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The effect of perception anisotropy on particle systems describing pedestrian flows in corridors

by

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The effect of perception anisotropy on particle systems describing pedestrian flows in corridors

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We consider a microscopic model (a system of self-propelled particles) to study the behaviour of a large group of pedestrians walking in a corridor. Our point of interest is the effect of anisotropic interactions on the global behaviour of the crowd. The anisotropy we have in mind reflects the fact that people do not perceive (i.e. see, hear, feel or smell) their environment equally well in all directions. The dynamics of the individuals in our model follow from a system of Newton-like equations in the overdamped limit. The instantaneous velocity is modelled in such a way that it accounts for the angle under which an individual perceives another individual.

We investigate the effects of this perception anisotropy by means of extensive simulations, very much in the spirit of molecular dynamics. We define a number of characteristic quantifiers (including the polarization index and Morisita index) that serve as measures for e.g. organization and clustering, and we use these to investigate the influence of anisotropy on the global behaviour of the crowd.

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I. INTRODUCTION

During the last two, three decades, the field of crowd dynamics has emerged as the natural sciences’ reaction to questions arising from social sciences, population biology and urban planning. The roots and philosophy of crowd dynamics are very much in the spirit of statistical mechanics, molecular dynamics, interacting particle systems methods and the theory of granular matter, as such treating individual humans nearly as non-living material (cf. e.g. [1–5] or the nice overview [6] and references cited therein). A justification for this approach lies in the fact that the individuals’ personal will is more or less averaged out if one looks at the crowd as a whole. From this perspective, it can be considered as (stochastic) noise, superimposed on some ‘clean’ (deterministic) dynamics.

An evident and important difference between people and molecules or grains is the fact that people clearly have own opinions, own irritations, front and back sides, etc. Our degree of perceiving our surroundings highly depends on the direction of looking. We mainly base our walking behaviour on what we see, and clearly what happens in front of us thus has more influence than what happens behind us. A modification or extension of physics-inspired models is needed to incorporate this kind of anisotropy in the interactions between individuals. This paper investigates the effect of anisotropy on the global behaviour of a group of pedestrians.

Our focus is on the simulation of a scenario where pedestrians move in a long corridor. We might relate this situation to evacuation of people from a building (cf. [7, 8]). It is a sane to assume that these evacuees have an intrinsic drive to move towards the exit (i.e. one side of the corridor), and moreover that there view is focused in the same direction. Investigating the effect of anisotropy on the large-scale behaviour of the crowd therefore relates to assessing the escape process.

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In Section II of this paper the model is presented and explained. Section III is the main part of the paper. It describes the exact scenario of our simulations and the definitions of the quantities we use for assessing the results (polarization index, projected density, Morisita index). Moreover, in this section the simulation results are presented and discussed. Conclusions and an outlook on possible future work are given in Section IV.

II. A MODEL FOR ANISOTROPIC INTERACTIONS BETWEEN PEDESTRIANS

We represent pedestrians by point particles having masses $m_i$. They are located in a long corridor of length $L$ and width $B$. Here, the word ‘long’ refers to the fact that at the time scales we focus on, the pedestrians are not able to reach the end of the corridor. Interactions between pedestrians are short-ranged. We therefore suppose that the correlation length in the system is less than or equal to a certain $L \ll L$ and we can subdivide the corridor in an array of rectangles (width $B$ and length $L$), which are all duplicates of each other. We thus have a scenario with periodic boundary conditions. Our domain of interest is therefore a rectangular box

$$\Omega := [-\frac{L}{2}, \frac{L}{2}] \times [-\frac{B}{2}, \frac{B}{2}],$$

(1)

with periodic boundary conditions in one direction and impermeable walls in the other direction. The corridor contains $N < \infty$ pedestrians. For all $i \in \{1, \ldots, N\}$ and $t \geq 0$, the vector $\vec{r}_i(t) = (x_i(t), y_i(t)) \in \Omega$ represents the position of the $i$-th pedestrian at time $t$. We denote its velocity by $\vec{v}_i(t)$.

