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In the field of service logistics for capital goods, generally, METRIC type methods are used to decide where to stock spare parts in a multi-echelon repair network such that a target availability of the capital goods is achieved. These methods generate a trade-off curve of spares investment costs versus backorders. Backorders of spare parts lead to unavailability of the capital goods. Inputs in the spare parts stocking problem are decisions on 1) which components to repair upon failure and which to discard, and 2) at which locations in the repair network to perform the repairs and discards. The level of repair analysis (LORA) can be used to make such decisions in conjunction with the decisions 3) at which locations to deploy resources, such as test equipment, that may be required to repair, discard, or move components. Since these decisions significantly impact the spare parts investment costs, we propose to solve the LORA and spare parts stocking problems jointly. We design an algorithm that finds efficient points, i.e., lower backorder levels cannot be achieved against the same (or lower) costs. In a computational experiment, we show that solving the joint problem is worthwhile, since we achieve a cost reduction of 5.1% on average and over 43% at maximum compared with using a sequential approach of first solving a LORA and then the spare parts stocking problem.

Key words: Service logistics, Level of repair analysis, Spare parts, Inventories

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1. Introduction

In this paper, we discuss the maintenance of capital goods. Examples of capital goods are baggage handling systems at airports, radar systems on board naval vessels, MRI-scanners in hospitals, and wafer steppers that are used in the semiconductor industry. Capital goods can be defined as expensive and technologically advanced systems that are used to manufacture products or services. Because they are critical in the primary process of their users, their unavailability may lead to high costs. In other cases, their unavailability may lead to dangerous situations, e.g., in the case of naval radar systems (military mission failure) or MRI-scanners (patients that cannot be treated).

To prevent downtime, capital goods are generally repaired by replacement, which means that a defective component is quickly taken out of the system and replaced with a functioning spare part. These spare parts may be located both close to the installed base, to reduce replacement times, and at more distant locations at a higher echelon level, to use risk pooling effects: one spare part can be used for various systems at various locations. In this paper, we consider two-echelon distribution networks, so networks consisting of one central depot and a number of bases (sometimes referred to as a one warehouse, multiple retailer system).

Some defective components can only be discarded and replaced with a newly purchased component. For example, mechanical parts that wear or small parts such as screws cannot be repaired economically. Other parts may be both repaired and discarded. Repairs may be performed by replacing a subcomponent for which spare parts may be stocked. To facilitate an exact analysis, we do not consider such subcomponents here (i.e., we consider single-indenture product structures). The tactical problem (solved when the product is designed or deployed in the field) of determining which components to repair upon failure and which to discard is referred to as the *level of repair analysis* (*LORA*) problem in the military world (see, e.g., MIL-STD-1388-1A, United States Department of Defense 1993). To be precise, it determines:

1. which components to repair upon failure and which to discard;
2. at which locations in the repair network to perform the repairs and discards; and
3. at which locations to deploy resources required to repair, discard, or move components.

We refer to this set of decisions as the *LORA decisions* and to the first two decisions as the *repair/-discard decisions*. The goal is to achieve the lowest possible costs, consisting of both fixed costs and costs that are variable in the number of failures. Fixed costs are due to the resources. They result from the LORA decisions, but do not depend on the annual number of failures. Examples are training of service engineers and depreciation of repair equipment. Variable costs may include transportation costs, working hours of service engineers, and usage of bulk items.

In the spare parts literature and in practice, the LORA is generally solved first and next the spare parts stocking problem is solved: which spare parts to stock at which locations in which amounts, in order to achieve a target availability of the installed base. In the context of capital goods, generally METRIC type models and methods are used. A key idea in the METRIC type models is that the focus is not directly on the maximization of the availability, but instead, the focus is on the minimization of the expected number of backorders of components at the bases. A backorder occurs if a component is requested, but cannot be delivered immediately. As a result of a backorder, a system is unavailable waiting for spares. When referring to optimality in the spare parts stocking problem, it is meant that efficient points are found in the sense that at the corresponding (or lower) cost levels, it is not possible to achieve a lower expected number of backorders.

A problem with the sequential approach of first performing a LORA and then solving a spare parts stocking problem is that the decision (in the LORA) to repair or discard is based purely on the costs to perform a repair or discard, disregarding the fact that the lead times of both decisions may differ. If the lead time for discard is much higher than the lead time for repair (i.e., if purchasing a new component takes more time than repairing a component), then choosing discard may lead to higher number of spare parts to stock to achieve the same availability of the installed base. Although discarding may seem interesting in the LORA, considering the spare parts costs may mean that repairing turns out to be better. Therefore, we propose to solve the problem of LORA and spare parts stocking jointly.

In Section 2, we give an overview of the related literature, and we outline our model in Section 3. In Section 4, we summarize key results on spare parts stocking that we need in Section 5, where we present our algorithm. In Section 6, we design a numerical experiment and discuss its results. We show that solving the two problems jointly opposed to solving them sequentially leads to a cost reduction of 5.1% on average and over 43% at maximum. Since a few percent cost reduction is already worth millions over the life time of capital goods, it is worthwhile to consider the problem of LORA and spare parts stocks jointly. In Section 7, we give conclusions and recommendations for further research.

2. Literature review

We first discuss the related literature on LORA, then that on spare parts stocking, and finally one paper on the joint problem of LORA and spare parts stocking.

