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Smits, R.M.M.

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INVESTIGATIONS ON THE POSITIVE COLUMN
OF A MEDIUM PRESSURE NEON DISCHARGE

PROEFSCHRIFT

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PROF. DR. IR. H. L. HAGEDOORN
EN
DR. P. W. SLUYTER
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In this thesis some properties of the medium pressure neon discharge are investigated. In some cases the medium pressure argon discharge is also discussed.

At first a general introduction is given on the properties of the positive column in a region where $0.50 < p_o R < 10$ torr m and $0.01 < i/R < 10$ A/m ($p_o$ = filling pressure, $i$ = discharge current, $R$ = tube radius). The electron energy distribution function is calculated from the Boltzmann-equation. This distribution function is used for the computation of coefficients appearing in a model set up for the description of the positive column. The properties of this model are discussed in relation with those of other models presented in the literature. The results of the model are compared with measurements. The abrupt transition from the diffuse into the constricted column is investigated. The connection between this constriction and the appearance of ionization waves (stripes) is discussed. Optical measurements on the positive column are presented. These are the measurements on the electron-atom bremsstrahlung continuum radiation and fluorescence measurements for the detection of atoms in an excited state. From these measurements radial profiles of the electron density, the electron temperature and the densities of the metastable $^3P_2 (1s5)$ and resonant $^3P_1 (1s4)$ states are found. For the fluorescence measurements a dye laser was used. The coupling coefficients of the 2p-levels in neon are found in dependence on the gas temperature.
CHAPTER 1  INTRODUCTION

1.1 Introduction

In the past the positive column of a neon discharge has been the subject of many studies. However, these studies were concerned mostly with the low pressure column ($p_0 < 5$ torr) or the high pressure arc ($p_0 > 1$ atm). Only in recent times the intermediate region has been examined in more detail. This medium pressure region is more complicated, but also more interesting, because many physical processes are of importance. Recently the understanding of the various processes has been growing. The electron energy distribution function (E.D.) which was formerly assumed to be maxwellian has now been calculated by several authors (Kag64, Woj65a, Wil70, Lya72, Wind72) from the Boltzmann equation, including the effects of the applied axial electric field, the elastic and inelastic collisions of electrons with neon ground state atoms and interelectronic collisions. Also much work has been done on the investigation of the striations in the positive column (Pok68). These striations are ionization waves and appear at several pressure-current regions (see fig. 1.1). Except for the fact that the column can be axially homogeneous and stationary or striated (character-

![Diagram](https://example.com/diagram.png)

**Fig. 1.1** Existence regions of several column states in the neon discharge (from Pf68). (Shaded area: our region of interest.) $p_0 =$ filling pressure, $i =$ discharge current, $R =$ tube radius.
ized by the appearance of ionization waves), there is also a division into the diffuse and the constricted state. In the diffuse state the column fills the entire tube and the axial electric field strength is high. In the constricted state the column is very narrow and the axial electric field strength is low. From fig. 1.1 it can be seen that the discharge is constricted only for high values of the filling pressure and the discharge current.

In this thesis we investigate the following pressure-current region: \(0.50 < p_o R < 10\) torr m and \(0.01 < i/R < 10\) A/m (the shaded area in fig.1.1). (The filling pressure \(p_o\), the discharge current \(i\) and the tube radius \(R\) are present only in the combinations \(p_o R\) and \(i/R\) conform the similarity rules (discussed in Pfa69).) In this region the diffuse column (low currents) is axially homogeneous and stationary, while the constricted column (high currents) is striated. The constriction occurs abruptly at a certain current value which is slightly dependent on the filling pressure (see fig.1.1). The investigation of the physical mechanisms responsible for this sudden constriction is one of the subjects of this thesis.

We present model calculations on the positive column, using the electron energy distribution function (E.F.) calculated from the Boltzmann-equation. The model is compared with other models presented in the literature. The connection between the constriction and the onset of the striations is discussed. Finally optical measurements on the electron-atom bremsstrahlung continuum and laser fluorescence measurements are presented. The calculations are compared with the data obtained from these measurements.

The optical measurements and measurements of the axial electric field strength have been performed on tubes which had a cataphoresis purification chamber on the anode and on the cathode side (see for such a purification chamber the thesis of Baghuis (Bag74)). The current was regulated by a stabilization unit (Pri75). The axial electric field strength was measured with the double-probe method (Bag74, Pri75).

1.2 The electron energy distribution function (E.F.)

The E.F. is calculated from the Boltzmann-equation using the methods given by Shkarofsky et al. (Shk66) and Lyaguschencko (Lya72).
In this calculation the E.F. is developed in spherical harmonics. After substitution of this expansion into the Boltzmann-equation one obtains a hierarchy of tensor equations (Shk66). The expansion is truncated after the zero-order term, yielding a scalar equation. In this equation the radial derivative term is nearly cancelled by the ambipolar field term. These terms therefore are neglected. Furthermore superelastic collisions are neglected because the plasma is far from thermodynamic equilibrium (Lya72).

With respect to earlier calculations for neon (see for instance Göl68a) we add the following alterations:
- an energy dependence of the elastic cross-section of neon is taken into account. This dependence is fitted to the experimental data given by Salop and Nakano (Sal70);
- the gas temperature term is taken into account for the calculation of the excitation frequency from the E.F.;
- the axial electric fieldstrength is calculated in a consistent way;
- for several correction factors, the dependences on the electron temperature and the electron density are taken into account;
- the inelastic region is split into two regions in stead of describing it as one single region;
- the particular solution in the elastic region is treated in a consistent way.

The calculations are presented in chapter 2. The calculated E.F. is used for the computation of the reaction coefficients appearing in our numerical model for the positive column, presented in chapter 3.

1.3 A numerical model for the positive column

The numerical model which we have set up for the positive column of a medium pressure neon discharge is an extension of the model given in a former publication (Sm74). In this thesis we also adapt the model to the case of argon (the E.F. for argon is also calculated in chapter 2). The differences with respect to the former version (Sm74) of the model are the following:
- for the calculation of the reaction coefficients, the E.F. calculated from the Boltzmann-equation is used (chapter 2) in stead of the Druyvesteyn E.F. used in the former publication;
- a more complex treatment of the balance equations for the excited atoms;
- an improved analytical approximation to the measurements on the elastic cross-section of Salop and Nakano (Sal70);
- inclusion of the radiation term in the calculation of the wall temperature;
- a consistent calculation of the axial electric field strength;
- inclusion of the effect of the presence of dead volumes on the pressure in the active discharge;
- a more refined numerical treatment.

In chapter 4, the model is compared with other models presented in the literature. Also the influence of the variation of some parameters (the E.R., the stepwise ionization frequency, the volume recombination coefficient, the frequency of interelectronic collisions) is investigated.

1.4 The constriction and the appearance of striations

In the pressure-current region of our interest the constriction is characterized by the appearance of striations. In chapter 4 we discuss this connection in more detail. There are indications that the onset and the nonlinear growth of the striations cause an instability which in its turn starts the constriction. This hypothesis is compared with mechanisms proposed by several other authors.

1.5 The electron-atom bremsstrahlung continuum

In the medium pressure inert gas discharge the optical line radiation is superimposed upon a rather strong continuum. While in the past most authors believed this continuum to be of molecular origin, nowadays many authors are convinced that the continuum is caused by electron-atom bremsstrahlung (Rut66, Go173, Ven73, Pfa76). In chapter 5 we present measurements of the radial profiles of the electron density and the electron temperature based on the assumption that the latter point of view is correct. The electron density has been obtained from the continuum intensity at a fixed wavelength. The electron temperature has been determined from the ratio of the continuum intensities at two fixed wavelengths. The measurements are compared with model calculations.
1.6 Fluorescence measurements

With the fluorescence technique the densities of the metastable $^3\text{P}_2$ ($|s_5\rangle$) and the resonant $^3\text{P}_1$ ($|s_4\rangle$) atoms have been measured. For this technique a dye-laser (Spectra-Physics 370, 5600 Å < λ < 6400 Å) has been used (Sm175, Ste75, Co076). In this way the relative radial density profiles of the $^3\text{P}_2$ and $^3\text{P}_1$ atoms have been measured for various values of the discharge current. The measured profiles are compared with calculated ones.

With the fluorescence technique the coupling coefficients of the $2p$-levels have been determined. These coupling coefficients have been measured on the axis of the discharge as a function of the discharge current. With the aid of our model the measured dependence of the coupling coefficients on the discharge current is transformed into a dependence on the gas temperature.
CHAPTER 2 THE ELECTRON ENERGY DISTRIBUTION FUNCTION IN THE MEDIUM PRESSURE DISCHARGE

Scope

In this chapter the electron energy distribution function is calculated from the Boltzmann-equation for the conditions in the medium pressure discharge. Expressions for the axial electric field strength and the excitation frequency are derived. The calculations are performed for neon (sections 2.2-2.5) and argon (section 2.6).

Introduction

For the calculation of the coefficients appearing in the equations of the model described in chapter 3, the electron energy distribution function (E.F.) must be known. In the medium pressure discharge the E.F. is determined by the electric field, the elastic and inelastic collisions with the gas atoms and by the Coulomb interaction between the electrons. When the Coulomb interaction prevails, the distribution function is maxwellian. When the elastic collisions and the field influence are dominant, one obtains the Druyvesteyn distribution (Shk66). In our pressure and current region (0.50 < p_o < 10 torr m, 0.01 < i/R < 10 A/m, see chapter 1) all mentioned processes can be of equal importance. Therefore the E.F. has been calculated from the Boltzmann-equation.

We apply the method given by Shkarofsky et al. (Shk66) and Lyaguschenko (Lyas72) to our case. In sections 2.4 and 2.5 expressions for the axial electric field strength and the excitation frequency are derived which will be used in the model described in chapter 3.

2.1 The Boltzmann-equation

The electron energy distribution function (E.F.) is obtained from the Boltzmann-equation for the stationary discharge:

\[ \mathbf{v} \cdot \nabla f - \frac{eE}{m} \cdot \mathbf{v} f = C(f), \]

where \( f = f(r, v) \) is the E.F., \( r \) position, \( v \) velocity, \( e \) elementary charge, \( m \) electron mass, \( E \) electric field strength, \( C \) collision operator.

The left-hand side of eq. (2.1) represents the flow processes in phase
space, the right-hand side represents the collision processes.

When the electric field is low, such that the drift velocity of
the electrons is much smaller than their thermal velocity, the distri-
bution function is nearly isotropic in velocity space. Then the E.F.
can be developed into spherical harmonics (Shk66). This development can
be written as (Shk66, Joh66):

$$f(x,v) = \sum_{\ell=0}^{\infty} f_{\ell}(x,v) \frac{v^\ell}{\ell!},$$

and

$$C(\ell) = \sum_{\ell=0}^{\infty} c_{\ell} \frac{v^\ell}{\ell!}.$$  \hspace{1cm} (2.2)

Here $f_{\ell}$ and $c_{\ell}$ are tensors of the $\ell$th rank and $v$ represents the repeated
inproduct (see Joh66).

Substitution of this expansion into the Boltzmann-equation re-
results in a hierarchy of tensor equations of increasing rank. Because
the ratio of an element of the tensor $f_{\ell+1}$ to an element of the tensor
$f_{\ell}$ is of the order of magnitude $\sqrt{18m/M}$ (M atomic mass) (see appendix
2.1), we have sufficient accuracy for our purpose when we replace
$f(x,v)$ by $f_{\ell}(x,v)$. The Boltzmann-equation then reduces to a scalar equa-
tion for $f_{\ell}$ (see appendix 2.1). In this equation the radial derivative
term nearly cancels the term containing the radial ambipolar field (see appendix 2.2). Therefore these two effects are neglected. Furthermore,
as the conditions in the column are far from thermodynamic equilibrium,
the super elastic collisions are negligible (see Lya72).

After a transformation from velocity coordinates to the dimen-
lessness energy coordinate $\varepsilon = U/U_e$ (U electron energy, $U_e$ scaling param-
eter, see below), the scalar equation for $f_{\ell}(\varepsilon)$ (using the same symbol
$f_{\ell}$ in the case of energy coordinates.) becomes according to
Lyaguschhenko (Lya72) (where we added the term containing the gas tem-
perature according to Shkarofsky (Shk66)):

$$\frac{1}{\varepsilon} \frac{d}{d\varepsilon} \left[ \dot{f}_{\ell}(\varepsilon) \frac{df_{\ell}}{d\varepsilon} + G(\varepsilon) f_{\ell} \right] =$$

$$\frac{\nu}{\nu_1} (\varepsilon) \sqrt{\varepsilon} f_{\ell}(\varepsilon) - \frac{\nu}{\nu_1} (\varepsilon + \varepsilon_1) \frac{\varepsilon + \varepsilon_1}{\sqrt{\varepsilon_1}} f_{\ell}(\varepsilon + \varepsilon_1),$$  \hspace{1cm} (2.3)

with $\nu = U/U_e$, $\nu_1 = U_1/U_e$, $U$ electron energy, $U_1$ excitation energy,
$\nu_1$ scaling parameter to be chosen as the electron temperature (see
below, $U$, $U_1$, $U_e$ all in Volt equivalents),
\[ H(\varepsilon) = v_{ee} A_1(\varepsilon) + v_{E}(\varepsilon)\varepsilon + \frac{T_g}{T_e} v_{g}(\varepsilon)\varepsilon^2, \]
\[ G(\varepsilon) = v_{ee} A_2(\varepsilon) + v_{g}(\varepsilon)\varepsilon^2, \]
\[ v_{ee} = \frac{8\pi e^4 N_e}{(4\pi\varepsilon_0\hbar)^{2/3} N_e} \ln \left( \frac{12\pi (kT_e)^{3/2}}{1/2} \right), \]
\[ v_{E} = \frac{4}{3} \frac{e^2 \varepsilon^2}{m} - \frac{1}{N_g Q_D w^3}, \]
\[ v_g = \frac{2m}{M} N_g Q_D w, \]
\[ v_{in} = N_g Q_t w, \]
\[ w = \sqrt{2eU_e/m}, \quad T_e = eU_e/k, \]
\[ N_e \text{ electron density, } N_g \text{ gas density, } T_g \text{ gas temperature,} \]
\[ Q_D \text{ momentum transfer cross-section,} \]
\[ Q_t \text{ total inelastic cross-section,} \]
\[ A_1(\varepsilon) = \frac{4\pi}{3} \left[ \int_{-\varepsilon}^{\varepsilon} \varepsilon^{1/2} f_0(\varepsilon')d\varepsilon' + \varepsilon^{3/2} \int_{-\varepsilon}^{\varepsilon} f_0(\varepsilon')d\varepsilon' \right], \]
\[ A_2(\varepsilon) = 2\pi \int_{-\varepsilon}^{\varepsilon} \varepsilon^{1/2} f_0(\varepsilon')d\varepsilon'. \]

After the transformation from \( v \) to \( \varepsilon \), the scaling parameter \( U_e \) still is a free parameter. In section 2.4 this parameter will be fixed, defining \( U_e \) as the electron temperature, according to

\[ \frac{3}{2} eU_e = \frac{1}{2} m < v^2 > . \]

This definition then yields a relation between \( U_e \) and \( E \).

Because \( E \sim N_g \) (see section 2.4), and the logarithm appearing in \( v_{ee} \) is a weak function of \( N_e \), the E.F. depends on \( N_e \) and \( N_g \) mainly, via the ratio \( N_e/N_g \), as can be seen from eq. (2.3).

In eq. (2.3) the term with \( H(\varepsilon) \) describes the influence of the electron energy gain processes on the E.F. These processes are the Coulomb interaction, the influence of the axial electric field and the thermal motion of gas atoms. The term with \( G(\varepsilon) \) contains the influence of the following electron energy loss processes on the E.F.: Coulomb interaction and elastic collisions. The right-hand side of eq. (2.3) describes the inelastic losses and the creation of slow electrons by inelastic collisions.

When \( v_{ee} \gg v_{E}, v_{g}, v_{in} \), hence when Coulomb interaction prevails, the solution of (2.3) is the Maxwell distribution: \( f_0(\varepsilon) = N e^{-\varepsilon} \). When \( v_{E}, v_{g} \gg v_{ee}, v_{in} \), one obtains the Druyvesteyn distribution:
\[ f_0(\varepsilon) = N \exp \left( -\frac{\varepsilon^2}{2\varepsilon_0^2} \right) \]  

In the following analytical calculations for neon, the term \( \frac{T_g}{T_e} \varepsilon^2 \) is neglected, because \( T_g \ll T_e \) and inclusion of this term gives cumbersome expressions (in section 2.5 a correction for this omission is given). For argon, however, the term can be maintained without difficulty (see section 2.6).

The integrals \( A_1 \) and \( A_2 \) have been calculated by Golubovskii et al. (Gol70a) for various distribution functions. It appeared that for all cases, within 15%: \( A_1(\varepsilon) = A_2(\varepsilon) = A(\varepsilon) \equiv \{ 0.385 \varepsilon \) for \( \varepsilon \leq 2.6 \), 

\[ \] for \( \varepsilon > 2.6 \).

2.3 Cross-sections for neon

To calculate the frequencies \( \nu_g \), \( \nu_E \) and \( \nu_{in} \) which appear in eq. (2.3), we have to know the cross-sections for the elastic and inelastic collision processes. In sections 2.2-2.5 the case for neon will be considered. The case of argon will be dealt with in section 2.6. For neon the total elastic cross-section has been measured by Salop and Nakano (Sal70). We approximate this cross-section by (see fig. 2.1)

\[ Q = \begin{cases} a_{el} U^2 & \text{for } U \leq U_1, \\ a_1 U_1 & \text{for } U > U_1, \end{cases} \]  

with \( a_{el} = 1.8 \times 10^{-20} \text{ m }^2 \text{ V}^{-1/4} \), \( n = 1/4 \) and \( U_1 = 16.6 \text{ V} \).

Here the momentum transfer cross-section is calculated. We take (McDo64):

\[ Q_D = \frac{a_D}{a_{el}} Q_{el} \]  

with \( \frac{a_D}{a_{el}} = 0.85 \).  

(2.5)

Then \( \nu_E \) and \( \nu_g \) become:

\[ \nu_E = \begin{cases} \nu_E^* \varepsilon^{-1/4} & \text{for } \varepsilon \leq \varepsilon_1, \\ \nu_E^* \varepsilon_1 & \text{for } \varepsilon > \varepsilon_1, \end{cases} \]

\[ \nu_g = \begin{cases} \nu_g^* \varepsilon^{1/4} & \text{for } \varepsilon \leq \varepsilon_1, \\ \nu_g^* \varepsilon_1 & \text{for } \varepsilon > \varepsilon_1, \end{cases} \]

with \( \nu_E^* = \frac{4}{3} \frac{a_e^N U^N g^e}{m^2} \frac{1}{a_D g^D g^e} \),

and \( \nu_g^* = \frac{2m}{N} a_D N g^N w \) with \( n = \frac{1}{4} \).

(2.6)
The total inelastic cross-section for neon has been measured by Schaper and Schelber (Sch69). We approximate these measurements by (fig. 2.2):

$$Q_t = \begin{cases} 
0 & 0 \leq U \leq U_1 \\
A_1(U-U_1) & U_1 \leq U \leq U_2 \\
A_2(U-U_2) + A_1(U_2-U_1) & U \geq U_2,
\end{cases} \quad (2.7)$$

with $U_2 = 17.0 \, \text{V}$, $A_1 = 6.10^{-22} \, \text{m}^2 \, \text{V}^{-1}$,
$A_2 = 0.5 \times 10^{-22} \, \text{m}^2 \, \text{V}^{-1}$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2_1}
\caption{Elastic cross-section for neon}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2_2}
\caption{Inelastic cross-section for neon.}
\end{figure}
2.3 Solution of the scalar equation

2.3.1 The elastic region

For the elastic region $\varepsilon < \varepsilon_1$, the inelastic cross-section $Q_\varepsilon = 0$ (see eq. 2.7). Because $\lim_{\varepsilon \to 0} A(\varepsilon) = 0$, and therefore $H(0) = G(0) = 0$, multiplication of eq. (2.3) by $\sqrt{\varepsilon}$ and integration over $\varepsilon$ yields:

$$H(\varepsilon) \frac{d^2 \varepsilon}{d \varepsilon} + G(\varepsilon) f_0 = -t(\varepsilon),$$

(2.8)

where $t(\varepsilon) = N \omega \int_{\varepsilon}^{\infty} (\varepsilon' + \varepsilon_1) Q_\varepsilon (\varepsilon' + \varepsilon_1) f_0 (\varepsilon' + \varepsilon_1) d\varepsilon'$,

(2.9)

is the number of electrons scattered in the low energy region by inelastic collisions (Lyap72). The formal solution of (2.8) is:

$$f_0 (\varepsilon) = NJ_1 (\varepsilon) + J_2 (\varepsilon),$$

(2.10)

with

$$J_1 (\varepsilon) = \exp \left( - \int_{\varepsilon}^{\infty} G(\varepsilon') H(\varepsilon') d\varepsilon' \right),$$

(2.11)

and

$$J_2 (\varepsilon) = \int_{\varepsilon}^{\infty} \frac{t(\varepsilon')}{H(\varepsilon') J_1 (\varepsilon')} d\varepsilon',$$

(2.12)

($N$ is a constant).

For an analytical solution we have approximated the integrals $A_1$ and $A_2$, appearing in $G(\varepsilon)$ and $H(\varepsilon)$ by a constant $C = 1$. Comparison with a numerical solution yields errors in the final E.P. smaller than 15%. Writing $J_1 (\varepsilon) = \exp (-M(\varepsilon))$, we find for $M(\varepsilon)$:

$$M(\varepsilon) = \frac{b}{a^{13/3}} \left( \frac{2}{5} y^{10} - \frac{4}{3} y^7 + y^4 + \right. \left. 4 \left( \frac{3}{2} \frac{a}{b} - 1 \right) y + \frac{1}{3} \ln \left( \frac{\sqrt{1+y^2}}{1+y} \right) - \frac{1}{\sqrt{3}} \arctg \frac{2y-1}{\sqrt{3}} \arctg \frac{1}{\sqrt{3}} \right),$$

(2.13)

where $y = a^{1/3} \varepsilon^{1/4}$ and

$$a = \frac{\nu_e^*}{\nu_{ee}}, \quad b = \frac{\nu_0^*}{\nu_{ee}} \quad \text{(see eqs. (2.3) and (2.6))}.$$  

(2.14)

(The expression for $M(\varepsilon)$ in the cases $n = 0$ and $n = 1/3$ in eq. (2.4), can be found in the references Gol68a and Duy75 respectively.)

For low electron densities ($N_e < 10^{15}$ m$^{-3}$), $a$ and $b$ become large, and $M(\varepsilon)$ becomes $M(\varepsilon) = \frac{2b}{5a} \varepsilon^{5/2}$ (Druyvesteyn limit). For high electron densities ($N_e > 10^{19}$ m$^{-3}$), $a$ and $b$ tend to zero, yielding
\[ M(\varepsilon) = \varepsilon \quad \text{(Maxwell limit)}. \]

![Graph showing M(U/U_e) as a function of U.](image)

**Fig. 2.3** \(-M(U/U_e)\) as a function of \(U\).

In fig. 2.3 \(M(\varepsilon)\) is plotted for several values of \(N_e\), with the scaling factor \(U_e = 3\) (taken such that \(\frac{3}{2} eU_e = \frac{1}{2} m < v^2 >\), see section 2.4). To calculate the constant \(a\), use has been made of the relation between the electric field strength and the electron temperature derived in section 2.4. For high electron densities, the distribution function becomes maxwellian indeed. For low electron densities the Druyvesteyn form is achieved.