We assume that the governing equation of motion is

$$\frac{m_i}{\tau_{\text{drive}}} (\vec{v}_i(t) - \vec{v}_{\text{des}}) = \vec{F}^\text{soc}_i + \vec{F}^\text{phys}_i.$$  

(2)

The equation describes the motion of the $i$-th individual, which has mass $m_i$ and which moves with velocity $\vec{v}_i(t)$. However, he/she tries to move according to its desired velocity $\vec{v}_{\text{des}}$. Here, $\tau_{\text{drive}}$ is the characteristic relaxation time related to attaining the desired velocity. Its actual velocity is moreover perturbed by two ‘forces’. The word ‘force’ is used since (2) can be regarded as an overdamped limit of a Newton-like equation (cf. [9] for this Newton-like way of modelling). There is a physical force $\vec{F}^\text{phys}_i$ that acts on the individual to describe the effect of the non-living environment (geometry). In this paper we only take into account the influence of walls on pedestrians, that is: $\vec{F}^\text{phys}_i = \vec{F}^\text{wall}_i$. Furthermore, pedestrian $i$ experiences a so-called social force $\vec{F}^\text{soc}_i$ due to the presence of other individuals, which influences the motion of this particular pedestrian $i$.

Individuals are influenced by the walls as soon as they come too close, say within a distance $R_{\text{wall}}$. We model these impermeable walls by means of a strong repulsive force $\vec{F}^\text{wall}_i$ acting on pedestrian $i$:

$$\vec{F}^\text{wall}_i = F_{\text{Wall}}(1 - \frac{R_{\text{wall}}}{d})\vec{n},$$

(3)

where $\vec{n}$ is the unit normal pointing from the corresponding wall into the corridor, $F_{\text{Wall}}$ is the strength of the repulsive force and $d$ is the distance to the wall for pedestrian $i$. The word ‘strong’ here implies that this force is not just a contact force, but has a longer range. Typically, this makes individuals avoid walls before touching them.

Furthermore, very much in the spirit of [9], we specify the social force by

$$\vec{F}^\text{soc}_i = \sum_{\vec{r}_j \in \Omega_i} -\nabla W(\vec{r}_i - \vec{r}_j),$$

(4)

where:

- $\Omega_i$ is the collection of the position vectors of all individuals which are within a distance $R_{\text{cut}}$ to pedestrian $i$. In other words, pedestrians interact only when they are close enough to each other;
- we assume that the interaction potential $W$ depends only on the relative position of the two pedestrians $i$ and $j$ and not on their relative velocity.

FIG. 1. Schematic drawing of the variables $\vec{r}_i$, $\vec{r}_j$, $\vec{r}_j - \vec{r}_i$, involved angle $\theta_{\vec{r}_i, \vec{r}_j}$, and unit vectors $\hat{e}_x$, $\hat{e}_y$. The vector $\vec{v}_{\text{des}}$ can in principle have arbitrary direction. For the sake of clarity in the picture, it is chosen to be parallel to $\hat{e}_x$. Here, $\theta_{\vec{r}_i, \vec{r}_j}$ is the angle under which an individual positioned in $\vec{r}_i$ and heading along $\vec{v}_{\text{des}}$, perceives location $\vec{r}_j$.

Specifically, $W$ takes the form,

$$W(\vec{r}_i - \vec{r}_j) = U((\vec{r}_i - \vec{r}_j)\cdot \frac{1}{2}(1 + \sigma) + \frac{1}{2}(1 - \sigma)\cos \theta_{\vec{r}_i, \vec{r}_j}).$$

(5)

The parameter $\sigma \in [0, 1]$ is called potential of anisotropy, for which $\sigma = 0$ means that the anisotropy effects are maximal, and $\sigma = 1$ means that the potential is isotropic.
The angle of perception \( \theta_{r,r_i} \) is the angle under which an individual positioned in \( \vec{r}_i \) perceives location \( \vec{r}_j \), see Fig. 1. The precise type of interaction is hidden in the structure of the function \( U \). In particular, we distinguish between two types of interactions, namely

- **only repulsive interaction** (for simplicity denoted by R case). Pedestrians repel each other when their separation distance is smaller than \( R^R_r \) (called repulsive radius in R case) and do not interact at larger distances;

- **both repulsive and attractive interaction** (denoted by AR case). For this type of interaction, individuals repel each other when their separation distance is smaller than \( R^AR_r \) (called repulsive radius in AR case). However, when they are separated by a distance between \( R^AR_r \) and \( R^AR_a \) (called attractive radius in AR case), they are attracted to one another. They do not interact outside these regions.

