Aeroacoustic power generated by multiple compact axisymmetric cavities: Effect of hydrodynamic interference on the sound production

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Aeroacoustic sound generation due to self-sustained oscillations by a series of compact axisymmetric cavities exposed to a grazing flow is studied both experimentally and numerically. The driving feedback is produced by the velocity fluctuations resulting from a coupling of vortex sheddings at the upstream cavity edges with acoustic standing waves in the coaxial pipe. When the cavities are separated sufficiently from each other, the whistling behavior of the complete system can be determined from the individual contribution of each cavity. When the cavities are placed close to each other there is a strong hydrodynamic interference between the cavities which affects both the peak amplitude attained during whistling and the corresponding Strouhal number. This hydrodynamic interference is captured successfully by the proposed numerical method. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.4718726]

I. INTRODUCTION

Pipe systems with axisymmetric cavities are often used in engineering applications. However, at critical conditions, the flow through such an axisymmetric cavity-pipe system causes self-sustained oscillations that lead to high-amplitude sound generation and associated mechanical vibration. Grazing flow over axisymmetric cavities in ducted flows has been investigated for a wide range of applications such as control valves, piping systems, and impedance walls.

Despite the geometric simplicity of axisymmetric cavity-pipe configurations, self-sustained cavity oscillations in such systems involve several complex fluid mechanics phenomena. The oscillations can be classified into three categories based on the nature of the feedback: fluid dynamic, fluid-resonant, and fluid-elastic oscillations. The current work deals with a fluid-resonant mechanism in which the feedback is produced by the velocity fluctuations at the upstream edge of the cavity resulting from a coupling of vortex shedding with a longitudinal acoustic pipe mode. In such a feedback loop, the shear layer instability in the mouth of the cavity and the longitudinal acoustic pipe modes can be considered as the amplifier and the filter of a feedback system, respectively. This results in stable self-sustained oscillations at discreet frequencies, which is called whistling.

A single cavity in a pipe line with an acoustic feedback due to a longitudinal standing wave has been the focus of a considerable number of experimental studies, in which the effect of various geometric parameters such as the depth of the cavity and the width of the cavity is addressed.

On the other hand, there are only a limited number of studies considering the hydrodynamic interaction between series of cavities/side branches adjacent to each other. A study by Ziada and Bühlmann has demonstrated that two deep resonating closed side branches in close proximity display some hydrodynamic interaction. A more spectacular effect was found by Lange and Ronneberger and Aurégan and M. Leroux when considering deep axisymmetric cavities separated by thin walls forming an acoustic liner. They stated that the hydrodynamic interaction coupled with

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acoustic cavity resonance leads to a flow instability. To evaluate the impact of hydrodynamic interaction in acoustic liners, Derks and Hirschberg studied the interaction in a tandem of two Helmholtz resonators exposed to grazing flow. They reported a significant hydrodynamic interaction for a distance between the resonator comparable or smaller than the width of the opening of the resonators.

The authors have recently been interested in the sound generation in corrugated pipes. Corrugated pipes provide local stiffness while keeping its global flexibility. This unique characteristic makes corrugated pipes suitable for a broad range of industrial applications, e.g., ventilation systems, domestic appliances, heat exchangers, and offshore natural gas transportation systems. The main drawback of corrugated pipes is that flow through these pipes can cause self sustained oscillation, which produces high-amplitude whistling sound. In corrugated pipes the width and the depth of the cavities are much smaller than the wavelength of the longitudinal acoustic pipe mode, i.e., the cavities are acoustically compact sound sources. One approach is to model a corrugated pipe as a system composed of a series of acoustically coupled axisymmetric cavities connected to each other with straight pipe segments. Neglecting a possible hydrodynamic interference between the individual elements of the series of cavities, such a system can be investigated by simulating a single cavity with appropriate boundary conditions. In an earlier work, the authors had proposed a numerical methodology to investigate the aeroacoustic response of a single confined compact cavity, exposed to a low Mach number grazing flow, to acoustic excitations. The method combines incompressible simulations with Vortex Sound Theory. Although this so-called hybrid method is a highly simplified approach compared to the current computational aeroacoustic algorithms, it has been quite successful in predicting the Strouhal number ranges of acoustic energy production/absorption and the nonlinear saturation mechanism responsible for the stabilization of the limit cycle oscillation. This computationally low cost numerical method also predicts the peak-whistling Strouhal number, at which the maximum amplitude in pressure fluctuations is registered, in close agreement with experiments, and explains the dependency of the peak-whistling Strouhal number on the momentum thickness of the velocity profile that the cavity is subjected to. There is, however, an inaccuracy in this approach due to the neglected hydrodynamic interference, particularly when the distance between successive cavities is small. The current study addresses hydrodynamic interaction between compact (non-resonating) cavities which, to the authors knowledge, has not yet been addressed in the literature.

Hydrodynamic interference considered here has some similarity with the phenomena observed in heat exchanger pipe bundles placed in cross flow. Coupling of vortex shedding with acoustic standing waves is strongly influenced by the hydrodynamic interference between successive cylinders. Flow-excited acoustic behavior of tandem cylinder configuration in a resonating duct is considerably different than that of a single cylinder. A strong effect of spacing ratio has been recorded both on the amplitude of the maximum acoustic pressure and the Strouhal number range of the resonance. This stresses the need of an assessment for the effect of the hydrodynamic interference, between successive cavities in corrugated pipe, on whistling.

