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Out-of-plane stability of roller bent steel arches – An experimental investigation

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ABSTRACT

This paper presents an experimental investigation of the elastic-plastic out-of-plane buckling response of roller bent circular steel arches subjected to a single force applied to the crown. The experiments are used to validate a finite element model described in a related paper. A series of 15 tests was performed on full-scale and model arches where the developed length of the arches was kept constant. The subtended angle was varied between 90° and 180°. Each full-scale arch configuration was tested at least twice to monitor experimental scatter and assess the repeatability of the tests. Special attention was paid to the boundary conditions: at the crown the load was introduced at the centroid of the arch-rib with the use of hydrostatic bearing to eliminate any torsional restraint. Loading was applied by means of a tension rod affixed to the centre of the baseline, rendering a directed load. The supports were designed such that they acted as hinges in-plane while they were fixed out-of-plane. Geometric imperfections were measured prior to loading. All arches failed by elastic-plastic out-of-plane buckling featuring the presence of plastic zones in the arch-rib and out-of-plane arch deformation.

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1. Introduction

In the past few decades, the use of single freestanding arches has increased. These arches do not have lateral bracing and can therefore fail in an out-of-plane buckling mode. A structure is an arch by the common definition where the member is curved in elevation, loaded in its plane, with spreading of the supports prevented, and with its rib primarily in compression (Fig. 1a). If outward spreading can occur at one support, the structure is considered a curved beam (Fig. 1b) and not an arch. This paper will delineate strictly between these two structures and is directed at arches only.

The increase in use of freestanding arches has not been fully accommodated by the design codes which do not provide design rules to check the out-of-plane buckling response. Out-of-plane buckling (Fig. 1(e)) is one of three buckling modes that can take place when an arch is subjected to a combination of bending and compression. The other buckling modes, snap-through buckling (Fig. 1(c)) and in-plane buckling (Fig. 1(d)) are more likely to occur in shallow arches or braced arches, respectively. During out-of-plane buckling, the deformation mode of the arch changes from in-plane to a combination of in-plane and out-of-plane. Prior to buckling the arch resists loading through a combination of compression and bending. During buckling the arch response is characterized by a combination of compression, bi-axial

bending (minor axis and major axis bending), torsion and restrained warping, causing a complex internal force distribution. The resistance of a freestanding arch to buckling is supplied mainly by its torsional resistance, out-of-plane bending stiffness and the out-of-plane support conditions. Often freestanding arches are made from hollow sections with sufficient torsional stiffness and are fixed out-of-plane at the abutments to provide adequate out-of-plane buckling resistance.

The lack of design rules has led to the initiation of a research project at Eindhoven University of Technology. It is aimed to obtain design rules through finite element analyses. The performance of a finite element model to capture the out-of-plane elastic-plastic buckling response is evaluated through comparison with experiments. Close coherence between both approaches permits the use of the finite element model to evaluate the structural response of arches not part of the experimental program. In the current paper the experiments are presented, and in a related paper by Spooenberg et al. [1] finite element analyses and design rules are presented. Hence the goal of the experiments is two-fold: to provide fundamental data concerning the out-of-plane elastic-plastic buckling response of freestanding arches under compression and bending and to use the data to arrive at design rules to check their stability.

2. State of the art of experimental testing

It has since long been recognized that a freestanding arch can fail in an out-of-plane buckling mode prior to reaching its in-plane load bearing capacity. The first tests to evaluate the out-of-plane buckling response were those by Stüssi [2], who tested a parabolic arch, loaded

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Fig. 1. Arch vs. curved beam and buckling modes for arches.