For these two cases \( R_{cut} = R^R_r \), respectively \( R_{cut} = R^AR_r \). The second case is an extension of the first one. The R case we might regard as a population of individualistic people that simply try to avoid each other. In the AR case there is also some social cohesion, as they try to keep the group together. In the following, we describe the precise structure of our potentials.

### 1. R case

In the repulsive case we take

\[
U(s) = \begin{cases} 
F^R(s - R^R_r - R^R_r \ln s / R^R_r) & \text{if } s < R^R_r; \\
0 & \text{if } s > R^R_r.
\end{cases} 
\]  

(6)

See Fig. 2 for an example of an interaction potential of the above form.

### 2. AR case

If we want to involve a both attractive and repulsive way of interactions, then we take

\[
U(s) = \begin{cases} 
F^AR(s - R^AR_r \ln s / R^AR_r) + C_2 & \text{if } s < R^AR_r; \\
\tilde{U}(s) & \text{if } R^AR_r < s < R^AR_a; \\
0 & \text{if } s > R^AR_a;
\end{cases}
\]

where

\[
\tilde{U}(s) = F^AR \left( \frac{s^3}{3} - \left( R^AR_a + R^AR_r \right) s^2 + R^AR_a R^AR_r s \right) + C_1.
\]

(7)

In the above, the constants \( C_1 \) and \( C_2 \) are such that \( U \) is continuous. Fig. 3 shows an example of this kind of interaction potentials.

![Fig. 2. Typical example of the interaction potential as given in (6).](image2)

![Fig. 3. Typical example of the interaction potential as given in (7)–(8).](image3)

### III. SIMULATION: SET-UP AND RESULTS

For all simulations, the number of pedestrians \( N \) is an integer multiple of 10. Initially, we place these individuals on a lattice of \( N_x = \frac{N}{10} \) rows of 10 pedestrians each. The distance between two succeeding rows is always \( \Delta x = \frac{L}{N_x} \). Note that there is an \( N \) here, not \( N_x \): the pedestrians are thus distributed initially in a domain about one tenth the length of the total corridor (see Fig. 4). The distance between two pedestrians in the same row is \( \Delta y = \frac{B}{10} \). Let \( (l_i, y_j) \) denote the position of pedestrian \( k := i + 10(j - 1) \), where \( i = 1, \ldots, 10 \) and \( j = 1, \ldots, N_x \). Then the coordinates are \( l_i = \frac{L}{2} + \Delta x(i - \frac{1}{2}) \) and \( y_j = \frac{B}{2} + \Delta y(j - \frac{1}{2}) \). We use these initial conditions for all simulations.

The simulation time corresponds with a real life time \( \tau_{obs} = 100 \text{ s} \). This time is large enough to witness a stable profile for all simulations. We give in Table I a summary of the parameters and their values used in both the ‘repulsive’ and ‘attractive and repulsive’ interaction potentials.

For more technical details about this kind of simulations, the reader is referred e.g. to [10, 11].
FIG. 4. A snapshot of the positions of the 100 pedestrians at time $t = 0$ s. This initial configuration is used in all simulations. The markers that indicate the individuals’ positions are smaller than in Figs. 5–10, just to avoid overlap here.

FIG. 5. A snapshot of the positions of the 100 pedestrians at time $t = 100$ s, plot for the R potential with $\sigma = 0.0$ (most anisotropic interactions).