The aim of the present work is to investigate the aeroacoustic sound generation by double and triple compact axisymmetric cavity configurations exposed to a grazing flow, as a model for the whistling behavior of multiple axisymmetric cavity systems, e.g., corrugated pipes, and to assess the effect of hydrodynamic interference between successive cavities on the whistling. Section II is dedicated to the experiments. The experimental setup (Sec. II A), sound generation in hydrodynamically separated cavities (Sec. II B), hydrodynamic interference among the cavities in close vicinity of each other (Sec. II C), and the effect of plateau length on the hydrodynamic interference (Sec. II D) are covered in this part. In Section III the numerical methodology is summarized and the predictions of hydrodynamic interference (Sec. III B) are discussed. In Sec. IV, the conclusions are stated.

II. EXPERIMENTS

A. Experimental Setup

An experimental setup has been constructed to investigate self-sustained oscillations due to the coupling of vortex sheddings in a series of axisymmetric cavities exposed to grazing flow with a
The tested cavity configurations. The dotted line in configuration A, indicates the spatial distribution of the pressure amplitude ($p'$) in the standing wave at the first longitudinal acoustic mode.

FIG. 1. The tested cavity configurations. The dotted line in configuration A, indicates the spatial distribution of the pressure amplitude ($p'$) in the standing wave at the first longitudinal acoustic mode.

longitudinal standing wave. The test section is composed of a straight pipe section and a number of axisymmetric cavities, which can be placed at either end of the pipe. The cavity configurations considered in this paper are presented in Figure 1, each configuration is referred with a letter, e.g., double cavity downstream of the pipe is configuration D.

All the experiments have been carried out at the first longitudinal acoustic mode. This is a standing wave of a half-wave length in the pipe ($L_{\text{tot}} \approx \lambda / 2$). The system whistles also for the higher acoustic modes (i.e., $L_{\text{tot}} \approx \lambda, \lambda / 2, \lambda / 3, \ldots$). Experiments have been performed at the lowest possible whistling mode in order to keep the sound source as compact as possible ($W \ll \lambda$), allowing an incompressible model for the flow in the cavity. Configuration A (single cavity at the downstream termination) is similar to the one used by Schachenmann and Rockwell.8, 9 In their work the first whistling mode was obtained at the third longitudinal acoustic mode $L_{\text{tot}} \approx 3\lambda / 2\lambda$. This is due to their configuration combining a cavity with sharp edges with a relatively long pipe. In the current study, cavities have rounded edges which promote the sound production considerably.18, 28 Furthermore, a relatively short pipe segment is used to limit the viscous losses. It should also be noted that in all the configurations the cavities are placed close to a pressure node in order to maximize the sound production.17, 18, 29

The mean flow velocity profile of the flow that is approaching the cavity has a strong influence on the whistling phenomenon.19, 20 A sand paper strip (ISO/FEPA grid designation - P40) with a width of 5 mm is placed on the inner pipe wall at the inlet of the pipe in order to trip the boundary layer from laminar to turbulent flow. This avoids uncertainties due to the natural transition from laminar to turbulent flow. The cavity in configuration A experiences a developed turbulent approach velocity profile with a displacement thickness of $\delta_1 = 1.7 \times 10^{-3}$ m and a momentum thickness of $\delta_2 = 1.2 \times 10^{-3}$ m. The cavity in configuration B, however, experiences a top-hat velocity profile with a thin boundary layer, i.e., $\delta_1 = 3.9 \times 10^{-4}$ m and $\delta_2 = 2.0 \times 10^{-4}$ m. These velocity profiles and their effects on the whistling of single cavity configurations A and B have been discussed in an earlier paper.30 In the current work attention is given to the multiple axisymmetric cavities. Yet the same two different velocity profiles, turbulent and top-hat, are experienced by cavities in other configurations considered here, e.g., the upstream and downstream cavities of configuration C experience top-hat and turbulent, velocity profiles, respectively.

The notation for the relevant geometric parameters are shown in Figure 2 for configuration D. The same notation is used for all the other configurations as well. There are two straight pipe sections, the long one is $L_1 = 850$ mm and the short one is $L_2 = 15$ mm. Depending on the configuration, see Figure 1, the positions of these pipes interchange, e.g., for configuration E, $L_1$ is downstream of the cavities and $L_2$ is upstream. The inner diameter of the pipe sections is $D = 44$ mm. The pipe is made of steel with a roughness height of $\epsilon \leq 4.0 \times 10^{-5}$ m (European Standard EN 10305-1) and has a wall thickness of 6 mm. Such a thick pipe is used to avoid wall vibrations. Measurements of
the reflection coefficient for the closed pipe have confirmed that there is no significant effect of pipe wall vibrations on the measurements. The depth of the axisymmetric cavity is \( H = 27 \text{ mm} \) and the width of the cavity is \( W = 40 \text{ mm} \), hence height to width ratio is \( H/W = 0.675 \). For such a deep cavity the whistling characteristics, i.e., Strouhal number and the amplitude of the fluctuations are independent of the depth.\(^{18, 30}\) The setup allows the placement of a second cavity and to vary the plateau (\( L_p \)) length, the constant diameter part between the two cavities. It is also possible to mount a third cavity. The radii of curvature of the upstream and downstream cavity edges are denoted by \( r_{\text{up}} \) and \( r_{\text{down}} \), respectively. The upstream edge radius of the first cavity and the downstream edge radius of the last cavity is 5 mm, all the other edge radii are 2.5 mm. Experiments have been performed for seven different plateau length to width ratios, namely, \( L_p/W = 0, 0.375, 0.625, 0.750, 0.875, \) and \( 1.375 \). It should be noted that the plateau length does not include the edge radii of the cavities, see Figure 2. Thus, when the plateau length is \( L_p = 0 \) there, still, exists a wall thickness (\( r_{\text{down}} + L_p + r_{\text{up}} \)) between the cavities, which is 5 mm. The total length of the system is denoted by \( L_{\text{tot}} \), which varies in the range of \( 915 \text{ mm} \leq L_{\text{tot}} \leq 1115 \text{ mm} \) depending on the configuration. All the components of the experimental setup have been manufactured with an accuracy of 0.1 mm.