To the best of our knowledge, Barros (1998) presents the first LORA model. Barros models the problem as an integer linear programming model that she solves using CPLEX, a commercial solver. She assumes that the same decisions are taken at all locations at one echelon level and that resources required to perform repairs are uncapacitated. Furthermore, all components at one indenture level require the same resource in order to be repaired. Saranga and Dinesh Kumar (2006) propose a model that differs from the model by Barros in that each component requires its own unique resource. Saranga and Dinesh Kumar solve the model using a genetic algorithm. Basten et al. (2009) propose a formulation that generalizes the models by Barros (1998) and Saranga and Dinesh Kumar (2006) in that components may require any number of resources and resources may be shared by components. Finally, Basten et al. (2010) generalize the model by Basten et al. (2009) in that different decisions may be taken at various locations at the same echelon level. Basten et al. (2010) model the problem as a minimum cost flow model with side constraints, which leads to a much lower computation time compared with Basten et al. (2009). In both papers, CPLEX is used to solve the model. For a more extensive overview of the literature on LORA we refer to Basten et al. (2010).

A vast amount of literature exists on the spare parts stocking problem. The seminal paper in this field is the work by Sherbrooke (1968) in which he develops the METRIC model (Multi-Echelon Technique for Recoverable Item Control). He considers a two-echelon distribution network with each location using one-for-one, or $(S-1, S)$ replenishment. He further considers a multi-item, single-indenture problem. Sherbrooke (1968) proposes an approximate evaluation of the number of backorders at the bases and a greedy heuristic to optimize the base stock levels. Muckstadt (1973) extends the work by Sherbrooke by allowing for two indenture levels, leading to the so-called MOD-METRIC model. Graves (1985) proposes a more accurate approximation for the two-echelon, single-indenture problem, the VARI-METRIC model, which Sherbrooke (1986) extends to two indenture levels. We refer to the books by Sherbrooke (2004) and Muckstadt (2005) for extensive overviews of these and related models and methods. There are also exact evaluations of these models: Simon (1971) considers the two-echelon, single-indenture, single-item problem, which is later extended to the general multi-echelon problem by Kruse (1979). Graves (1985) gives another exact evaluation of the same model. Axsäter (1990) provides an exact evaluation and enumerative, but relatively efficient optimization of the same model, but with penalty costs instead of a service level constraint. Rustenburg et al. (2003) give both an exact and approximate evaluation for the general multi-echelon, multi-indenture problem. They also give an extensive overview of the related literature.

We are aware of one paper in which a method is presented to solve the two problems of LORA and spare parts stocking jointly: Alfredsson (1997). The author considers two-echelon, single-indenture problems. Each component requires one specific tester (resource), which is required by one component only. Furthermore, one multi-tester exists that can be used for the repair of one component. Adapters can be added in a fixed order to enable the multi-tester to be used for the repair of additional components. Alfredsson's resources have finite capacity, so system downtime includes waiting times for resources, repair times, and waiting times for spares. We base our optimization method on the method by Alfredsson.

3. Model

We model the joint problem of LORA and spare parts stocking. The goal is to achieve the lowest possible costs, subject to a constraint on the availability. The costs include the variable and fixed LORA costs, as mentioned in Section 1, and the spare parts holding costs. In Section 3.1, we give our assumptions and in Section 3.2, we present the mathematical model formulation.

3.1. Assumptions

We use the assumptions underlying the METRIC type models (see, e.g., Muckstadt 2005, Sherbrooke 2004):

- components fail according to a Poisson process with constant rate;
- there are no lateral transshipments between bases, or emergency shipments from the central warehouse;

- for each component at each location, an $(S-1,S)$ continuous review inventory control policy (one for one replenishment) is used;
- replacement of a defective component by a functioning component takes zero time;
- the repair lead time includes the time used for sending the defective component to the repair location and for diagnosing the failure cause;
- the repair or replenishment lead times for each component are i.i.d. random variables (implying that resources are uncapacitated);
- the move lead times (to move a functioning, repaired or newly purchased, component from the depot to a base) are deterministic.

To ease the presentation in the remainder of this paper and to decrease the problem size, we make some additional assumptions, which are not critical for our algorithm:

- repairs are always successful;
- each base is identical (same costs, demand rates, lead times to central warehouse, et cetera);
- we take the same decisions at each base for each component, spare part, and resource. With identical bases, this is an optimal strategy, except that the so-called overshoot increases (see Section 4 for an explanation of overshoot);
- since resources that are required to enable discard or movement do not occur frequently in practice, we assume that resources may be required to enable repair only;
- since the discard costs mainly consist of the costs of acquiring a new component, we assume that discard costs for a certain component are equal at all echelon levels. Since the costs to move a component to another echelon level are relatively low, we consider discard at the highest echelon level only.

3.2. Mathematical model

Let C be the set of all components and $E=\{1,2\}$ the set of two echelon levels, with the bases being at echelon level 1 and the central warehouse being at echelon level 2. The set D consists of the possible decisions that can be made, so $D=\{\text{discard},\text{repair},\text{move}\}$. We define $D_1=D$ and $D_2=D\setminus\{\text{move}\}$ as the possible decisions at echelon level 1 and 2, respectively. Let R be the set of resources. $\Omega_r\subseteq C$ is the set of components that require resource $r\in R$ in order to be repaired.

We define the following decisions variables:

$$X_{c,e,d}=\begin{cases}1, & \text{if for component } c\in C \text{ at echelon level } e\in E \text{ decision } d\in D_e \text{ is made,}\\ 0, & \text{otherwise;}\end{cases}$$

$$Y_{r,e}=\begin{cases}1, & \text{if resource } r\in R \text{ is located at echelon level } e\in E,\\ 0, & \text{otherwise;}\end{cases}$$

$$S_{c,e}=\text{the number of spare parts of component } c \text{ located at each location at echelon level } e.$$

Furthermore, $\mathcal{X}$ is a three-dimensional vector with entries all variables $X_{c,e,d}$ and $\mathcal{S}$ is a two-dimensional vector with entries all variables $S_{c,e}$.