with eight equally spaced concentrated loads. The arch was cut from aluminum with a rectangular cross section and was subjected to gravity loading (load does not change direction as the arch deforms out-of-plane). The objective of these tests was to verify a proposed method of calculating the elastic buckling load. Godden [3] and Kee [4] investigated the elastic out-of-plane stability of parabolic arches with cross sections comprising either steel solid bars or hollow tubes. Load was applied by pushing the supports closer together. At the same time 11 wire hangers were fixed in place on the chord of the arch. As the arch buckled out-of-plane, the wire hangers changed direction, which is also referred to as directed loading. Kee [5] reported on tests of aluminum arches that were loaded into the elastic-plastic range. Klöppel and Protte [6] tested the elastic stability of circular steel arches, by means of horizontal loads at the supports. The arches had a subtended angle of 90° or 180° . The cross sections were either a rectangular plate or an I-section. These tests were performed to verify analytical work. Tokarz [7] and Tokarz [8] studied the elastic stability of 13 free standing arches, parabolic and circular, of which seven were tested with gravity loading and six with tilting loads. Load was applied through 14 equally spaced wire-hangers and the supports were either fixed or hinged. The purpose of these tests was to supply additional experimental verification of lateral buckling loads of curved members based on linear buckling theory. Di Tomasso and Viola [9] tested the elastic stability of a circular arch with fixed supports and a single load at the crown. The arch was made of a thin sheet of aluminum and had a subtended angle of 130° . The experiment was performed for comparison with analytical results. Papangelis and Trahair [10] tested the elastic stability of circular curved beams with a single concentrated load at the crown. Both supports were rollers and the cross section was an aluminum I-section. The experiments were set-up to evaluate the difference between analytical solutions by Timoshenko and Gere [11] and Vlasov [12] on the one hand with those presented by Yoo [13] on the other. The lateral elastic-plastic buckling performance of braced and freestanding model arches made from welded hollow sections was investigated by Sakimoto et al. [14]. A total of 12 circular and parabolic arches were subjected to different loading types. In addition to nine braced arches, three freestanding arches were tested. The arch configurations were chosen to represent existing full-scale arches with a rise-to-span ratio (f/L) fixed at 0.2. The freestanding arches were subjected to a directed uniformly distributed load or a combination of a directed uniformly distributed load and a uniformly distributed load on half the span. It was found that the freestanding arches displayed out-of-plane buckling when subjected to symmetric load, but in-plane buckling was observed for the arch with unsymmetric loading.

As part of a large experimental study on braced arches, Sakata and Sakimoto [15] investigated the elastic-plastic buckling response of five freestanding circular arches. The five tested specimens consisted of one circular and four parabolic arches. The cross-sections were made from welded I-sections. For the circular and two parabolic freestanding arches a gravity load was adopted. For the remaining two freestanding parabolic arches directed loading was adopted. It was found that the arch failed by out-of-plane elastic-plastic buckling prior to the attainment of the plastic collapse load.

It can be concluded that nearly all experiments were setup to verify calculation techniques, the older ones for hand calculations, the more recent ones by Sakimoto et al. [14] Papangelis and Trahair [10] and Sakata and Sakimoto [15] for finite element analyses. Most experiments were carried out on model arches and were aimed at initiating elastic buckling. Rectangular or circular cross sections were often selected and only a few tested arches comprised I-sections see Fig. 2(a). For the majority of tests the aim was to achieve uniform compression in the arch-rib by selecting a loading type whose funicular line would match the arch shape. Arches subjected to significant bending in addition to compression were investigated by Klöppel and Protte [6], Di Tomasso and Viola [9] and Papangelis and Trahair [10]. Sakimoto et al. [14] and Sakata and Sakimoto [15] investigated circular arches with relatively low rise-to-span ratios subjected to uniformly distributed loading for which compressive action in the arch-rib is dominant and bending insignificant.

Experiments aimed at elastic buckling do not describe the full interaction between material non-linearities and buckling and therefore can only reveal part of the out-of-plane buckling response. Elastic buckling occurs mainly in slender freestanding arches, but arches with intermediate slenderness or stocky arches will fail by elastic-plastic buckling. Only Kee [5], Sakimoto et al. [14], and Sakata and Sakimoto [15] reported on the elastic-plastic buckling response, although load-displacement graphs were not made available for all tests. Sakata and Sakimoto [15] presented load-displacement curves for parabolic arches subjected to a uniformly distributed load resulting in uniform compression. To the knowledge of the authors no load-displacement graphs have been published featuring the full elastic-plastic buckling response of freestanding circular arches subject to compression and bending.

3. Experiments

3.1. Experimental program

During out-of-plane buckling, the internal force distribution in the arch is a combination of different components: compression, bi-axial bending, torsion and restrained warping. In order to obtain experimental

Fig. 2. Cross sections in previous and current experiments.

data where all components are present, wide flange sections were selected in preference to rectangular or circular hollow sections. Since the ratio between the minor moment of inertia (I_z) and the major moment of inertia (I_y) for wide flange sections considerable has influence on the out-of-plane buckling response, wide flange sections with extreme inertia ratios were considered most suitable for further investigation. Commercially available HE 100A and HE 600B sections in steel grade S235 were used for the arches, which have a ratio of I_z/I_y of approximately 0.4 and 0.1 respectively.