The setup used for the experiments is shown in Figure 3. The upstream termination of the test section is connected to the high-pressure air supply system, which is composed of, from upstream...
to downstream, a compressor, a constant pressure vessel, a control valve, a buffer vessel, a turbine flow meter, and an expansion chamber muffler. The expansion chamber muffler has a length of 1.5 m and a diameter of 0.6 m. It is covered internally with sound absorbing foam with a thickness of 100 mm in order to avoid cavity resonances. The downstream termination is open to the laboratory, a large room of 15 m × 4 m × 4 m (not an anechoic chamber).

The acoustic pressure in the long straight pipe segment ($L_1$) is measured using seven acceleration compensated piezo-electric pressure transducers, which are flush mounted. The transducers are calibrated together with their adapter pieces at the end-wall of a closed pipe both for the relative amplitude and relative phase difference. These microphones have a separation distance of 108.33 mm in axial direction and an angular position difference of 30° from each other in azimuthal direction, see Figure 3. The first and the last transducer are positioned 100 mm from the pipe terminations. The signals from the microphones are amplified by charge amplifiers. These amplifiers are connected to a combined data acquisition-personal computer (PC) system. The frequencies of whistling are in the range of 150−250 Hz and the sampling rate of the experiments was 5 kHz. Thus, the sampling rate satisfies the Nyquist criteria. By sampling for long enough time, a discrete time signal can be obtained whose discrete Fourier transform roughly represents the Fourier transform of the continuous time signal.31 For that purpose a sampling duration of 10 s has been used for each data point.

Knowing the pressure fluctuations at seven different spatial positions using the multi-microphone method32 traveling acoustic plane waves were reconstructed. The term fluctuation amplitude is used to specify the maximum dimensionless sound amplitude that is attained in the standing wave. The fluctuation amplitude is defined as

$$\left| \frac{p'_{\text{max}}}{p_0 c_0 U} \right| = \left| \frac{u'_{\text{max}}}{U} \right|,$$

where $|p'_{\text{max}}|$ is the amplitude of the standing pressure wave at a pressure anti-node inside the pipe and $|u'_{\text{max}}|$ is the amplitude of acoustic velocity at a pressure node. Since the term fluctuation amplitude always refers to this maximum dimensionless sound amplitude; the subscript max will be dropped for convenience. The details of the data processing are provided in an earlier work.30

The temperature of the air is measured at the pipe termination with an accuracy of 0.1 °C using a digital thermometer. A turbine flow meter is used to measure the average velocity ($U$) through the volumetric flow rate. The turbine flow meter is connected to a pulse shaper and a counter. The acquisition system of the turbine flow meter and the piezo-electric pressure transducers are synchronized by means of a trigger pulse. The simultaneous measurement of flow velocity and pressure fluctuations allows a waterfall representation of the data, in which the frequency spectra of the whistling at different flow velocities are presented in a single graph. Using this waterfall diagram consecutive modes that appear simultaneously with the dominant hydrodynamic mode can easily be identified.12, 33 Such secondary modes were not observed for the flow range studied in the current work and unlike the case of wall cavities, such as the ones studied by Delprat,34 nonlinear interaction between modes has not been observed. The details of the instrumentation are provided in an earlier paper.30

B. Sound generation in hydrodynamically separated multiple cavities

In Figure 4 measured dimensionless fluctuation amplitude $|u'/U|$ is plotted against Strouhal number $Sr = f (W + r_{\text{up}})/U$ for configurations A, B, and C. Considering the acoustics there is only a minor difference between configurations A (single cavity downstream) and B (single cavity upstream). This difference is caused by cavity having a finite width and the upstream and downstream edge of the cavity not being acoustically symmetric in the sense that the vortex is shed from the upstream edge of the cavity. Thus, the spatial position of the vortex with respect to the standing wave is not identical for these two configurations. The cavity width is, however, much smaller than the wavelength of the standing wave $W \ll \lambda$. Hence, this difference between configurations A and B is very small. The variation in the whistling behavior of these two configurations is mainly due
to the difference in the velocity profile that the cavities are subjected to, which is discussed in an earlier work of the authors. It is seen from Figure 4 that the peak-whistling Strouhal number $S_{r_{p-w}}$, the Strouhal number at which the maximum amplitude in pressure fluctuations is registered, is $S_{r_{p-w}} = 0.73$ and $S_{r_{p-w}} = 0.82$ for configurations A and B, respectively. For configuration C, where there is a cavity both at the upstream and downstream termination, the system whistles at a peak-whistling Strouhal number of $S_{r_{p-w}} = 0.77$, which is a compromise between the peak-whistling Strouhal numbers obtained in configurations A and B.