For each component $c\in C$, we define $\lambda_c(>0)$ as the total annual failure rate over all bases. We define three cost types. For component $c\in C$ at echelon level $e\in E$, $vc_{c,e,d}(\geq 0)$ are the variable costs of making decision $d\in D$. Since we have chosen, without loss of generality, to minimize the total annual costs with our definition of λ_c, we define $fc_{r,e}(\geq 0)$ to be the annual fixed costs to locate resource $r\in R$ at echelon level $e\in E$ and we define $hc_{c,e}(>0)$ to be the annual costs of holding one spare of component $c\in C$ at each location at echelon level e.

We define our model as follows:

$$\text{minimize} \sum_{c \in C} \sum_{e \in E} \sum_{d \in D_e} vc_{c,e,d} \cdot \lambda_c \cdot X_{c,e,d} + \sum_{r \in R} \sum_{e \in E} fc_{r,e} \cdot Y_{r,e} + \sum_{c \in C} \sum_{e \in E} hc_{c,e} \cdot S_{c,e} \tag{1}$$

subject to:

$$\sum_{d \in D_1} X_{c,1,d} = 1 \ , \forall c \in C \tag{2}$$

$$X_{c,1,\text{move}} \leq \sum_{d \in D_2} X_{c,2,d} \ , \forall c \in C \tag{3}$$

$$X_{c,e,\text{repair}} \leq Y_{r,e} \ , \forall r \in R \ , \forall c \in \Omega_r \ , \forall e \in E \tag{4}$$

$$\text{availability}(\mathcal{X}, \mathcal{S}) \geq \text{target availability} \tag{5}$$

$$X_{c,e,d}, Y_{r,e} \in \{0,1\} \tag{6}$$

$$S_{c,e} \in \mathbb{N} \tag{7}$$

Constraints 2 to 4 are the 'LORA constraints' and define the same model as Basten et al. (2010) use, except that they do not necessarily take the same decision at each base and they model any number of echelon levels and indenture levels. Constraint 2 assures that for each component a decision is made at the base. If a component is moved, Constraint 3 assures that a decision is made for that component at the central warehouse. Some options are only available if all resources are present, which is guaranteed by Constraint 4. Finally, Constraint 5 assures that the target availability is met. This is the only 'spare parts stocking constraint' and it takes into account the various lead times (for which we have not introduced notation). Since the availability is a non-linear function of all repair/discard decisions $\mathcal{X}$ and all spare parts decisions $\mathcal{S}$, we have a large non-linear integer optimization problem that cannot be solved using standard optimization software such as CPLEX. We therefore propose a different algorithm in Section 5.

4. Spare parts stocking

As mentioned in Section 1, the goal in the METRIC type methods is to locate spare parts such that a certain target availability is achieved against the lowest possible costs. This is achieved indirectly, by locating spare parts such that for a given cost level, the expected number of backorders (EBO) at the bases is minimized. Since having a backorder at a base means that a capital good is not available, minimizing EBO at bases is approximately equal to maximizing the availability of the capital goods (see, e.g., Sherbrooke 2004, Muckstadt 2005).

In a single-location problem, if increasingly more spare parts are stocked, the EBO will keep decreasing, but at a decreasing rate. In other words, the EBO is a decreasing, convex function of the number of spare parts (see, e.g., Sherbrooke 2004, Muckstadt 2005). If the spares investment costs are proportional to the number of spare parts or if the annual spare parts holding costs are proportional to the number of spare parts, which we assume, then plotting the EBO as a function of those costs leads to a decreasing, convex (discontinuous) EBO-curve. The points on such a curve may represent annual spare parts holding costs for spare parts stocked at *multiple locations* and for *multiple components* in the spare parts stocking problem. Specifically for our problem of joint LORA and spare parts stocking, we may also take into account the costs resulting from the LORA decisions in these points. This results in the following definition.

DEFINITION 1. An *EBO-curve* B_l (presented as an ordered set) consists of points $b_{l,i}$ ($i \in \{1, \ldots, |B_l|\}$) that indicate the total costs (annual LORA costs and spare parts holding costs), denoted by $\text{costs}(b_{l,i})$, and expected number of backorders, denoted by $\text{EBO}(b_{l,i})$, that result from certain LORA decisions and spare parts stocking decisions for a set of components. These points are efficient points, meaning that if $b_{l,i} \in B_l$, then it is not possible to find a point $b_{l,j}$ such that $\text{costs}(b_{l,j}) \leq \text{costs}(b_{l,i})$ and $\text{EBO}(b_{l,j}) < \text{EBO}(b_{l,i})$. Next, they are ordered such that if $\text{costs}(b_{l,i}) < \text{costs}(b_{l,j})$, then $i < j$ (if $\text{costs}(b_{l,i}) = \text{costs}(b_{l,j})$, then $i = j$).