To illustrate simulation results, in Figs. 5–10 snapshots of the position of $N = 100$ pedestrians at time $t = 100$ s are shown in both the R case and the AR case for $\sigma = 1.0, \sigma = 0.5$ and $\sigma = 1.0$ (fully isotropic), respectively.

Excepting Fig. 8, all figures depict a completely symmetric profile with respect to the line $y = 0$. This is a natural result of the symmetry in the initial data and symmetry w.r.t. the direction of $\vec{v}_{des}$ in the interactions [This is just a property of the hyperbolic conservation laws to govern the mean-field behaviour [12].]. Note also the strong tendency of the system to maintain TABLE I. Model parameters, symbols, units and reference values. We do simulations in two cases. The AR case: pedestrians interact in a both attractive and repulsive way and the R case: pedestrians only interact in a repulsive way. NB: $M$ and $S_x \cdot S_y$ relate to the Morisita index and are specified in Section III A 3.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Reference value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>-</td>
<td>20, 40, 60, 80, 100</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>-</td>
<td>0, 0.5, 1.0</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>s</td>
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<td>$\tau_{drive}$</td>
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<td>$\tau_{obs}$</td>
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<tr>
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<td>kg</td>
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</tr>
<tr>
<td>$B$</td>
<td>m</td>
<td>10</td>
</tr>
<tr>
<td>$L$</td>
<td>m</td>
<td>40</td>
</tr>
<tr>
<td>$M$</td>
<td>-</td>
<td>64</td>
</tr>
<tr>
<td>$S_x \cdot S_y$</td>
<td>m$^2$</td>
<td>2.5-2.5</td>
</tr>
<tr>
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<tr>
<td>$F_{AR}$</td>
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<td>m</td>
<td>4.0</td>
</tr>
<tr>
<td>$R_{AR}$</td>
<td>m</td>
<td>1.5</td>
</tr>
<tr>
<td>$R_{AR}$</td>
<td>m</td>
<td>3.0</td>
</tr>
</tbody>
</table>

(or produce) organized patterns. These issues will be addressed in more detail in Section III B.

Fig. 8 is an exception in the sense that the distribution of the crowd is not symmetric around the line $y = 0$. This cannot be explained from the model equations, as these imply that the distribution should be symmetric. This effect must be due to discretization errors and their
propagation with respect to time.
In the AR case for σ = 1.0 (cf. Fig. 10,) we see that the people move in a crystal-like formation parallel to the \( \vec{c_x} \) - axis, which remotely resembles the 2D crystallization patterns at low temperature pointed out in [13]. The group as a whole is very compact. In the sequel we will use the words clusters/clustering for a situation in which pedestrians move close together.
However, if we compare Fig. 10 to Figs. 8 and 9, we observe that introducing anisotropy by setting \( \sigma < 1 \) decreases the amount of structure and clustering. In the R case, all three values of \( \sigma \) allow for a well-structured way of moving, but there is no clear clustering. It is evident from the figures that the people are spread over the whole corridor.

Now, we introduce a number of measurable quantities to help quantify the above statements.

A. Definitions of the measured quantities

1. Polarization index

Inspired by [14], we define the (time-dependent) polarization index \( p \) of a group of people as the average angular deviation from the mean propagation direction. Here, the average is taken over all individuals. Note that zero polarization means that all the people move in parallel. This definition in words translates into

\[
p(t) := \frac{1}{N} \sum_{i=1}^{N} ||\theta_i(t), \theta(t)||,
\]

for the instantaneous polarization. We are also interested in the time average of \( p \), which is defined as

\[
P := \langle p(t) \rangle_t = \frac{1}{N} \sum_{i=1}^{N} ||\theta_i(t), \theta(t)||_t.
\]

In the above we use that

- \( \langle q \rangle_t := \frac{\sum_{i=1}^{M} q(t_j)}{M} \) denotes the time average of an arbitrary quantity \( q \), based on the values \( q(t_j) \) at time \( t_j, j \in \{1,2,\ldots,M\} \);
- \( \theta_i(t) \in (-\pi,\pi] \) is the direction of motion of the pedestrian \( i \) at time \( t \). It is defined to be the angle \( \theta_i(t) \) such that

\[
\tan(\theta_i(t)) = \frac{\vec{v}_i(t) \cdot \vec{e}_y}{\vec{v}_i(t) \cdot \vec{e}_x};
\]