In configuration A there exists only one range of Strouhal numbers, $0.65 \leq S_r \leq 0.80$, for which the whistling is observed, whereas for configuration B there exist two distinct ranges of Strouhal numbers with whistling, $0.74 \leq S_r \leq 0.92$ and $1.30 \leq S_r \leq 1.50$. The lower and the higher ranges of whistling Strouhal numbers around $S_r = 0.76$ and $S_r = 1.40$ belong to the second and third hydrodynamic modes, respectively. At the second hydrodynamic mode there exist two vortices at the same moment inside the cavity mouth: one traveling and one forming. Similarly for the third hydrodynamic mode there are three vortices appearing simultaneously at the cavity opening: one forming and two traveling. The disappearance of the whistling for the higher Strouhal number range ($1.30 \leq S_r \leq 1.50$) for configuration A is explained by using the linearized theory of an inviscid quasi-parallel free shear layer. Configuration C has also two distinct ranges of Strouhal numbers in which the system whistles. It is evident from Figure 4 that for the second hydrodynamic mode the range of whistling for configuration C, $0.66 \leq S_r \leq 0.89$, is almost equal to the combined range of whistling for configurations A and B, $0.65 \leq S_r \leq 0.92$. For the third hydrodynamic mode the whistling range of configuration C is identical to the range of whistling of configuration B, $1.30 \leq S_r \leq 1.50$, because configuration A is silent for the third hydrodynamic mode.

Considering the dimensionless fluctuation amplitude $|u'|/U$ a parallel behavior with the Strouhal number is observed for configuration C. The peak-whistling amplitude obtained in configuration C for the second hydrodynamic mode is $|u'|/U = 9.3 \times 10^{-2}$, which is almost equal to the summation of the peak-whistling amplitudes obtained with configuration A, $|u'|/U = 4.7 \times 10^{-2}$ and B, $|u'|/U = 4.3 \times 10^{-2}$. For the third hydrodynamic mode the peak-whistling amplitude of configuration C is
identical to that of configuration B, $|u'|/U = 2.1 \times 10^{-2}$. This is expected because configuration A is not whistling at the third hydrodynamic mode.

It is concluded from this set of experiments that for a system of multiple cavities, in which the cavities are coupled to the same standing wave, when two cavities are placed at different pressure nodes such that there is no hydrodynamic interference, the whistling behavior of the system can be determined from the individual contributions of each cavity.

C. Hydrodynamic interference

In this section a possible hydrodynamic interference between cavities, which are adjacent to each other, is investigated. In Figure 5 the dimensionless fluctuation amplitude $|u'|/U$ is plotted against Strouhal number $Sr = f(W + r_{up})/U$ for configuration D (double cavity downstream) and for configuration E (double cavity upstream) with a plateau length of $L_p = 0$ (see Figure 2) together with configuration C, which has already been discussed in Sec. II B. It is seen that for the second hydrodynamic mode ($0.65 \leq Sr \leq 0.91$) the difference in the peak-whistling Strouhal number between configuration D, $Sr_{p-w} = 0.77$, and E, $Sr_{p-w} = 0.79$, is not as large as between configurations A and B, which are the single cavity versions of the same configurations. This is expected to be due to the fact that the downstream cavities in configurations D and E are experiencing similar velocity profiles.

Configuration D has the same peak-whistling Strouhal number, $Sr_{p-w} = 0.77$, as configuration C for the second hydrodynamic mode. This can be explained through the different velocity profiles that the two cavities of configuration D are subjected to. The upstream cavity of configuration D experiences a turbulent velocity profile, as in configuration A (turbulent, see Sec. II A). Assuming that the velocity profile is redeveloping on the plateau between the cavities, the downstream cavity of configuration D experiences a velocity profile with a thin boundary layer as in configuration B (top-hat, see Sec. II A). Thus, in configurations C and D each of the two cavities experience similar velocity profiles. As a consequence, it is not surprising that the peak-whistling Strouhal numbers of these two configuration are the same. A weak third hydrodynamic mode is observed for configuration D between $1.23 \leq Sr \leq 1.30$. The appearance of the third hydrodynamic mode also
indicates that the downstream cavity in configuration D experiences a boundary layer thinner than that in configuration A.

For configuration E, the third hydrodynamic mode has a dimensionless amplitude of $|u'|/U = 4.4 \times 10^{-2}$, which is two times higher than that of the third hydrodynamic mode observed in configurations B and C $|u'/U = 2.1 \times 10^{-2}$. As discussed in Sec. II B, only cavities experiencing a top-hat approach velocity profile contribute to the third hydrodynamic mode. This also indicates that both the downstream and the upstream cavities of configuration E experience a thin boundary layer. This also supports the idea that the velocity profile is redeveloping on the plateau. The focus in this work is, however, given to the second hydrodynamic mode, at which the peak-whistling is observed. Thus, the rest of the paper will be limited to the second hydrodynamic mode.

It is seen that both configurations C and D whistle in the same Strouhal number range so the convective acoustic losses at the downstream open pipe termination are similar for these two configurations. The acoustic boundary conditions both upstream and downstream are also identical in these configurations. Thus, the total acoustic losses are comparable for configurations C and D. In earlier papers it has been explained that the dimensionless fluctuation amplitude obtained in a system can be determined through an energy balance between the acoustic sources and acoustic losses. Since configurations C and D have comparable acoustic losses and configuration D has a 30% higher dimensionless fluctuation amplitude $|u'|/U = 1.2 \times 10^{-1}$ than configuration C, it can be concluded that the acoustic source power produced in configuration D with cavities in close proximity is higher than the one produced by configuration C.