Figure 1 **Multiple EBO-curves (data from Sherbrooke 2004, Table 3.3)**

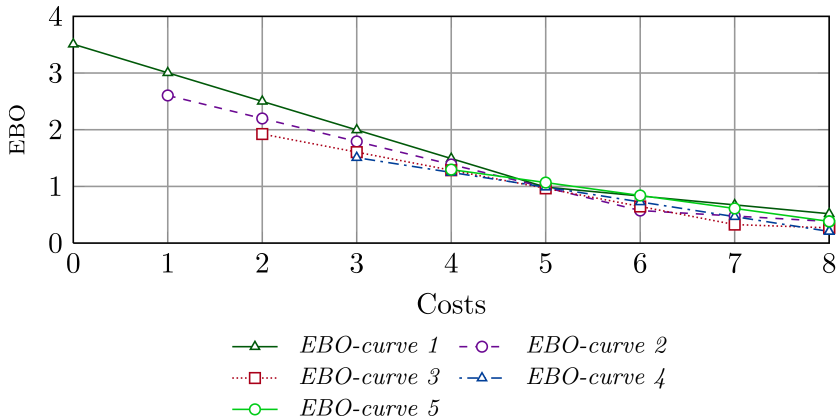
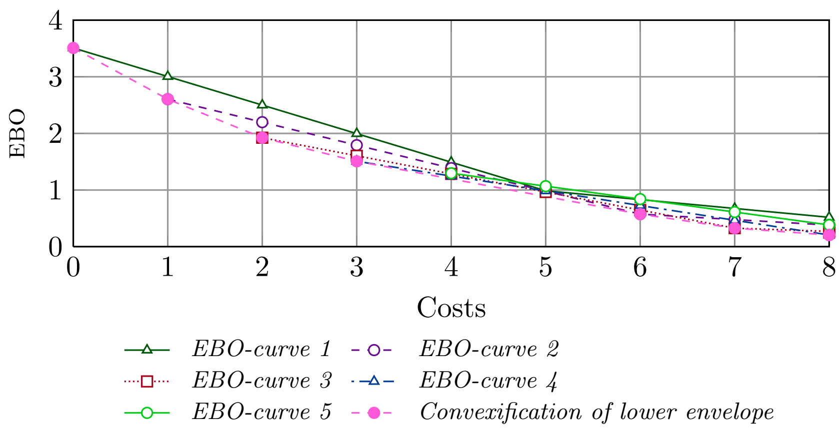


Figure 2 **Multiple EBO-curves, including the convexification of the lower envelope**



To improve the readability of the remainder of this paper, we introduce $\Delta_{l,i,j} = \frac{\text{EBO}(b_{l,i}) - \text{EBO}(b_{l,j})}{\text{costs}(b_{l,j}) - \text{costs}(b_{l,i})}$. We are interested in *convex* EBO-curves B_l only, meaning that if there are three points $b_{l,i}, b_{l,j}, b_{l,k} \in B_l$ such that $i < j < k$, then $\Delta_{l,i,j} \geq \Delta_{l,j,k}$.

If there are *two echelon levels*, generating an EBO-curve is done as follows. Spare parts are first stocked at the central depot. For each spare parts level at the central depot (a point on the resulting EBO-curve), spare parts are stocked at the bases, resulting in as many EBO-curves as there are points on the curve at the central depot. Graves (1985) shows how to calculate the backorder levels at the bases, taking into account the backorder levels at the central warehouse. We introduce the *lower envelope* of a set of functions $f_i(x)$ as the function given by their pointwise minimum: $f(x) = \min_i f_i(x)$. Taking the *lower envelope* of the EBO-curves and removing all non-convex points (convexification) results in one convex EBO-curve for the two-echelon problem. See Figures 1 and 2 for an example. Lemma 1 formalizes these results, without proof.

LEMMA 1. *It is possible to construct a convex* EBO*-curve for two-echelon, single-item problems.*

If there are *multiple components*, generating an EBO-curve is done using marginal analysis. We formalize how this is done in the proof of Lemma 2. In words, the idea is to start with constructing an EBO-curve per component as described above. Next, an EBO-curve is constructed for the total problem, with the first point having costs and EBO that are the total of the costs and EBO of the first points on each of the curves per component. Then, look at each of the EBO-curves per component and add that spare part to stock that leads to the largest backorder reduction per invested dollar ('biggest bang for the buck'). Because of the convexity of the curves, such a myopic approach will lead to one convex EBO-curve.

LEMMA 2. *Applying marginal analysis to a set of convex* EBO*-curves results in one convex* EBO*-curve.*

Proof Consider two convex EBO-curves (B_1 and B_2) that are merged using marginal analysis. The first point on the resulting EBO-curve (B_3) is $b_{3,1}$ with $\text{costs}(b_{3,1}) = \text{costs}(b_{1,1}) + \text{costs}(b_{2,1})$ and $\text{EBO}(b_{3,1}) = \text{EBO}(b_{1,1}) + \text{EBO}(b_{2,1})$. Next, consider the second point on each of the two original curves. Two cases can be distinguished:

- If $\Delta_{1,1,2} \geq \Delta_{2,1,2}$, then the second point on the resulting EBO-curve is $b_{3,2}$ with $\text{costs}(b_{3,2}) = \text{costs}(b_{1,2}) + \text{costs}(b_{2,1})$ and $\text{EBO}(b_{3,2}) = \text{EBO}(b_{1,2}) + \text{EBO}(b_{2,1})$. Of course, $\Delta_{3,1,2} = \Delta_{1,1,2}$. In the next step, compare $\Delta_{1,2,3}$ with $\Delta_{2,1,2}$.
- Otherwise, the second point on the resulting EBO-curve is $b_{3,2}$ with $\text{costs}(b_{3,2}) = \text{costs}(b_{1,1}) + \text{costs}(b_{2,2})$ and $\text{EBO}(b_{3,2}) = \text{EBO}(b_{1,1}) + \text{EBO}(b_{2,2})$. Then, $\Delta_{3,1,2} = \Delta_{2,1,2}$. And in the next step, compare $\Delta_{1,1,2}$ with $\Delta_{2,2,3}$.