As the HE 600B arches were not expected to buckle within the range of the available load actuator, the sections were scaled with geometric similarity, Hosdorf [16]. The scaled cross section is called the model and is marked with an asterix, i.e. HE 600B*. The model section was scaled to the depth of a HE 100A (Fig. 2b), rendering a scale factor of 1/6.25. The scaling process will be described in Section 3.3. For the test program, it was decided to vary the subtended angle 2γ between 90° and 180° . The developed length S was kept constant at 6 m for the full scale arches and 2.5 m for the model arches. The geometrical properties of the arches are shown in Fig. 3. All arches were fixed to end plates of 20 mm thickness (t_b).

The full-scale arches with minimum and maximum subtended angles of 90° and 180° were tested three times to determine the repeatability of these tests (Table 1).

The range of subtended angles in between was covered by three angles, for which tests were repeated once. The numbering of the tests increases with decreasing subtended angles. The appended capital letters A to C distinguish test specimens with the same configuration. Due to the high production cost of the HE 600B* model arches each configuration was only tested once. To these test numbers, the letter M was appended to clearly indicate that these were model tests.

Fig. 3. Arch geometry definitions.

3.2. Load case

A single force applied at the crown was selected for this study as it offers the possibility to investigate the combined influence of bending and compression on the behavior of arch structures. Earlier experimental studies on elastic-plastic buckling were mainly focused on arches for which the arch-rib was subject to only compression. The load will be introduced by means of a tension rod, where a prescribed displacement applied at the baseline of the arch will result in a concentrated force F acting at the crown (Fig. 4a).

The displacement is applied in the positive Z-direction of the global coordinate system causing an in-plane displacement w at the crown. The arch is considered in-plane pin-ended which means that translations in local x - and z -directions are restrained at the supports. Rotations with respect to the local x -, and z -axes and translations in the y -direction are suppressed at the supports making it an out-of-plane fixed boundary condition. Warping deformations are prevented at the supports. During out-of-plane buckling arch deformation is featured by a lateral displacement v and a rotation ζ . The tension rod and hence the load thus will undergo a directional change ϕ_1 (Fig. 4b). The tension rod remains directed to the baseline of the arch and at the crown it rotates ϕ_2 independently from the arch-rib without introducing a restraining moment.

At low load levels, the bending moment and normal force distributions shown in Fig. 4(c) and (d) can be determined with linear elastic calculations. The maximum bending moment is at the crown (M_{crown}) and the second largest bending moment ($M_{2\text{nd}}$) occurs on both sides of the crown. The crown moment introduces compressive stresses in the top flange and the second largest bending moment causes compressive stresses in the bottom flange. The maximum compressive force (N_{max}) occurs at the locations of the second largest bending moment. These locations coincide with the points where the tangents to the funicular lines equal the tangent to the arch-rib. Displacement and strains will be recorded at the locations of M_{crown} and $M_{2\text{nd}}$.

Table 1
Dimensions of test specimen.

Section	HE 100A					HE 600B*		
	1	2	3	4	5	6 M	7 M	8 M
Test	A B C	A B	A B	A B	A B C			
2γ [°]	180	160	135	110	90	180	135	90
R [mm]	1910	2149	2546	3125	3820	796	1061	1592
S [mm]	6000	6000	6000	6000	6000	2500	2500	2500
f [mm]	1978	1835	1620	5143	5431	864	703	495
L [mm]	3820	4240	4720	5143	5431	1592	1976	2280
ϕ_{crown} [°]	8.4	7.5	6.3	5.1	4.2	18.8	14.1	9.4

Fig. 4. Load application for experiments and internal force distribution.

Assuming that the arch-rib is incompressible, formulae published in Stahl im Hochbau [17] were used to determine the bending moment and normal force distribution in the full-scale and model arches. The salient internal forces for the investigated arches are tabulated in Table 2. The angle at which the bending moment is zero and the angle at which the second largest bending moment occurs are denoted by $\phi_{M=0}$ and ϕ_{2nd} , respectively. It can be seen that as the subtended angle decreases, the horizontal reaction force F_h and maximum compressive force N_{max} increase. For constant radius the bending moment at the crown and the second largest bending moment decrease when the subtended angle decreases. This illustrates the diminishing influence of bending moment in more shallow arches.