It has been shown that the spatial position of the cavity with respect to the coupling longitudinal standing wave is important for the sound production. The sound production is maximized when the cavity is placed close to a pressure node. It is already known that the difference in acoustic source power produced by a cavity at the downstream pipe termination (configuration A) and at the upstream pipe termination (configuration B), due to a difference in the velocity profile, is rather small, see Figure 4. Neglecting a possible hydrodynamic interference in configuration D, one would expect configuration C to produce more acoustic source power than configuration D, because in configuration C both cavities at the upstream and downstream termination are very close to pressure nodes. In configuration D, however, both of the cavities are placed at the downstream pipe termination. Thus, in configuration D, the upstream cavity is further away from the pressure node than the downstream cavity. As a consequence the upstream cavity is expected to be less efficient for acoustic power generation than the downstream cavity. It is observed, however, that in the experiments configuration D produces a higher acoustic source power than configuration C. This suggests that in configuration D there exists a constructive hydrodynamic interference between the cavities for the plateau length of $L_p = 0$. This also holds for configuration E, for which the double cavity is placed at upstream termination.

D. Effect of plateau length

To assess the extent of this hydrodynamic interference and its dependence on the plateau length $L_p$ a series of experiments was performed with configuration D with various plateau lengths. In Figure 6 the dimensionless fluctuation amplitude $|u'|/U$ is plotted against Strouhal number

$$Sr = f(W + r_{wp})/U$$

for configuration D with plateau length to cavity width ratios of $L_p/W = 0, 0.375, 0.625, 0.750, 0.875$, and 1.375.

As shown already in Figure 5 when the two cavities are very close to each other, i.e., $L_p/W = 0$, there is a single peak (primary peak). With increasing plateau length this peak shifts to lower Strouhal numbers and decreases in amplitude. At a plateau length of $L_p/W = 0.675$ a secondary peak appears at a higher Strouhal number. As the plateau length is further increased the secondary peak shifts to lower Strouhal numbers with increasing amplitude. At a plateau length of $L_p/W = 1.375$ the secondary peak replaces the primary peak at the same Strouhal number and almost at the same amplitude as found for $L_p/W = 0$. In Figure 6 the primary and the secondary peaks are indicated with open and solid arrows, respectively. The peak-whistling Strouhal numbers $Sr_{p-w}$ and the peak-whistling amplitudes $|u'|/U$ for primary and secondary peaks for configuration D for all plateau length to cavity width ratios $L_p/W$ are summarized in Table 1.
FIG. 6. Dimensionless fluctuation amplitude $|u'|/U$ plotted against Strouhal number $Sr = f(W + r_{up})/U$ for plateau length to cavity width ratios of $L_p/W = 0$ (a), 0.375 (b), 0.625 (c), 0.750 (d), 0.875 (e), and 1.375 (f) for configuration D, see Figure 1.

Considering the two extremes of the plateau length $L_p/W = 0$ and $L_p/W = 1.375$, the two configurations whistle at the same peak-whistling Strouhal number and almost at the same amplitude. The small decrease in the dimensionless amplitude in the case of $L_p/W = 1.375$ is expected to be due to the fact that the upstream cavity has been moved further away from the pressure node compared to the case of $L_p/W = 0$. In between these two extremes of the plateau length, however, there is a range of plateau lengths around $L_p/W = 0.750$, such that the peak-whistling amplitude of the system is half the peak-whistling amplitude recorded at the extremes ($L_p/W = 0$ and $L_p/W = 1.375$). Thus, if the cavities are close to each other, the sound production characteristics of a double cavity system depend strongly on the hydrodynamic interference between the cavities. This hydrodynamic interference depends on the plateau length and the Strouhal number. The physical interpretation of this dependence is explained in Sec. III B together with the numerical predictions.

As explained in the Introduction one of the motivations of the current study is to understand the whistling behavior in corrugated pipes, which can be considered in first approximation, as a series of axisymmetric cavities placed along a resonating duct. As a consequence, it is important to assess whether the hydrodynamic interference observed for double cavities can be scaled up to many cavity configurations. In Figure 7 dimensionless fluctuation amplitude $|u'|/U$ is plotted against Strouhal

TABLE I. Peak-whistling Strouhal numbers $Sr_{p-w}$ and the peak-whistling amplitudes $|u'|/U$ for primary and secondary peaks for configuration D at all the plateau length to cavity width ratios $L_p/W$.

| $L_p/W$ | $Sr_{p-w}$ | $|u'|/U \times 10^{-2}$ |
|---------|-------------|-------------------|
|         | Primary     | Secondary         | Primary     | Secondary         |
| 0       | 0.77        | N/A               | 11.5        | N/A               |
| 0.375   | 0.74        | N/A               | 12.3        | N/A               |
| 0.625   | 0.72        | 0.82              | 9.8         | 2.2               |
| 0.750   | 0.71        | 0.79              | 6.4         | 5.6               |
| 0.875   | 0.67        | 0.78              | 3.7         | 7.4               |
| 1.375   | N/A         | 0.77              | N/A         | 10.7              |
Strouhal number \( Sr = \frac{f(W + r_{up})}{U} \)

Dimensionless fluctuation amplitude \( \left| \frac{u'}{U} \right| \)

FIG. 7. Dimensionless fluctuation amplitude \( \left| \frac{u'}{U} \right| \) plotted against Strouhal number \( Sr = \frac{f(W + r_{up})}{U} \) for plateau length to cavity width ratios of \( L_p/W = 0, 0.875, \) and \( 1.375 \) for configuration F, see Figure 1.

number \( Sr = \frac{f(W + r_{up})}{U} \) for configuration F, a triple cavity placed close to the downstream termination, see Figure 1, for plateau length to cavity width ratios of \( L_p/W = 0, 0.875, \) and \( 1.375 \).