In either case, because of the convexity of both curves, it is guaranteed that both fractions that we are comparing in the next step are smaller than $\Delta_{3,1,2}$. Therefore, proceeding in this way, we are certain to get a convex EBO-curve for the total problem. Notice furthermore that marginal analysis is an *associative* operator. This means that applying marginal analysis to $n+1$ components leads to the same result as applying it first to n components and next to that result and the $n+1^{\text{th}}$ component. Therefore, the result holds for any number of components. □

We formalize a well known, important result (see, e.g., Sherbrooke 2004, Muckstadt 2005).

THEOREM 1. *Given a set of components, a two-echelon network structure, and* LORA *decisions, applying marginal analysis to spare parts stocking results in a convex* EBO*-curve.*

This follows directly from Lemmas 1 and 2. □

Since a discrete set of solutions is found, there is usually some *overshoot* over the target availability, meaning that a point is found that corresponds to an availability level that is somewhat higher than the target availability.

We restrict ourselves in this paper to two-echelon, single-indenture problems since enumeration is required to find efficient points for two-indenture problems and for general multi-echelon problems. This is too time-consuming to be a realistic approach for any but very small problems (see Appendix A to see what may go wrong using a greedy approach).

5. Algorithm

The basic idea of our algorithm is to decompose the problem in a smart way and to aggregate the results. In this way, we can enumerate all possible solutions and find a convex EBO-curve for the total problem. We first explain our algorithm for the case of one component and no resources, then we go to the multi-item problem, and finally we introduce resources.

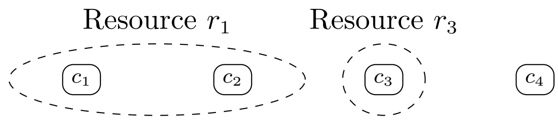
5.1. One component, no resources

If a problem consists of one component only and there are no resources, the algorithm works as follows:

- Consider each of the three possible repair/discard decisions (repair at base, repair at depot, or discard) and apply marginal analysis to stock spare parts for each of them, resulting in three EBO-curves.
- Take the convexification of the lower envelope of these curves: this is a convex EBO-curve.

Notice that this is analogue to what we explained in Section 4 and Figures 1 and 2.

Figure 3 **Example: one resource per component**



THEOREM 2. *Given a problem instance of the joint problem of* LORA *and spare parts stocking, consisting of one component only and no resources, applying the algorithm results in a convex* EBO-*curve.*

Proof Theorem 1 states that given the LORA decisions, spare parts stocking leads to an EBO-curve. Since there are three possible LORA decisions for the component, there will be three EBO-curves: B_1, B_2, B_3. Taking the convexification of the lower envelope leads to the set $B_4 = \{b_{4,j} \mid b_{4,j} \in B_1 \cup B_2 \cup B_3, \text{costs}(b_{4,i}) < \text{costs}(b_{4,j}) < \text{costs}(b_{4,k}), \Delta_{4,i,j} \geq \Delta_{4,i,k}, \forall b_{4,i}, b_{4,k} \in B_1 \cup B_2 \cup B_3\}$. If there exist two points $b_{4,i}, b_{4,j} \in B_4$ such that $\text{costs}(b_{4,i}) = \text{costs}(b_{4,j})$ and $\text{EBO}(b_{4,i}) = \text{EBO}(b_{4,j})$, we remove one at random. The set B_4 meets the definition of a convex EBO-curve. □

5.2. Multiple components, no resources

If there are multiple components, but still no resources, we can solve a subproblem per component as described above and use marginal analysis to find the total solution.

THEOREM 3. *Given a problem instance of the joint problem of* LORA *and spare parts stocking, consisting of one or more components and no resources, applying the algorithm results in a convex* EBO-*curve.*

Proof If we split the problem into problems per component, then Theorem 2 states that the result holds for each subproblem. Lemma 2 states that applying marginal analysis to the set of resulting EBO-curves leads to the desired result. □

5.3. Multiple components, at most one resource per component

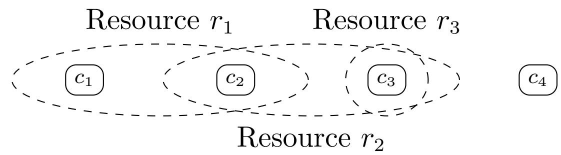
If there are multiple components and each component requires at most one resource, as in the example in Figure 3, we divide the problem into subproblems per resource (in our example three subproblems, one consisting of resource r_1 and components c_1 and c_2, one consisting of resource r_3 and component c_3, and one consisting of component c_4). In each of these subproblems, the resource can be located at one of the two echelon levels, at both echelon levels, or not at all, so there are four *scenarios*. In each of these scenarios, one or more repair/discard options may be available for each of the components (the discard option is always available). Each scenario can be solved similar to how problems are solved if there are no resources involved. Taking the convexification of the lower envelope results in one EBO-curve per resource. Applying marginal analysis leads to an EBO-curve for the complete problem.

THEOREM 4. *Given a problem instance of the joint problem of* LORA *and spare parts stocking, consisting of one or more components and each component requiring at most one resource, applying the algorithm results in a convex* EBO-*curve.*

Proof We show that the theorem holds, by showing that we can enumerate all options and that this would result in the same EBO-curve.

Enumeration works as follows: Consider all combinations of resource locations for all resources (in the example, 4 possibilities for resource r_1 and 4 for r_3, resulting in $4^2 = 16$ possible combinations of resource locations). Fix the resource locations and solve the problem as if there are no resources involved (the only difference is that some repair options are not available if required resources are

Figure 4 Example: multiple resources per component



not available), resulting in an EBO-curve as stated in Theorem 3, and finally take the convexification of the lower envelope of all (16, in our example) EBO-curves to find an overall EBO-curve.