The constant \(N\) in eq. (2.10) is obtained from the normalization condition:

\[ \int_0^\infty f_0(v) 4\pi v^2 \, dv = 2\pi \alpha^3 \int_0^\infty f_0(\varepsilon) \sqrt{\varepsilon} \, d\varepsilon = 1. \quad (2.15) \]

Because \(f_0(\varepsilon)\) rapidly decreases for increasing \(\varepsilon\), (see section 2.3.3), we replace this condition by:

\[ 2\pi \alpha^3 \int_0^{\varepsilon_1} (N J_1 + J_2) \sqrt{\varepsilon} \, d\varepsilon = 1. \quad (2.16) \]
Numerical calculations show that $J_2(\varepsilon) \ll NJ_1(\varepsilon)$, except for $\varepsilon \to \varepsilon_1$ (see fig. 2.4 below). Therefore neglecting $J_2$, we take:

$$N = \frac{1}{2m^3\omega^3 \int_0^{\varepsilon_1} J_1(\varepsilon) \sqrt{\varepsilon} \, d\varepsilon} \tag{2.17}$$

To fulfill the continuity conditions (see section 2.3.3), we need to know $J_2(\varepsilon)$ in the neighbourhood of $\varepsilon_1$. A good approximation $J_2^*(\varepsilon)$ for $J_2(\varepsilon)$ at $\varepsilon \to \varepsilon_1$ is derived in appendix 2.III, where it is shown that:

$$J_2(\varepsilon) = J_2^*(\varepsilon) = -t(\varepsilon_1)/G(\varepsilon). \tag{2.18}$$

In fig. 2.4, $NJ_1(\varepsilon)$, $J_2(\varepsilon)$ and $J_2^*(\varepsilon)$ are plotted for $U_e = 3$, $N_e = 10^{17} m^{-3}$. $J_2(\varepsilon)$ is indeed negligible compared to $NJ_1(\varepsilon)$ for $\varepsilon \ll \varepsilon_1$. For $\varepsilon \to \varepsilon_1$, $J_2^*(\varepsilon)$ is a good approximation for $J_2(\varepsilon)$.

![Fig. 2.4 A comparison of $NJ_1$ with $J_2$ and $J_2^*$.
$N_g = 3 \times 10^{24} m^{-3}$, $N_e = 10^{17} m^{-3}$, $U_e = 3 V.$](image)

**2.3.2 The inelastic region**

In the inelastic region where $\varepsilon > \varepsilon_1$, the second term on the right-hand side of eq. (2.3) is negligible because of the rapid decay of $f_0(\varepsilon)$ in this region. Then eq. (2.3) becomes:

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\[
\frac{d^2 f_0}{dc^2} + P(\varepsilon) \frac{df_0}{dc} + S(\varepsilon)f_0 = 0,
\]
where \( P(\varepsilon) = \frac{dH}{dc} + G(\varepsilon)/H(\varepsilon), \)
and \( S(\varepsilon) = \frac{dG}{dc} - \frac{N_0}{g} w_0^0(\varepsilon) / H(\varepsilon). \)
(2.20)

Here \( v_g(\varepsilon) \) and \( v_e(\varepsilon) \) have to be taken constant (see eqs. 2.5, 2.6). After the transformation \( f_0(\varepsilon) = \Psi(\varepsilon) \exp(-\frac{1}{2} \varepsilon_1 P(\varepsilon')d\varepsilon') \) eq. (2.19) becomes:
\[
\frac{d^2 \Psi}{dc^2} + q(\varepsilon) \Psi = 0,
\]
where \( q(\varepsilon) = -\frac{1}{2} \frac{dP}{dc} - \frac{1}{4} \varepsilon^2 + S. \)
Using eqs. (2.19) and (2.20) we get:
\[
q(\varepsilon) = -\frac{(4v_0 a_0^2 + 1) + 4b_0 \varepsilon + (4a_0 b_0 + 2b_0) \varepsilon^2 + 2v_0 \varepsilon^3 + b_0 \varepsilon^4}{4(1 + a_0 e + v_b e^2)}
+ \frac{[v_g(\varepsilon)/v_e(\varepsilon)^2]}{(1 + a_0 e + v_b e^2)},
\]
(2.22)
with \( v = T_g/T_e, \) \( a_0 = a_{e1}^{-1/4}, \) \( b_0 = b_{e1}^{-1/4}. \)
Numerical evaluation of \( q \) gives a curve consisting of two nearly straight lines. Therefore we rewrite (2.22) retaining only linear terms (Duy75):
\[
q(\varepsilon) = -a_1^2 (\varepsilon - \varepsilon_1 + b_1) \quad \varepsilon_1 \leq \varepsilon \leq \varepsilon_2 \equiv \frac{U_2}{U_e},
- a_2^2 (\varepsilon - \varepsilon_2 + b_2) \quad \varepsilon > \varepsilon_2,
\]
where
\[
a_1^2 = \frac{N_0 w_0^0}{H(\varepsilon_1)}, \quad b_1 = \left( \frac{(1 + b_0 \varepsilon_1^2)^2 - 4b_0 \varepsilon_1 (1 + a_0 \varepsilon_1)}{4H(\varepsilon_1)^2} \right) \frac{v_e e}{a_1^2},
\]
\[
a_2^2 = \frac{N_0 w_0^0}{H(\varepsilon_2)},
\]
\[
b_2 = \left( \frac{(1 + b_0 \varepsilon_2^2)^2 - 4b_0 \varepsilon_2 (1 + a_0 \varepsilon_2)}{4H(\varepsilon_2)^2} \right) \frac{v_e e + A_1 (\varepsilon_2 - \varepsilon_1) N_0 w_0^2}{4H(\varepsilon_2)} \frac{1}{a_2^2}.
\]
(2.23)

The exponent in the transformation factor can be written in good approximation as (Duy75):
\[
-\frac{1}{2} \varepsilon_1 P(\varepsilon')d\varepsilon' = -\frac{1}{2} P(\varepsilon_1) (\varepsilon - \varepsilon_1),
\]
(2.24)
because \( P(\varepsilon) \) is nearly constant around \( \varepsilon_1. \)
Then the solution of eq. (2.19) becomes (with the requirement \[ f_0(\varepsilon) = 0 \]):

\[
f_0(\varepsilon) = \begin{cases} 
A\Phi_1(\varepsilon) + B\Phi_2(\varepsilon) & \varepsilon_1 \leq \varepsilon \leq \varepsilon_2, \\
C\Phi_3(\varepsilon) & \varepsilon \geq \varepsilon_2,
\end{cases}
\]  

(2.25)

(2.26)

where \( \Phi_1 = A_1 (a_1 (\varepsilon - \varepsilon_1 + b_1)) U(\varepsilon) \),
\( \Phi_2 = B_1 (a_1 (\varepsilon - \varepsilon_1 + b_1)) U(\varepsilon) \),
\( \Phi_3 = A_2 (a_2 (\varepsilon - \varepsilon_2 + b_2)) U(\varepsilon) \),
\( U(\varepsilon) = \exp(-\frac{1}{2}P(\varepsilon)(\varepsilon - \varepsilon_1)) \),

(Ai and Bi are Airy functions, see Abr68).

### 2.3.3 Continuity conditions

The solutions (2.10) for the elastic region and (2.25), (2.26) for the inelastic region have to be matched at \( \varepsilon = \varepsilon_1 \) ((2.10) and (2.25)) and \( \varepsilon = \varepsilon_2 \) ((2.25) and (2.26)). This determines the constants \( A, B \) and \( C \) in (2.25) and (2.26) as \( N \) has already been given in (2.17). Continuity for \( f_0(\varepsilon) \) and \( f_0'(\varepsilon) \) at \( \varepsilon_1 \) and \( \varepsilon_2 \) gives four equations:

\[
NJ_1(\varepsilon_1) + J_2(\varepsilon_1) - A\Phi_1(\varepsilon_1) + B\Phi_2(\varepsilon_1),
\]

(2.27)

\[
NJ_1'(\varepsilon_1) + J_2'(\varepsilon_1) = A\Phi_1'(\varepsilon_1) + B\Phi_2'(\varepsilon_1),
\]

(2.28)

\[
A\Phi_1(\varepsilon_2) + B\Phi_2(\varepsilon_2) = C\Phi_3(\varepsilon_2),
\]

(2.29)

\[
A\Phi_1'(\varepsilon_2) + B\Phi_2'(\varepsilon_2) = C\Phi_3'(\varepsilon_2).
\]

(2.30)

When eq. (2.27) is fulfilled, eq. (2.28) is not an independent equation, because it has been assumed already in the calculation of \( J_2(\varepsilon) \) (eq. 2.12) from eq. (2.8). This can be seen by replacing \( t(\varepsilon) \) (eq. 2.9) via the transformation \( \varepsilon'' = \varepsilon' + \varepsilon_1 \) by:

\[
t(\varepsilon) = N \int_{\varepsilon_1}^{\varepsilon_1 + \varepsilon} \varepsilon''Q_\varepsilon(\varepsilon'') f_0(\varepsilon'') d\varepsilon''.
\]

(2.31)

Neglecting the last term in (2.3) (see section 2.3.2), this integral can be written (by integration of eq. (2.3)) as:

\[
t(\varepsilon) = [H(\varepsilon)f_0'(\varepsilon) + G(\varepsilon)f_0]_{\varepsilon_1 + \varepsilon}^{\varepsilon} - [H(\varepsilon)f_0'(\varepsilon) + G(\varepsilon)f_0]_{\varepsilon_1}.
\]

(2.32)
Because of the rapid decay of $f_0(\varepsilon)$ for $\varepsilon > \varepsilon_1$, the first term in (2.32) can be neglected when $\varepsilon = \varepsilon_1$. Then eq. (2.8) becomes for $\varepsilon = \varepsilon_1$, when (2.25) is substituted in (2.32):

$$
H(\varepsilon_1) (NJ_1'(\varepsilon_1)+J_2'(\varepsilon_1)) + G(\varepsilon_1) (NJ_1+J_2) = \\
H(\varepsilon_1) (A\Phi_1'(\varepsilon_1)+B\Phi_2'(\varepsilon_1)) + G(\varepsilon)(A\Phi_1(\varepsilon_1)+B\Phi_2(\varepsilon_1)) .
$$

(2.33)

Substitution of (2.27) into (2.33) yields (2.28). Calculation of $J_2(\varepsilon)$ from (2.8) with $t(\varepsilon)$ from (2.9), thus yields (2.28), so that the latter equation has already been used in the calculations and is superfluous. We then retain eqs. (2.27), (2.29) and (2.30) for the determination of $A$, $B$ and $C$.

We replace $J_2$ by $J_2 = -t(\varepsilon_1)/G(\varepsilon)$ (see appendix 2.III).

Introducing $I = \frac{\phi_{12}' \phi_{32}'' - \phi_{12}'' \phi_{32}'}{\phi_{32}'' \phi_{22} - \phi_{32}' \phi_{22}'}$,

(where $\phi_{ij} = \phi_i'(\varepsilon_j)$ and $\phi_{ij}' = \phi_i'(\varepsilon_j)$), we get

$$
A = -\frac{G(\varepsilon_1) \times NJ_1'(\varepsilon_1)}{H(\varepsilon_1) \times \phi_{11}'' + \phi_{21}'' I} ,
$$

$$
B = AI , 
C = (A\Phi_{12}+B\Phi_{22})\phi_{32}^{-1} .
$$

(2.34)

With eqs. (2.10-2.14), (2.17), (2.23-2.26) and (2.34) the distribution function is determined for the entire energy region. In fig.2.5 $f_0(\varepsilon)$ is plotted at $U_e = 3$ for various values of the electron density. The inelastic collisions cause a depletion in the tail of E.F.

2.4 The axial electric field strength

In this section a relation between $E$ and $U_e$ is derived, which will be used in the model described in chapter 3.

After the transformation of the Boltzmann-equation, which contains the parameters $E$, $N_e$, and $N_g$, to the dimensionless energy coordinate $\varepsilon = U/U_e$, an extra (scaling) parameter $U_e$ was introduced. By most authors (Gol168a, Gol74, Woj67) $U_e$ is coupled to $E$ by solving the electron energy equation:

$$
eb \varepsilon^2 = P_{el} + P_{in} ,
$$

(2.35)

where the left-hand side is the energy gain per electron ($b_e =$ electron mobility), and the right-hand side represents the elastic and inelastic
energy losses per electron. This relation is obtained by multiplication of the Boltzmann-equation by $\frac{1}{2} m v^2$ and integration over $4\pi v^2 dv$ (Lyam72). The Coulomb terms then cancel each other. The authors mentioned above solve the Boltzmann-equation without the gas term $\frac{T_0}{T_e} \nu_g E^2$ (in $H(\varepsilon)$). This term introduces the extra energy gain term $P_{\text{gas}}$:

$$e \nu E^2 = P_{\text{el}} - P_{\text{gas}} + P_{\text{in}}.$$  

Because (2.36) is obtained from the Boltzmann-equation, substitution of $f_0$ into (2.36) yields an identity. Because the maxwellian distribution function does not contain $E$, it is often substituted into (2.35), because $E$ then can be solved from (2.35). As (2.35) is an identity this procedure is not valid. The E.F. can only be maxwellian when the electric field and the inelastic collisions are negligible. In that case the electrons relaxate to the gas temperature (Shk66). Therefore an expression which connects $U_e$ with $F$ can principally not be obtained from (2.35). When the E.F. is close to the maxwellian E.F., still the terms in the Boltzmann-equation which cause deviation from the maxwellian E.F., are the ones that constitute eq.(2.35) (or (2.36)). Therefore the calcula-
tion of \( E \) from (2.35), through substitution of the maxwellian E.F. is incorrect, and deviates in general considerably from the realistic calculation.

We obtain the relation between \( E \) and \( U_e \) from the requirement that the scaling parameter \( U_e \) is the electron temperature, according to the definition:

\[
\frac{3}{2} U_e = \frac{3}{2} k T_e = \frac{1}{2} m \langle \nu^2 \rangle .
\]  

(2.37)

Because the contributions to integrals over the distribution function of both \( f_2(e) \) and of \( f_0(e) \) in the inelastic region are very small (see figs. 2.4, 2.5), eq.(2.37) can be written as:

\[
\frac{\int f_1 e^{3/2} e^{-M_1} \, de}{\int f_1 e^{-1/2} e^{-M_1} \, de} = \frac{3}{2} .
\]  

(2.38)

When \( E_1 \) is replaced by infinity in (2.38), this equation is automatically satisfied for a Maxwell distribution, corresponding to the fact that for that case no electric field can be present (the electron temperature then must be equal to the gas temperature). In our region the E.F. is still far from maxwellian. Eq.(2.38) has been evaluated numerically, computing \( M(e) \) numerically according to eq.(2.11), so that the gas term was included in the calculations. Substitution of eq.(2.13) into (2.38) yields

\[
\begin{align*}
E &= C \left( \frac{N_e}{N_g} \right) \sqrt{\frac{18 m}{M} a N_e U_e} \frac{5/4}{},
\end{align*}
\]  

(2.39)

with

\[
C \left( \frac{N_e}{N_g} \right) = \begin{cases} 
1 & \text{for } \frac{N_e}{N_g} < 0 , \\
1.4 & \text{for } \frac{N_e}{N_g} = \infty .
\end{cases}
\]

The numerical calculations showed that within our region, \( C \) can be approximated within 5% by:

\[
C = \begin{cases} 
1 & \text{for } \frac{N_e}{N_g} < 0.33 \times 10^{-7} , \\
1 + \frac{2}{15} \left( 10 \log \frac{N_e}{N_g} + 7.48 \right) & 0.33 \times 10^{-7} \leq \frac{N_e}{N_g} \leq 0.33 \times 10^{-4} ,
1.4 & \text{for } \frac{N_e}{N_g} > 0.33 \times 10^{-4} .
\end{cases}
\]

From (2.39) we have \( \frac{b}{a} = \frac{1}{3c^2} \) which has been used in fig. 2.3.

2.5 The excitation frequency

For the model calculations given in chapter 3 we need the excitation frequency. This is given by:
\[ Z = \int_0^\infty N_1 \nu v \phi_0^2(v) dv = 2\pi \nu^3 \int \phi_0^2(v) dv, \quad (2.40a) \]

\[ (\text{with } \nu = \sqrt{\frac{2eU_1}{m}}). \]

From multiplication of eq. (2.3) by \( v^2 \) and integration over \( \nu \) we obtain (neglecting the last term in eq. (2.3), see 2.3.2):

\[ Z = 2\pi \nu^3 \int v \phi_0^2(v) dv = -2\pi \nu^3 \left| H(\nu_1) \left( \frac{df_0}{d\nu} \right) \bigg|_{\nu = \nu_1} + G(\nu_1) \right| F_0 F_1, \quad (2.40b) \]

Substitution of \( f_0(\nu) \) according to eq. (2.28) into (2.40b) yields:

\[ Z = N_1 J_1(\nu_1) G(\nu_1) F_1 F_2, \quad (2.41) \]

with \( N_1 = 2\pi \nu^3 N = \frac{1}{\nu_0^2} \int_0^{\nu_0} M(\nu) \nu^2 d\nu \),

\[ F_1 = \frac{\phi_2 + \frac{H(\nu_1)}{G(\nu_1)} \phi_2'}{\frac{1}{G(\nu_1)} \phi_2 + \frac{H(\nu_1)}{G(\nu_1)} \phi_2'} \]

\[ \phi_{ij} \text{ and } \phi'_{ij} \text{ according to eq. (2.34)).} \]

\[ F_2 = \text{correction factor for the gas temperature (see below eq. (2.43)).} \]

Numerical calculation of the factor \( N_1 F_1 \) shows that \( N_1 F_1 \) is a slowly varying monotonic function of \( U_e \) and \( N_e/N_g \), the lower bound being 0.2 and the upper bound being 0.6. This function has been approximated for our region \( 1 < U_e < 4, 0.33 \times 10^{-2} < N_e/N_g < 0.33 \times 10^{-5} \) within 10% by:

\[ N_1 F_1 = 0.105 U_e - 0.1 + 0.044 \left( \frac{N_e}{N_g} \right)^{0.1} + 8.48. \quad (2.42) \]

Furthermore, a correction factor \( F_2 \) for the replacement of \( A_1(\nu) \) and \( A_2(\nu) \) by 1, and for the omission of the gas temperature term in \( H(\nu) \) has been calculated numerically. In the region \( 0 < T_g < 2000 \text{ K}, 1 < U_e < 4 \text{ V}, 0.33 \times 10^{-6} < N_e/N_g < 0.33 \times 10^{-5} \), numerical calculations show that this factor \( F_2 \) can become very large \( (10^6 \text{ to } 10^7) \) for small values of \( N_e/N_g \) and \( U_e \) and large values of \( T_g \). The excitation frequency is, however, used in the numerical model for the positive column, discussed in chapter 3. In the positive column at the axis, small values of \( N_e/N_g \) are coupled to small values of \( T_g \) and large values of \( U_e \), while large values of \( N_e/N_g \) are coupled to large values of \( T_g \) and small \( U_e \). There-
fore the value of $F_2$ at the axis of the column remains limited to values in the neighbourhood of 1. A good approximation to the numerical values in this region appeared to be:

$$F_2 = \exp\left( \frac{1.23 \times 10^{-2} T}{\nu_e} \left( 10 \lg \frac{N_e}{N} + 0.557 \right)^{-1} \right).$$

(2.43)

In the positive column, $F_2$ can become large for increasing radial distance (near the wall). However, for these values of the radial distance, the influence of $F_2$ on the model results is negligible because the excitation frequency is almost zero near the tube wall. Therefore we use (2.43) in the entire region in our calculations of chapter 3.

In fig.2.6 the excitation frequency (which will be used for the model described in chapter 3) is given as a function of $N_e$ for several values of $\nu_e$. For low electron densities, the Coulomb terms cancel in the Boltzmann-equation, and the E.F. (hence $Z$) is not dependent on $N_e$. For high electron densities, the E.F. becomes maxwellian, and $Z$ becomes again electron density independent. In the intermediate region $Z$ is a strong function of $N_e$.

2.6 The distribution function for argon

To compare experimental data on the argon discharge (Woj67) with model calculations we need the electron energy distribution in the argon discharge.

For argon the momentum transfer cross-section is approximated by (Woj65b):

$$Q_D = a_D U, \quad U \leq U_1 = 11.5 \, \text{V},$$

$$= a_D U_1, \quad U > U_1,$$

with $a_D = 1.4 \times 10^{-20} \, \text{m}^2 \, \text{V}^{-1}$.

The inelastic cross-section is approximated by (Woj65b):

$$Q_I = A_I (U-U_1), \quad U \geq U_1,$$

with $A_I = 0.5 \times 10^{-21} \, \text{m}^2 \, \text{V}^{-1}$. 

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Fig. 2.6 \( \varepsilon \) as a function of \( N_e \).

In this case \( G(\varepsilon) \) and \( H(\varepsilon) \) become:

\[
H(\varepsilon) = \nu_{ee}^* \nu_E^* + \frac{\nu_g^*}{T_e} \nu_{ee}^* \varepsilon^3,
\]

\[
G(\varepsilon) = \nu_{ee}^* \nu_{ee}^* \varepsilon^3,
\]

with \( \nu_E^* \) and \( \nu_C^* \) according to eq. (2.6),

with \( n = 1 \) and \( a, b, \) and \( M \) for argon.

In this case the gas temperature term can easily be maintained in \( H(\varepsilon) \)
for the analytical calculations. Then \( M(\varepsilon) \) becomes:

\[
M(\varepsilon) = \frac{C_1}{C_2} \left( \frac{\nu b}{C_3} \right) + \frac{1}{3} \left( 1 - \frac{b}{C_2} \right) \left( \ln \left( \frac{1 + u}{V1 - u + u^2} \right) + \sqrt{3} \left( \arctg \left( \frac{2u-1}{\sqrt{3}} \right) + \arctg \frac{1}{\sqrt{3}} \right) \right),
\]

where \( C_1 = (1+a)^{-1} \), \( C_2 = (\nu b(1+a))^{-1} \), \( \nu = T_g/T_e \),

\[
a = \nu_E^*/\nu_{ee}, \quad b = \nu_g^*/\nu_{ee}, \quad u = C_2 \varepsilon \quad \text{and} \quad \nu = T_g/T_e.
\]
When $T_g \to 0$ this expression reduces to the one given by Golubovskii et al. (Gol70b)

$$M(\varepsilon) = \frac{1}{1 + a} (\varepsilon - \frac{1}{4} \varepsilon^4).$$

In the inelastic region $Q_0$ is constant. Because $Q_0$ is one linear function in the entire inelastic region of interest, we get for argon one single solution $\phi_1$ in the inelastic region. Similar to the treatment given in 2.3.2 for neon, we obtain for argon:

$$\phi_0(\varepsilon) = A_1 \phi_1(\varepsilon) \quad \text{for } \varepsilon \geq \varepsilon_1,$$

with

$$A = -\frac{G(\varepsilon_1)}{H(\varepsilon_1)} \frac{N J_1(\varepsilon_1)}{\phi_1'(\varepsilon_1)},$$

$$\phi_1(\varepsilon) = \exp\left(-\frac{1}{2} P(\varepsilon_1)(\varepsilon - \varepsilon_1)\right) A_1 \left(a_1^{2/3} (\varepsilon - \varepsilon_1 + b_1)\right),$$

and

$$a_1^2 = \frac{A G_{\text{new}} J_1}{H(\varepsilon_1)},$$

$$b_1 = \frac{(1 + b_1)^3}{4 H(\varepsilon_1)^2} \left(\frac{\varepsilon - \varepsilon_1}{a_1^2}\right) \left(\frac{\varepsilon - \varepsilon_1}{b_1}\right).$$

$$P(\varepsilon_1) = \left[H'(\varepsilon_1) + G(\varepsilon_1)\right] H(\varepsilon_1)^{-1} \text{ (see eq. (2.2)).}$$

Then

$$Z = 2 \pi w \frac{3}{2} N G(\varepsilon_1) J_1(\varepsilon_1) F_1 F_2 \text{ (section 2.5)},$$

with

$$F_1 = 1 + \frac{G(\varepsilon_1)}{H(\varepsilon_1)} \frac{\phi_1(\varepsilon_1)}{\phi_1'(\varepsilon_1)},$$

hence

$$F_1 = \frac{2 a_1^{2/3} (1 + a) A_1^{2/3} b_1 + (1 + b_1)^3 A_1 (a_1^{2/3} b_1)}{2 a_1^{2/3} (1 + a) A_1 (a_1^{2/3} b_1) + (1 + b_1)^3 A_1 (a_1^{2/3} b_1)}.$$

(see also Gol70b).