- \( \theta(t) \in (-\pi,\pi] \) is the mean direction of motion of the pedestrians group. It is formally defined as the angle such that

\[
\tan(\theta(t)) = \frac{\langle \vec{v}(t) \cdot \vec{e}_y \rangle_N}{\langle \vec{v}(t) \cdot \vec{e}_x \rangle_N},
\]

where \( \langle \vec{v}(t) \cdot \vec{e}_x \rangle_N \) denotes averaging over the total number of \( N \) individuals:

\[
\langle \vec{v}(t) \cdot \vec{e}_x \rangle_N := \frac{\sum_{j=1}^{N} \vec{v}_j(t) \cdot \vec{e}_x}{N}, \quad \xi \in \{x, y\};
\]

- \( \parallel \theta_i(t), \theta(t) \parallel := \min_{k \in \mathbb{Z}} |\theta_i(t) - \theta(t) + 2k\pi| \) denotes the angle in \([0,\pi]\) between \( \vec{v}_i(t) \) and the average direction of motion.

2. Projection of the pedestrian density on the \( \vec{e}_x \) axis

To examine the distribution of our crowd in the direction that corresponds to the desired velocity, we can consider the number of pedestrians that are located in

\[
S_{m}^\epsilon := \{(x, y) \in \Omega : -\frac{B}{2} \leq y \leq \frac{B}{2}, |\eta - x| \leq \epsilon\}.
\]

Here, \( 0 < \epsilon \ll 1 \), and \( S_{m}^\epsilon \) denotes a narrow strip parallel to the \( \vec{e}_x \) axis, centered at \( x \)-position \( \eta \) and of width \( \epsilon \). We could simply plot the number of people in \( S_{m}^\epsilon \) as a function of \( \eta \). Since pedestrians are point masses in our model, this procedure would produce a discontinuous, histogram-like graph. Moreover, the number of discontinuities would highly depend on the value of \( \epsilon \).

In order to smoothen the results, the individuals’ coordinates have been projected on their corresponding \( x \)-coordinates. Next, we assign an ‘induced density’ \( \rho_{\vec{e}_x,i} \), to each individual \( i \), which can be considered as a mollified Dirac delta distribution, see Fig. 11. It was constructed in such a way that it has support of width \( \frac{L}{4} \).

The total density in a point \( \eta \in [-\frac{L}{2}, \frac{L}{2}] \) is obtained by adding all individual contributions:

\[
\rho_{\vec{e}_x}^N(\eta) := \sum_{i=1}^{N} \rho_{\vec{e}_x,i}(\eta),
\]

![Figure 11](image.png)
where the induced density $\rho_{x,i}$ is given by,

$$\rho_{x,i}(\eta) := \begin{cases} \frac{10}{BL} \frac{L}{8} - \frac{2}{BL} & \text{if } |x_i - \eta| \leq \frac{L}{16}; \\ 0 & \text{if } \frac{L}{16} < |x_i - \eta| \leq \frac{L}{8}; \\ \text{otherwise}. \end{cases} \quad (15)$$

Note that this density distribution is normalized:

$$\int_{\Omega} \rho_{x,i}^N d\Omega = \sum_{i=1}^{N} B \int_{-\frac{L}{2}}^{\frac{L}{2}} \rho_{x,i}(x) dx = \sum_{i=1}^{N} 1 = N. \quad (16)$$

3. Morisita index

We subdivide the domain $\Omega$ in $M$ equally sized rectangular boxes of dimensions $S_x \cdot S_y$. Following [14], we define the Morisita index $I_M$, which is a measure for the degree of clustering in our crowd. The index is the probability of having any two pedestrians together in any of the $M$ boxes, divided by this probability had the individuals been distributed uniformly:

$$I_M := M \frac{\sum_{i=1}^{M} n_i(n_i - 1)}{N(N - 1)}, \quad (17)$$

where $n_i$ equals the number of pedestrians in box $i$. See Fig. 12 for an example that illustrates the use of the Morisita index. The figure also shows that the value of the Morisita index depends on the number of boxes $M$ and their distribution.