Our approach is to split the problem into subproblems, each consisting of one resource and a number of components. Next, we construct the convex EBO-curve for each subproblem by constructing an EBO-curve for each resource location and taking the convexification of the lower envelope of the resulting (four) EBO-curves. If a point $b_{l,j}$ is removed in this last step, then there exist two points ($b_{l,i}$ and $b_{l,k}$) such that costs$(b_{l,i}) <$ costs$(b_{l,j}) <$ costs$(b_{l,k})$ and $\Delta_{l,i,k} \geq \Delta_{l,i,j}$. $b_{l,j}$ can then not be part of a solution in the enumeration, since replacing $b_{l,j}$ by $b_{l,k}$ would lead to a better solution. This reasoning can also be reversed, so that it is clear that first solving each subproblem and then applying marginal analysis to find the overall result, will lead to the same curve as enumeration. □

5.4. Multiple components, any number of resource per component

In general, one component may require multiple resources, so that resources cannot be considered independently of each other. In the example in Figure 4, component c_2 requires resources r_1 and r_2 in order to be repaired, and to repair component c_3, resources r_2 and r_3 are required. As a result, the decision where to install resource r_2 depends on the decision where to install resource r_1 (and vice versa) as well as the decision where to install resource r_3 (and vice versa).

Since there are four scenarios per resource, there are $4^3 = 64$ scenarios if we consider all combinations of resource locations in our example. Since the number of scenarios explodes if the number of interacting resources increases, we decompose the problem into subproblems. In the example, we observe that once the location of resource r_2 is fixed, resources r_1 and r_3 may be treated independently of each other. Therefore, we make four main scenarios in which the location of resource r_2 is fixed (at none, one, or both echelon levels). For each of these main scenarios, there are two subproblems: the first consists of resource r_1 and components c_1 and c_2, the second consists of resource r_3 and component c_3. For each of these subproblems, there are four scenarios, resulting in $4 \cdot (4+4) = 32$ scenarios. Notice that in each of the 32 scenarios, we consider one or two components as opposed to three components for each of the 64 original scenarios.

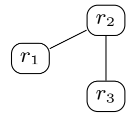
Above, we explained how to obtain the results for each of the subproblems. Using marginal analysis, we merge the results of the two subproblems to get a result for each main scenario. This leads to the final result:

THEOREM 5. *Given a problem instance of the joint problem of* LORA *and spare parts stocking, consisting of one or more components and each component requiring any number of resources, applying the algorithm results in a convex* EBO*-curve.*

Proof The proof is analogous to the proof of Theorem 4. □

The only problem that remains is how to decompose a general problem with shared resources into independent subproblems so that the total problem can be solved in an efficient way. To this end, we represent the interaction between the resources in a graph, which we will call a *resource graph*: a vertex represents a resource, and an edge between two vertices exists if there exists a component that uses both resources. Figure 5 represents the resource graph for the example. Below, we first give a number of definitions, then we explain how we use the graph representation of the

Figure 5 Graph of three resources



interaction between the resources. We assume a familiarity with basic graph theory; for further definitions, we refer to any book on general graph theory (e.g., Godsil and Royle 2001).

A graph $G = (V, E)$ consists of vertices $v \in V$ and edges between vertices $(v, w) \in E$. A graph $G' = (V', E')$, with $V' \subseteq V$ and $(v, w) \in E' \iff (v, w) \in E, v \in V', w \in V'$, is called an *induced subgraph* of G. A graph is *connected* if any two of its vertices are linked by a path (the graph consisting of one vertex is connected as well). A *maximal connected subgraph* $G' = (V', E')$ is an induced subgraph of $G = (V, E)$, such that adding any more vertices $v \in V \setminus V'$ (and the required edges $(v, w) \in E \setminus E'$ to keep an induced subgraph) leads to a disconnected graph. A maximal connected subgraph is also called a *connected component* or just component; we will use the term *graph component* to avoid confusion. A depth first search can be used to identify the graph components in a graph (see, e.g., Hopcroft and Tarjan 1973). A graph is *complete* if an edge exists between any two vertices: if for all vertices $v_1, v_2 \in V$ with $v_1 \neq v_2$ it holds that $(v_1, v_2) \in E$, then the graph $G = (V, E)$ is complete.

We now define a *resource group* as the set of all resources represented by the vertices in one graph component in the resource graph. Let $|R^{\text{group}}|$ be the number of resources in such a resource group. Since fixing the locations for one resource leads to 4 scenarios, we know that the number of possible combinations of locations for the resources in a resource group is $4^{|R^{\text{group}}|}$. The goal is to decompose any resource group in such a way that the smallest number of scenarios remains.[1] Notice that decomposition of the resource group is useful only if it is represented by a graph component that is not complete. We decompose the problem using a recursive approach on the graph representation of the resource group as follows.

1. Check whether the graph component, representing a resource group, is complete. If so, the number of scenarios is $4^{|R^{\text{group}}|}$. Otherwise, go to step 2.
2. For each vertex in the graph component, representing a resource, remove the vertex from the graph component.[2] Using a depth-first search, find the graph components in the new subgraph. For each of the new graph components, go to step 1. The total number of scenarios is 4 times the summation of the number of scenarios in each new graph components. Go to step 3.
3. Over all the vertices that can be removed, choose the vertex that leads to the smallest number of scenarios. This vertex represents the resource that should be fixed.

