of the optimal series $(n, N_p, N_x) = (n, 2n - 1, 3n)$, it stretches the interfaces optimally. In this case it concerns a (2, 3, 6) mixer that, with 6 cross bars, increases the interface length from $1D$ to $6D - 1$, etc.
3. Comparing the interface multiplication after one element using the results of the mapping method, two schemes resulted: Scheme 1 with idealized stretching, and Scheme 2 with more realistic stretching, see Table 1. Usually Scheme 2 results in a lower interface multiplication factor than Scheme 1, but there are two exceptions. The Kenics design produces an extra interface at the trailing edge of the twisted bar, stacking “white” on top of “black” fluid making use of the two asymmetric co-rotating vortices. The SMX designs give, during interface multiplication, an extra fold in the interfaces formed, resulting in “hairy” structures. This folding is by the way the (only) working principle of the Ross designs.
4. Based on these findings, a new mixer design is proposed, the SMX (1, 1, 4, $\theta = 135^\circ$) which creates like the Kenics two co-rotating vortices, which give the extra interface, and combines that with extra interface folding. Being not a member of the optimal series $(n, N_p, N_x) = (n, 2n - 1, 3n)$, its interface stretch is insufficient. This is solved by increasing the crossing angle of the X bars from 90° to 135° . Usually increasing the crossing angle has no influence on interface stretch in SMX flows, but there is an exception close to the walls. The (1, 1, 4) only has close-to-the-walls stretch, and that is what makes it unique.
5. In compact mixing, the most “busy” geometries with the largest number of bars inside the cross section are to be preferred. As such, the standard Sulzer performs much better than the Kenics or Ross designs. However, interface stretching in the standard Sulzer is not optimal. Therefore the series $(n, N_p, N_x) = (n, 2n - 1, 3n)$ was designed. The $n = 3$ and $n = 4$ versions of this series perform indeed more than twice as good as the standard SMX, both in their circular and square designs.
6. Although higher order SMX(n) elements, like the $n = 4$, SMX (4, 7, 12), or the $n = 5$, etc. versions, of course yield continuously more compact mixers but at cost of high pressure losses, the $n = 3$, SMX (3, 5, 9) is a good compromise in the series of compact motionless mixers. The building block of the SMX(n) designs can be injection molded, see [26], and thus can any mixer be build up from staggered building blocks inside a square hollow pipe, and can be made disposable. Stacking these building blocks in a hierarchical sequence, gradually changing from overall mixing to local mixing, can further improve the mixer compactness, but a too early switch to local mixing limits the final mixing performance, see [26] for further explanation.
7. In efficient mixing aiming at low pressure drops, the more “simple” mixers with only a few bars or blades inside are to be preferred. The Kenics is the most efficient motionless mixer in its RL 140° design, being shorter and better than the standard RL 180° . Interesting is that the circular or square version of the SMX(n) (1, 1, 3, $\theta = 90^\circ$) follows shortly. This mixer can be easily injection molded, and thus made disposable, using a small axial offset between the crossing bars.
8. Most interesting though is that the new design, the SMX (1, 1, 4, $\theta = 135^\circ$) that combines extra interfaces with folded interfaces, behaves better than Kenics RL 180° and its performance in energy efficiency is identical to that of the Kenics RL 140° . The higher pressure drop per element needed to guide the flow through the crossing bars X as compared to that needed to pass the twisted blades, compare, e.g. the dimensionless pressure drops per element ΔP^* of the SMX(n) (1, 1, 3) with that of the Kenics in Table 1, is apparently completely compensated for by the extra interface folding, increasing the distributive mixing power of the SMX (1, 1, 4, $\theta = 135^\circ$).
9. As a consequence, the on crossing bars X based SMX designs (SMX means: Static Mixer using crossbars X) are at least equal to and usually outperform all others in all design criteria.

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