![Fig. 12. Illustration of the Morisita index. In this example, the corridor is subdivided into 32 boxes of dimensions $S_x \cdot S_y = \frac{L}{8} \cdot \frac{L}{4}$, and 5 pedestrians are distributed over the boxes. This results in a Morisita index of $\frac{64}{15}$.](image)

B. Measured quantities: results

1. Polarization

To show the kind of information we can deduce from the polarization index, we start by examining the evolution of $p(t)$ for $N = 100$. The combined results are shown in Figs. 13 and 14.

In each of the six cases we identify a relatively short period of time just after $t = 0$, during which the polarization decreases rapidly. Afterwards, there is a state in which $p$ fluctuates around a certain average level; this state might be an equilibrium. For each $\sigma$ in the R case, and for $\sigma = 1.0$ in the AR case, this average level is zero (we needed to zoom in in Fig. 13 to verify that these curves really decay below the level as in Fig. 14).

The scenarios $\sigma = 0.0$ and $\sigma = 0.5$ in the AR case are different in the sense that they do not decay to zero. Moreover, if one would zoom in, one would see that the oscillations in the R case are less rapid and of smaller amplitude than in the AR case. This holds especially for $\sigma = 0.5$ and $\sigma = 1.0$. Comparing R to AR requires some extra care, however. It is difficult to compare them in a fair way, because of their intrinsically different nature, and because of the (in)compatibility of the tested values for the interaction radii (cf. Table 1).

Let us focus on the initial rapid decay of $p$: this suggests that the initial configuration is not a favourable
state for the system to be in. An immediate relaxation takes place, implying spreading of the individuals in all possible directions until a more preferable situation is reached. Figs. 5–9 support this statement. Most clearly in Figs. 8 and 9 the system evolved away from the initial configuration. The same statement is true for Figs. 5, 6 and 7, although this is less evident. Once we realize however that the particles move in what seem to be six horizontal rows, we must indeed conclude that the particles have deviated from the initial situation in which there were ten rows. Fig. 10 is somewhat different, even though we recognize relaxation, as the initial occupation was only about one tenth of the corridor length. Spreading in a direction parallel to the mean direction of motion does not explain the peak in just after $t = 0$ however. This is because fluctuations in the magnitude of the velocity (that is: the speed) do not affect the polarization if all individuals move in the same direction. The peak shows that there must have been some vertical displacement too.

We remark that in the AR case $p$ does not necessarily decay to zero, while it does (or at least: seems to do up to fluctuations and noise) in the R case. Regarding Figs. 5–10, we can distinguish the graphs for $\sigma = 0.0$ and $\sigma = 0.5$ in the AR case from the four others, since they do not seem to possess the degree of order and structure that the other graphs do have. Strikingly, these are exactly the cases in Fig. 14 where $p$ does not tend to zero. From this we conclude that there is a strong relation between the polarization tending to zero, and the preservation or favouring of patterns and organization in the system. The cases in which $p$ oscillates around a non-zero average in the long run, are exactly those in which the initial ordered configuration has disappeared after some time. Moreover, the fact that the polarization remains positive is an indicator that the configurations in Figs. 8 and 9 are not stable.

FIG. 15. The time average of the polarization $P$ as a function of the number of pedestrians $N$. Results in the R case, for several different values of $\sigma$.

In Figs. 15 and 16 the time average polarization index $P$ is shown. For each $N$, we recognize in $P$ the same ordering with respect to $\sigma$ as the ordering we have seen before in $p$ (just for $N = 100$, cf. Figs. 13 and 14). The ordering between $\sigma = 0.5$ and $\sigma = 1.0$ (R case) is even much clearer when considering the time average. Note that, comparing R and AR, the absolute differences in $P$ are much smaller than those in $p$. This is mainly because the $p$-curves in the AR case have smaller range but larger domain where they are non-negligible.