6. Computational experiment

In this section, we use an extensive computational experiment to compare solving the joint problem using the integrated algorithm with solving the two problems sequentially. In Section 6.1, we give the most important characteristics of the problem instances that we use; a more extensive explanation of how the problem instances are generated can be found in Appendix B. Section 6.2 gives the results. We have implemented our work in Delphi 2007 and solve problems instances on an Intel Core 2 Duo P8600@2.40 GHz, with 3.5 GB RAM, under Microsoft Windows XP SP 3.

[1] Notice that some scenarios may consist of many components, whereas others may consist of a few components only. One may want to incorporate this in the search for the best way of decomposing the resource group, but for sake of simplicity, we do not do that.

[2] Do not remove a vertex that is connected to one other vertex only, since that does not help in decomposing the graph component.

Table 1 Fixed values

Parameter	Value(s)
# Bases	5
# components	100
# Resources	10

Table 2 Values that vary over a range

Parameter	Range(s)
Annual demand of component	$[0.01; 0.1]$
Net cost of component	$[1,000; 10,000]$ & $[1,000; 100,000]$
Discard costs	$[75\%; 125\%]$
Repair costs	$[25\%; 75\%]$
Move costs	$[1\%; 1\%]$
Annual holding costs	$[20\%; 20\%]$ & $[20\%; 40\%]$
Annual cost of resource	$[10,000; 100,000]$ & $[10,000; 500,000]$
Discard time (in years)	$[1/10; 1/2]$ & $[1/4; 1/2]$
Repair time (in years)	$[0.5/52; 4/52]$ & $[2/52; 4/52]$
Move time (in years)	$[2/365; 4/52]$ & $[1/52; 4/52]$
# Components per resource	$[2; 3]$ & $[2; 6]$

Table 3 Overview of the results

Algorithm used	Optimization time in seconds		Cost reduction compared with sequential		Average availability
	average	maximum	average	maximum	
Sequential	0.02	0.13	–	–	95.14%
Integrated	0.11	0.64	5.07%	43.26%	95.11%

6.1. Generator

There are seven parameters that get two different values or ranges in our set of problem instances, the other parameters get a fixed value or range (see Tables 1 and 2 for the exact values). We generate ten problem instances per parameter combination to avoid basing conclusions on one exceptional case only. In total this leads to $10 \cdot 2^7 = 1{,}280$ problem instances in the computational experiment.

6.2. Results

Table 3 gives an overview of the results: compared with solving the two problems sequentially, solving them using the integrated algorithm results in a cost reduction of 5.07% on average and 43% at maximum. The two key reasons why cost reductions result are:

- Some components that require resources in order to be repaired, are repaired in the solution of the integrated approach, whereas they are discarded in the solution of the sequential approach. As the repair lead time is considerably less than the resupply lead time in our experiments, we need less spare parts if we repair. In rare cases, components are repaired at the bases instead of at the central warehouse in order to reduce the lead time.
- Components that do not require resources in order to be repaired, are always repaired at the base in the sequential solution, because there is no reason to induce move costs in order to repair the components at the central warehouse. However, in our model (and generally in the METRIC type models), spare parts may only be stocked at locations where the spare parts are repaired or downstream in the network, which means that if repairs are performed at the bases, spare parts may only be stocked there. In the integrated solution, quite some of these repairs are performed at the central warehouse so that risk pooling effects can be used by stocking spare parts there.

Table 4 % of demand that is repaired

# Components per resource	Sequential Ech. 1	Sequential Ech. 2	Sequential Total	Integrated Ech. 1	Integrated Ech. 2	Integrated Total
[2; 3]	78.6%	0.1%	78.8%	62.7%	16.8%	79.5%
[2; 6]	67.1%	1.1%	68.2%	65.5%	4.3%	69.8%

The second reason seems to be the most important reason why cost reductions are achieved. To see this, look at the results for the problem instances in which each resource is required by 2 to 3 components ([2; 3]) in Table 4. Using the integrated approach, more repairs are performed at the central depot (echelon level 2) and in total, compared with using the sequential approach. However, the increase in the total number of repairs is only minor (the increase in installed number of resources is also minor; this is not shown in the tables); the main difference results from changing the repair location from echelon level 1 to echelon level 2 for many components.

Since there are more components that do not require any resource when 2 to 3 components require each resource, than when 2 to 6 components require each resource (the number of resources is not changed), the repair strategy of many more components is changed in the former case than in the latter case, leading to an average cost reduction of 9.2% and 0.9%, respectively.

Another interesting thing to notice is that the cost reduction that may be achieved is especially large if the move lead time is low. If this lead time is drawn from a uniform distribution on the interval [2/365; 4/52], the average cost reduction is 7.6%, whereas it is only 2.5% when the interval [1/52; 4/52] is used. The reason is that if the move lead time is low, then the increase in lead time is small when a component is repaired at echelon level 2 (in the solution of the integrated algorithm) instead of at echelon level 1 (in the solution of the sequential approach). This means that the advantage of being able to use risk pooling effects more easily outweighs the disadvantage of a higher lead time.

The effects of the other parameter settings are much smaller. The next biggest effect results from a change in the repair lead time. If it is drawn from a uniform distribution on the interval [0.5/52; 4/52], then the average cost reduction is 4.3%, whereas it is 5.8% if the interval is [2/52; 4/52].

7. Conclusions and further research

We have presented an algorithm that finds optimal solutions for the joint problem of LORA and spare parts stocking. Solving the problem using our integrated algorithm leads to a cost reduction of 5.07% on average and more than 43% at maximum in our numerical experiment compared with solving the two problems sequentially, which is done usually in both the literature and in practice. Therefore, we conclude that it is worthwhile to solve this problem integrally, especially if the lead time between the echelon levels is low. The cost reduction is achieved mainly by performing repairs at the central depot instead of at the bases (for parts that do not require resources) so that spare parts may be stocked at the central depot and thus risk pooling effects may be used. Performing repairs instead of discards and performing repairs at base instead of at the central depot has a much smaller effect on the achieved cost reduction.