The factor $N_1 F_1$ is numerically approximated by:

$$N_1 F_1 = 0.40 + 0.067 (10^8 \frac{N_e}{N_g} + 8.3)^2.$$

The correction $F_2$ now contains only the $A(\varepsilon)$ correction as the gas temperature term in $M(\varepsilon)$ has already been taken into account (see above). The correction for $A(\varepsilon)$ appeared to be 20% in our region of interest. Thus $F_2 = 1.2$ has been taken.

A similar calculation of $E$ as in section 4 results for argon in:

$$E = \sqrt{\frac{36.6 m}{M}} a_b N g_e^2 C \quad \text{and} \quad \frac{a_b}{b} = 9.4 \text{ C}^2,$$

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where $C$ is approximated (see section 2.4) by:

$$C = 1.0 + (0.049-0.006 U_e)(10 \log \frac{N_e}{N_g})^2,$$

(for $U_e < 5, 0.5 \times 10^{-9} < \frac{N_e}{N_g} < 0.5 \times 10^{-4}$).

Appendix 2.1 Truncation of the expansion

When radial derivatives are neglected (see appendix 2.2), the equation for $\lambda = 0$ becomes (Shk66):

$$\frac{1}{3v^2} \frac{d}{dv} (v^2 \Delta f_1) - C_{el} (f_{el}) + C_{in} (f_{in}) + C_{ee} (f_{ee}),$$

(2.1-1)

where $e = -\frac{eZ^2}{m} e_z$, $e$ elementary charge, $m$ electron mass, $E$, axial electric field strength, $e_z$ unit vector in axial direction,

$C_{el}$ elastic collision operator, $C_{in}$ inelastic collision operator,

$C_{ee}$ collision operator for the Coulomb interaction between the electrons ($C_e$ operates on $f_e$).

The equation for $\lambda = 1$ becomes, when the term containing $f_2$ is neglected (see below):

$$\frac{d}{dv} C_{el} (f_{el}) + C_{in} (f_{in}) + C_{ee} (f_{ee}),$$

(2.1-2)

($C_{el}$ operates on $f_{el}$).

For neon and argon the elastic cross-section is about a hundred times larger than the inelastic cross-section in the energy region of interest, hence $|C_{in}| \ll |C_{el}|$. The ratio $|C_{ee}|/|C_{el}|$ can be estimated by substituting a Maxwellian distribution function in the expression for $C_{ee}$ given by Shkarofsky et al. (Shk66). Replacement of derivatives $\frac{d}{dv}$ by $\frac{1}{w}$, where $w$ is the thermal velocity, gives that the ratio $|C_{ee}|/|C_{el}|$ is of the order of magnitude $10^3 N_e/N_g$ (N_e electron density, $N_g$ gas density). The ionization degree is less than $10^{-5}$ in our region, hence $|C_{ee}| \ll |C_{el}|$.

The term $C_{el}$ is (Shk66):

$$C_{el} = -1/\nu_{el},$$

(2.1-3)

where $\nu_{el} = N_g Q_{el}$ ($Q_{el}$ = momentum transfer cross-section).
Then eq. (2.1-2) becomes:

$$f_1 = -\frac{\partial f_o}{\partial v},$$

(2.1-4)

For an order of magnitude estimation, we substitute for \(f_o\) the maxwellian distribution function \(f_o = C e^{-v^2/\sigma^2}\) (\(C\) constant, \(\sigma = \sqrt{2mV_e}/m\), \(V_e\) electron temperature in Volts, (see section 2.4)). This yields \(f_1 = \frac{\partial f_o}{\partial v}\).

With the approximate expression for the drift velocity

\[v_d = \frac{e\phi}{mNQ_D} = \frac{e\phi}{mv} \text{ (McD64)}\]

this gives for the order of magnitude of \(|v_1/v_o|\):

\[\frac{|f_1|}{|f_o|} = 2 \frac{v_d}{w}.
\]

Approximating the expression for the axial electric fieldstrength of section 2.4 by:

\[E = \sqrt{\frac{18m}{M}} N Q_D U e,\]

we obtain:

\[\frac{|f_1|}{|f_o|} = 0 \left(\sqrt{\frac{18m}{M}}\right) \quad (M \text{ atomic mass}).
\]

From the general relation for \(f\) given by Johnston (Joh66), a same kind of approximation gives for the ratio of an element of the tensor \(f\) to an element of the tensor \(f_{-1}\):

\[\frac{|f_1|}{|f_{-1}|} = 0 \left(\sqrt{\frac{18m}{M}}\right) \quad \text{for all } i \geq 1.
\]

Because \(\sqrt{\frac{18m}{M}} \approx 10^{-2}\), the replacement of \(f\) by \(f_o\) gives sufficient accuracy in the calculation of the coefficients needed for the discharge model given in chapter 3. However, as will be shown now, the value of \(f_1\) is necessary to calculate \(f_o\), whereas we do not need \(f_2, f_3, \ldots\) to calculate \(f_1\).

The value of \(f_o\) is obtained from eq. (2.1-1). In this equation the elastic collision term is equal to (Shk66):

\[\nu_{el} = \frac{1}{v^2} \frac{m}{M} \left(\frac{d}{dv} \left(\frac{2kT_g}{m} \frac{df}{dv^2} \right) \right),
\]

(2.1-5)

with \(T_g\) = gas temperature.

Here the elastic collision term contains the factor \(m/M\), which does not appear in the elastic collision term in the \(i = 1\) equation (2.1-3). This is so because the direction of the velocity can change
much during an elastic collision, while the magnitude changes little, so that the anisotropy in velocity space is small (Shk66). As the term containing $\mathbf{a} \cdot \mathbf{E}$ in eq. (2.1-1) is of the order of magnitude $\frac{eE}{M} \omega_0$ (see the expression for $E$ given above), this term is of the same order as $C_{\text{el}}$ and can therefore not be neglected in the equation for $f_\theta$. For all $\ell \geq 1$ the term containing $\frac{\mathbf{E}}{\ell+1}$ can be neglected in the $f_\theta$ equation, as $C_{\text{el}}$ does not contain the factor $\frac{m}{M}$ (see eq. (2.1-3) for $\ell = 1$, and Shk66, Joh66 for $\ell \geq 1$).

In the $\ell = 0$ equation the terms $C_{\text{on}}$ and $C_{\text{ee}}$ also are not longer negligible compared to $C_{\text{el}}$ as they were in the $\ell = 1$ equation. The term $C_{\text{on}}$ here is a factor hundred larger than $C_{\text{el}}$ in the inelastic region. The Coulomb term becomes important here when $\frac{N_{e}}{N_{g}} > \frac{3}{M} \times 10^{-3} = 10^{-7}$, a normal ionization degree for our region. Therefore this term must be taken into account in our region.

Then we obtain, after substitution of (2.1-4) into (2.1-1) the scalar equation for $f_\theta$:

$$\frac{1}{3} \frac{e^2 \nu_0^2}{m^2} \frac{1}{v^2} \frac{d}{dv} \left( \frac{v^2}{\nu_m} \frac{df}{dv} \right) = C_{\text{el}} + C_{\text{on}} + C_{\text{ee}},$$

(2.1-6)

with $C_{\text{el}}$ from (2.1-5), $C_{\text{on}}$ from Shk66 and $C_{\text{on}}$ from Lya72.

Appendix 2.11. The radial derivative term and the ambipolar field term

In this appendix the order of magnitude of the radial derivative term and the ambipolar field term is compared with the order of magnitude of other terms.

When the radial derivative term $v \cdot \nabla f_\theta$ is taken into account, the scalar equation for $f_\theta$ (eq. (2.1-6)) becomes:

$$\frac{v^2}{3v_m} \frac{\partial^2}{\partial r^2} f_\theta + \frac{1}{3} \frac{e^2 \nu_0^2}{m^2 v^2} \frac{d}{dv} \left( \frac{v^2}{\nu_m} \frac{df}{dv} \right) =$$

$$\frac{1}{v^2} \frac{m}{M} \frac{d}{dv} \left( v^4 v_0 \left( f_\theta + \frac{2kT}{m} \frac{df}{dv} \right) \right) + C_{\text{on}} + C_{\text{ee}}.$$

The first term is of the order of magnitude $\frac{v^2 f_\theta}{3v_m \nu_0^2}$, with $r_1$ the column radius. The first term at the right-hand side is of the order of magnitude $\frac{m}{M} v_m f_\theta$. Thus the ratio $A$ of these two terms is of the order:
\[ A = \frac{M}{3m(N^2 \sigma_D)^2} \]

For neon, \( N = 3.10^{22} \text{p}_o \) (\( \text{p}_o \) in torr) and \( \sigma_D = 3.10^{-20} \). Then \( A \) becomes:

\[ A = \frac{10^{-2}}{(\text{p}_o \sigma_D)^2} \]

For \( \text{p}_o = 100 \) torr and \( \sigma_D = 0.01 \) m: \( A = 10^{-2} \).

The actual effect of the derivative term is, however, even smaller because this term cancels according to the Schottky theory (Sch24) nearly the radial ambipolar field term contributing to the term \( a \cdot \nabla f \) in the Boltzmann-equation. The total contribution of these two terms is therefore smaller than the value of \( A \) estimated above. Therefore these two effects are neglected.

Appendix 2.III The approximation \( J_2^+ \) for \( J_2 \)

The term \( J_2(\varepsilon) \) in the E.F. for \( \varepsilon < \varepsilon_1 \) is only important for \( \varepsilon = \varepsilon_1 \), where we need \( J_2(\varepsilon) \) for the calculation of \( A, B \) and \( C \) (see fig. 2.4 and section 2.3.3). We therefore derive an expression that is an approximation for \( J_2 \) in the neighbourhood of \( \varepsilon = \varepsilon_1 \).

From eq. (2.12) of section 2.3.1 we have:

\[ J_2(\varepsilon) = J_1(\varepsilon) \int_0^{\varepsilon} \frac{t(\varepsilon') d\varepsilon'}{H(\varepsilon') J_1(\varepsilon')} \]  \hspace{1cm} (2.III-1)

Because \( f_\circ(\varepsilon) \) is a rapidly decreasing function for \( \varepsilon > \varepsilon_1 \), \( t(\varepsilon) \) assumes its final constant value very soon. (Numerical calculations show that \( t(\varepsilon) = t(\varepsilon_1) \frac{\varepsilon}{\varepsilon_0} \) for \( \varepsilon < \varepsilon_0 \) and \( t(\varepsilon) = t(\varepsilon_1) \) for \( \varepsilon > \varepsilon_0 \), with \( \varepsilon_0 = 0.06 \varepsilon_1 \). Therefore we take \( t(\varepsilon) \) out of the integral:

\[ J_2(\varepsilon) = -J_1(\varepsilon) t(\varepsilon_1) \int_0^{\varepsilon} \frac{d\varepsilon'}{H(\varepsilon') J_1(\varepsilon')} \]  \hspace{1cm} (2.III-2)

Introducing

\[ J_1(\varepsilon) = \exp\left(-\int_0^\varepsilon \frac{G(\varepsilon')}{H(\varepsilon')} d\varepsilon'\right), \]

and applying partial integration, yields

\[ J_2(\varepsilon) = -J_1(\varepsilon) t(\varepsilon_1) \left[ \frac{1}{G(\varepsilon) J_1(\varepsilon)} - \frac{1}{G(0) J_1(0)} \right] \]

\[ + \int_0^\varepsilon \frac{1}{G^2} \frac{dG}{d\varepsilon'} \frac{1}{J_1(\varepsilon')} d\varepsilon' \]  \hspace{1cm} (2.III-3)
(Here $A(t) = 1$ must be taken in $G(t)$, to avoid the difficulty at $t = 0$, which is caused by taking $t \cdot (t)$ out of the integral $(t(0) = 0)$). Because $J_1(t) \ll J_1(0) G(t)/G(t)$ for $t + t_1$, the second term in (2.III-3) is negligible. The third term is negligible if $t \cdot \frac{dG}{dt} \ll \frac{G}{H}$ (Lya72). After substitution of $G(t)$ and $H(t)$, this inequality becomes (with $\nu_0$, $\nu$, taken constant):

$$\frac{2 \nu_0^2}{\gamma} \ll \frac{\nu_0 + \nu \epsilon \epsilon}{\nu_0 + \nu \epsilon \epsilon}.$$ 

In the Maxwell limit ($\nu_0 \gg \nu_0 \epsilon, \nu_0 \gamma$), this condition is satisfied. In the Druyvesteyn limit ($\nu_0 \ll \nu_0 \epsilon, \nu_0 \gamma$) it yields:

$$\epsilon \gg \sqrt{2} \frac{\nu_0 \epsilon}{\nu_0 \gamma} \approx \sqrt{6},$$

(see eqs. (2.14) and (2.39)).

Hence if $U \ll 16.6/\sqrt{6} \approx 6.9$, the third term can be neglected in respect to the first in eq. (2.III-3). We then obtain as an approximation for $J_2(\epsilon)$ in the neighbourhood of $\epsilon_1$:

$$J_2^*(\epsilon) = -t(\epsilon_1)/G(\epsilon). \quad (2.III-4)$$

In fig.2.4 of section 2.3.1 we see that this approximation is quite sufficient.
CHAPTER 3 MODEL CALCULATIONS ON THE POSITIVE COLUMN OF A MEDIUM PRESSURE INERT GAS DISCHARGE

Scope

In sections 3.1-3.4 of this chapter a model for the positive column of the medium pressure neon discharge is presented. In appendix 3.1 the model is adapted to the case of argon. The model is based on the particle balance equations, the temperature equations and the equation for the discharge current. For the calculation of the reaction coefficients the electron energy distribution function calculated in chapter 2 has been used. The equations have been solved numerically. The results of the calculations are compared with experimental data.

Introduction

For the positive column of inert gas discharges several theoretical models have been presented in the literature. For low pressure columns \( p_R < 10^{-2} \) torr m the Schottky theory (Sch42) in a slightly generalized form (Pra76) is often suitable. For high pressures \( p_R > 10 \) torr m, the arc model holds. At intermediate pressures there are several complications which require a more sophisticated model. Effects like the non-uniform gas heating and the dependence of the electron energy distribution function on the electron density have to be taken into account.

In this chapter a model for the medium pressure neon and argon column is presented. The model is compared with other models from the literature in chapter 4.

The positive column is supposed to be infinitely long, axially homogeneous and stationary. Effects due to the electrodes or the detailed tube geometry are neglected. It has been experimentally verified that the main features of the column do not change as a function of the tube length, provided this length is much greater than the tube radius. Because of the atomic densities considered here \( N_a > 3 \times 10^{23} \text{ m}^{-3} \), several assumptions hold. These are:

- the molecular ion density is much greater than the atomic ion density so that the latter can be neglected. Hence the molecular ion density equals the electron density everywhere in accordance with quasi-neutrality (Bag74);
- direct ionization is negligible with respect to stepwise ionization (Pra68);
- diffusion of excited atoms is negligible;
- the radiation-diffusion is described by the Holstein theory in the high pressure limit (Hol74,51);
- the process of dissociative recombination mainly determines the volume recombination process (Maa74);
- heat conduction by electrons is negligible (Go169);
- super-elastic collisions are negligible (Lya72).

3.1 The elementary processes

For the analysis of the stepwise ionization process the excited states are divided into several groups (Kag71). Group I contains the four Is-levels (Paschen notation, see fig. 3.1).

Ionization level IV

Group III

Group II (containing 2p)

Group I (Is)

1

2

3

4

1P1 (s2) T

3P1 (s3) S

3P1 (s4) R

3P2 (s5) M

Ground state 1S0

Fig. 3.1 Simplified level scheme of neon: division into several important groups (see text)(s2-s5 Paschen notation).

Group II contains the levels which can be excluded from the ionization process (such as the ten 2p-levels) because their life-time is very short (the 2p-levels radiate back to the Is-levels in about 10⁻⁷ s). Following Kagan we introduce a group III which consists of the higher
excited levels in which the quenching by atomic collisions is so strong, that there is a high rate of associative ionization via these levels (Kag71).

Group IV consists of the continuum above the ionization level.

In fig. 3.2 a scheme of the several processes occurring in the medium pressure discharge is given.

Fig. 3.2 Scheme of stepwise ionization and recombination processes (\(e^-\) = electron, Ne = ground state atom, Ne* = excited atom).

The production of ions is described by a stepwise ionization process in the following way. The levels in group I are produced by inelastic electron collisions with ground state atoms. An atom in group I can be excited to higher levels belonging to groups II, III or IV. When such a higher level belongs to group II, it radiates back to group I very rapidly. Calculations show that atoms in group II can be neglected in the ionization process. These atoms are only important for a redistribution over the s-levels in group I. Atoms excited into levels belonging to group III are rapidly ionized because of the high rate of associative ionization. Therefore in our model excitation to a level in group III is taken as an ionization process (Kag71).
For the determination of the rate of the stepwise ionization we need to know the densities of the atoms in group I. These densities are determined by the excitation of ground state atoms, the transfer of energy between these levels by atomic and electronic collisions, the destruction of the metastable atoms \( ^3P_0 \) and \( ^3P_2 \) by three-body collisions with neutral atoms, and the decrease of the resonant atom densities \( \| ^3P_1 \) and \( ^1P_1 \) by the radiation-diffusion process (Ho55,56).

The atomic ions produced by the stepwise ionization process are transformed very rapidly into molecular ions by three-body collisions with gas atoms. This process is so fast that the atomic ion density can be neglected with respect to the molecular ion density (Bag74).

The loss of molecular ions occurs by ambipolar diffusion and dissociative recombination. The latter process yields excited atoms in group II (Maa74).

All processes are described by particle balance equations. For the calculations of the reaction coefficients we use the electron energy distribution function calculated in chapter 2. The reaction coefficients depend on the electron temperature and the gas temperature. Therefore the equations for the electron temperature and the gas temperature have to be taken into account. Furthermore the electron density profile must be in accordance with the equation for the discharge current. A complete set of equations results. This set has been solved numerically.

### 3.2 The particle balance equations

#### 3.2.1 The excited atoms

The balance equations for the excited atoms of group I are:

\[
2 \| e + A_i N_i + \sum k_{ji} N_j = N_i \left[ (c_{12} + c_{13} + c_{14}) N_0 + V_i + \sum k_{ji} \right],
\]

where index \( i = 1, 2, 3, 4 \) refers to the \( ^3P_0(s2), ^3P_1(s4), ^3P_2(s3) \) and \( ^1P_1(s2) \) state respectively. In this equation we have represented the levels in group II by one single level \( H \) with density \( N_H \). Furthermore \( \nu_i \) is the frequency of excitation to level \( i \) by electron collisions with ground state atoms, \( A_i \) is the frequency with which level \( H \) decays to level \( i \), \( k_{ji} \) is the coefficient for coupling due to atomic and electronic collisions between levels \( i \) and \( j \) within group I, \( c_{12}, c_{13}, \) and \( c_{14} \) are excitation frequencies from level \( i \) to group II, III and IV respectively.
\( \nu_i \) is the destruction frequency for level \( i \) by other processes; for the metastable atoms (\( i=1,3 \)) \( \nu_i = \gamma_i N e^2 \) (three body destruction of metastable atoms), for the resonant atoms (\( i=2,4 \)) \( \nu_i = \frac{0.205}{\tau_i} \left( \frac{\lambda_i^2}{R} \right)^{1/2} \) (Holstein radiation-diffusion frequency for \( p_o > 10 \) torr, where \( \tau_i = \) natural life-time of level \( i \) and \( \lambda_i = \) wavelength of the emitted radiation, \( R = \) tube radius (Ph59)).

The density of atoms in level \( H \) is obtained from the balance equation:

\[
\sum_{i=1}^{4} c_{i2} N_i e^2 + aN_e^2 = \sum_{i=1}^{4} A_i N_i 
\]

where \( a \) is the volume-recombination coefficient given by (Mas74):

\[
a = \frac{2.5 \times 10^{-14}}{U_e^{1/2}} \left( 1 - \exp\left( -\frac{900}{T_g} \right) \right)
\]

(\( T_g = \) gas temperature, \( U_e = \) electron temperature in Volts). Solving \( N_H \) from (3.2) and substitution into (3.1) yields a \( 4 \times 4 \) matrix equation for the excited atom densities \( N_i \).

From chapter 2 we have \( i \sum_i Z_i = Z \), where \( Z \) is the total excitation frequency (eq.2.41). By lack of data we take all \( Z_i = \frac{1}{4} Z \). The values of \( k_{ij}, \nu_i, c_{12} = c_{14} \) and \( A_i \) are given in Appendix 3.IX. The equations (3.1) and (3.2) are solved numerically yielding the values of \( N_i \).

Then the ionization frequency, necessary for the solution of the electron balance, can be calculated from:

\[
Z_{\text{ion}} = \sum_{i=1}^{4} (c_{i3} + c_{14}) N_i e^2 N_i
\]

From eq. (3.1) it can be seen that \( Z_{\text{ion}} \) is a linear function of the total excitation frequency \( Z \), which is calculated from eq.(2.41).

(See also appendix 3.I for argon.)

3.2.2 The electron balance and the current equation

The electrons are produced by stepwise ionization and lost by dissociative recombination (coefficient \( \alpha \)) and ambipolar diffusion to the tube wall (coefficient \( D_a \)). The particle balance equation for the electron density \( N_e \) then becomes:

\[
\frac{1}{r} \frac{d}{dr} \left( r D_a \frac{dN_e}{dr} \right) + Z_{\text{ion}} N_e - aN_e^2 = 0
\]

(3.5)

where \( r = \) the radial distance.
Here \( D = C_D b_{mi} U_e \), where \( b_{mi} \) is the mobility of the molecular ions, and \( C_D \) a factor close to unity depending on the shape of the electron energy distribution function. \( C_D \) and \( b_{mi} \) are given in appendix 3.III.

The solution of eq. (3.5) must satisfy the boundary conditions:

\[
N_e(R) = 0, \quad \left( \frac{dN_e}{dr} \right)_{r=R} = 0, \quad \text{and} \quad N_e(0) = N_{eo} \tag{3.6}
\]

where \( N_{eo} \) is obtained from the equation for a given discharge current (see appendix 3.III):

\[
N_{eo} = \frac{C_N}{3meU_{eo}} \left[ \frac{e^2}{M U_{eo}} \right]^{1/2} S_1 R^\frac{3}{2} \tag{3.7}
\]

Here \( i \) = discharge current, \( e \) = elementary charge, \( M \) = atomic mass, \( U_{eo} \) = electron temperature at the axis in Volts, \( R \) = tube radius.

\[
S_1 = \int_0^1 t(x)^{2/5} n(x) x dx \tag{3.8}
\]

\( x = r/R, \quad t(x) = T_e(r)/T_e(0), \quad n(x) = N_e(r)/N_e(0), \quad C_N = \) correction factor close to unity, depending on the shape of the electron energy distribution function. \( C_N \) is given in Appendix 3.III.

3.3 The temperature equations

3.3.1 The electron temperature

The equation for the electric field strength derived in chapter 2 (eq. 2.39) is:

\[
E = \sqrt{\frac{\varepsilon_0}{M}} \frac{\rho_a N U^{5/4}}{C(N_e/N_g)} \tag{3.9}
\]

The axial electric field strength \( E \) is radially constant. Because \( C \) is only weakly dependent on \( N_e/N_g \), we replace in (3.9) \( C(N_e/N_g) \) by \( C(N_{eo}/N_g(0)) \). Then we obtain from (3.9):

\[
\frac{U_e(r)}{U_e(0)} = \left( \frac{N_e(r)}{N_g(0)} \right)^{-4/5} = \left( \frac{T_e(r)}{T_g(0)} \right)^{4/5} \tag{3.10}
\]

where use has been made of the ideal gas law.

The value of the electron temperature at the axis \( U_e(0) \) is fixed
by the demand \( N_e (R) = 0 \) (see section 3.4). The gas temperature profile \( T_g (r) \) is obtained from the Heller-Elenbaas equation.