Preliminary finite element analyses showed that the selected arch configurations would fail in an elastic-plastic buckling mode prior to the attainment of the in-plane plastic capacity, thereby meeting the desired requirements for the experimental output. The finite element analyses also showed that the arches would fail at an ultimate load well below the capacity of the load actuator (150 kN).

3.3. Producing the test specimens

The full scale arches were made from HE 100A sections as delivered. The model arches were made from HE 100A sections through a process of sequential planing. Before planing the members were exposed to heat treatment to relieve residual stresses. This was deemed necessary as the presence of residual stress can cause deformation of the member during planing. The planing process comprised a sequence of material removing operations where the flanges are shortened and the thickness of the flanges and web is reduced in stages until the scaled HE 600B* section is obtained.

A scaled HE 600B* section as planed from a HE 100A section is shown schematically in Fig. 5.

Straight HE 100A en HE 600B* members were bent into circular arches at ambient temperature by means of a roller bending process. In the roller bending process the member is placed in the machine and curved between three rolls at ambient temperature, see Fig. 6a.

Permanent curvature in the member is achieved by movement of the right roller along a prescribed path and subsequent rolling of all rolls (Fig. 6b–c), inducing a process of continuous plastic deformations. The members were curved in excess of the desired arch length S and later cut to the desired length. The elongated flange and shortened flange are referred to as the top flange and bottom flange, respectively. It is known that the roller bending process alters the residual stresses, mechanical properties and cross-sectional properties of the section. Residual stresses were measured in roller bent wide flange section by Spoorenberg et al. [18]. The changes in cross-sectional dimensions (flange widths, web height etc.) were measured by La Poutré [19], however the influence on the cross-section properties was found negligible. A maximum deviation in minor moment of inertia I_z was found to be 1.50%.

From the curved leftovers, coupons were cut and tested to obtain the mechanical properties of the roller bent arch. The coupons were removed from various parts of the flanges and the web as it was expected that the distribution of the mechanical properties is non-uniform over the cross-section. For multiple tested arches, coupons were removed from the leftovers of a single specimen. The cross-section of the full scale arches and model arches was divided into 7 different zones (Fig. 7). The experimental stress-strain curves were converted to true-stress-true strain curves applicable to the specific zones of the cross-section. The true stress-true strain curves

Table 2
Bending moments, normal forces and support forces based on linear elastic computations as function of central load F and radius R .

2γ [°]	F_h	M_{crown}	ϕ_{2nd} [°]	M_{2nd}	$\phi_{M=0}$ [°]	N_{max}
180	0.318 F	0.1817 FR	57.5	−0.0927 FR	25.0	0.593 F
160	0.403 F	0.1596 FR	51.1	−0.0797 FR	22.3	0.642 F
135	0.533 F	0.1329 FR	43.1	−0.0649 FR	18.9	0.731 F
110	0.709 F	0.1072 FR	35.2	−0.0514 FR	15.4	0.868 F
90	0.910 F	0.0871 FR	28.8	−0.0413 FR	12.6	1.038 F

Fig. 5. Planing of model section out of HE 100A.

Fig. 6. Roller bending process.

were approximated by seven discrete stress-strain points. Fig. 8 shows the seven true stress-true strain curves for arch 1 and arch 6 M for the specific zones of the cross-section. The discrete stress-strain curves per zone for all tested arches are presented in La Poutré [24].

3.4. Test rig

A test rig was built to withstand all loads exercised on the arches, Fig. 9. This way the rig functions as a tie in a tied arch. As there was no strong floor available in the laboratory, a test rig was necessary for testing the arches. The tie consisted of two stiffened HE 300B sections which were placed parallel and spaced 200 mm apart. An actuator and ram were placed underneath the tie which was connected to a tension rod running through the space between the HEB 300 members. The tension rod was attached to a load introduction frame from which a force was exerted on the arch.