2. Projection of the pedestrian density on the $\vec{e}_x$ axis

Figs. 17 (R case) and 18 (AR case) show the projected mollified density that was introduced in Section III A 2. The graphs show that in the isotropic case ($\sigma = 1.0$) the density profile is completely symmetric around the center of mass. Note that this is only the case if the initial conditions are symmetric. For $\sigma = 0.0$ and $\sigma = 0.5$ in the AR case symmetry is no longer present. This is perfectly sane, since anisotropy (i.e. $\sigma < 1$) was introduced to incorporate asymmetric interactions in our model: a pedestrian is more influenced by an other individual in front of him than by one behind him. The graphs corresponding to $\sigma = 0.0$ and $\sigma = 0.5$ in the R case are not given. They possess the same oscillatory behaviour as Fig. 17, but without being symmetric around the center of mass. They do not provide any further information or insight, and thus are omitted.

The highly oscillatory behaviour in Fig. 17 and the omitted graphs reflects the ordered, lattice-like, structures we already observed in Figs. 5–7. Following this reasoning, one would expect the same kind of oscillations also in the AR case for $\sigma = 1.0$ (cf. the ordered pattern in Fig. 10). The reason for not seeing this in Fig. 18 is simple: The average distance in $\vec{e}_x$ direction is smaller than the width of the support of our induced density. The total density therefore smoothen out the periodic structure of the
individuals’ positions. What we do see is the fact that in this situation the individuals do not occupy the whole corridor, but are confined to a certain section of it.

What we observe is that including attraction in the interaction has some regularizing effect. For each of the three choices for $\sigma$, there is a core of high density around the center of mass, which forms the heart of our crowd. This core is present due to the initial condition that was concentrated on a part of the corridor. Without attraction the repulsive interactions would drive it apart. However, attraction is not able to completely diminish the effect of repulsion. Especially when $\sigma$ decreases, the projected density of the core also decreases, while there is a tail of mass just behind it (that is, in the graph on the left of the center). This effect can be explained by the fact that individuals in the anisotropic case are driven backwards if they are too close together (like in the high density core). There is no (sufficient) compensation driving the individual forward as this effect is decreased/switched off by lowering $\sigma$.

Fig. 19 shows the Morisita index as a function of the number of individuals $N$. The corridor is subdivided in $M = 6400$ boxes. Note that in the graph the results of the AR case and R case are combined; each case has its own scaling on the vertical axis.

The value at $N = 100$ for $\sigma = 0.0$ in the AR case is omitted, since this is very big. This is exactly the case of Fig. 8, in which symmetry is no longer present in the distribution of the individuals. We expect this to be related to the unphysically high Morisita index. We should not use the results corresponding to this situation for basing our conclusions on. Note however, that in Fig. 16 the graph for $\sigma = 0.0$ does not arouse any suspicion at $N = 100$.

3. Morisita index

Fig. 19 shows the Morisita index $I$ as a function of the number of pedestrians $N$. Results in the AR case (vertical axis left hand side, black) and R case (vertical axis right hand side, grey), for several different values of $\sigma$ at time $t = 100$s.
The increase in the Morisita index as \( N \) increases (in the R case) can be explained. For repulsive interactions, the individuals have a tendency to move as far apart as possible (if possible until they are a distance \( R_i^H \) apart). This was illustrated by Figs. 5–7. As \( N \) increases, they are however packed together more and more, thus leading to an increase in Morisita index. If we assume (to obtain an approximate result) that the individuals are distributed uniformly, then \( n_i = \frac{N}{M} \) for all \( i \). Taking into consideration the ordered distribution in Figs. 5–7, this assumption is justifiable. It follows that \( I_M = \frac{N - M}{N - 1} = 1 - \frac{M - 1}{N - 1} \to 1 \) as \( N \to \infty \) for fixed \( M \). Moreover, this implies that \( I_M \) tends to its limit value from below. This matches with the increase of the curves in the R case (Fig. 19), and we conjecture that the Morisita index will tend to 1 for \( N \) increasing beyond \( N = 100 \).