3.3.2 The gas temperature

In the Heller-Elenbaas equation, the heat-flux to the wall equals the heat dissipation of the electric current:

\[
\frac{1}{r} \frac{d}{dr} \left( r \lambda(T_g) \frac{dT_g}{dr} \right) = -JE , \tag{3.11}
\]

where \( J = eN_e (r) b_e \) is the current density \((b_e = \text{electron mobility})

Using experimental data given by Sah (Sax68) the thermal conductivity can be written as:

\[
\lambda(T_g) = \lambda_0 T_g^{2/3} , \tag{3.12}
\]

with \( \lambda_0 = 10^{-3} \text{W/mK}^{5/3} \).

The boundary conditions for eq. (3.11) are:

\[
\frac{dT_g}{dr} \bigg|_{r=R} = 0 \quad \text{and} \quad T_g (R) = T_{\text{wall}} .
\]

The wall temperature \( T_{\text{wall}} \) is obtained from the condition that the dissipated power must be equal to the heatflow by convection and radiation from the wall to the environment. Hence:

\[
JE = 2\pi R (\alpha \left( T_{\text{wall}} - T_{\text{env}} \right) + \sigma (T_{\text{wall}} - T_{\text{env}}^4)) , \tag{3.13}
\]

where \( T_{\text{env}} = \text{temperature of the environment} \),

\( \alpha \) = heat transfer coefficient, equal to:

\[
\alpha = 7L^{-1/4} \left( \frac{T_{\text{wall}} - T_{\text{env}}}{T_{\text{wall}} + T_{\text{env}}} \right)^{1/4} \tag{Jac57} ,
\]

\( L = \text{tube length} \),

\( \varepsilon = \text{transmittivity of glass} \),

\( \sigma = \text{Stefan-Boltzmann const} \).

3.3.3 The pressure in the discharge

The gas density \( N_g \) is obtained from the gas temperature \( T_g \) via the ideal gas law. When the discharge is connected with a large dead volume, with temperature equal to the room temperature, the operating
pressure \( p_1 \) is equal to the filling pressure \( p_0 \) (both not radial dependent): \( \frac{p_1}{p_0} = N_g(r)kT_g(r) \). (3.14)

On the other hand, when the dead volume is small, we get

\[
p_2 = N_g(r)kT_g(r)
\]

where \( p_2 \) is obtained from the equation for the conservation of the total number of gas atoms:

\[
\int_0^R 2\pi r N_g(r)dr = N_gbr \pi R^2
\]

(3.16)

where \( N_g = \frac{p_0}{kT_{\text{env}}} \) is the gas density in the discharge-off-situation.

This yields

\[
p_2 = \frac{p_0}{T_{\text{env}} s_2}, \quad \text{with} \quad s_2 = \frac{2}{R^2} \int_0^R rdr \int_0^R N_g(r)dr
\]

(3.17)

For an arbitrary value of the fraction of the dead volume, we can obtain from the ideal gas law a general formula for \( p \) containing the volumes \( V_1 \) and \( V_2 \) of the discharge tube and the dead volume, and the average temperatures \( T_1 \) and \( T_2 \) in the cases of small respectively large dead volume as parameters. In our region where \( T_1 < 2T_2 \) this expression can be approximated within 10% by:

\[
p = \frac{V_2}{V_1+V_2}p_1 + \frac{V_1}{V_1+V_2}p_2
\]

(3.18)

3.4 The numerical treatment

The equations used for the numerical treatment have been given in sections 3.2 and 3.3. They are the balance equations for the excited atoms (3.1) and (3.2), the one for the electrons (3.5), the current equation (3.7) and the Heller-Elenbaas equation (3.11). The equations are coupled, and eq. (3.5) is nonlinear. Therefore they have been solved numerically with an iterative procedure. The flow diagram of the program is given in fig. 3.3. The two second order differential equations (3.1) and (3.11) have been solved explicitly with forward discretization with \( k \) discrete intervals. After the calculation it was checked whether a sufficient large number of intervals \( k \) was used. If not, \( k \) was enlarged and the calculation was repeated.

The iteration procedure is started with chosen profiles obtained from former calculations. The value \( V^{(p+1)} \) of \( E, N_g(r)/N_0, T_g(r)/T_g(0) \) and \( N_g \) calculated in the \( p+1 \)th iteration loop is compared with the value \( V^{(p)} \) of the foregoing iteration. When
Fig. 3.3 Scheme of the numerical procedure.
the iteration is stopped, otherwise it is repeated (see fig. 3.3).

In the loop for the calculation of eq. (3.5), the value of $U_{eo}(=U_e(0))$ is obtained iteratively from the demand $N_e(R) = 0$ in the following way (see fig. 3.4). When the electron temperature at the axis $U_{eo}$ is too small, the profile $N_e(r)$ can become zero for $r > R$ or may even have no zero point at all, but increase for increasing $r$ (see fig. 3.4).

When $U_{eo}$ is too high the zero point of the profile is reached already for $r < R$. The value of $U_{eo}$ is adjusted iteratively with a regula falsi procedure such that a value of $U_{eo}$ is reached which gives a profile according to profile 2 in fig. 3.4. The calculation of $U_{eo}$ is stopped when $|N_e(R)/N_e(0)| < 10^{-2}$.

However, when the discharge current and the pressure are so high that the column is near the constriction region, the electron density profile becomes so narrow, that the calculation of this profile becomes

![Diagram](image.png)

**Fig. 3.1** Normalized electron density profiles obtained in the determination of $U_{eo}$:

1 $U_{eo}$ too small; 2 $U_{eo}$ right; 3 $U_{eo}$ too high.

very sensitive to extremely small variations of $U_{eo}$. Then the discharge is not longer wall dominated, but is in the volume regime (Rak/0). Then the method to determine $U_{eo}$ given above gives great difficulties. We then apply the following method. The profiles 1 and 3 in fig. 3.4 start to differ from the ideal profile (profile 2) for values of $r$ for which
Fig. 3.5 Approximation for $N_e(r)/N_e(0)$ for $r > r_1$ when $U_e^{(3)} - U_e^{(1)} < 10^{-4}$.

$r > r_1$. From numerical calculations it appeared that when the difference in the values of $U_e$ belonging to profiles 3 and 1, $U_e^{(3)} - U_e^{(1)}$, was smaller than $10^{-4}$, the profiles 1, 2 and 3 differed not more than 1% from each other for $r < r_1$, while for all profiles the normalized densities had dropped under 0.1 at $r = r_1$. Therefore the normalized density profile calculated with a value of $U_e$ for which $U_e^{(1)} < U_e < U_e^{(3)}$ and $U_e^{(3)} - U_e^{(1)} < 10^{-4}$ is nearly equal to profile 2 (the ideal profile) for $r < r_1$. According to profile 2, the normalized density is small for $r_1 < r < R$. In this outer region the $N_e(r)/N_e(0)$ profile has been approximated by:

$$N_e(r) = N_{eo} C_1 \exp(-0.2 \frac{r}{R})$$

where the value of $C_1$ is chosen such as to guarantee continuity with the inner region at $r = r_1$.

When in the outer region the ionization frequency $Z_{ion}$ is small (because $N_e(r)$ and $U_e(r)$ are small in that region), and $\alpha N_e^2$ is small, eq. (3.5) reduces to

$$\frac{1}{r} \frac{d}{dr} (D_a N_e \frac{dN_e}{dr}) = 0$$

with the analytical solution

$$N_e(r) \sim \ln r \left( D_a = \text{constant} \right)$$.  (3.21)

We have used eq. (3.20) in stead of (3.21) because more precise numerical calculations proved that (3.20) is a better approximation to the real situation in that region. Hence in the outer region the production term $Z_{ion}$ and the volume recombination term $-\alpha N_e^2$ still have to be taken in-
Fig. 3.6 $E$ as a function of $i$ for neon. $R = 32$ mm.
- 200 torr measured, $\times$ 100 torr measured, $\times$ 50 torr measured
--- calculations, --- constricted discharge.

to account in the solution of eq. (3.5).

In all cases the thus calculated profile fulfilled the criterion $|N_e(R)/N_e(0)| < 10^{-2}$ (see fig. 3.5). In this way eq. (3.5) could be solved without numerical difficulties.

3.5 Numerical results and comparison with experimental data

For a few cases calculated values of the axial electric field-strength are given together with experimental data in fig. 3.6 for a neon discharge ($R=32$ mm, $p_0=50, 100, 200$ torr, $V_2/V_1 = 0.15$). For the diffuse discharge the agreement between calculations and experiment is very good. To obtain this agreement we had to take $a_{el} = 1.5 \times 10^{-20}$ instead of $a_{el} = 1.8 \times 10^{-20}$ in eq. (2.4). This corresponds to the lowest measured curve for the elastic cross-section (see Sal70). Because the axial electric field-strength $E$ is proportional to $a_{el}$, the value $a_{el} = 1.8 \times 10^{-20}$ (see eq. 2.4) gives calculated values of $E$ which are 20% higher than the measured ones. With the model given here the transition into the constricted state can not be calculated. A discussion on the constringtion phenomenon is given in chapter 4. In fig. 3.7 a few calculated values for $E$ are given for argon, together with experimental data and calculations by Wojaczek (Woj77). Here also, our calculations are in agreement with experiment for the diffuse discharge. (For argon no alteration of $a_{el}$ was necessary.) The model of Wojaczek is discussed
In section 4.2.2. In Fig. 3.8 some calculated and observed radial electron density profiles are given for neon \((R = 32 \text{ mm}, P_0 = 100 \text{ torr})\) for various values of the discharge current. The experimental values have been obtained from measurements on the electron-atom bremsstrahlung continuum (see chapter 5).

Considering the experimental inaccuracy (see chapter 5) here too we can conclude to a good agreement between model and experiment. Calculated electron temperature profiles are compared with experiments in chapter 5. In chapter 6 calculated density profiles for the excited atoms are compared with observed ones. In that chapter the model will also be used in the determination of the dependence of the coupling coefficients of the 2p-levels in neon on the gas temperature.

![Fig. 3.8 E in dependence on i for argon, \(p_0 = 76 \text{ torr}, R = 10 \text{ mm}\).](image)

1. our model calculation (appendix 3.II),
2. measurements of Wojaczek (Woj87) (x),
3. calculations of Wojaczek (Woj87).

**Conclusion**

The model presented here gives a fair description of the diffuse discharge. The transition to the constricted state, however, is not included in this model. Possible mechanisms responsible for an onset of the constriction are discussed in the next chapter. There we also give a comparison with other models presented in the literature.
Appendix 3.1 The positive column of the argon discharge

In the same way as it has been done for neon, a model for the argon discharge can be set up. We give here some data which are needed in that model. Our treatment of the balance equations of the excited atoms in the argon discharge differs from the treatment for the neon discharge. In group I we have (by lack of data) only taken into account the metastable $^3P_2(5S)$ level. As this level has the largest population within group I this assumption can not very seriously affect the results. For neon this is also the case. However, for neon we have calculated the excited atom balances in full detail, to obtain the densities of the excited 1s-atoms, as these densities have been measured with the fluorescence method described in chapter 6.

Then the total ionization frequency for argon is (cf. eq. (3.1) for neon):

$$Z_{\text{ion}} = \frac{c_{\text{mi}}N_e}{c_{\text{mi}}N_e + \gamma N_2} Z,$$  

(3.1-1)

where $Z$ = the total ionization frequency (see section 2.6),

$\gamma$ = three-body collision coefficient for the destruction of atoms in the $5S$-state,

$c_{\text{mi}}$ = coefficient for excitation from the $5S$-level in group I to levels in the groups III and IV.

Group II is ignored here. For neon, group II has been taken into account because group II may be of influence for redistribution (after
recombination) over the ls-levels. As we cannot measure the ls-levels in the case of argon, because of the restricted wavelength region of the dye-laser, a detailed knowledge of the distribution over these levels is not of importance here.

The coefficient $c_{mi}$ has been estimated using the classical Thomson cross-section (Mit73). Numerical evaluation then shows that $c_{mi}$ can be given by:

$$c_{mi} = \begin{cases} 5.3 \times 10^{-14} & (U_e \leq 2.5), \\ (1.8U_e - 3.8) \times 7.5 \times 10^{-14} & (U_e > 2.5). \end{cases} (3.1.2)$$

From Massey and Burhop (Mas74) we obtain:

$$\gamma = 3.6 \times 10^{-45} \text{m}^6/\text{s}. \quad (3.1.3)$$

For the calculation of $D_a$ we refer to appendix 3.I. The volume recombination coefficient is (Mas74):

$$\alpha = \frac{8.6 \times 10^{-14}}{U_e^{2/3}} (1 - \exp(-650/T_g)). \quad (3.1.4)$$

From the expression for $E$ given in section 2.6, we have:

$$\frac{U_e(r)}{U_e(0)} = \left(\frac{T_g(r)}{T_g(0)}\right)^{1/2}. \quad (3.1.5)$$

Because of the different energy dependences of the elastic cross-section of argon we have here the exponent 1/2 in stead of 4/5, as is the case for neon. Then $N_{eo}$ can be written as (see appendix 3.III):

$$N_{eo} = \frac{C_N}{2\pi e V^{12.5e} \mu U_e^{1/2} S_1 R^2}. \quad (3.1.6)$$

where $S_1 = \int_0^1 t(x)^{1/4} n(x)x dx$, and $C_N$ is given in appendix 3.III.

Finally we have for $\lambda(T)$, needed in the Heller-Ellenbaas equation (Vru76, Woj76):

$$\lambda(T) = 4.17 \times 10^{-4} T_g^{2/3}. \quad (3.1.7)$$

Appendix 3.II Data for the excited atom balances

The results obtained with our model are very insensitive to the treatment of the balance equations for the excited atoms. In fact treating the case of neon similar to the argon case gives already very good results. However, because in our laboratory we are able to measure the densities of the ls-atoms with the dye-laser, we want a more detailed
knowledge of the distribution over the 1s-levels. We give here the coefficients necessary to solve eqs. (3.1) and (3.2). From data given by Phelps (Phe59), we obtain:

\[ v_1 = 9.6 \times 10^{-50} N_{\frac{2}{3}} T_{\frac{3}{2}}^{-1/2}, \quad v_2 = 3 \times 10^2 N R_{-1/2}, \]

\[ v_3 = 1.3 \times 10^{-50} N_{\frac{2}{3}} T_{\frac{3}{2}}^{-1/2}, \quad v_4 = 4 \times 10^4 N R_{-1/2}. \]

The coefficients \( k_{ij} \) can be written as:

\[ k_{ij} = k_{ij}^a 10^{-26} N_{\frac{2}{3}} T_{\frac{3}{2}}^{5/2} + k_{ij}^e 10^{-14} N_{\frac{2}{3}} T_{\frac{3}{2}}^{1/2}, \]

with \( k_{ij}^a \) and \( k_{ij}^e \) from table 3.II-1, with \( k_{ij}^a = k_{ij}^a g_{ij} e^{-E_{ij}/Tg} \) and \( k_{ij}^e = k_{ij}^e g_{ij} e^{-E_{ij}/Tg} \) (\( E_{ij} \) = energy gap between level \( i \) and level \( j \) in Kelvin, \( g_{ij} = \frac{2J_i + 1}{2J_j + 1} \)).

<table>
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<th>i</th>
<th>j</th>
<th>( k_{ij}^a )</th>
<th>( k_{ij}^e )</th>
<th>( g_{ij} )</th>
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<td>2.0</td>
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<td>0.55</td>
<td>0.5</td>
<td>3.0</td>
<td>1540</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.II-1 Parameter values for coupling coefficients between the 1s-levels (group I).

We define \( a_i = A_i / \bar{A}_i \). From Coolen (Coo76) we obtain:

\( a_1 = 0.40, \quad a_2 = 0.27, \quad a_3 = 0.08, \quad a_4 = 0.25. \)

The cross-section for excitation from group I to group IV is

(3.1):

\[ Q_{14} = \begin{cases} 1.2 \times 10^{-20} (U-5) & 5 \leq U \leq 10 \\ (6.44-0.0444U) \times 10^{-20} & 10 \leq U \leq 100 \end{cases} \]

The coefficients \( \tilde{Q}_{14} \) are then obtained according to (where we take \( Q_{1,4} = \frac{1}{4} Q_{14} \)).
\[ c_{i4} = \int_{a}^{\infty} Q(v) f_o(v) 4\pi v^2 dv = \frac{1}{4} N_1 \int_{a}^{\infty} Q_{i4}^1(\varepsilon) \sqrt{\varepsilon} d\varepsilon, \quad (3.11-2) \]

(for all \( i = 1, 2, 3, 4 \)),

with \( \varepsilon_a = \frac{2mv_a^2}{U_e} = \frac{U_a}{U_e} \) (\( U_a = 4.9 \)), where \( f_o(\varepsilon) = N_{i4}(\varepsilon) \) has been substituted \((N_i = N_{2i})\) according to chapter 2. Numerical calculations show that \( c_{i4} \) can in our region be approximated by:

\[ c_{i4} = \begin{cases} 0.7 \times 10^{-14} & U_e \leq 2.5, \\ (1.8U_e - 3.8) \times 10^{-14} & U_e > 2.5. \end{cases} \quad (3.11-3) \]

By lack of data on the cross-sections for excitation from group I to the groups II and III, these cross-sections have been approximated by the classical Thomson formula (Mit73):

\[ Q = \frac{9.64}{U_1} 10^{-18} \frac{U-U_1}{U^2} \quad \text{for } U \geq U_1, \quad (3.11-4) \]

where \( U_1 \) is the energy gap in Volt (for the excitation from group I to IV this formula gives \( Q_{14}^{1} \approx 1.5 \cdot Q_{14} \), with \( Q_{14} \) from (3.11-1). This yields \( c_{i2} = 10c_{i4} \) and \( c_{i3} = 5c_{i4} \) with \( c_{i4} \) from (3.11-3). The inaccurate knowledge of the above mentioned cross-sections has no serious consequences for the model results. For instance when \( c_{i4} \) is multiplied by a factor 100, the value of the axial electric field strength decreases with 20%. Other discharge quantities show a similar weak dependence on the values of \( c_{i2}, c_{i3} \) and \( c_{i4} \).

Appendix 3.111 Calculation of \( D_a \) and \( N_{e0} \)

3.111-1 The ambipolar diffusion coefficient

The ambipolar diffusion coefficient \( D_a \) can be written as (because the electron temperature is much higher than the gas temperature, (McD64)):

\[ D_a = \frac{D_e b_{m1}}{b_{e}}, \quad (3.111-1) \]

where \( D_e \) = the electron diffusion coefficient, \( b_{e} \) = the electron mobility and \( b_{m1} \) = the mobility of the \( N_{2}^+ \) molecular ion, which can be written as (McD64):

\[ b_{m1} = b_{m1} \left( \frac{T}{273} \right) \left( \frac{760}{p} \right), \quad (3.111-2) \]
with \( b_{1D} = 6.10^{-4} \) for neon and \( 2.10^{-4} \) for argon. The electron mobility is (Shk66):

\[
b_e = \frac{4\pi e^2 \omega^2}{3 m n g} \int_0^\infty \frac{dE}{Q_D} d\xi,
\]

with \( f_\infty = \frac{N_1}{2\pi^{1/2}} e^{-N_1(\xi)} \) and \( Q_D = n_D U_0^2 \) (see chapter 2 for the definition of the parameters).

For argon \( n=1 \), and we get:

\[
b_e = \frac{1}{3} \sqrt{\frac{2m}{\pi}} \frac{N_1}{n_D U_0 3/2}.
\]

For neon we get after partial integration of (3.III-3):

\[
b_e = \frac{1}{3} \sqrt{\frac{2m}{\pi}} \frac{N_1 (1-n)}{a_D U_0 ^{1/2+n}} J(1-n).
\]

Here \( J(p) = \int_0^\infty E e^{-M(\xi)} d\xi \), and \( N_1 = \frac{1}{J(1/2)} \).

The electron diffusion coefficient is (Shk66):

\[
D_e = \frac{2\pi b_e}{3 N e} \int_0^\infty \frac{dE}{Q_D} f d\xi.
\]

This can be written as:

\[
D_e = \frac{a_D}{3 N e} J(1-n).
\]

Substituting \( n=1/4 \) for neon into (3.III-5) and (3.III-6) we get:

\[
\frac{D_e}{b_e U_e} = \frac{3}{4} \frac{J(3/4)}{J(-1/4)} \equiv C_D(U_e, N_e/N_g).
\]

(for argon we have to use eq. (3.III-4) and eq. (3.III-6) with \( n=1 \).)

Numerical calculations show that in our region \( C_D \) can be well approximated by:

**Neon:** \( C_D = \begin{cases} 
1.36 & N_e/N_g < 3.10^{-8}, \\
1.36 + c_1 & N_e/N_g \geq 3.10^{-8}, 
\end{cases} \)

where \( c_1 = 0.07 \log \frac{N_e}{N_g} + 9.48 \).

**Argon:** \( C_D = \begin{cases} 
2.2 & N_e/N_g < 3.10^{-10}, \\
2.2 + c_2 & N_e/N_g \geq 3.10^{-10}, 
\end{cases} \)

where \( c_2 = 0.08 \log \frac{N_e}{N_g} + 9.48 \).
The expression for the discharge current:

\[ i = \int_{0}^{R} e N \frac{d}{e(0)} \frac{N_e(x)}{2\pi r dr} \]  

(3.III-9)

Hence

\[ N_{eo} = \frac{1}{2\pi e^2 b_e(0) S_1} \]  

(3.III-10)

where

\[ S_1 = \int_{0}^{1} b_e(x) n(x) dx, \quad n(x) = \frac{N_e(x)}{N_e(0)} \]

and \( x = r/R \).

From (3.III-4) and (3.III-5) we obtain for neon

\[ \frac{b_e(x)}{b_e(0)} = t(x) \frac{2}{2} \]  

and for argon

\[ \frac{b_e(x)}{b_e(0)} = t(x) \frac{1}{4} \]  

(3.III-11)

Substituting \( E \) from chapter 2 and \( b_e \) from section 3.III-1 (eqs. (3.III-4) and (3.III-5)) into (3.III-10) we obtain for neon:

\[ N_{eo} = \frac{C_N}{3 \pi e^2 \frac{E}{M \cdot e_0} \frac{1}{2} \frac{S_1}{R^2} i} \]  

(3.III-12)

where

\[ C_N = \frac{J(1/2)}{C(N_{eo}/N_{go})^3(-1/4)} \]

and for argon:

\[ N_{eo} = \frac{C_N}{2 \pi e^2 \frac{12.6 e}{M \cdot e_0} \frac{1/2}{S_1 R^2} i} \]

\[ C_N = \frac{J(1/2)}{C(N_{eo}/N_{go})} \]

The factor \( C_N \) has been calculated numerically and could be approximated by:

\[
\begin{align*}
\text{neon} & \quad C_N = 0.93 + 0.4c_1, & \quad N_{eo}/N_{go} < 3.10^{-8}, \\
& \quad N_{eo}/N_{go} < 3.10^{-8}, & \quad N_{eo}/N_{go} > 3.10^{-8}, \\
& \quad c_1 \text{ as in (3.III-7)}, &
\end{align*}
\]
\[
\text{argon } C_N = \frac{2.3}{1 + c_3} \quad \begin{array}{l}
N_{eo}/N_{go} < 3 \cdot 10^{-10}, \\
N_{eo}/N_{go} \geq 3 \cdot 10^{-10},
\end{array}
\]

\[
c_3 = 0.004 \left(10^{19} \frac{N_{eo}}{N_{go}} + 9.48\right)^{4.6}.
\]
CHAPTER 4  PHILOSOPHY ON THE CONSTRUCTION OF THE POSITIVE COLUMN

Scope

In this chapter we discuss the constriction phenomenon. In section 4.1 some physical mechanisms which are of importance for the constriction are discussed. On the basis of this discussion a review of some models presented in the literature is given in section 4.2. In section 4.3 a number of deviations from our model (given in chapter 3) are discussed. From these calculations arguments follow for a connection between the constriction and the appearance of striations in the considered pressure/current range. In section 4.4 the development of the striations is investigated.

Introduction

The constriction of the positive column of inert gas discharges sets in above certain values of the discharge current and the filling pressure (see fig. 1.1). This means that the constriction is caused by processes which become more important for higher values of the gas density and the electron density. These processes are: the maxwellization of the electron energy distribution function (E.F.) by the increasing Coulomb interaction of the electrons at increasing electron density, the increasing influence of volume-recombination with respect to ambipolar diffusion and the non-uniform gas heating, causing a radial dependence of gas and electron temperature. These processes, which are of equal importance in our region, are discussed in section 4.1.