3.5. Design of boundary conditions

For stability experiments specific attention needs to be paid to the boundary conditions, i.e. supports and load introduction. The physical boundary conditions should resemble as closely as possible their 'ideal' behavior as shown in Fig. 4(a)–(b). As stability experiments are in general dominated by sudden directional change of displacements during the course of loading, the boundary conditions must accommodate this behavior without introducing restraining forces, see Birkemoe [20]. This applies specifically to the load introduction at the crown, which must permit rotations and out-of-plane deformation of

the arch-rib. The objective is to apply a pure point load at the crown without accidental restraining forces.

3.5.1. Load introduction at crown

In an attempt to minimize the resistance in the point of load application a load introduction system was designed which consisted of a solid sphere and matching socket (Fig. 10). The radius of the sphere was designed such that the point of load application in the arch coincided with the centroid of the cross section.

Zero resistance to rotation can only be obtained by the absence of friction in the contact area between sphere and socket. This was achieved by a hydrostatic bearing; O'Donoghue et al. [21]. Therefore an oil film was supplied between the socket and bearing. Fig. 10 shows the hydrostatic bearing that was specifically designed and built for the experiments. The top part of the bearing, the socket, was divided into three recesses. These were fed with oil through orifice restrictions, allowing self centering of the socket. The three parameters governing self centering are: the viscosity of the oil used (1), the pressure drop over the orifice restriction (2), and the oil flow (3); Rowe [22]. The viscosity depends on the type of oil and operating temperature.

The oil was circulated and a heat exchanger was inserted in the circuit to keep the temperature within a prescribed range. The temperature of the oil was measured to be able to rule out malfunctioning of the bearing system due to reduced viscosity. With pressure transmitters in each recess and in the oil supply line, the pressure drop in each recess was measured. The flow was measured as well and always surpassed the minimum required value. The oil formed a film between the socket and sphere and ran off at the perimeter into a receptacle. A disc with inclined edges was bolted to the bottom side of the sphere to limit the contact area to 25 mm times the width of the arch rib. Two strips, bolted to the bottom side of the disc, prevented the sphere from sliding off the arch in the out-of-plane direction. The bearing was not fixed in-plane but stayed in place due to the friction between the bottom of the disc and top surface of the arch rib.

Two preliminary tests, reported elsewhere La Poutré et al. [23] La Poutré [19], were required to fine tune the three self centering parameters. To monitor the working of the bearing, the electrical resistance of the oil film was measured. This showed that sphere and socket were insulated. Additional details of the working of the bearing have been reported earlier by La Poutré [24]. Fig. 11 shows the bearing in operation near the end of a test with a substantial angle between the flat bottom of the sphere and the socket.

3.5.2. Supports

The arch is considered to be in-plane pin supported and out-of-plane fixed. Warping deformations and torsional rotations are suppressed at the support. The support design for the test setup is shown in Fig. 12.

Fig. 7. Zonal division of the cross-section.

Fig. 8. Discrete true stress-true strain curves for 7 zones in the cross-section for arch 1 and arch 6 M. Initial true stress-true strain curve (left) and full true stress-true strain (right).

A base-plate was bolted onto a thick notched support platen to which two axes were mounted to achieve in-plane pinned support conditions, see Fig. 12(b). The bolted base plate prevented out-of-plane rotations and warping deformations (Fig. 12(a)). The axes were supported by two inline, double-row spherical roller bearings with self-aligning capabilities. These bearings were able to carry small axial loads (R_v), which correspond to lateral loads V from the arches, without losing their in-plane rotational capability. Opposite forces in the two support base plates (R_N) can sustain the out-of-plane bending moments M , in addition to the compressive force N and shear force V (Fig. 12(c)).

3.6. Geometric imperfections

Prior to testing, the lateral (v_{imp}), radial (w_{imp}) and twist (ζ_{imp}) imperfections were measured. The imperfections were measured at regular intervals along the developed arch length yielding discrete imperfection values at specific points along the arch-rib (Fig. 13).

In order to make the measurements suitable for implementation in finite element models, they were approximated by polynomial expressions. A polynomial of the sixth degree was used to render a closed-form equation of the distribution of the lateral, radial and

Fig. 9. Test setup with 90° arch.

Fig. 10. Hydrostatic bearing for load introduction at the crown.

Fig. 11. Hydrostatic bearing in operation.

Fig. 12. Support for arch-rib.

Fig. 13. Arch geometric imperfections.

Fig. 14. Measurements and polynomial fit of lateral imperfections.