In the experiments with a triple cavity, similar to the ones with a double cavity, the plateau length to width ratios of \( L_p/W = 0 \) and \( L_p/W = 1.375 \) have a single peak with the same peak-whistling Strouhal number, seen in Figure 7. The difference in the dimensionless fluctuation amplitude, \( \left| \frac{u'}{U} \right| \), between the cases \( L_p/W = 0 \) and \( L_p/W = 1.375 \) is higher in configuration F than in configuration D as expected, because using a longer plateau length \( (L_p/W = 1.375) \) pushes the cavities further away from the pressure node, thus decreasing the acoustic source power. For the configurations with three cavities this effect is more pronounced than for two cavities. For the case of \( L_p/W = 0.875 \), again similar to the double cavity configuration, there is a primary peak at a low Strouhal number, \( Sr_{p-w} = 0.65 \) with a lower amplitude and a secondary peak at a high Strouhal number, \( Sr_{p-w} = 0.78 \) with a higher amplitude. Thus, the hydrodynamic interference observed in double cavity configurations can be scaled up to triple cavity configurations and a similar phenomenon is expected to appear in corrugated pipes.

It should be noted that some corrugated pipes have helical corrugations. It is expected that the flow inside a helical corrugated pipe has some longitudinal swirl due to the spiraling wavy walls of the pipe. Experiments performed on helical corrugated pipes have shown that their global whistling characteristics are similar to that of regular corrugated pipes with axially symmetric cavities. To address the effect of swirl particularly on the hydrodynamic interaction considered here, a series of experiments have been performed with the same setup but imposing a swirling inflow. It appeared that a similar hydrodynamic interaction prevails, however the amplitude and the Strouhal number of the peaks have been altered.

III. NUMERICAL PREDICTIONS

A. Numerical method

The authors had proposed in Ref. 19 a numerical methodology to investigate the aeroacoustic response of low Mach number confined flows to acoustic excitations. The method combines incompressible flow simulations with Vortex Sound Theory to estimate the strength of an acoustic
Furthermore, only low Mach numbers (\( Ma \leq 0.05 \)) are considered. These correspond to the assumption that the flow is locally incompressible.\(^{38}\) Therefore, incompressible 2D-axisymmetric flow simulations were performed. The simulations were carried out at low Reynolds numbers, \( Re = 4 \times 10^3 \), without turbulence modeling. The diameter of the pipe \((D)\) and the geometry of the cavity \((W, H, r_{up}, r_{down}, L_p)\) are identical to the ones in the experiments. The inlet is located at 0.175\( W\) upstream of the first cavity; such a short inlet pipe section is chosen to insure that the imposed inlet mean velocity profile does not evolve significantly before reaching the cavity. The outlet of the numerical domain is placed at a reasonably distant location, 9\( W\) downstream, from the cavity.

The finite volume commercial code FLUENT 6.3 is used. A pressure-based segregated solution algorithm is employed, the details of the simulation parameters are provided in Ref.\(^{19}\). At the inlet, a fluctuating acoustic axial velocity is imposed, \( u'(t) \), uniformly over the time-averaged velocity profile, \( u(r) \), at the inlet. The acoustic velocity is a sinusoid with a frequency \( f \) and an amplitude \( |u'|\):

\[
u'(t) = |u'| \sin(2\pi f t),\]

where \( |u'| \) is the amplitude of the acoustic velocity induced by the longitudinal standing wave at the position of the cavity.

Note that acoustic velocity fluctuation, \( u'(t) \), changes with time but it is uniform over the profile of the inlet. Time-averaged inlet velocity profile \( u(r) \) on the other hand is fixed over time but it changes with the distance from the axis of the pipe. Thus, it has been implicitly assumed that the radial change of velocity and the temporal acoustic fluctuation of the velocity can be decoupled as follows:

\[
u(r, t) = u(r) + u'(t).
\]

In all experiments performed, the cavity is placed close to a pressure node. Thus, throughout the paper \( |u'| \) stands for the maximum amplitude of the acoustic velocity in the standing wave. The mean velocity profile \( u(r) \) is determined experimentally by means of hotwire measurements, as explained in Sec. \( II \) A. The outlet boundary condition of \( \partial u_r / \partial x = 0 \) is used. For the majority of simulations 30 periods of the excitation frequency turn out to be enough to dissipate transient response due to computational methods and initial conditions. For some simulations, however, a steady periodic state is reached only after 70 periods. In each simulation after the steady periodic state is reached, simulations were continued for 10 additional periods. These 10 periods were then used to calculate the time-averaged acoustic source power. The time step is chosen as \( \Delta t = 0.01W/U \).

The computational domains contain around \( 1.5 \times 10^5 \) quadrilateral cells. The cells are clustered close to the opening of the cavity and to the walls, where there are high gradients of velocity due to the shear layer and boundary layers, respectively. In the domain between 6\( W\) and 9\( W\) downstream of the cavity, cells with high aspect ratio \((\Delta x / \Delta y \gg 1)\) are employed. By doing so problems that can arise due to reverse flow at the outlet boundary condition are avoided. A study on mesh dependency has been carried out. In a test case the same computation was performed with two times more densely meshed domains, producing differences in the calculated acoustic source power of less than \( 5\% \).