In the literature several models based on one or more of the effects discussed above have been set up for the constriction. We discuss these models in section 4.2.

As none of these models is capable of describing the abrupt transition of the diffuse into the constricted discharge, and this transition is not described by our model (given in chapter 3) either, we searched for processes which may be the cause of the constriction by a variation of parameters in our model (section 4.3). There are arguments that the onset of the constriction is connected with the abrupt generation of ionization waves (striations) above certain pressure and current values.

In section 4.4 we therefore investigate these striations in more detail.
4.1 Physical mechanisms for the constriction

The influence of the three processes mentioned in the introduction on the constriction phenomenon is discussed in this section.

4.1.1 Coulomb relaxation

From experiments it follows that in the constricted column the radial electron density profile $N_e(r)/N_e(0)$ is much narrower than in the diffuse column at the same current. From the equation for the discharge current (eq. (3.7)) it follows that in the constricted column at the axis the electron density $N_e(0)$ must be higher than in the diffuse column. From the electron density equation (eq. (3.5)) we obtain that the ionization frequency on the axis $Z_{ion}(0)$ also must be enhanced in the constricted column. Because the ionization frequency is nearly equal to the excitation frequency $Z$ in this region (see eqs. (3.1), (3.4)), it follows that $Z$ increases during the transition of the column from the diffuse into the constricted state.

This occurs simultaneously with an observed decrease of the axial electric field strength $E$ (fig. 1.6). This decrease is connected with a decrease of $U_{eo}$, because $E$ and $U_{eo}$ are coupled according to eq. (3.9):

$$E = \sqrt{\frac{T_e}{M}} a N_{eo} U_{eo}^5 \approx 4 (N_{eo}/N_{go}).$$

As $C$ increases with increasing $N_{eo}$ (eq. (2.39)), and $N_{go}$ is higher in the constricted column than in the diffuse one (because the power input per unit length is lower in the constricted column), the decrease of $E$ can only be explained with a simultaneous decrease in $U_{eo}$.

The excitation frequency on the axis $Z(0)$ can only increase for decreasing $U_{eo}$ when it depends strongly on $N_{eo}$. When $Z(0)$ depends strongly on $N_{eo}$, that

$$\frac{\delta (Z_{ion} N_e)}{dN_e} \bigg|_{r=0} > \frac{d}{dN_e} \left[ N_e^{2} \nabla \cdot (\mathbf{D} \nabla N_e) \right]_{r=0},$$

an instability in the electron density production is amplified which causes an avalanche in the electron density at the axis that can be balanced by ambipolar diffusion and volume recombination only when $N_e(r)/N_e(0)$ is sufficiently narrow and $N_e(0)$ is sufficiently high. These two conditions are coupled via the equation for $N_e(0)$, see eq. (3.7). This means that the column constrits.
A model which does not take into account the dependence of the excitation frequency on the electron density is not capable of describing the abrupt transition from the diffuse into the constricted column. As for increasing electron density this dependence is caused by the increasing influence of the Coulomb terms in the Boltzmann-equation (see chapter 2), an accurate calculation of the electron energy distribution function (E.P.) is necessary for the description of the constriction (see section 4.1.4 for a further discussion).

4.1.2 Volume recombination and ambipolar diffusion

When ambipolar diffusion is neglected in the electron density equation (eq.(3.5)), the electron density profile has the same shape as the radial profile of the ionization frequency, which is nearly equal to that of the excitation frequency:

\[
\frac{N_e(r)}{N_e(0)} = \frac{Z_{\text{ion}}(r)}{Z_{\text{ion}}(0)} = \frac{Z(r)}{Z(0)}.
\]  

(4.2)

For large values of the electron density, the expression for \( Z \) (see chapter 2), can be written as (see also Woj66):

\[ Z = C_1 e^{-C_2/N_e}, \]

where \( C_2 \) depends solely on \( U_e \), and \( C_1 \) depends only weakly on \( N_e \) and \( U_e \). Therefore for large \( N_e \) eq.(4.2) approaches to

\[
\frac{N_e(r)}{N_e(0)} = \exp \left( \frac{C_2}{N_e(0)} \left( 1 - \frac{N_e(0)}{N_e(r)} \right) \right),
\]

(4.3)

where we take \( C_2 \) constant, because its radial dependence is weak. Because of the boundary condition \( N_e(R)=0 \) and the exponential dependence at the right-hand side of eq.(4.3) the stable solution is:

\[
N_e(x) = \begin{cases} 
N_e(0) & r = 0, \\
0 & 0 < r < R. 
\end{cases}
\]

(4.4)

The condition \( N_e'(0)=0 \) is violated because of the neglect of ambipolar diffusion. Numerical solution of the model of chapter 3 with \( D_a=0 \) gives indeed electron density profiles which narrow more and more when the number of discretization intervals \( k \) is increased. From model
calculations for the diffuse column we find that the volume recombination represents about 60% of the electron losses at the axis in the region where the constriction can be expected. It may thus be conjectured that volume-recombination is essential for the constriction. However, ambipolar diffusion never may be neglected in the constricted column. This can be seen from the equation for the discharge current (eq. (3.7)), from which we obtain as an approximation:

\[ n_{e0} r_1^2 = C i, \]

where \( C \) - a constant, \( r_1 \) - a characteristic radius for the column, and \( i \) = discharge current. Approximating \( \nabla_r \cdot (D_n \nabla N_e)_{r=0} \) by \( \frac{D_n n_{e0}}{r_1^2} \), we have:

\[ \nabla_r \cdot (D_n \nabla N_e)_{r=0} \approx \frac{D_n n_{e0}^2}{C i}. \]

Hence the dependence is quadratically on \( n_{e0} \), as is the volume recombination. Therefore as \( \nabla_r \cdot (D_n \nabla N_e)_{r=0} \) and \( n_{e0}^2 \) have the same order of magnitude in the diffuse discharge near the constriction, this will remain the case in the constricted discharge as long as the current has not been altered. This is confirmed by the numerical calculations.

4.1.3 Heat dissipation

At low current values, the effects of electron-electron interaction and volume-recombination are still negligible in the diffuse column. However, also for these current values the radial electron density profile can differ considerably from the Bessel-profile resulting from the original Schottky-theory (Sch24). This is caused by the radial dependence of the electron temperature, in its turn due to the radial dependence of the gas temperature (see chapter 3). The \( U_e(r) \) dependence causes a strong radial dependence of the ionization frequency \( Z_{ion}(r) \), which is responsible for the deviation from the Bessel-profile.

4.1.4 The combination of the three effects

The maxwellization of the E.F. sets in when the ionization degree \( N_e/N_g \) is sufficiently high. The volume-recombination becomes important with respect to ambipolar diffusion when \( N_e/N_g \) is large enough. The effect of the gas heating also sets in at high values of \( N_e/N_g \). These three effects are connected with each other. When the gas density and the
electron density are high, the non-uniform gasheating causes a strong radial dependence of the ionization frequency $Z_{\text{ion}}(r)$. Hence the electron density profile in the diffuse column narrows (section 4.1.3), so that $N_e(0)$ becomes high. For higher values of $N_e(0)$ the volume-recombination becomes important (section 4.1.2), which in its turn causes an extra narrowing of $N_e(r)$, hence an extra increase of $N_e(0)$. Then $N_e(0)$ may become so high that the maxwellization of the E.F. sets in (section 4.1.1), which gives an extra contribution to the electron production. For low current values the $Z_{\text{ion}}(N_e)$ dependence is not so strong that this results in an instability. However, for high currents we may have (see eq. (4.1)):

$$\frac{d}{dr} \left( ZN_e + \frac{q}{8} (D \nabla^2 N_e) - \alpha N_e^2 \right) > 0$$

so that constriction sets in.

Therefore constriction is expected to start when the three effects mentioned in sections 4.1.1-4.1.3 are so strong that they enhance each other. This is the case when the ionization degree $N_e/N_g$ and the product $N_eN_g$ are large enough. Because $N_e$ is coupled to the current (eq. (3.7)) and $N_g$ to the pressure (eq. (3.15)), the constriction will take place at sufficiently high currents while the pressure has to be in the right interval (see also fig. 1.1).

The model described in chapter 3, however, contains the three mentioned effects but does not describe the constriction. It is likely that the constriction is an instability effect which can not be described by a stationary model. In section 4.3 we have laid a connection between this instability and the onset of striations. The striation development is discussed in section 4.4. First, however, we discuss some models from the literature using the considerations given in this section.

4.2 Discussion of some models for the positive column

The models presented in the literature for the medium pressure column can be divided into three groups.

4.2.1 Thermal models

Many authors (Bar69, Rak70, Mou71, Uly73), have considered the constriction as a purely thermal effect (described in section 4.1.3). They
have not taken into account the other effects mentioned in section 4.1. All these authors assume a Maxwellian E.F. Furthermore they neglect ambipolar diffusion and their calculations are based on direct ionization. From sections 4.1.1-4.1.4 it is clear that the abrupt transition from the diffuse to the constricted column can not be explained on the basis of these assumptions. Moreover, these assumptions not even hold in the constricted column itself. Because the estimated ionization degree on the axis of the constricted column is about $10^{-6}$ to $10^{-5}$ (see section 4.3), the depletion of the high energy tail of the E.F. still is very large (see fig.2.5), and the assumption of direct ionization calculated with the help of a Maxwellian distribution can not be correct. Furthermore, the neglect of ambipolar diffusion is not justified (see section 4.1.2). Therefore the models from this category can not be expected to give quantitative agreement with experiments.

4.2.2 Ionization models

The strong dependence of the excitation frequency on the electron density as a possible cause for the constriction has been considered by Kagan et al. (Kag65). The ionization has been assumed to take place stepwise, where the ionization frequency has been taken equal to the excitation frequency. (Near the constriction region this assumption is reasonable.) Furthermore, the authors neglect volume-recombination and the radial dependence of the gas temperature and the electron temperature. These assumptions do not lead to a realistic description of the medium pressure column. A more sophisticated version of this model is discussed in section 4.2.3.

Wojaczk has set up a model for the constricted argon discharge which is of the same kind as the one of Kagan et al. discussed above (Woj66). In three publications he then considers subsequently the separate influence of the $T_g(r)$ dependence, the $U_e(r)$ dependence and volume-recombination (Woj67, 68, 69). He concluded that they all three can be neglected. In reality the effects are interrelated and enhance each other, so that their combined effect can not be neglected.

4.2.3 A more sophisticated model

In a later version of the model of Kagan the effects of gas-heating and volume-recombination have been incorporated (Gol74). Below we discuss a number of properties of their model.
-Kagan et al. solve the Heller-Elenbaas equation assuming a parabolic profile $T_g(r) = T_g(0)(1-c\frac{r^2}{R^2})$. The coefficient $c$ has been obtained from the condition:

$$\left(\frac{dT_g}{dr}\right)_R = -\frac{iE}{2\pi a(T)}$$

This condition has been obtained from multiplication of the Heller-Elenbaas equation by $2\pi r$ and integration from 0 to $R$. However, because $T_g(r)$ is not parabolic for $r \geq R$, (which can be seen from $J.E = 0$ for $r = R$ so that $T_g(r)S^3 \sim -\ln r$ in that region, (see chapter 3)), in general the thus calculated $T_g(r)$ profile differs considerably from the one obtained in our model from the Heller-Elenbaas equation itself (see fig. 4.1).

![Fig. 4.1 Calculation of the gas temperature profile.](image)

1 Method of Golubovskii et al.; 2 Exact solution.

$p_o = 100$ torr, $R = 32$ mm, $i = 100$ mA.

-Kagan et al. take the integral $S_1$ in the equation for $N_{eo}$ (eq. (3.7)) constant ($S_1 = 0.22$). This is more or less a self-fulfilling prophecy, because when $S_1$ is kept constant, $N_{eo}$ increases only linear with the discharge current, so that volume-recombination and maxwellization of the E.P. can not increase rapidly. Then the $N_c(r)$ profile will not narrow, so that $S_1$ does not change much with increasing discharge current. In reality, when $S_1$ is not kept constant, we get a non-
Fig. 4.2 The dependence of the integral $S_1$ (in the current equation) and of $N_{eo}$ on the discharge current, in our model. Golubovskii et al. take $S_1=0.22$ and consequently, the dashed curve gives their values of $N_{eo}$ calculated with $S_1=0.22$.

linear dependence of $N_{eo}$ on the discharge current (see fig.4.2), which cause a sharp decrease of $S_1$ hence a still larger increase of $N_{eo}$ with increasing current.

- The axial electric fieldstrength $E$ is calculated from the electron energy equation. Arguments against this procedure have been discussed in section 2.4.

- In the balance equations for the excited atoms in group I (see section 3.2) only the resonant $^3P_1(s4)$-state has been taken into account. The metastable $^3P_2(s5)$-state has, however, a much larger density so that the stepwise ionization takes place mainly over this level. Furthermore the effects of redistribution of the 1s-levels via group II, and the production of excited atoms in group II via volume-recombination have been neglected.

- The elastic cross-section of neon has been taken constant in the calculation of the E.F., which is not realistic (see fig.2.1).
-The gas temperature term has been neglected in the calculation of the E.F., which is of importance for the calculation of the excitation frequency (see section 2.5). At the constriction region, where the gas temperature is high (see table 4.1 (section 4.3)), this term becomes non-negligible.

- The wall temperature has been taken equal to the room temperature which is not correct for the currents and pressures near the constriction region.

- The correction factors $C_N$ and $C_D$ in $N_{e0}$ and $D_a$ have not been taken into account. (The importance of these factors, however, is not very large.)

Especially the first two mentioned assumptions cause the large deviations between this model and our results. Their calculated $T_g(r)$ profiles are much too flat, and their calculated values of $N_{e0}$ too low (see figs 4.1, 4.2).

4.2.4 Discussion

Because of the assumptions made in the models described in sections 4.2.1-4.2.3), these models do not describe the medium pressure column accurately. Though our model gives an accurate description of the diffuse discharge it does not include the transition to the constricted state without special precautions. In the next section we investigate by a variation of some parameters in our model whether we can find methods to account for the constriction.

4.3 Variation of some parameters of the model

4.3.1 Parameters with negligible influence

In this section we investigate the sensitivity of our model to variations in the various reaction coefficients. All results are for $p_0 = 100$ torr, $R = 32$ mm. Numerical calculations showed that a variation in the coefficients have the most important influence on the value of the axial electric fieldstrength. Therefore we discuss in detail the influence of the variations on the $E-I$ curve. A variation of the coefficients of the balance equations for the excited atoms did not have much influence on the model results. For instance multiplication of the stepwise ionization coefficient by a factor 100 resulted in a decrease of $E$ of 20%. The stepwise ionization frequency has an inaccuracy of about a factor of 3. Variation of the elastic cross-section factor $d_o$ with a
The two-temperature model and influence of the volume-recombination coefficient.

certain factor results in a same relative variation of $E$. The variance of values found in the literature for $\alpha_D$ is about 15%. The precise value of the inelastic cross-section is less important, because of the slowly varying function $F_1$ in the expression for the excitation frequency $\tilde{\gamma}$ (section 2.5; when $Q_e$ is changed a factor 10, the variation of $F_1$ is approximately 25%).

1.3.2 Parameters with a significant influence

We discuss first the volume-recombination coefficient $\alpha$. (For a review of the results of the variation analysis see figs.4.3, 4.4 and table 4.1.)

Increase of the volume-recombination coefficient $\alpha$ with a factor of 3 results in an increase of $N_e$ (see section 4.1.2 and table 4.1), consequently in a stronger Coulomb-interaction in the E.F., which in its turn results in a decrease of $E$ (section 4.1.1). It appears not to be possible to get constriction by enlargement of $\alpha$.

In previous calculations the number of discretization intervals in the numerical elaboration of the electron density equation (eq. (3.5))
was restricted to \( k = 32 \). In that case both the sharp constriction and the hysteresis were calculated with good values of \( E \). The calculated electron density profiles, however, were much too narrow. In the normal calculations and the variations, it is checked whether a sufficient large number of intervals \( k \) is used. The value of \( k \) is altered if necessary.

If in stead of using the solution of the Boltzmann-equation, we use a chosen E.F. according to a maxwellian E.F. with a bulk temperature and a tail temperature, an abrupt transition of the diffuse into the constricted state can be obtained. This distribution function has been suggested by Vriens in another context (Vri73). For our purpose \( \frac{U_e}{U_T} \) was calculated not from the method of Vriens but according to

\[
\frac{U_e}{U_T} = \frac{c_1 c_2 + N_e \bar{N}}{c_2 + N_e \bar{N}}/g .
\]

In this model, \( c_1 \) is chosen such that the calculated \( E \) fits the measured one at 10 mA. The value of \( c_2 \) is chosen such that the constriction takes place around 120 mA. The values of \( c_1 \) and \( c_2 \) belonging to fig.4.3 were: \( c_1 = 2.66, \quad c_2 = 6.5 \times 10^{-7} \). We have introduced this version of the model to demonstrate that a calculation of the constriction is possible with a steady state model. Though the choice of \( \frac{U_e}{U_T} \) is arbitrary, and this model does not calculate the hysteresis loop, it seems to indicate that a variation in the parameters of the Boltzmann-equation is necessary to explain the constriction.

In fig.4.4 the influence of some variations in the used E.F. are shown.

The normal calculation (chapter 3) fits well the experimental curve for low currents. For high currents the experimental values tend to the Maxwellian case (constricted column). An increase of the Coulomb terms in the Boltzmann-equation with a factor of six, gives a curve which goes smoothly from the diffuse to the constricted one. As in the case of the volume-recombination coefficient, it appears impossible to obtain a sharp constriction by a change of \( U_{ee} \).

### 4.3.3 A striation model

In section 4.4 a criterium for the onset of striations is derived from the dispersion relation for these waves. When the striations develop, the values of \( N_e N_0, \quad E \) and \( U_{ee} \) at the top of the striation are related to the average values in the following way:
This follows from non-linear calculations which we have carried out for our case using the method of Grabec (Gra74). The value of $N_{eo}$ is in agreement with electron density measurements in the constricted-striated column performed by us with the method described in chapter 5. We therefore introduce the following striation-model. When the striations start to develop, the values of $N_{eo}$, $E$ and $U_{eo}$ must be replaced by their values at the top (see above). With these values, a region for $N_{eo}$ and $U_{eo}$ is reached in which the constriction criterium (4.1) (given in section 4.1.1) is reached, so that constriction sets in. The numerical calculations show, however, that the onset criterium for the striation development (derived in section 4.4), is not fulfilled for the conditions in the diffuse discharge for our current values. (For the conditions in the constricted discharge, however, the criterium is fulfilled for currents higher than 90 mA ($P_o = 100$ torr, $R = 32$ mm), assuming a homogeneous column with changes in the values of $N_{eo}$, $E$ and $U_{eo}$ as given above. This corresponds with the observed fact that the constriction disappears below 90 mA (see fig.4.4)). Under the conditions of
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<tr>
<td>(\nu_{ce} \times 6)</td>
</tr>
<tr>
<td>(\alpha \times 3)</td>
</tr>
</tbody>
</table>

Table 4.1: Calculated column parameters for various versions of the model \((p_o = 100 \text{ torr}, R = 32 \text{ mm})\).  
* chapter 3,  
** average value.

The diffuse discharge the onset of the constriction at 120 mA has been forced on the system ad hoc substituting the above given changes in \(N_{eo}\), \(E\) and \(U_{eo}\). Probably local fluctuations in the densities in the column make it possible to meet the striation criterium at 120 mA.

From table 4.1 and figs. 4.3 and 4.4 it appears that the striation model gives the best agreement with the constricted column experiments.
As a matter of fact for the diffuse discharge the striation model is identical with the normal model because there are no striations in the diffuse column. We thus have presented strong indications that the con-
striction can be regarded as a striation generated process. The striations will be investigated in more detail in the next section.

4.4 The striation development

4.4.1 Introduction

The physical mechanism of the striation development can be understood by considering the equations for the conservation of electron density, momentum and energy.

\[
\frac{\partial N_e}{\partial t} = \frac{3}{3x} \left( \frac{3\partial N_e}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial N_e}{\partial r} \right) + z_{\text{ion}} N_e - \alpha N_e^2, \tag{4.5}
\]

\[
N_e \frac{\partial v}{\partial x} - b N_e \left( \frac{2}{3} \frac{\partial N_e}{\partial x} \right) = - b \frac{\partial}{\partial \nu} N_e \frac{E}{e}, \tag{4.6}
\]

\[
\frac{\partial}{\partial x} \left( N_e \frac{e}{m_e} \right) = - \frac{N_e \nu}{t_1} - N_e \frac{E}{e} \left( \frac{1}{t_1} \right), \tag{4.7}
\]

where \(x = \) axial distance, \(t = \) time, \(t_1 = \) a characteristic time for elastic energy losses, \(v = \) electron drift velocity (defined positive from anode to cathode).

An index zero refers to the stationary and homogeneous situation. In eq. (4.7) the time derivative has been neglected, as the striation fre-
quency (about 2 to 3 kHz) is negligible with respect to the frequencies of the fundamental processes (about \(10^6 \) Hz). Furthermore the heat con-
duction of the electrons has been neglected in eq. (4.7) (Go69).

For a qualitative description of the striations we follow the argu-
mentation of Pekárek (Pek68). Let us assume a local fluctuation in the electron density in the column. Then an ambipolar field will arise,
changing the local axial fieldstrength according to eq. (4.6). According to eq.(4.7), the electron temperature will also change, but this change is temporally and spatially delayed because of the electron drift. This will influence the ionization frequency \(z_{\text{ion}}\), generating a new temporally and spatially delayed electron density fluctuation (conform eq.(4.5)) having opposite sign with respect to the initial one. If this chain of processes repeats, the wave develops. The striations may be damped, how-
ever, if the ambipolar diffusion is sufficiently important (see eq.(4.5))
and/or by too large (in)elastic energy losses (see eq. (4.7)).

The eqs. (4.5-4.7) have been calculated numerically by Grabec (Gra74). He neglects volume recombination and assumes a Maxwellian electron energy distribution function. Then eq. (4.5) is linear, and allows separation, yielding the term \(-\frac{D_n}{a} \frac{\partial^3 N_e}{\partial r^3}\) in stead of \(\frac{1}{r} \frac{\partial}{\partial r} (rD_n \frac{\partial N_e}{\partial r})\), with \(r_1 = R/2.405\) according to a zero order Bessel function for the radial electron density profile. (R is the tube radius.) Substituting (4.6) into (4.7), and treating the latter as a first order inhomogeneous differential equation, the equations have been solved numerically by Grabec (Gra74).

In the medium pressure discharge volume-recombination can not be neglected (chapter 3 and section 4.1.2). Moreover the electron energy distribution function is strongly non-Maxwellian, and depends on the electron density (chapter 2). Therefore eq. (4.5) is non-linear and can not be separated. (The fact that the column constricts simultaneously with the striation development introduces extra difficulties.) Calculations which we performed using the method of Grabec for the medium pressure discharge where the term \(\frac{1}{r} \frac{\partial}{\partial r} (rD_n \frac{\partial N_e}{\partial r})\) is replaced by \(-D_n \frac{\partial^2 N_e}{\partial r_1^2}\), with \(r_1\) a characteristic column radius, and with volume-recombination and the non-Maxwellian E.F. taken into account give large deviations from the experimental values for the frequency of the waves. The calculations are also very sensitive to the numerical treatment. In fact the possibility that the thus calculated values are mere numerical instabilities can not be excluded. Therefore these calculations are not presented here. Instead we will derive analytically in the next section the dispersion relation for the waves by means of a linearization of eqs. (4.5-4.7).