Table 3
Ultimate loads and geometric imperfections of full-scale arches.

Test	1A	1B	1C	2A	2B	3A	3B	4A	4B	5A	5B	5C
2γ [°]	180			160		135		110		90		
F_{ult} [kN]	104.7	103.2	104.8	104.9	104.3	100.0	99.2	99.4	96.3	97.3	95.2	95.0
$v_{imp}(crown)$ [mm]	4.9	−3.5	12	9.3	−1.1	16.2	7.1	−16	−12.6	−4.6	0.3	−2.2
$ v_{imp}(max) /S$ [−]	$\frac{1}{1052}$	$\frac{984}{1}$	$\frac{387}{1}$	$\frac{594}{1}$	$\frac{5455}{1}$	$\frac{323}{1}$	$\frac{392}{1}$	$\frac{370}{1}$	$\frac{476}{1}$	$\frac{1000}{1}$	$\frac{1764}{1}$	$\frac{1200}{1}$
v_{h-b} [mm]	1.3	−1.6	0.5	1.2	1.5	−0.1	1.5	−0.6	−1.0	−0.1	−0.8	−1.6

twist imperfections given by Eqs. (1), (2) and (3), respectively as a function of the angular coordinate θ .

$$v_{imp}(\theta) = \alpha_0 + \alpha_1\theta + \alpha_2\theta^2 + \alpha_3\theta^3 + \alpha_4\theta^4 + \alpha_5\theta^5 + \alpha_6\theta^6 [mm] \quad (1)$$

$$w_{imp}(\theta) = b_0 + b_1\theta + b_2\theta^2 + b_3\theta^3 + b_4\theta^4 + b_5\theta^5 + b_6\theta^6 [mm] \quad (2)$$

$$\zeta_{imp}(\theta) = c_0 + c_1\theta + c_2\theta^2 + c_3\theta^3 + c_4\theta^4 + c_5\theta^5 + c_6\theta^6 [^\circ] \quad (3)$$

The measured lateral imperfections for two arches are shown in Fig. 14 in addition to the polynomial approximations. The coefficients of the polynomials for all tested arches are presented in La Poutré [19]. The coefficients are based on linear regression analyses. The

expressions will be used in a related paper to define the initial state of the arch in the finite element environment. The lateral imperfection at the crown and the maximum lateral imperfection are presented in Table 3 and Table 4 for the full-scale arches and model arches, respectively.

In addition to the geometric imperfections of the arch-rib, a discrepancy in the alignment of the tension-rod was measured. The discrepancy (v_{h-b}) is defined as the difference between the measured position of the tension rod and the center of the baseline between both supports in the horizontal plane perpendicular to the plane of arch. Values of the misalignments are given in Table 3 and Table 4 for all tested arches as these are necessary for further study, Spoorenberg et al. [1].

3.7. Loading phase

Before loading the sphere of the hydrostatic bearing was placed on the crown of the arch and aligned with the centroid of the arch-rib. A load introduction frame was lifted over the sphere but contact between socket and sphere was not yet made.

Oil was supplied between the sphere and socket. After approximately half an hour the loading phase commenced. The loading frame was placed on the sphere and subsequently the tension rod was pulled downwards. For the full-scale arches, the load was applied through a prescribed displacement w of the ram at a rate of 1.67 mm/min. For

Table 4
Ultimate loads and geometric imperfections of model arches.

Test	6 M	7 M	8 M
2γ [°]	180	135	90
F_{ult} [kN]	66.8	59.8	58.9
$v_{imp}(crown)$ [mm]	−4.3	8.6	4.3
$ v_{imp}(max) /S$ [−]	$\frac{500}{1}$	$\frac{258}{1}$	$\frac{568}{1}$
v_{h-b} [mm]	−0.7	0.7	0.2

Fig. 15. Tilting load.

the model arches a displacement rate of 0.267 mm/min was used. As soon as the arch displayed out-of-plane deformations, the direction of the load changed as illustrated by ϕ_1 in Fig. 15.

The loading frame weighed 78 kg and the sphere was 16.7 kg causing a total extra load of approximately 0.93 kN on the arch prior to testing. This load was not recorded by the load cell. However, as this load constitutes less than 1% of the failure load of the full-scale

arches and less than 2% of the failure load of the model arches it was omitted in the evaluation of the experimental results.