For further research, it would be interesting to extend the model to multi-echelon, multi-indenture problems and find good heuristic procedures. The integrated algorithm that we have presented will not find efficient points anymore, but may still serve as a benchmark.

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Appendix A: Counter example optimality single-site, multi-indenture problem

To understand the calculations in the example below, a basic knowledge of METRIC models is required. We refer to the books by Sherbrooke (2004) or Muckstadt (2005) for a detailed explanation. We also require some notation: Let $\mathbf{P}\{X_c \leq k\}$ denote the probability that over the repair or replenishment lead time of component c, the number of demands for component c is less than or equal to k. These Poisson probabilities are easily calculated.

Consider a single stock point where spare parts are stocked for a component (c_1) and its two subcomponents (c_2 and c_3). So, we aim to construct a curve of EBO of component c_1 versus total spare parts costs resulting from stocking components c_1, c_2, and c_3. The failure rate per unit time of component c_1 is 4. Each failure can be repaired by replacing either one of the two subcomponents. Out of the 4 failures, on average 1.5 are caused by component c_2 and 2.5 are caused by component c_3. Replacement of the defective subcomponent (so, repair of component c_1) takes zero time. Repair of components c_2 and c_3 both take 1 unit time. If no spare parts are stocked of any of the three components, we will thus have expected number of backorders of component c_1 of $1 \cdot 1.5 + 1 \cdot 2.5 + 0 \cdot 4 = 4$.

Consider two possible spare parts stocking solutions for the subcomponents, having the same holding costs per unit time (the holding costs per unit time for component c_2 are two third of those for component c_3):

1. Stock 1 spare component c_2 and 4 spare components c_3, resulting in expected number of backorders for the subcomponents of 0.8939 (0.7231 and 0.1708 for component c_2 and c_3, respectively). These backorders delay repairs of the parent component c_1.
2. Stock 4 spare components c_2 and 2 spare components c_3, resulting in expected number of backorders for the subcomponents of 0.8935 (0.0242 and 0.8694 for component c_2 and c_3, respectively).

Constructing a convex EBO-curve would lead to removal of solution 1 (since it has a higher EBO than solution 2 and the same costs). If we do not stock any spare components c_1, this is fine. However, if we stock one spare component c_1, solutions 1 and 2 would lead to an expected number of backorders of component c_1 of 0.39 and 0.43, respectively. Solution 1 clearly dominates solution 2. This shows that generally, it is not possible to solve a subproblem for subcomponents first and use the resulting convex EBO-curve to solve the problem for the parent component. Instead, we should enumerate all possible solutions for the two subcomponents, construct an EBO-curve for the parent for each resulting solution, and take the lower envelope of the resulting curves. The computation time clearly explodes if the number of subcomponents per component increases.

A similar problem occurs in the case of general multi-echelon problems (more than two echelon levels). Although these results are often mentioned, both for the multi-indenture and general multi-echelon problems, we have never seen an example such as we have provided here.

Appendix B: Problem instances generator

We explain how we generate the problem instances that we use in our experiments. For each parameter that we use to generate these instances, we use the default setting in the text. Tables 1 and 2 give a complete overview of the possible settings. Some values are set to a certain value, others are drawn from a given distribution. These random values are the same for all settings of the other parameters. We use a full factorial design and we generate 10 problem instances for each combination of parameters to decrease the risk of basing conclusions on one odd problem instance. As a result, there are 1,280 problem instances in total.

We use a two-echelon repair network that is completely symmetrical in the cost factors, the demand rates, and the throughput times. It consists of a central depot and five bases. The product structure consists of 100 components. The annual demand for a component is drawn from a uniform distribution on the interval $[0.01; 0.1]$. The component's price is drawn from a shifted exponential distribution with shift factor 1,000 and rate parameter $7/(10{,}000 - 1{,}000)$. As a result, we do not have components with a price below 1,000, since they are typically discarded by default. Furthermore, there are considerably more cheap components than expensive ones. On average 1‰ of the components get a value larger than 10,000, but we draw a new price for these components to avoid odd problem instances. Using the calculated prices, we calculate the variable costs as follows:

- Repair costs as a fraction of the component price are drawn from a uniform distribution on the interval $[0.25; 0.75]$.
- The discard costs as a fraction of the component price are drawn from a uniform distribution on the interval $[0.75; 1.25]$. These costs include the costs of purchasing a new component and the disposal costs or residual value of the defective component.

- The move costs as a fraction of the component price are 1%.
- The annual costs of holding one spare part of a component are 20% of the gross component price.

For each component, we draw a repair time from a uniform distribution on the interval $[2/52; 4/52]$. The discard time, the time it takes to order a component and receive it at the central depot, is drawn from a uniform distribution on the interval $[1/10; 1/2]$. Both the discard and the repair times vary over the components, but are the same at both echelon levels. The replenishment lead time between the two echelon levels is drawn from a uniform distribution on the interval $[2/362; 4/52]$. This value is the same for all components.

There are ten resources and their annual costs are drawn from a shifted exponential distribution with shift factor 10,000 and rate parameter $7/(100{,}000 - 10{,}000)$. We draw a new price for the resources that get a price higher than 100,000 to avoid odd problem instances. The number of components that requires a certain resource is drawn from a uniform distribution on the range $[2; 3]]$. These components are chose randomly.

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