2. Time-averaged acoustic source power

Following the Vortex Sound Theory of Howe\(^{21,22}\) and an exact energy corollary of Myers\(^{39,40}\) for a high Reynolds number flow, the time-averaged acoustic source power produced by a single or series of axisymmetric cavities, \( \langle P_{source} \rangle \), can be determined by a surface integral of the product of fluctuating total enthalpy and the mass flux through the boundary of the control volume, provided that
FIG. 8. Numerically predicted dimensionless time-averaged acoustic source power ($P_{\text{source}}$)/($\rho U S_p |u'|^2$) is plotted against Strouhal number $Sr = f(W + r_p)/U$ for plateau length to cavity width ratios of $L_p/W = 0$ (a), 0.375 (b), 0.625 (c), 0.750 (d), 0.875 (e), and 1.375 (f) for configuration D at a fluctuation amplitude of $|u'|/U = 0.05$.

the control volume encloses the compact source region. The details of this procedure are explained in Nakiboğlu et al. 19, 20

**B. Prediction of hydrodynamic interference**

In this section, the hydrodynamic interference observed in configuration D, double cavity downstream configuration corresponding to measurement data in Figure 6, is addressed numerically. In Figure 8 the predicted dimensionless time-averaged acoustic source power ($P_{\text{source}}$)/($\rho U S_p |u'|^2$) is plotted against the Strouhal number $Sr = f(W + r_p)/U$ for six different plateau length to cavity width ratios, namely, $L_p/W = 0$ (a), 0.375 (b), 0.625 (c), 0.750 (d), 0.875 (e), and 1.375 (f) for configuration D, for a fluctuation amplitude of $|u'|/U = 0.05$.

It should be noted that the numerical method provides a time-averaged acoustic source power. To predict the dimensionless fluctuation amplitude $|u'|/U$, which is measured in the experiments, an energy balance is necessary in which the time-averaged acoustic losses of the system are balanced against the predicted time-averaged acoustic source power. 19,20 This approach requires a considerable amount of additional simulations in order to cover the full range of possible dimensionless fluctuation amplitudes. In an earlier work of the authors 30 an attempt was made to predict the peak-whistling dimensionless fluctuation amplitude for a single cavity at the downstream termination (configuration A). The numerical method overestimates the dimensionless fluctuation amplitude by a factor of 2. This approach is not repeated in the current study for the double cavity configurations.

For all the cases considered here for configuration D, the system whistles in the same range of Strouhal numbers $0.6 \leq Sr \leq 0.9$. Thus, given that the system has the same upstream and downstream acoustic boundary conditions, the time-averaged acoustic losses of the system remain more or less the same for all the plateau lengths considered. As a consequence, the predicted time-averaged acoustic source powers can be used on a qualitative basis for comparison with the measured dimensionless fluctuation amplitude $|u'|/U$ in the experiments. The numerical predictions, shown in Figure 8, capture most of the aspects observed in the corresponding experiments, see Figure 6. As in the experiments:

- The two extreme values of the plateau lengths, $L_p/W = 0$ and $L_p/W = 1.375$, display a single peak, at the same peak-whistling Strouhal number and at the same time-averaged acoustic source power.
• With increasing plateau length a secondary peak appears at a higher Strouhal number than Strouhal number of the primary peak. As the plateau length further increases both the primary and the secondary peak shift to lower Strouhal numbers. However, the primary peak does this with decreasing acoustic power, whereas the secondary peak with increasing acoustic power.

• In between these two extremes of the plateau lengths, $L_p/W = 0$ and $L_p/W = 1.375$, there is a critical plateau length around $L_p/W = 0.625$, for which the time-averaged acoustic source power of the system is about half of the value recorded at the extremes.

The predicted peak-whistling Strouhal numbers $S_{p-w}$ and the peak-whistling time-averaged acoustic source powers $(P_{source})/(\rho U_S|u'|^2)$ for the primary and the secondary peaks for configuration D at all the plateau length to cavity width ratios $L_p/W$ considered are summarized in Table II. It is seen by comparison of Tables I and II that the numerical method predicts the peak-whistling Strouhal number $S_{p-w}$ for both the primary and the secondary peaks with an accuracy of 3%.

Following the Vortex Sound Theory of Howe,\textsuperscript{21} as explained for a single cavity/side branch in detail in Refs. 17 and 18, the spatial position of the vortex in the cavity mouth and the moment in time with respect to the oscillation period of the coupling standing wave determine whether the vortex produces or absorbs sound. In a system of multiple cavities in which the cavities are coupled with the same longitudinal standing wave, the vortex shedding at the cavity mouths are synchronized so that their sound production/absorption are constructive. When the cavities are close to each other, as in configuration D, the vortex shed by the upstream cavity reaches the upstream edge of the downstream cavity and affects the vortex shedding there. This hydrodynamic interference between successive cavities can be constructive for sound production/absorption, if it is enhancing the synchronized vortex shedding. If the hydrodynamic interference is counteracting the acoustic synchronization, then it will have a destructive effect for sound production/absorption characteristic of the double cavity system. The nature of the hydrodynamic interference (constructive/destructive) depends both on the Strouhal number and on the travel time of the vortex between cavities, which is determined by the plateau length separating the cavities.