4.4.2 The dispersion relation

The moment of onset for the striation development is derived from a linearization of eqs. (4.5-4.7). The linearization yields the dispersion relation, from which the initial growth rate of the waves can be found. Assuming that the radial variations are not large in this linear treatment, the term \(\frac{1}{r} \frac{\partial}{\partial r} (rD_n \frac{\partial N_e}{\partial r})\) is replaced by \(-D_n \frac{\partial N_e}{\partial r_1^2}\) here. Substituting eq. (4.6) in eq. (4.7) and using the stationary and homogeneous column conditions:

\[
N_e = D_n \frac{\partial^2 N_e}{\partial x_1^2}
\]

(4.5a)

The moment of onset for the striation development is derived from a linearization of eqs. (4.5-4.7). The linearization yields the dispersion relation, from which the initial growth rate of the waves can be found. Assuming that the radial variations are not large in this linear treatment, the term \(\frac{1}{r} \frac{\partial}{\partial r} (rD_n \frac{\partial N_e}{\partial r})\) is replaced by \(-D_n \frac{\partial N_e}{\partial r_1^2}\) here. Substituting eq. (4.6) in eq. (4.7) and using the stationary and homogeneous column conditions:
and
\[ \frac{D}{\varepsilon_0} \frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \left( \varepsilon_0 \varepsilon N \right) = \frac{\partial}{\partial x} \left( \varepsilon_0 \varepsilon N \right) + \frac{\partial}{\partial x} \left( \varepsilon_0 \varepsilon N \right), \]  
(4.7a)

(\text{where } \varepsilon_0 = \varepsilon \text{ has been taken). eqs. (4.5) and (4.7) become (} \text{D} \text{ is assumed to be constant):}

\[ \frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial x^2} + (Z - Z_0) N - \varepsilon_0 \varepsilon N (N - N_0), \]  
(4.8)

\[ \frac{\partial U}{\partial x} = - \frac{3}{\varepsilon_0} \frac{\partial N}{\partial x} + \frac{2}{\varepsilon_0} \frac{\partial N}{\partial x} + \varepsilon_0 \varepsilon U + \frac{N}{\varepsilon_0} \frac{\partial U}{\partial x} + \frac{N}{\varepsilon_0} \frac{\partial U}{\partial x} (Z - Z_0 \varepsilon U). \]  
(4.9)

Substituting \( N = N_0 + N_0 e^{i(kx - \omega t)} \) and \( U = U_0 + U_0 e^{i(kx - \omega t)} \), with \( N_0 \) and \( U_0 \) complex, into (4.8) and (4.9), retaining only linear terms in \( N_0 \) and \( U_0 \), gives two linear and homogeneous equations in \( N_0 \) and \( U_0 \).

This system has only a nontrivial solution when its determinant vanishes. This condition yields the dispersion relation:

\[ (D \varepsilon - (Z_n - \alpha) N_0 (ik - k_1)) = 2 N_0 (2 + \frac{P_{in}}{P_{el}} \frac{\partial \varepsilon_0}{\partial \varepsilon_0}) \left( \frac{\varepsilon_0}{\varepsilon_0} \right)^2 i k \frac{U_0}{E_n}, \]  
(4.10)

where \( k_1 = \frac{\varepsilon_0}{U_0} \left( 1 + \frac{P_{in}}{P_{el}} (\partial \varepsilon_0 - 1) \right), \)

\[ \zeta_u = \frac{\varepsilon_0}{U_0}, \quad \zeta_n = \frac{\varepsilon_1}{N_0}, \]

\[ \psi_u = \frac{u_0}{Z_0}, \quad \psi_n = \frac{N_0}{Z_0} \]

and \( P_{el} = \varepsilon_0 E_0^2 \) and \( P_{in} = Z_0 U_1 \) are the elastic and inelastic energy losses respectively.

We take \( \omega = \omega + i \omega_1 \) and \( k \) real, because spatial damping can not be present because the column is assumed to be infinitely long. The dispersion relation (4.10) is split into real and imaginary parts. Then \( \omega \) and \( \omega_1 \) expressed in terms of \( k \) become:

\[ \omega = \frac{T_1 \varepsilon_1}{k^2 + k_1^2}, \]  
(4.11)

\[ \omega_1 = - \frac{k^2 \varepsilon_1}{2} - \frac{T_1 \varepsilon_1}{k^2 + k_1^2}, \]  
(4.12)

where \( T_1 = \frac{Z_0 E_0}{3} \left( \frac{\partial \varepsilon_0}{\partial \varepsilon_0} \frac{\varepsilon_0}{\varepsilon_0} \right)^2 + \frac{P_{in}}{P_{el}} \left( \frac{\varepsilon_0}{\varepsilon_0} + \frac{2}{3} \frac{\partial \varepsilon_0}{\partial \varepsilon_0} - \frac{2}{3} \right) \).
and \[ T_2 = \left( \frac{2}{3} \zeta_u + \zeta_n - \frac{\alpha N_{eo}}{2} \right) T_0 \].

When inelastic energy losses are absent \((P_{in} = 0)\), then \( k_1 = E_0 / U_{eo} \), which is the reciprocal of the electron temperature relaxation length due to elastic collisions \((\text{Gra55})\).

From eq. (4.12) the onset criterion for the striations is derived, requiring \( \omega_i > 0 \). This gives:

\[ (k^2 + \frac{b_1^2}{4}) + c - \frac{b^2}{4} < 0, \]

with \( b = k_1^2 - \frac{T_2}{D_a} \) and \( c = \frac{k_1(T_1 - k_1 T_2)}{D_a} \).

In our parameter domain, \( b < 0 \) and the striation development will start as soon as \( b^2 \) becomes larger than \( 4c \). The striations then start with a wavenumber \( k = \sqrt{-\frac{b}{2}} \).

From table 4.2 it can be seen that the criterion \( b^2 > 4c \) is fulfilled when \( N_{eo} \) becomes sufficiently large.

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<th>( b^2 - 4c (\text{m}^{-4}) )</th>
<th>( \omega_i (\text{s}^{-1}) )</th>
<th>( \zeta_u )</th>
<th>( \zeta_n )</th>
<th>( P_{in}/P_{el} )</th>
<th>( k )</th>
<th>( k_1 (\text{m}^{-1}) )</th>
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</tbody>
</table>

Table 4.2 Calculated values from the dispersion relation for \( P_0 = 100 \) torr. The striation development starts when \( N_{eo} > 4.10^{18} \text{m}^{-3} \) \((\omega_i > 0)\). (Tube radius 32 mm.)

From eq. (4.11) the phase velocity \( v_p \) and the group velocity \( v_g \) are obtained:

\[ v_p = \frac{\omega_i}{k} = \frac{T_1}{k^2 + k_1^2} > 0, \]
In our case \( k >> k_1 \) (see table 4.2) thus \( v_g < 0 \) (\( |v_g| < |v_p| \)). According to the notation of Pekářek (Pek68), the waves are of the B-type, B referring to \( v_p/v_g < 0 \), the minus sign referring to \( v_g < 0 \) (opposite to the current direction which is from anode to cathode).

Eqs. (4.11) and (4.12) are of the same form as those derived by Pekářek (Pek68).

To get some insight in the physical mechanisms determining eqs. (4.11) and (4.12), we discuss the case \( \alpha = 0 \), \( P_{in} = 0 \), \( \xi_n = 0 \) (from the data in table 4.2, and considering that \( v_{g0} < Z_0 \), this appears to be a realistic approximation). From eq. (3.5) we get approximately: \( Z_{0g}/Z_{0e} = 1 \). Because \( E_{eo}/U_{eo} = 5 \), we get (see also table 4.1): \( U_{eo}/E_0 \gg x_1 \), hence \( (E_{0}/U_{eo})^2 << Z_0/D_a \) in the region considered here. Then \( k_1^2 << T_2/D_a \), and

\[
\begin{align*}
\lambda &= \frac{2}{3} \frac{Z U_{eo}}{D_a} \quad \text{and} \quad c = \frac{2}{3} \frac{Z U_{eo}}{D_a \lambda} \\
\end{align*}
\]

where \( \lambda = U_{eo}/E_0 \) is the electron temperature relaxation length.

The criterion \( b^2 > 4c \) then becomes:

\[
Z_{0g} U_{eo} > 18 \frac{D_a}{\lambda} .
\]

This formula shows the ionization mechanism as a creation process and ambipolar diffusion and electron temperature relaxation as striation damping processes. The wavenumber with which the striation development starts is:

\[
k = \sqrt{\frac{b}{2}} \sqrt{\frac{1}{3} \frac{Z U_{eo}}{D_a}} = \sqrt{b} k_1 = \frac{\sqrt{b}}{\lambda} .
\]

Thus the wavelength \( \lambda \) becomes:

\[
\lambda = \frac{2\pi}{\sqrt{b}} \lambda \sim \frac{1}{N_g} \quad (\text{see chapter 3}).
\]

The frequency \( f \) becomes in this case:

\[
f = \frac{\omega_p}{2\pi} = \frac{4\sqrt{b}}{21\pi} \frac{Z U_{eo}}{N_g} .
\]
The group velocity \( v_g \) becomes finally:

\[
v_g = -\frac{40}{147} \frac{Z u_0 \cos \frac{\lambda}{r}}{v_p} \approx v_p \quad \text{(independent of } N_g).\]

In Table 4.3 the calculated values for the general case \((\alpha, \Pi_n, \tau_n \neq 0)\) are compared with experimental data (see next section). The calculated relationship of \( \lambda, f, \) and \( v_g \) on the gas pressure is in reasonable agreement with experiment. The large deviations in the calculated frequencies are attributed to the approximations inherent to the linear treatment. This linear treatment is given here because we are only interested in the onset of the constriction (see Section 4.3.3) and not in a detailed calculation of the striation development itself.

<table>
<thead>
<tr>
<th>( p_0 ) (torr)</th>
<th>( \lambda \cdot 10^{-2} ) (m)</th>
<th>( f ) (kHz)</th>
<th>( v ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>exp.</td>
<td>calc.</td>
<td>exp.</td>
</tr>
<tr>
<td>50</td>
<td>2.9</td>
<td>0.86</td>
<td>1.7</td>
</tr>
<tr>
<td>100</td>
<td>1.0</td>
<td>0.45</td>
<td>2.8</td>
</tr>
<tr>
<td>200</td>
<td>0.9</td>
<td>0.22</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Table 4.3 Comparison of calculated values with experimental data.
(Tube radius 32 mm, current 130 mA.)

4.4.3 Experiments

For the experimental study of the striations we have used the discharge tubes and the electric circuit as mentioned in Section 1.1. The striations were observed optically. For this purpose, one photodiode, positioned at a fixed place along the tube axis, was used as trigger signal, whereas a second one could be moved axially. Both signals were displayed on a double ray oscilloscope. Wavelength, frequency and velocity of the striations were measured (see Table 4.3). Using a monochromator with a photomultiplier, various line transitions were investigated. In this case a movable photodiode was used as trigger signal, while the photomultiplier was positioned at a fixed place. The striations did not depend observably on the wavelength of the line radiation. (In the low pressure column, Rutscher has observed a slight dependence (see Ol068).)
The modulation of the line radiation, which is nearly proportional to the ionization function $Z N_e$ (see chapter 3), is about 90%. The modulation of the continuum intensity, which is nearly proportional to the electron density (see chapter 5), is about 50%. (Hence a nonlinear treatment is necessary to calculate the striation development accurately.)

The waves travel from cathode to anode, i.e. opposite to the current direction.

The stability of the waves depends rather strongly on the geometry of the tube. Thermal turbulence disturbed the measurements at higher pressures ($p_o > 5$ torr m). In case of irregular fluctuations of the frequency and the amplitude of the waves around their average values, the waves could be stabilized by modulating the reference voltage of the current regulator with the mean frequency (or the first harmonic component) of the striations.

4.5 Conclusion

From the calculations given in this chapter based on various versions of our model, it is very likely that the constriction is caused by the onset of the striations. The model which incorporates the changes in the electron density, the electron temperature and the axial electric field strength in the top of the striation, gives good agreement with the measurements on the constricted column. To give a full description of the transition of the diffuse into the constricted state, a description in $r$, $z$ and $t$ coordinates would be necessary for the electron density, momentum and energy equations. As the stationary model (only radial dependences) is already complicated enough, such a complete description would give large numerical complications. The description of the transition with the striation model described in section 4.3.3 is a simple and rather accurate way to circumvent these difficulties.
CHAPTER 5 THE ELECTRON-ATOM BREMSSTRAHLUNG CONTINUUM

Scope
Measurements of radial profiles of the electron density and the electron temperature have been performed using the electron-atom bremsstrahlung continuum. The profiles are compared with model calculations.

Introduction
The optical spectrum emitted by medium pressure inert gas discharges consists of a line spectrum superimposed on a rather strong continuum. In the past this continuum was explained as a molecular effect (Pri66, Ken67, Gol68b). More recently Rutscher and Pfau (Rut68) have ascribed the continuum to electron-atom bremsstrahlung. In the last few years many authors have taken the latter point of view as the correct one (Gol73, Ven73). Though there is still some reason to believe that molecular radiation gives a non-negligible contribution to the continuum (Smi75b), in this chapter we will confine ourselves to the electron-atom bremsstrahlung continuum. The electron density and the electron temperature are determined from measurements on the continuum, assuming that the continuum is caused by electron-atom bremsstrahlung.

5.1 The electron-atom bremsstrahlung theory
The spectral intensity of the electron-atom bremsstrahlung at wavelength \( \lambda \) is (Rut68):

\[
i(\lambda) = \sqrt{\frac{2e}{N_e}} \int \frac{hc}{\lambda^2} \int_{0}^{\infty} Q_B(U) U^{1/2} f(U) dU.
\]

(5.1)

Here \( f(U) \) is the isotropic part of the electron energy distribution function, and \( Q_B \) is the electron-atom bremsstrahlung cross-section. From the quantum-mechanical calculation of Kas'yanov and Starostin (Kas65, see also Ned32, Fir61), we have:

\[
Q_B(U) = \frac{4e^3}{3\pi^4 \hbar c^2} U \sqrt{1 - \frac{hc}{\lambda eU}} \left(1 - \frac{hc}{2\lambda eU}\right) Q_D(U),
\]

(5.2)
where \( Q_D = a_D v^n \) (see section 2.2).

From section 2.1 we have

\[
F(U) dU = f(v) 4\pi v^2 dv = 2\pi w^3 \frac{1}{2} f_0(\varepsilon) d\varepsilon ,
\]

(5.3)

Because the electron energies involved here lie mostly in the elastic region, we take (see section 2.3):

\[
f_0(\varepsilon) = \frac{N_1}{2\pi w^3} e^{-M(\varepsilon)\varepsilon},
\]

with

\[
N_1 = \frac{1}{\int_0^\infty \sqrt{\varepsilon} e^{-M(\varepsilon)} d\varepsilon}.
\]

(5.4)

Substitution of eqs. (5.2-5.4) into eq. (5.1) yields:

\[
i(\lambda) = C_1 \frac{N_e N_g}{U_e} \frac{e^2}{\lambda} \frac{S_D}{\varepsilon c m c^2} J(U_e, N_e/N_g, \lambda),
\]

(5.5)

with

\[
C_1 = \frac{4e^3}{3w} N_1 \sqrt{\frac{2e}{m}} \frac{S_D}{\varepsilon c m c^2} \frac{h c}{e} \frac{3+n}{2},
\]

\[
J = \int_1^\infty \frac{z^n}{(z - \frac{1}{2}) e^{-M(xz)}} dx ,
\]

(5.6)

and \( x = \frac{hc}{e U_e \lambda} \), and \( z \) is dimensionless integration variable.

The integral \( J \) depends on \( N_e/N_g \) because the distribution function depends on \( N_e/N_g \) via the exponent \( M(\varepsilon) \) (see chapter 2). The dependence of \( J \) on \( N_e/N_g \), \( U_e \) and \( \lambda \) is not very strong. Therefore in first order we obtain from eq. (5.5):

\[
i(\lambda) \sim N_e N_g.
\]

Hence from the continuum intensity the electron density can be determined. This is discussed in section 5.2. The determination of the electron temperature from the continuum is discussed in section 5.3.

### 5.2 Determination of the electron density

From eq. (5.5) we can derive the radial profile of the electron density from the measured intensities:

\[
\frac{N_e(r)}{N_e(0)} = \frac{i(\lambda, r)/i(\lambda, 0)}{C_0(0)/C_0(r)},
\]

(5.7)
with \( C_0(r) = \frac{C_0}{U_e(r)^{3/2} 5+n} \) \( J(U_e(r), N_e/N_g, \lambda) \). \hspace{1cm} (5.8)

The radial profiles of the continuum intensity have been measured at \( \lambda = 4000 \text{ R} \). The profiles \( i(\lambda,r) \) were obtained from the measured profiles \( \varepsilon(\lambda,r) \) by means of the Abel-transformation (Rom67).

The numerical Abel-transformation is rather sensitive to measurement inaccuracies. Therefore the measured profiles were curve-fitted first with an analytical function containing a few parameters.

The gas density profiles \( N_g(r)/N_g(0) \) were obtained from the model calculations (see chapter 3).

Because the factor \( C_0(0)/C_0(r) \) in eq. (5.7) depends rather weakly on \( U_e(r) \) and \( N_e/r(N_g(r)) \), here also the profiles following from the numerical model (see chapter 3) were substituted. The relative behaviour of \( C_0 \) is given in fig. 5.1. In fig. 5.2 a measured intensity profile \( \varepsilon(\lambda,r) \) is given (curve 1). After Abel-transformation curve 2 is found. Finally curve 3 represents the relative electron density profile. It can be seen that the difference between curves 2 and 3 is very small, so that the influence of the correction (eq. (5.7)) is small. In fig. 5.3 some measured profiles are compared with model calculations (see also chapter 3). The calculation of the constricted profile is obtained with the striation model discussed in chapter 4. The agreement between measured and calculated profiles is satisfying. This may, however, not be regarded as a clear cut confirmation of the bremsstrahlung hypothesis, because the molecular theory can yield the same result (Smi75b).

5.3 Determination of the electron temperature

The electron temperature can be determined from the bremsstrahlung intensity according to a method given by Golubovskii et al. (Gol73). From eq. (5.5) we get for the ratio \( R_a \) of the continuum intensities at two different wavelengths \( \lambda_1 \) and \( \lambda_2 \):

\[
R_a(U_e, N_e/N_g) = \frac{i(\lambda_1)}{i(\lambda_2)} = \left( \frac{\lambda_2}{\lambda_1} \right)^{5+n} \frac{J(U_e, N_e/N_g, \lambda_1)}{J(U_e, N_e/N_g, \lambda_2)}. \hspace{1cm} (5.9)
\]

From the measurement of this ratio the electron temperature \( U_e \) can be derived when the electron density \( N_e \) is known. We choose \( \lambda_1 = 2500 \text{ R} \) and \( \lambda_2 = 4000 \text{ R} \). In fig. 5.4 the relationship between \( R_a \) and \( U_e \) is given with \( N_e/N_g \) as parameter. In the diffuse column, where \( N_e/N_g < 3 \times 10^{-7} \).
Fig. 5.1 Correction factor $C_0(U_e, N_e/N_0, \lambda)$ for the determination of $N_e$ from the bremsstrahlung intensity.

Fig. 5.2 Measured profile of the continuum intensity (1), profile after Abel-transformation (2), and corrected profile, according to the factor $C_0$ and $N_0(r)$ eq.(5.7) (3).
Fig. 5.3 Measured and calculated electron density profiles for some current values at $p_o = 100$ torr, $R = 32$ mm (see text). 

The dependence of $R_a$ on $N_e$ is weak. Therefore the determination of $U_e(r)$ with this method is not very sensitive to the precise value of $N_e(r)/N_g(r)$. In the constricted column $N_e/N_g \approx 3 \times 10^{-6}$ on the axis, but $N_e/N_g$ falls off very rapidly at increasing radial distance. Then the determination of the electron temperature from the ratio $R_a$ will be less accurate on the axis, but becomes better with increasing radial distance. We therefore substitute for $N_e/N_g$ in eq. (5.9) the values calculated from the model of chapter 3 for the diffuse column, and the striation model discussed in chapter 4 for the constricted column. (As was shown in section 5.2 the calculated relative profiles for $N_e(r)$ were in good agreement with the measured ones.) Measurement of $R_a = i(\lambda_1)/i(\lambda_2)$ then yields $U_e$.

The measurements for the determination of the electron temperature had to be performed with high accuracy and stability to obtain reproducible results. Temperature stabilization of the amplifier unit was applied. At each radial position the intensity was measured at the two wavelengths 2500 and 4000 Å with a 25 cm Jarrell Ash monochromator. This was more stable than scanning the radial position first for 2500 Å and then for 4000 Å. A calibration for the relative wavelength sensitivity of the optical system was performed with a synchrotron radiation calibrated standard deuterium lamp.

In fig. 5.5 the thus obtained electron temperature profiles are
given for a 150 torr neon discharge (tube radius 32 mm), for various values of the discharge current. They are compared with model calculations (chapters 3 and 4). The discrepancies between the calculated and measured profiles are too big for high currents (the accuracy of the measurements is approximately 0.1 V).

The calculated values follow from the relation $U_e(r)/U_e(0) = (T_g(r)/T_g(0))^{4/5}$ (see eq. (3.10)). The gas temperature $T_g$ follows from the Heller-Blendaas equation (eq. (3.11)). The calculated $T_g(r)$ profiles are not very sensitive to the coefficients used in the model of chapter 3. Furthermore the agreement between the calculated and measured electron density profiles gives us reason to assume that also the calculated $T_g(r)$-profiles will be in agreement with the actual ones. Therefore the deviation cannot be explained with our model, although this model takes into account most of the physical mechanisms of importance, and is in good agreement with other experimental results. A part of the continuum radiation may be ascribed to molecular radiation (Smi75b). This gives a possible explanation of the above found discrepancies. The discrepancies
Fig. 5.5 Measured and calculated electron temperature profiles, 150 Torr, tube radius 32 mm. Curve parameter: discharge current.

in the constricted discharge may be due to larger experimental errors because of the appearance of striations and the strong radial dependencies.
CHAPTER 6 Fluorescence Measurements

Scope

Radial density profiles of the $^3P_2$ and $^3P_1$ 1s-states have been measured in a neon discharge using the fluorescence technique. The measured profiles are compared with model calculations.

Reaction coefficients for atom induced transitions between the 2p-levels have been determined by measuring the fluorescence spectra for different laser input wavelengths. These coefficients have been measured at different values of the discharge current. Herefrom the dependence of the coupling coefficients on the gas temperature has been obtained, using the dependence of the gas temperature at the discharge axis on the discharge current, calculated with our discharge model (see chapter 3).

Introduction

The densities of excited atoms in a discharge are often measured by means of the radiation-absorption technique (Lad33, Beh71). Here, we present measurements of the radial profiles of the densities of the $^3P_2$ and $^3P_1$ 1s-atoms (see fig.6.1) in a neon discharge ($p_0 = 100$ torr, $R = 32$ mm), which have been performed with the fluorescence technique (Smi75a, Ste75, Coa76), using a tunable CW dye-laser. The fluorescence light was measured perpendicular to the laser beam with a spatial resolution better than 1 mm. The discharge current was varied between 20 mA (diffuse column) and 120 mA (diffuse and constricted column). Excitation of the $^3P_2$ 1s-atoms with laser light of 5892 Å yields an extra contribution to the population of the $p_2$-level (see fig.6.1). The $p_2$-level decays to the four 1s-levels emitting light with wavelengths 5882, 6030, 6163 and 6598 Å. The density of the $^3P_2$ (s$_p$) level is determined by measuring the 6598 Å fluorescence signal with a monochromator to eliminate the scattering of the input light. The intensity of the fluorescence signal is proportional with the $^3P_2$ atom density when sufficiently low laser photon fluxes are used (this has been treated in Smi75a). A correction has been applied for the transition to the other p-levels discussed below. In the same way the density of the $^3P_1$ level has been measured by using laser light of 6030 Å.

When the $^3P_2$ (or $^3P_1$) state is excited to the $p_2$-level by photons from the dye laser, the fluorescence spectrum is not only composed of
the four lines originating from this level (5882, 6030, 6163 and 6598 Å), but also from lines originating from other 2p-levels. This is caused by transfer of excitation between these levels by collisions with gas atoms. We have recorded the spectra of the fluorescence light induced by laser excitation to each of the 2p₂-2p₉ states at various values of the discharge current. Herefrom the reaction coefficients for the atom induced transitions between the 2p-levels have been obtained in dependence on the discharge current. With the aid of our model for the positive column, we know the relation between the discharge current and the gas temperature so that the dependence of the reaction coefficients on the gas temperature can be found. The values of the transition
coefficients for \( T_g = 300 \) K are found by extrapolation from the measured values to 300 K. We have compared these values with those measured by Coolen (Coo76) in a proton-generated plasma at 300 K. The derivation of the gas temperature dependence of the coupling coefficients from the measurements with the aid of our model, can be considered as an application of our model to measurements on medium pressure discharges.