3.8. Displacement measurements

The displacements of the arch-rib were measured from a separate frame which is shown in Fig. 16. The displacements were measured at

Fig. 16. Independent measuring frame (in gray) for semicircular arch.

Fig. 17. Load-displacement graphs of test 1A–1C, 2A, 2B and 3A, 3B.

three points along the arch: two at M_{2nd} (see also Section 3.2) and the third point was close to the crown, giving a good indication of the arch deformation during loading.

Placing a measurement point exactly at the crown was not possible due to the presence of the hydrostatic bearing and load introduction frame. Therefore the measurements were placed away from the crown at an angle ϕ_{crown} . This angle is specified in the Table 1 for all tested arches. At each of the three points, in-plane displacements were measured in the global coordinate system (X, Z plane). The out-of-plane displacements were measured at both the top and bottom flange (Y, Z plane). Dividing the difference between both out-of-plane displacements by the distance between them produces the twist rotations.

3.9. Strain measurements

Strain gages were attached to the arches to monitor their in-plane and out-of-plane response to the acting loads. A total of 4 strain gages were applied at the centre of the top flange and another 4 strain

gages were applied at the centre of the bottom flange to obtain the strain readings from in-plane actions. The out-of-plane response of the arch was recorded by strain gages on the tips of the flanges for a selected group of arches. Out-of-plane response was recorded at either a location near the support at approximately 40 mm from the base plate or at the location of the second largest bending moment M_{2nd} (Fig. 4c). Similar to the displacement measurements, the strain gages were placed away from the crown as represented by angle ϕ_{crown} due to placement requirements of the hydrostatic bearing and the load introduction frame.

4. Experimental results and discussion

4.1. Load-displacement graphs

Only the displacements of the crown will be presented as they were larger than at other points. For all tests, three load-displacement graphs are presented. The load is plotted versus the in-plane displacement w , the out-of-plane displacement v and the twist rotation ζ .

Fig. 18. Load-displacement graphs of test 4A, 4B, 5A, 5C and 6 M–8 M.

All displacements and rotations are shown positive. The load-displacement graphs for tests 1, 2 and 3 are shown in Fig. 17. Test 4 and 5 are presented in Fig. 18 in addition to tests 6 M, 7 M and 8 M.

From the in-plane displacement graph for the full-scale arches it can be seen that the displacements initially increase linearly with the load. After reaching approximately 60 % of the ultimate load the load-displacement curve deviates from this loading path and the arch continues to resist loading at a reduced stiffness. After reaching the ultimate load (maximum on load-displacement curve) the arch is subjected to a decrease in load carrying capacity. Termination of the test is featured by a rapid decrease in loads accompanied by arch movement in the opposite direction compared to the loading stage. Due to a malfunctioning displacement recorder, the load-displacement graphs of the in-plane displacement for tests 6 M and 7 M are incomplete.

The out-of-plane buckling response of the full-scale arches represented by the lateral displacement and twist rotations is initially small. However lateral displacements and twist rotations start to increase rapidly after the applied load exceeds approximately 60% of the ultimate load. After reaching the ultimate load the out-of-plane

displacements and rotations show post-buckling behavior similar to the in-plane displacements. Load-displacement curves from tests with identical cross section and subtended angle show largely similar results, illustrating the robustness of the testing procedure. The model arches display a displacement behavior that is similar to that of the full-scale arches.

4.2. Load-strain graphs

Load-strain graphs are presented for a number of tested arches. The load-strain graphs for all tested arches are reported in La Poutré [19]. A distinction is made between the strain readings to monitor the in-plane response and out-of-plane response.

4.2.1. In-plane response

Load-strain graphs are presented for arch 1C and arch 5A in Fig. 19 to show the in-plane response. The strain recordings from gages near the crown are presented in addition to strains at positions of the second largest bending moments. An elastic limit ϵ_y is defined to identify the possible presence of plastic strains at $f_y/E = 290/200\,000 = 0.145$

Fig. 19. Load-strain graphs for in-plane response.

%, which is an arbitrary value as it ignores the presence of residual stress and gradual yielding. Residual stress measurements by Spoorenberg et al. [18] and tensile testing by Spoorenberg et al. [25] have shown that both phenomena are present in a roller bent arch. It can be seen that for both arches, the strains measured at the crown remain within the elastic limit.