In the AR case, we observe that the Morisita index is not monotonic in \( \sigma \) for \( N \) smaller than \( N \approx 45 \) where the graphs for \( \sigma = 0.0 \) and \( \sigma = 0.5 \) intersect. This point of intersection is hard to explain and requires further investigation. Moreover, it is hard to draw any conclusion about the precise dependence of the Morisita index on \( N \) or \( \sigma \) in this case. One could argue that increasing \( \sigma \) corresponds to lower Morisita index. At least this is the case if one compares \( \sigma = 1 \) to \( \sigma < 1 \).

IV. CONCLUSIONS AND OUTLOOK

The behaviour of a crowd of pedestrians inside a corridor, in which the individuals interact via an anisotropic way, can be distinguished clearly from the case in which pedestrians interact in a completely isotropic way. In particular, we observe the following differences compared to the isotropic case:

1. the polarization index increases with increasing anisotropy (i.e. decreasing \( \sigma \));

2. the projected density along the \( \vec{e}_x \) axis shows a symmetric profile around the center of mass for the isotropic case. However, increasing anisotropy implies loss of symmetry;

3. the Morisita index, as a measure of clustering, depends clearly on the anisotropy. It increases (with increasing anisotropy) in the R case and, roughly speaking, decreases in the AR case.

4. In case of repulsive interactions, the crowd tends to fill the whole corridor. If attraction is included, the group stays compact. Decreasing \( \sigma \) however seems to diminish this kind of social cohesion as individuals do not look behind.

As a result of the study presented in this framework, many new questions arose. Future research should be concentrated on the following three directions:

- Most obviously: what is the effect of a further increase of the number of pedestrians? Do the observed relations between the measured quantities and the number of pedestrians still hold? How do the observed limiting values of the polarization for large \( t \) depend on \( N \)? What about the limit \( N \to \infty \) in this case?

- Can we gain more insight in the strange issues of the AR case (cf. Fig. 19: intersecting curves for \( \sigma = 0.0 \) and \( \sigma = 0.5 \); extremely high Morisita index at \( N = 100 \) for \( \sigma = 0.0 \)) by further measurements of the Morisita index for increasing \( N \)? E.g. can we extrapolate information for \( N > 100 \) back into the interval \([0, 100]\)?

- If \( N \) increases, the natural thing to do is to consider the discrete-to-continuum limit (i.e. construct educated procedures to derive mean-field limit equations). Does such limit exist, can we derive it, and can we compare the effect of anisotropy in the limit to the observations of the current work?

- How much does the large time behaviour of the crowd depends on the initial conditions? The initial distribution of pedestrians in this thesis is not a realistic situation. People starting to enter a corridor are in real life never distributed in a crystalline structure manner. However, for an escape situation (for example in the case of fire) it seems reasonable to assume that a group of people starts, being clustered, at one side of a corridor. Therefore, as an extension of this research, we propose to use as an initial distribution a more realistic configuration in which people are placed at one side of the corridor, with their positions slightly perturbed from the grid points. Averaging over a large collection of such perturbed initial distributions, will lead to effective results. Are these averaged results comparable to the ones presented in this paper? In other words: is averaging the results basically the same as removing the fluctuations from the initial conditions?

With respect to random initial conditions, we are also interested in whether these do or do not allow order and patterns to be formed (cf. Figs. 5–7 and 10). Do random initial conditions lead to ‘preservation of chaos’?

- What happens if we try to make our model more realistic: e.g. change the shape of the domain \( \Omega \), or allow variation in the direction and magnitude of individuals’ desired velocity? Including more sophisticated active parts in the boundary (doors) or impermeable objects within the domain, automatically leads to questions about the efficiency of the flow. Which geometry leads to the fastest evacua-
tion? First steps in this direction have been made in [15].

The issues addressed in this paper show that anisotropy related to perception has nontrivial effects on the global dynamics of a crowd. Certainly, these effects cannot be neglected. More work, both numerically and analytically, is needed to extend and formalize our results.

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