6.1 The measuring equipment

The experimental setup which has been used in these measurements is shown in fig. 6.2. An argon-ion laser supplies the power input for the

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**Fig. 6.2** The experimental setup. The arrow indicates the direction of translation of the discharge tube for the measurements of the radial profiles.

---

CW dye-laser. The dye-laser beam is chopped periodically for synchronous detection. The diameter of the laser beam and the optical properties of the detection system are such that the spatial resolution is better than 1 mm, so that deconvolution is not necessary. The intensity and the wavelength of the fluorescence light of the discharge are detected via a monochromator with a photomultiplier. In order to check and control the laser stability, half the laser beam intensity is used for a reference neon discharge. The radial profiles of the excited atom
densities have been measured by translation of the discharge tube perpendicular to the laser beam.

6.2 Measurement of the radial density profiles of the $^3P_2$ and $^3P_1$ ls-atoms

In figs. 6.3-6.5 the relative radial density profiles of the $^3P_2$ and $^3P_1$ ls-atoms (Paschen notation) are shown for various values of the discharge current. The calculated $^3P_2$ profiles are always identical to the calculated $^3P_1$ profiles. In the diffuse discharge the measured $^3P_2$ profiles are equal to the measured $^3P_1$ profiles. For the 20 mA diffuse discharge the agreement between the calculated and the measured profiles is good. For the 120 mA diffuse discharge the measured profiles are somewhat broader than the calculated ones. For this current value the excitation frequency is a strong function of $u_e(r)$ and $N_e(r)$ so that small errors in the calculation of these quantities may be responsible for the observed deviation. In the 120 mA constricted column, the measured $^3P_1$ profile is less narrow than the measured $^3P_2$ profile. The

![Graph showing the relative $^3P_2$ and $^3P_1$ densities for a 20 mA diffuse neon discharge.](image)

Fig. 6.3 $^3P_2$ and $^3P_1$ profiles for a 20 mA diffuse neon discharge ($p_o = 100$ torr, $R = 32$ mm).
Fig. 6.4 Same conditions as in fig.6.3, but \( i = 120 \text{ mA (diffuse)} \).

Fig. 6.5 Same conditions as in fig.6.3, but \( i = 120 \text{ mA (constricted)} \).

The latter is in good agreement with the calculations for small radial distances. For larger values of \( r \) the calculated profile decreases very
rapidly, probably due to the same factors as discussed above. The measured \( ^3P_1 \) profile is broader than the measured \( ^3P_2 \) profile. This deviation must be ascribed to our treatment of the radiation-diffusion term. In the constricted discharge the densities of the excited atoms decrease very rapidly at increasing radial distance. Then the approximations made in the treatment of the Holstein-radiation term are not valid, and the original radiation-diffusion equation must be taken into account to obtain a realistic calculation of the radial density profile of the \( ^3P_1 \) atoms.

6.3 Excitation transfer between the 2p-levels of neon by collisions with neon atoms

The coupling between the neon 2p-levels by collisions with neutral neon atoms has been studied experimentally by several authors (Par65, Gra75, Coo76). Here we investigate the temperature dependence of the coupling coefficients, by measuring these coefficients at various values of the discharge current. The relation between the gas temperature and the discharge current is obtained from the model described in chapter 3.

6.3.1 The determination of the coupling coefficients

When atoms in one of the 1s-levels are excited to a level in the 2p-state by light from the dye-laser, other 2p-levels are also extra populated because of the collisional transfer of excitation. Hence the fluorescence light is not only composed of radiative transitions from the originally excited 2p-level, but also from other 2p-levels. To determine the reaction-coefficients for the coupling between the 2p-levels, each level has to be excited by the laser light and all the spectral lines of the levels have to be measured. When there are \( N \) levels, each level couples to \( N-1 \) other levels, so that there are \( N(N-1) \) coupling coefficients.

When level \( 2p_j \) is excited by the laser, level \( 2p_i \) is also populated via collisional coupling, so that the balance equation for level \( 2p_i \) is (\( i \neq j \)):

\[
\sum_{l \neq i} k_{l \rightarrow i} N_l \frac{N_i}{N} = \sum_{l \neq i} k_{i \rightarrow l} N_l \frac{N_i}{N} + N_i \left( \nu_i \right). \tag{6.1}
\]

Here \( \nu_i \) = total decay frequency of level \( i \) by radiation,
\[ k_{li} = \text{coefficient for coupling of level } l \text{ to level } i. \]

\[ N_{ij} = \text{density of atoms in the } 2p_i \text{ level, arising from laser} \]

\[ \text{excitation to the } 2p_j \text{ level.} \]

Eq. (6.1) is derived by subtraction of the density equation for level i for the laser-off situation from the equation for level i for the laser-on situation; \( N_{ij} \) is the difference of the densities of level i in the laser-on and laser-off situations, measured with the synchronous detection technique. In this way the term representing the production of \( 2p_i \) atoms by electrons in the discharge cancels. The optical signal \( S_{ij} \) of all lines arising from the decay of level \( 2p_i \) is:

\[ S_{ij} = v_i N_{ij} \quad (6.2) \]

The values of \( v_i \) have been obtained from Wiese et al. (Wic66). Hence the values of \( N_{ij} \) are obtained from the emission spectrum caused by laser excitation to level j. After renumbering the coupling coefficients according to \( k_1 = k_{12}, k_2 = k_{13}, \) etc., eq. (6.1) can be written as:

\[ A \cdot k = v, \quad (6.3) \]

where the \( N(N-1) \times N(N-1) \) matrix \( A \) consists of elements \( N_{ij} \), and the vector \( v \) consists of elements \( v_i N_{ij} \). Eq. (6.3) has been inverted numerically according to:

\[ k = A^{-1} \cdot v. \quad (6.4) \]

The measured spectra have been corrected for the relative wavelength sensitivity of the optical system in the way described by Coolen (Coo76).

Because of the restricted wavelength region of the dye-laser, we were not able to excite the \( 2p_i \) and \( 2p_{10} \) levels. Because the coupling of the \( 2p_{10} \) level to the \( 2p_1-2p_9 \) levels is very weak (because of the large energy gap), and the coupling of the \( 2p_2-2p_{10} \) levels to the \( 2p_1 \) level is also weak (for the same reason), we are able to determine the coupling coefficients between levels \( p_2-p_{10} \) by laser excitation to the levels \( p_2-p_9 \) (we insert \( k_{10,1} = 0 \) into eq. (6.1)). Then we have 64 equations with 64 unknown parameters \( k_{li} \) (as \( k_{ii} \) is not appearing in the equations, because \( j \neq i \)).
In this way we measured the set of values $k_{k_1}$ for different values of the discharge current on the axis of a neon discharge ($p_0 = 100$ torr, $R = 32$ mm). The dependence of the coefficients $k_{k_1}$ on the discharge current must be ascribed to a dependence of $k_{k_1}$ on the gas temperature, which increases at increasing current, as the electron coupling effect (mentioned in Sau75a) is very improbable as proven by the maximum value of the theoretically calculated cross-section for this effect (Mas69). That the coupling coefficients may depend on the gas temperature is shown in the next section. In section 6.3.3 the experimentally obtained dependences are given.

6.3.2 The temperature dependence of the coupling coefficients

The cross-section for excitation transfer at small energy gaps depends on whether the potential energy curves of the initial and final states of the atoms have a crossing point or not (Mas71). In the first case the cross-section depends on the values of the derivatives of the potential energy curves at the crossing point. Without direct knowledge of these potential energy curves for the 2p-levels of neon no estimation can be given for this case. In the case that there is no crossing point several approximations are possible (Mas71). As for the xenon molecule both cases are possible, we assume that also for neon the approximation that there is no crossing point will hold in several relevant situations. In that case Massey gives for the cross-section (Mas71):

$$Q = 2\pi \int_0^\infty 2p(1-p) \, ds,$$  \hfill (6.5)

where $p$ = the probability of a transition and $s$ = the impact parameter.

Here, $p = F(s)^2 \sin^2 \Omega_c$,

with

$$F(s) = \frac{2}{\Gamma(\frac{m-1}{2})} \left( \frac{s \Delta E}{h \nu} \right)^{\frac{m-1}{2}} k_{m-1} \left( \frac{s \Delta E}{h \nu} \right)^{\frac{m-1}{2}},$$

and

$$\Omega_c = \left( \frac{\alpha}{h \nu s} \right)^{1/2} \frac{\Gamma(\frac{m-1}{2})}{\Gamma(\frac{m}{2})},$$  \hfill (6.6)
where $\Delta \xi = $ energy gap, $v = $ relative velocity of the atoms, $\hbar = $ Dirac constant, $\gamma = $ gamma function, $K = $ modified Bessel-function, $\alpha = $ a constant depending on the kind of atom, $m = $ an integer depending on the kind of transition.

(this approximation holds when $\frac{\Delta \xi}{hv} = $ not so large that $F(s) \ll 1$).

In our case (neon 2p-states, $m = 5$), $Q_o$ is so large for all relevant values of $s$, that $\sin^2 Q_o$ in the integral can be replaced by its mean value $\frac{1}{2}$. Then eq.(6.5) can be written as:

$$Q = Q_c \frac{v^2}{\Delta \xi^2} \quad (C = \text{a constant}). \quad (6.7)$$

The coupling coefficient for coupling between two levels is:

$$k = \int_0^\infty v Q_f(v) 4\pi v^2 dv. \quad (6.8)$$

Substitution of a maxwellian distribution function for the velocity distribution of the gas atoms, and eq.(6.7) for $Q$ yields:

$$k \sim \frac{T_g^{3/2}}{\Delta \xi^2}. \quad (6.9)$$

Hence the expected dependence of the coupling coefficients on the gas temperature is of the form (6.9). We point out, however, that this expression is derived from approximation (6.5), (6.6) for $Q$, which is only valid for the case of no crossing point.

6.3.3 Measured values of the coupling coefficients

From the measurements, we obtain after a numerical analysis according to eq.(6.4) the values of $k_{\xi i} N_i$ for $i = 10, 40, 60, 90$ and $110 \text{ mA}^{(\text{diffuse column})}$. From the model of chapter 3, we obtain the gas density $N_g$ and the gas temperature $T_g$ in dependence of the discharge current (fig. 6.6). Hence we can derive from the measurements, using the model calculations, the dependences of the various $k_{\xi i}$ on $T_g$. It appeared that all $k_{\xi i}$ could be written as:

$$k_{\xi i} = k_{\xi i}^* \times \left( \frac{T_g}{300} \right)^n. \quad (6.10)$$

The values $k_{\xi i}^*$ are the (extrapolated) values for $T_g = 300 \text{ K}$, which can be compared with those of Coolen (Co076) (see table 6.1).
<table>
<thead>
<tr>
<th>j</th>
<th>i</th>
<th>n</th>
<th>( k_{ji}^* \times 10^{17} ) (this work)</th>
<th>( k_{ji}^* \times 10^{17} ) (Coo76)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
<td>0.040 ± 0.010</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0.25 ± 0.15</td>
<td>0.15 ± 0.07</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>0.030 ± 0.030</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2</td>
<td>0.026 ± 0.020</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2</td>
<td>0.034 ± 0.020</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0.10 ± 0.03</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>15.0 ± 2.0</td>
<td>14.4 ± 3.2</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1</td>
<td>0.80 ± 0.30</td>
<td>0.62 ± 0.20</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>0.50 ± 0.20</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>2.5±1.0</td>
<td>0.10 ± 0.07</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>0.05 ± 0.02</td>
<td>0.08 ± 0.04</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2.6 ± 1.0</td>
<td>1.7 ± 0.4</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1.7 ± 0.5</td>
<td>2.1 ± 0.4</td>
</tr>
<tr>
<td>6</td>
<td>1.5</td>
<td>0.17 ± 0.07</td>
<td>0.29 ± 0.10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.5</td>
<td>0.85 ± 0.35</td>
<td>0.22 ± 0.08</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0.3 ± 0.2</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2.5</td>
<td>0.015 ± 0.015</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>0.15 ± 0.05</td>
<td>0.09 ± 0.04</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td>1.9 ± 1.0</td>
<td>2.3 ± 0.4</td>
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<tr>
<td>6</td>
<td>2</td>
<td>0.03 ± 0.01</td>
<td>-</td>
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<tr>
<td>7</td>
<td>1</td>
<td>0.40 ± 0.20</td>
<td>0.54 ± 0.09</td>
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<tr>
<td>6</td>
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<td>2.5</td>
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<tr>
<td>7</td>
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<td>0.90 ± 0.20</td>
<td>0.85 ± 0.16</td>
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<td>8</td>
<td>0.5</td>
<td>0.30 ± 0.20</td>
<td>0.28 ± 0.09</td>
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<tr>
<td>9</td>
<td>2.5</td>
<td>0.025 ± 0.012</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0.08 ± 0.07</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>7</td>
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<td>2</td>
<td>0.12 ± 0.03</td>
<td>-</td>
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<tr>
<td>5</td>
<td>1</td>
<td>0.12 ± 0.03</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1.4 ± 0.4</td>
<td>0.60 ± 0.13</td>
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</tr>
<tr>
<td>8</td>
<td>1</td>
<td>0.80 ± 0.20</td>
<td>0.63 ± 0.12</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>0.70 ± 0.20</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0.40 ± 0.15</td>
<td>0.27 ± 0.09</td>
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</table>
Table 6.1 (continued)

<table>
<thead>
<tr>
<th>j</th>
<th>i</th>
<th>n</th>
<th>$k_{ji} \cdot 10^{17}$ (this work)</th>
<th>$k_{ji} \cdot 10^{17}$ (Co76)</th>
</tr>
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<tbody>
<tr>
<td>8</td>
<td>4</td>
<td>2.5</td>
<td>$0.030 \pm 0.025$</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td></td>
<td>$0.26 \pm 0.05$</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>2.5</td>
<td></td>
<td>$0.05 \pm 0.03$</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td></td>
<td>$2.6 \pm 0.6$</td>
<td>$1.9 \pm 0.4$</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td></td>
<td>$0.6 \pm 0.2$</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>1</td>
<td>$0.06 \pm 0.01$</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td></td>
<td>$0.90 \pm 0.20$</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>1.5</td>
<td></td>
<td>$1.05 \pm 0.15$</td>
<td>$0.55 \pm 0.10$</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td></td>
<td>$4.4 \pm 0.3$</td>
<td>$2.6 \pm 0.4$</td>
</tr>
</tbody>
</table>

Figure 6.6 The calculated gas temperature $T_g$ and the gas density $N_g$ as a function of the discharge current in a 100 torr neon discharge at the tube axis. (Tube radius 32 mm.)
In Figs. 6.7 and 6.8 some dependences are given derived from the measured points.

The agreement with the values of Coolen is satisfying (see Table 6.1). In some cases ($k_{47}$, $k_{76}$, $k_{98}$, $k_{910}$) there exist some discrepancies. In view of the rather large standard deviations these particular values must be obtained from the weighted average (with weighting factors chosen according to the largeness of the standard deviations) over the values of Coolen and our values.

The gas temperature dependence of the coupling coefficients is in reasonable agreement with the predicted behaviour for many coefficients (following eq. (6.9) an exponent $n = 1.5$ is expected). The deviations may be due to the assumption which has been used here, that the potential curves have no crossing point.

In conclusion we may say that the dependence of the coupling coefficients on the discharge current must be due to this gas temperature.
Fig. 6.8 Measured dependence of $k_{ji}$ on $T_g$ for $j = 5$.

dependence rather than the electron induced transitions suggested by us in an earlier publication (Smi75a).
Concluding remarks

1 The model presented here gives a fair description of the diffuse column. The constriction can be described by incorporating the striation development above certain current and pressure thresholds. In the constricted column the E.F. is still strongly non-maxwellian in the inelastic region. Furthermore ambipolar diffusion cannot be neglected in the constricted column.

2 From the calculated quantities which have been experimentally verified, the axial electric fieldstrength is most sensitive to variations in the coefficients of the model. Especially the value of the elastic electron-atom cross-section is important, because the axial electric fieldstrength is proportional to this cross-section. To fit the calculated values of the axial electric fieldstrength to the measured ones, it was necessary to diminish the measured curve for the elastic cross-section of Salop and Nakano (Sal70) with 20%. This corresponds to the lowest measured values for this cross-section given in the literature (see Sal70). The other data from the literature which have been used for the model have less influence on the model results, and have not been altered.

3 In the calculation of the E.F. several effects have been taken into account which are commonly neglected. The influence of the gas temperature term in the Boltzmann-equation on the excitation frequency has been taken into account. The axial electric fieldstrength has been calculated in a consistent way via the definition of the electron temperature. Also the particular integral $J_2$ has been treated more consistently than is usually done.

4 The radial profiles of the electron density and the electron temperature could be measured from the electron-atom bremsstrahlung continuum. In the determination of the electron temperature there may have been a disturbing influence from the molecular radiation.

5 The reaction coefficients for atom induced transitions between the 2p-levels on neon were measured as a function of the discharge current. Herefrom the dependence on the gas temperature was determined. The relation between the gas temperature and the discharge current was calcu-
lated with our numerical model. A theoretical estimate showed that the coupling coefficients should be proportional to $T_g^{3/2}$ for a special case. The measured dependences are $T_g^n$, with $0 < n < 2.5$. 
### Appendix I

#### List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1) adaptation constant for the E.F. in the inelastic region, 2) order of magnitude of the quotient of the radial derivative term and the elastic collision-term in the Boltzmann-equation.</td>
</tr>
<tr>
<td>A(ε)</td>
<td>approximation for $A_1(ε)$ and $A_2(ε)$.</td>
</tr>
<tr>
<td>$A_1(ε), A_2(ε)$</td>
<td>Coulomb integrals.</td>
</tr>
<tr>
<td>$A_1, A_2$</td>
<td>constants in the approximation for the inelastic cross-section.</td>
</tr>
<tr>
<td>$A_i$</td>
<td>transition probability for line radiation from level $i$.</td>
</tr>
<tr>
<td>$a$</td>
<td>acceleration due to the applied electric field ($a = \frac{eE}{m}$).</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>quotient of field frequency and Coulomb frequency ($\omega / \nu_ee$).</td>
</tr>
<tr>
<td>$a_{1/2}$</td>
<td>parameter in dispersion relation.</td>
</tr>
<tr>
<td>$a_1, a_2$</td>
<td>constants in approximation for $q(ε)$.</td>
</tr>
<tr>
<td>$a_D, a_{el}$</td>
<td>constants in approximation for the momentum transfer cross-section and the elastic cross-section.</td>
</tr>
<tr>
<td>$a_i$</td>
<td>$A_i/\sqrt{A_4}$.</td>
</tr>
<tr>
<td>$A$</td>
<td>adaptation constant for the E.F. in the inelastic region.</td>
</tr>
<tr>
<td>$b$</td>
<td>1) quotient of elastic collision frequency and Coulomb frequency ($\omega / \nu_ee$). 2) parameter in dispersion relation.</td>
</tr>
<tr>
<td>$b_1, b_2$</td>
<td>constants in approximation for $q(ε)$.</td>
</tr>
<tr>
<td>$b_{1/2}$</td>
<td>electron mobility.</td>
</tr>
<tr>
<td>$b_{mi}$</td>
<td>mobility of the molecular ions.</td>
</tr>
<tr>
<td>$C$</td>
<td>adaptation constant for the E.F. in the inelastic region.</td>
</tr>
<tr>
<td>$C(ε)$</td>
<td>collision operator in the Boltzmann-equation.</td>
</tr>
<tr>
<td>$C(N_e/N_g)$</td>
<td>correction factor in the equation for the axial electric fieldstrength.</td>
</tr>
<tr>
<td>$c$</td>
<td>1) constant in the parabolic approximation for the radial gas temperature profile, 2) parameter in dispersion relation, 3) light velocity.</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$C_{el}$, $C_{in}$, $C_{ee}$</td>
<td>Collision operators in the $l=0$ equation for the elastic, inelastic and electron-electron collisions respectively.</td>
</tr>
<tr>
<td>$C_0(r)$</td>
<td>Correction factor to obtain the electron density from the bremsstrahlung intensity.</td>
</tr>
<tr>
<td>$C_1$, $C_2$, $C_3$, $C_4$</td>
<td>Collision operators in the $l=1$ equation for the elastic, inelastic and electron-electron collisions respectively.</td>
</tr>
<tr>
<td>$C_1$</td>
<td>1) Constant in the expression for the bremsstrahlung intensity. 2) Constant in the numerical approximation for $N_{e}(r)$ for $r &gt; r_1$ (outer region).</td>
</tr>
<tr>
<td>$C_1$, $C_2$</td>
<td>Constants in approximate expression for $Z$ in high electron density limit.</td>
</tr>
<tr>
<td>$C_1$, $C_2$</td>
<td>1) Parameters in expression for $Z$ for argon, 2) Constants in two-temperature model.</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Correction factor in the equation for the ambipolar diffusion coefficient.</td>
</tr>
<tr>
<td>$C_{2i}$, $C_{3j}$, $C_{4i}$</td>
<td>Reaction coefficients for excitation from level $i$ in group I to a level in group II, III or IV respectively by electrons.</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$l$th-order tensor in the development of $C(f)$.</td>
</tr>
<tr>
<td>$C_{mi}$</td>
<td>Reaction coefficient for electron excitation from the metastable $3p_2$ level (group I) to group IV for argon.</td>
</tr>
<tr>
<td>$C_N$</td>
<td>Correction factor in the equation for $N_{eo}$</td>
</tr>
<tr>
<td>$D_a$</td>
<td>Diffusion coefficient of the electrons.</td>
</tr>
<tr>
<td>$D_e$</td>
<td>Axial electric fieldstrength.</td>
</tr>
<tr>
<td>$e$</td>
<td>Elementary charge.</td>
</tr>
<tr>
<td>$E_{ij}$</td>
<td>Energy gap between levels $i$ and $j$.</td>
</tr>
<tr>
<td>$e_z$</td>
<td>Unit vector in axial direction.</td>
</tr>
<tr>
<td>$\Delta_e$</td>
<td>Energy gap.</td>
</tr>
<tr>
<td>$F_1$</td>
<td>Correction factor in expression for $Z$.</td>
</tr>
<tr>
<td>$F_2$</td>
<td>Correction factor for the gas temperature in the expression for $Z$.</td>
</tr>
<tr>
<td>$F(s)$</td>
<td>Factor in the expression for the cross-section for excitation transfer between the 2p-levels</td>
</tr>
<tr>
<td>$F(U)$</td>
<td>Isotropic part of the E.F. (energy coordinates).</td>
</tr>
<tr>
<td>$f$</td>
<td>1) Electron energy distribution function, 2) Striation frequency.</td>
</tr>
</tbody>
</table>
\[ J_1(\varepsilon) \] first order term in the development of \( f \).

\[ J_2(\varepsilon) \] second order tensor in the development of \( f \).

\[ G(\varepsilon) \] expression in the scalar equation.

\[ S_{ij} \] expression in the scalar equation.

\[ n \] number of discretization intervals.

\[ h \] Planck constant.

\[ \hbar \] Dirac constant.

\[ i(\lambda) \] measured bremsstrahlung intensity after Abel transformation.

\[ J \] current density

\[ J(p) \] integrals over the E.F.

\[ J(U,e,N_e/N,g,\lambda) \] expression in the bremsstrahlung intensity.

\[ J_1(\varepsilon) \] homogeneous solution of the scalar equation for \( \varepsilon < \varepsilon_1 \).

\[ J_2(\varepsilon) \] particular solution of the scalar equation for \( \varepsilon < \varepsilon_1 \).

\[ J_2^*(\varepsilon) \] approximation for \( J_2(\varepsilon) \) in the neighbourhood of \( \varepsilon = \varepsilon_1 \).

\[ J_1,J_2 \] total quantum numbers of levels \( i \) and \( j \).

\[ k \]

1) Boltzmann constant,

2) number of discretization intervals,

3) wave number.

\[ k_{ij}, k_{ij}^* \] parameter in dispersion relation.

\[ k_{ij}^d, k_{ij}^e \] coupling coefficient for transfer of excitation between levels \( i \) and \( j \) (\( k_{ij}^* \) at 300 K) (group I, chapter 3 and group II, chapter 6).

\[ k_{ij} \] atomic and electronic part of \( k_{ij} \) (group I, chapter 3).

\[ K_n(x) \] modified Bessel function of \( n \)th order.

\[ L \] tube length.

\[ \ell \] rank of the tensors.

\[ M \] atomic mass.

\[ M(\varepsilon) \] exponent in the expression for \( J_1(\varepsilon) \)

\[ m \]

1) electron mass,

2) integer constant in the expression for the cross-section for excitation-transfer between the 2p-levels.

\[ N \] normalization constant.

\[ N_1 \] \( 2\pi \hbar^3N \).
1) exponent in the expression for the elastic cross-section,
2) exponent in the gas temperature dependence of the measured coupling coefficients for excitation transfer between the 2p-levels.