In Figure 9 normalized vorticity contours given for four different points in time within a single oscillation period in the region around the cavities for configuration D for a zero plateau length ($L_p = 0$) and at its peak-whistling Strouhal number $S_{p-w} = 0.77$ are shown. The moment in time is indicated for each frame in terms of the corresponding acoustic velocity period. The origin of time (the first frame) corresponds to the change in the sign of the acoustic grazing velocity from upstream to downstream. It is seen that the vortices for the upstream and the downstream cavity have a perfect synchronization, which results in a strong amplification of the sound production, see Figure 8(a).

In Figures 10 and 11, instantaneous normalized vorticity contours are given for four different moments in time within a single oscillation period in the region around the cavities for configuration D for a larger plateau length ($L_p/W = 0.750$) and at Strouhal numbers of $Sr = 0.75$ and $Sr = 0.82$, respectively. It appears that for $Sr = 0.75$ the vortex shed by the upstream cavity counteracts the synchronization imposed by the acoustic standing wave. This destructive hydrodynamic interference

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TABLE II. Numerically predicted peak-whistling Strouhal numbers $S_{p-w}$ and the peak-whistling time-averaged acoustic source powers $(P_{source})/(\rho U_S|u'|^2)$ for primary and secondary peaks for configuration D at all the plateau length to cavity width ratios $L_p/W$ at a fluctuation amplitude of $|u'|/U = 0.05$.

| $L_p/W$ | $S_{p-w}$ Primary | $S_{p-w}$ Secondary | $(P_{source})/(\rho U_S|u'|^2)$ Primary | $(P_{source})/(\rho U_S|u'|^2)$ Secondary |
|--------|--------------------|--------------------|--------------------------------------|--------------------------------------|
| 0      | 0.77 N/A           |                    | 3.86 N/A                             | N/A                                  |
| 0.375  | 0.73 0.86         | 3.43 0.29          |                                      |                                      |
| 0.625  | 0.72 0.82         | 2.25 1.98          |                                      |                                      |
| 0.750  | 0.71 0.81         | 1.69 2.51          |                                      |                                      |
| 0.875  | 0.70 0.80         | 1.04 3.05          |                                      |                                      |
| 1.375  | N/A 0.77         | 3.46               |                                      |                                      |
FIG. 9. Instantaneous normalized vorticity contours given for four different moments in time within a single oscillation period in the region around the cavities for configuration D for $L_p/W = 0, Sr = 0.77, |u'/U| = 0.05$ (enhanced online) [URL: http://dx.doi.org/10.1063/1.4718726.1].
FIG. 10. Instantaneous normalized vorticity contours given for four different moments in time within a single oscillation period in the region around the cavities for configuration D for $L_p/W = 0.750$, $Sr = 0.75$, and $|u'|/U = 0.05$ (enhanced online) [URL: http://dx.doi.org/10.1063/1.4718726.2].
FIG. 11. Instantaneous normalized vorticity contours given for four different moments in time within a single oscillation period in the region around the cavities for configuration D for $L_p/W = 0.750$, $Sr = 0.82$, and $|u'|/U = 0.05$ (enhanced online) [URL: http://dx.doi.org/10.1063/1.4718726.3].
suppresses the vortex shedding in the downstream cavity, which results in a local minimum in the produced time-averaged acoustic source power, see Figure 8(d).

As seen in Figure 11 for a Strouhal number of $Sr = 0.82$, although the plateau length is same as in the previous case, there is some synchronization of the vortex motions in the two cavities. This increases the time-averaged acoustic source power, as seen in Figure 8(d), as a matter of fact this point corresponds to a local maximum.

It is concluded that the synchronized vortex shedding observed in a double cavity system, in which the cavities are coupled to the same standing wave, can be effected by the hydrodynamic interaction if the cavities are close to each other. This effect can be enhancing or counteracting in nature depending on the Strouhal number and the plateau length between the cavities.

IV. CONCLUSIONS

Aeroacoustic sound generation due to self-sustained oscillations, whistling, by a series of compact axisymmetric cavities exposed to a grazing flow has been studied both experimentally and numerically. The feedback effect is produced by the velocity fluctuations resulting from a coupling of vortex sheddings at the upstream cavity edges with acoustic standing waves in the coaxial pipe.

When the acoustic sources, i.e., the axisymmetric cavities, are placed at different pressure nodes, which are hydrodynamically separated from each other, the total acoustic source power of the system can be determined from the addition of the individual contributions of each cavity.

When the cavities are placed adjacent to each other, i.e., around the same pressure node, a strong hydrodynamic interference between the cavities can be observed for $L_p/W = O(1)$. The hydrodynamic interference depends strongly on the distance between the cavities, i.e., $L_p/W$. This affects both the dimensionless peak amplitude, $|u'|/U$, and the Strouhal number, $Sr_{p-w}$, at which this peak amplitude is observed.

The proposed numerical method successfully captures the observed hydrodynamic interference between adjacent cavities. It provides excellent predictions of the values of $Sr_{p-w}$ (an accuracy of 3%). The numerical method shows that for adjacent cavities the synchronization of the travelling vortices at the cavity openings can be enhanced or hindered due to hydrodynamic interference. The synchronization amplifies the sound production of the multiple cavity system, which are coupled to the same standing wave. The synchronization between adjacent cavities depends both on the plateau length and the Strouhal number. The numerical method also demonstrates that the hydrodynamic interference can be so strong that it can even suppress the vortex shedding for the downstream cavity, which decreases the sound production/absorption considerably.

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