\( n(x) \)
relative electron density profile \((N_e(x)/N_e(0))\).

\( N_e(x) \)
electron density as a function of the radial distance.

\( N_e(0) \)
electron density at the axis.

\( N_{g0} \)
gas density.

\( N_{gO} \)
gas density at the axis.

\( N_{gO} \)
gas density in the discharge-off situation.

\( N_R \)
density of the atoms in level \( R \), representing group II.

\( N_i, N_j \)
densities of atoms in levels \( i \) and \( j \).

\( N \)
density of atoms in the 2p\(_i\)-level arising from laser excitation to the 2p\(_j\)-level.

\( C(x) \)
order of magnitude \( x \).

\( p \)
1) pressure in the active discharge,
2) argument of \( \tilde{v}(p) \),
3) probability in the expression for the cross-section for the coupling coefficients of the 2p-levels.

\( P(\xi) \)
expression in the scalar-equation for \( \xi > \xi_1 \).

\( P_0 \)
filling pressure.

\( P_1, P_2 \)
pressure in the discharge when the dead volume is large respectively small.

\( P_{el}, P_{in} \)
elastic and inelastic energy losses per electron.

\( P_{gas} \)
gas term in the electron energy equation.

\( Q \)
cross-section.

\( q(\xi) \)
expression in the scalar-equation for \( \xi > \xi_1 \).

\( Q_s \)
bremsstrahlung cross-section.

\( Q_0 \)
momentum transfer cross-section.

\( Q_{el} \)
elastic cross-section.

\( Q_{i,j} \)
cross-section for electronic excitation from level \( i \) in group I to group IV.

\( Q_{j,4} \)
cross-section for electronic excitation from a level in group I to group IV.

\( Q_0 \)
total inelastic cross-section.

\( R \)
tube radius.

\( r \)
radial distance.
Characteristic column radius.

\( r_1 \)

Ratio of the bremsstrahlung intensities at the wavelengths 2500 A and 4000 A.

\( R_a(U_0/N_g/N_g) \)

Impact parameter.

\( s \)

Expression in the scalar equation for \( \varepsilon > \varepsilon_1 \).

\( S(\varepsilon) \)

Integral in the equation for \( N_{e0} \).

\( S_1 \)

Integral in the equation for \( p \).

\( S_2 \)

Optical signal of all lines arising from the decay of level 2p_j in the case of laser excitation to level 2p_j time.

\( t(x) \)

Relative gas temperature profile \( (T_g(r)/T_g(0)) \).

\( t(\varepsilon) \)

Term in the scalar equation.

\( T_0 \)

Gas temperature at the discharge axis.

\( T_1, T_2 \)

1) Average temperatures in the discharge with small respectively large dead volume,
2) Terms in the dispersion relation.

\( t_1 \)

Characteristic time constant for elastic collisions.

\( T_e \)

Electron temperature.

\( T_{env} \)

Temperature of the environment.

\( T_g \)

Gas temperature.

\( T_{wall} \)

Wall temperature.

\( U \)

Electron energy.

\( U(\varepsilon) \)

Expression in the calculation of the E.F. for \( \varepsilon > \varepsilon_1 \).

\( u \)

Parameter in the E.F. for argon.

\( U_1 \)

Energy gap between group I and groups II or III.

\( U_{e0} \)

Energy values in the approximation for the inelastic cross-section of neon.

\( U_2 \)

Energy gap between group I and group IV.

\( U_e \)

Electron temperature in volt equivalents.

\( U_{eo} \)

Electron temperature at the discharge axis.

\( U_T \)

"Tail-temperature" in the two temperature models.

\( v \)

1) Electron velocity,
2) \( T_g/T_e \).

\( V_1, V_2 \)

Values of the volume of the discharge tube and the dead volume respectively.

\( v_a \)

Electron velocity belonging to the value of \( U_a \).

\( v_d \)

Electron drift velocity.

\( v_g \)

Striation group velocity.
electron velocity belonging to the value of $v_1$.

$v_\|$, striation phase velocity.

$v_p$, thermal velocity of the electrons.

$x$ 1) $r/R$,

2) axial distance,

3) $\frac{hc}{eU_j}$ in the bremsstrahlung expression.

$y$ expression in $M(\varepsilon)$.

$z$ excitation frequency (total).

$z$ dimensionless integration variable.

$Z_i$ excitation frequency (for excitation to level $i$ in group $I$).

$z_{ion}$ ionization frequency.

$z_{n}, z_{u}$ expressions in the dispersion relation.

$\alpha$ 1) volume-recombination coefficient,

2) constant in the expression for the cross-section for excitation transfer between the $2p$-levels.

$\beta_0$ heat transfer coefficient.

$\Gamma(n)$ gamma-function.

$\nabla_r, \nabla_v$ gradient in the geometrical respectively velocity space.

$\varepsilon$ 1) $U/U_e$,

2) emission coefficient.

$\varepsilon(\lambda, \tau)$ measured bremsstrahlung intensity without Abel correction.

$\varepsilon_0$ permittivity of vacuum.

$\varepsilon_1, \varepsilon_2$ $U_1/U_e$, $U_2/U_e$.

$\varepsilon_4$ $U_a/U_e$.

$\phi_1, \phi_2, \phi_3$ expressions in the E.P. for $\varepsilon > \varepsilon_1$.

$\phi_{1j}, \phi_{1j}'$ $\phi_i(\varepsilon_j)$, $d\phi_i/d\varepsilon$ $\varepsilon_j$.

$\gamma, \gamma_i$ three body collision coefficient for argon respectively the neon $^3P_2$ and $^3P_0$ atoms.

$\lambda(T_g)$ heat conductivity.

$\lambda$ 1) wavelength of light,

2) wavelength of striations.

$\lambda_0$ coefficient in $\lambda(T_g)$.

$\lambda_1, \lambda_2$ $2500$ resp. $4000$ Å.

$\lambda_i$ wavelengths of the radiation emitted by the $^3P_1$ and $^1P_1$ atoms.

$\lambda_r$ relaxation length of the electron temperature.

$\nu_\varepsilon$ frequency representing the field influence in the Boltzmann-equation.
$v_E^z$ coefficient in $v_E$.

Coulomb-interaction frequency.

$\nu_{ee}$ elastic collision frequency.

$\nu_g^x$ coefficient in $\nu_g$.

$\nu_i$

1) decay frequency of level $i$ in group I.

2) frequency of the emitted light at the fluorescence measurements.

$\nu_{in}$ inelastic collision frequency.

$\omega$ striation frequency in the dispersion relation.

$\omega_i$ imaginary part of $\omega$.

$\omega_r$ real part of $\omega$.

$\Omega_0$ expression in the cross-section for excitation transfer between the $2g$-levels.

$\zeta_n, \zeta_u$ expressions in the dispersion relation.

$\sigma$ Stefan-Boltzmann constant.

$\tau_i$ decay time of level $i$ in group I.
Appendix II Compilation of data used for the model

Several coefficients and cross-sections have been used for the calculations on the E.F. and for the numerical model. These values have been obtained from measurements reported in the literature. Here we give a compilation of these data.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value or formula</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i-a_j$</td>
<td>0.40, 0.27, 0.08, 0.25</td>
<td>Coo76</td>
</tr>
<tr>
<td>$b_{mi}$</td>
<td>$6 \times 10^{-4}$</td>
<td>McD64</td>
</tr>
<tr>
<td>$k_{ij}$ (ch.3)</td>
<td>see appendix 3.11</td>
<td>Phe59</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>0.95 $Q_{el}$</td>
<td>McD64</td>
</tr>
<tr>
<td>$Q_{el}$</td>
<td>$\begin{cases} 1.8 \times 10^{-20} U^{1/4} &amp; U \leq U_1 \equiv 16.6V \ 1.8 \times 10^{-20} U_1^{1/4} &amp; U &gt; U_1 \end{cases}$</td>
<td>analytical approximation on data given by Sal70</td>
</tr>
<tr>
<td>$Q_e$</td>
<td>$\begin{cases} 0 &amp; 0 \leq U \leq U_1 \ 6.10^{-22} (U-U_1) &amp; U_1 &lt; U &lt; U_2 \equiv 17.0V \ 6.10^{-22} (U_2-U_1) + 0.5 \times 10^{-22} (U-U_2) &amp; U &gt; U_2 \end{cases}$</td>
<td>analytical approximation on data given by Sch69</td>
</tr>
<tr>
<td>$Q_{i4}$</td>
<td>$\begin{cases} 1.2 \times 10^{-20} (U-5) &amp; 5 \leq U &lt; 10V \ (6.44-0.0444U) \times 10^{-20} &amp; 10 \leq U &lt; 100 V \end{cases}$</td>
<td>analytical approximation on data given by Dix73</td>
</tr>
<tr>
<td>$Q_{i,4}$</td>
<td>$\frac{1}{2} Q_{i4}$</td>
<td></td>
</tr>
<tr>
<td>$Q_{1,2}'$</td>
<td>$Q_{i,3}$</td>
<td>$\begin{cases} 0.64 \times 10^{-18} \frac{U-U_1}{U_1^2} &amp; 0 \leq U &lt; U_1 \ 0 &amp; U &gt; U_1 \end{cases}$</td>
</tr>
<tr>
<td></td>
<td>with $U_1 = 2$ for $Q_{i,2}'$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$U_1 = 3$ for $Q_{i,3}'$</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$2.5 \times 10^{-14} \frac{1}{U_{c}^{1/2}} (1-exp(-\frac{U}{T_s}))$</td>
<td>Mas74</td>
</tr>
<tr>
<td>$\lambda(T_s)$</td>
<td>$10^{-3} \frac{T_s^{2/3}}{T_s}$</td>
<td>Sax68</td>
</tr>
<tr>
<td>$\nu_1$ (ch.3)</td>
<td>see appendix 3.11</td>
<td>Phe59</td>
</tr>
<tr>
<td>$\nu_1$ (ch.6)</td>
<td>see reference</td>
<td>Wie66</td>
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</table>
### Argon

<table>
<thead>
<tr>
<th>Quantity</th>
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<th>Reference</th>
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<tbody>
<tr>
<td>$b_{mi}$</td>
<td>$2.10^{-4}$</td>
<td>McD64</td>
</tr>
<tr>
<td>$c_{mi}$</td>
<td>$5.3 \times 10^{-14}$</td>
<td>calculated from classical:</td>
</tr>
<tr>
<td></td>
<td>$(1.8u_e - 3.8) \times 7.5 \times 10^{-14}$</td>
<td>Thomson cross-section (Mit73)</td>
</tr>
<tr>
<td></td>
<td>$u_e \leq 2.5v$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$7.5 \times 10^{-14}$</td>
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</tr>
<tr>
<td></td>
<td>$u_e &gt; 2.5v$</td>
<td></td>
</tr>
<tr>
<td>$Q_D$</td>
<td>$1.4 \times 10^{-20}u_1$</td>
<td>Woj65b</td>
</tr>
<tr>
<td></td>
<td>$0 \leq u \leq u_1 \leq 11.5v$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1.4 \times 10^{-20}u_1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$u &gt; u_1$</td>
<td></td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$0$</td>
<td>Woj65b</td>
</tr>
<tr>
<td></td>
<td>$0 \leq u \leq u_1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.5 \times 10^{-21}(u-u_1)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$u &gt; u_1$</td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>$8.6 \times 10^{-14}$</td>
<td>Mas74</td>
</tr>
<tr>
<td></td>
<td>$u_0^{2/3} \left(1 - \exp\left(-\frac{550}{T_g}\right)\right)$</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$3.6 \times 10^{-45}$</td>
<td>Mas74</td>
</tr>
<tr>
<td>$\lambda(T_g)$</td>
<td>$4.17 \times 10^{-14}T_g^{2/3}$</td>
<td>Vru76, Woj67</td>
</tr>
</tbody>
</table>
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Summary

In this thesis we describe a model for the positive column of a medium pressure neon discharge, together with optical measurements to test the model. Furthermore the constriction of the positive column above certain thresholds of the current and the filling pressure is studied. The pressure-current region of our concern is: \(0.50 < \frac{p_o}{R} < 10\) torr m and \(0.01 < \frac{i}{R} < 10\) A/m \((p_o =\) filling pressure, \(i =\) discharge current, \(R =\) tube radius).

We present a numerical model for the positive column of the neon discharge in this parameter domain. For this model we need the electron energy distribution function, which is calculated from the Boltzmann-equation. From this calculation we obtain an expression for the axial electric field strength via the definition of the electron temperature in a consistent way. Furthermore the excitation frequency is calculated, taking into account the effect of the gas temperature term in the Boltzmann-equation.

The axial electric field strength and the excitation frequency are used in the numerical model set up for the positive column. The results based upon the model are in good agreement with experiments for the diffuse discharge. The constriction of the column which occurs abruptly above certain thresholds of the discharge current and the filling pressure can not be calculated without modifications. From a variation of the parameters of the model it appears that the constriction is started by the onset of ionization waves (striations). A comparison with other models presented in the literature is made.

The results of the model are compared with experimental values for the electron density, the electron temperature and the densities of the metastable \(^3P_2\) \((1S_g)\) and resonant \(^3P_1\) \((1s_4)\) atoms (Paschen notation). The radial profiles of the electron density and the electron temperature have been obtained from measurements of the electron-atom bremsstrahlung continuum. The measured electron density profiles are in good agreement with the calculated ones, both for the diffuse and for the constricted column (the latter calculated with the striation model). Between the calculated and the measured electron temperature profiles there is an increasing discrepancy with increasing current. Possibly this discrepancy is due to a contribution of molecular radiation to the continuum. The density profiles
of the $^{3}P_2$ and $^{3}P_1$ atoms have been measured with the fluorescence technique, using a dye-laser. The measured profiles are in reasonable agreement with the calculated ones for the diffuse discharge. For the constricted column, the measured $^{3}P_1$ profile is less narrow than the calculated profile. This must be due to our simplified treatment of the radiation-diffusion term (this treatment is, however, sufficiently accurate for the calculation of other data of the column).

The numerical model has been used for the evaluation of measurements of the reaction coefficients for atom induced transitions between the 2p-levels in neon. These measurements were performed for several values of the discharge current. Herefrom the coupling coefficients have been determined as a function of the gas temperature, where the relation between the gas temperature and the discharge current was calculated with the help of our model. Theoretical estimates show that the dependence of the coupling coefficients on the gas temperature $T_g$ can in some cases be written as $T_g^{3/2}$. The measurements show a dependence according to $T_g^n$, with $0 \leq n \leq 2.5$ (dependent on the levels between which the transition takes place).
Samenvatting

In dit proefschrift wordt een model voor de positieve kolom van een neon-ontlading van gematigde druk beschreven, samen met optische metingen ter toetsing van dit model. Verder wordt de contractie van de positieve kolom boven bepaalde waarden van stroomsterkte en vuldruk bestudeerd. Het onderzochte stroomdrukgebied is: $0.50 < p_0 R < 10$ torr m en $0.01 < i/R < 10$ A/m ($p_0 =$ vuldruk, $i =$ ontladingstroomsterkte, $R =$ buisstraal).

Voor dit gebied is een numeriek model voor de positieve kolom van de neon-ontlading opgesteld. Ten behoeve van dit model is kennis nodig van de energieverdelingsfunctie voor de electronen. Deze is berekend uit de Boltzmann-vergelijking. Hieruit wordt een consistente manier van een uitdrukking voor de elektrische veldsterkte afgeleid, via de definitie van de electronenenergieverdeling. Ook is de excitatie frequentie berekend: daarbij wordt rekening gehouden met de term in de Boltzmann-vergelijking, die de invloed van de gastemperatuur beschrijft.

De axiale elektrische veldsterkte en de excitatie-frequentie worden gebruikt in het numerieke model dat is opgesteld voor de verschijnselen in de positieve kolom. De resultaten van het model zijn in goede overeenstemming met experimenten voor de diffuse ontlading. De abrupte contractie van de kolom boven bepaalde drempelwaarden van vuldruk en stroomsterkte kan niet zonder modificaties in het model berekend worden. Uit een variatie van de modelparameetrs lijkt de conclusie gerechtvaardigd dat de contractie gestart wordt door de ontwikkeling van ionisatiegolven (striaties). Het model wordt vergeleken met andere modellen uit de literatuur.

De modelresultaten worden vergeleken met experimentele waarden van de electronendichtheid, de electronenenergieverdeling en de dichtheden van de metastabiele $^3P_2$ ($1s_5$) en resonante $^3P_1$ ($1s_4$) atomen (Paschen notatie). De radiale profielen van de electronendichtheid en de electronenergieverdeling zijn verkregen uit metingen van het electron-atoom remstralingscontinuum. De gemeten electronendichthedsprofielen zijn in goede overeenstemming met de berekende, zowel voor de diffuse als voor de gecontraheerde kolom (de laatste is berekend met het striaties-model). Er is een toenemende afwijking tussen gemeten en berekende electronenergieverdeling- profielen bij toenemende stroomsterkte. Het is mogelijk dat deze afwijking te verklaren valt uit een eventuele bijdrage van moleculaire straling tot het continuum.

De dichthedsprofielen van de $^3P_2$ en $^3P_1$ atomen zijn gemeten met de fluorescentietechniek door middel van een kleurstof laser. De gemeten pro-
fielen zijn in redelijke overeenstemming met de berekende voor de diffuse ontlading. Bij de gecontraheerde kolom is het gemeten $^3P_1$ profiel breder dan het berekende profiel. Dit is te wijten aan onze vereenvoudigde behandeling van de stralingstransport term (deze behandeling heeft echter geen merkbare invloed op de berekening van andere kolomgrootheden).

Net numerieke model is gebruikt bij de interpretatie van metingen van de reactie-coëfficiënten voor atoomgeïnduceerde overgangen tussen de 2p-niveaus in neon. Deze metingen zijn verricht bij verschillende stroomsterkten. Hieruit zijn de koppelingscoëfficiënten in afhankelijkheid van de gastemperatuur bepaald, waarbij de relatie tussen de gastemperatuur en de ontladingstroomsterkte uit het model berekend is. De afhankelijkheid van de koppelingscoëfficiënten van de gastemperatuur $T_g$ is theoretisch geschat op $T_g^{3/2}$. De metingen tonen een afhankelijkheid volgens $T_g^n$, met $0 \leq n \leq 2.5$ (een en ander afhankelijk van de niveaus waartussen de overgang plaatsvindt).
Nawoord


Gedurende deze periode heeft M. Prins aan het onderzoek meegewerkt. Verder zijn deelonderzoeken verricht door de volgende studenten:

De numerieke berekeningen hebben naar schatting 100 uur reken tijd gekost op de Burroughs B6700 en B7700 machines. De dye-laser is gedurende ongeveer 300 uur gebruikt. De totale kosten van apparatuur voor dit onderzoek bedroegen ongeveer 65 kf.

Een bijzonder woord van dank wil ik allereerst richten tot Maarten Prins, waarmee de samenwerking deze vier jaar uitermate plezierig, stimulerend en nuttig is geweest. Vervolgens dank ik Hans van Duyn in hoge mate voor zijn berekeningen aan de Boltzmann-vergelijking. Tevens dank ik H.J. Hagebeuk voor zijn assistentie bij de numerieke berekeningen aan de striaties. Het typewerk is zeer bekwam verricht door Francien Duifhuis, terwijl het tekenwerk met veel zorg is verricht door Ruth Gruyters. Bij den hiervoor oprechte dank.

Tot slot wil ik alle M.H.-medewerkers bedanken die direct of indirect hebben bijgedragen tot dit proefschrift en alle mensen van het cyclotrongebouw voor de plezierige samenwerking en sfeer binnen het gebouw.
Levensloop

10 juli 1950 geboren te Oosterhout (N.Br.).

juni 1967 eindexamen H.B.S.-b aan het Monseigneur Frencken College te Oosterhout.

september 1967 aanvang natuurrkunde studie aan de T.H.E.

juni 1973 doctoraal examen natuurkundig ingenieur.

oktober 1973-
augustus 1977 wetenschappelijk medewerker in de groep cyclotron-
toepassingen, afdeling der Technische Natuurkunde, T.H.E.

vanaf augustus 1977 leraar natuurrkunde aan het Bisschoppelijk College te Weert.
STELLINGEN

behorende bij het proefschrift van

R.M.M. Smits

Eindhoven, 21 oktober 1977
Er zijn principiële argumenten aan te voeren tegen het berekenen uit de lokale energiebalans van de axiale elektrische veldsterkte in een gasontlading.

_Dit proefschrift, hoofdstuk 2._

2

In een gecontraheerde ontlading van gematigde druk en stroom (voor neon: $10 < p_0 R < 100$ torr m, $5 < i/R < 20$ A/m) zijn de elektronenverliezen op de as van de ontlading ten gevolge van ambipolaire diffusie en volume-recombinatie van vergelijkbare orde van grootte.

_Dit proefschrift, hoofdstuk 4._

3

De ionisatiegolven in een edelgasontlading van gematigde druk leiden een mechanisme in dat tot contractie van de positieve kolom voert.

_Dit proefschrift, hoofdstuk 4._

4

Benadering van het gastemperatuurprofiel in een ontlading van gematigde druk door een parabool of een Lorentzprofiel kan ontoelaatbare afwijkingen ten opzichte van de numeriek bepaalde oplossing van de Beller-Elenbaasvergelijking geven. Deze afwijkingen hebben ten gevolge dat ook het elektronendichtheidprofiel en de axiale elektrische veldsterkte verkeerd worden berekend.

_Dit proefschrift, hoofdstuk 4._

5

Het gebruik van de klassieke logica door Wittgenstein, Popper en c.s. ter bestrijding van hun inziens onjuiste theorieën is aanvaardbaar op grond van de niet in termen van de klassieke logica formaliseerbare kwantummechanica, die zijn bruikbaarheid als fysische theorie bewezen heeft.

L. Wittgenstein, "Tractatus Logico-Philosophicus".
K. Popper, "What is dialectics".
F. Mittelaasdt, "Philosophische Probleme der modernen Physik".
M. Jammer, "The philosophy of quantum mechanics".
Indien kortlevende radio-isotopen belangrijk worden voor medische toepassingen, is een samenwerkingsverband tussen regionale cyclotronlaboratoria nodig om te voldoen aan leveringsgaranties.

Bij de huidige stand van de rekenmachine-techniek is het zinvol om het gebruik van analogonmethoden die duur en omslachtig zijn, af te wegen tegen een numerieke simulatie.

De recente ontwikkeling in computer on-line technieken maakt het noodzakelijk dat studenten in deze technieken steeds meer training krijgen. Hiermee dient in de onderwijsbegroting rekening gehouden te worden.

Het is wenselijk om elektronische zakrekenmachines op middelbare school-examens toe te laten; tevens zou dan echter het onderwerp "numerieke schattingmethoden" in het wiskundepakket verplicht moeten worden behandeld.

Aanwezigheid van bewaakte gratis rijwielstallingen in de steden heeft als voordeel dat rijwieliefstal wordt bemoeilijkt, zodat het rijwielgebruik wordt bevorderd.