Based on a linear elastic calculation it can be expected that the onset of yielding will take place at the crown, prior to yielding at the location of the second largest bending moment. The strain gauges applied at the second largest bending moment show strain values in excess of the

yield limit. Therefore it is most likely that yielding at the crown takes place in a segment of the arch-rib between the angles ϕ_{crown} on both sides of the crown (see Fig. 16c).

The strain readings from arch 1C show that at a specific load the magnitude of compression strains is similar to tensile strains, indicating that the arch resistance is dominated by bending. For arch 5A larger compressive strains are found in comparison to tensile strains. Compressive action in the arch-rib is present to a larger extent for arches with smaller rise-to-span ratios in comparison to arches with larger rise-to-span ratios.

Fig. 20. Load-strain graphs in the vicinity of the supports for out-of-plane response.

The strain readings for both arches at the location of the second largest bending moment on the left side of the crown and on to the right of the crown are of similar magnitude. This indicates that the arch deforms in a symmetric mode under the applied loading. However, strain values start to deviate after the ultimate load has been reached, indicating sway-displacements in the post buckling regime.

4.2.2. Out-of-plane response

The strains recorded at the top flange tips and bottom flange tips in the vicinity of the supports are shown in Fig. 20 for arch 1A and arch 5A. It can be seen that the strains grow rather slowly as long as the acting load remains below 60 % of the ultimate load. As the load exceeds this value the strains increase more rapidly in tension and compression. The strain readings in the vicinity of the supports are the result of out-of-plane bending action, axial compression, in-plane bending and warping deformations, although the latter two to a considerably smaller extent. It can be seen that the strain values do not enter the plastic regime. This was observed in all experiments.

Strain recordings at the location of the second largest bending moment are shown for arch 6 M and 5B, respectively in Fig. 21. These strain readings have similar characteristics to the strain values taken near the support, with the exception that most gages record strain value well in excess of the elastic limit.

4.3. Ultimate loads

Based on the significant out-of-plane displacements and rotations which were observed consistently from all tested arches in Section 4.1 and the presence of yielding in specific portions of the arch from Section 4.2 it can be stated that all arches failed by out-of-plane

elastic-plastic buckling. No local buckling was observed during testing. The ultimate loads are identified as the maximum force on the load-displacement graphs. A full overview of the ultimate loads for all tested full-scale arches is presented in Table 3. For the model arches, the ultimate loads are given in Table 4. The lateral imperfection at the crown, the maximum lateral imperfection normalized against the arch length and the misalignment of the tension rod are given as well. The model arches fail at a significantly lower buckling load and also the accompanying displacements are significantly smaller than for the full-scale arches (Fig. 18). When looking at the failure loads of full scale arches and model arches it can be seen that within each group there is little difference between the failure loads of the individual arches. It can be stated that the effect of the subtended angle on the failure load is small for the investigated group of arches. In general an increase in subtended angle will result in a minor increase of the failure load. It can be seen that the maximum imperfection $v_{imp}(max)/S$ shows great scatter between arches with identical subtended angle and cross-section. As the ultimate load for each configuration shows small differences between individual tests it can be stated that the lateral imperfection at the crown has little influence on the ultimate load. The small effect of the geometric imperfections on the elastic-plastic buckling response in the tests is remarkable as earlier finite element studies by others [26] on arches subjected to a central concentrated gravity load revealed a significant influence of these imperfections on arch buckling. The measured imperfections are significantly larger compared to values recorded by Sakata and Sakimoto [15] on welded wide flange arches with similar geometry. The misalignment of the tension rod (v_{h-b}) is significantly smaller than the lateral imperfection at the crown for most arches. Therefore it is stated that the misalignment of the tension rod has minor influence on the ultimate load.

Fig. 21. Load-strain graphs at second largest bending moment for out-of-plane response.

5. Conclusions

This paper presents experiments conducted on freestanding roller bent circular steel I-section arches to study the elastic-plastic out-of-plane buckling response. The arches were loaded by a single force applied at the crown. A total of 15 specimens were tested with subtended angles varying between 90° and 180° . Straight HE 100A and scaled down HE 600B sections of steel grade S235 were bent into circular arches at ambient temperature by means of the roller bending process. Geometric imperfections were measured prior to testing and showed considerable scatter in magnitude between the arches.

In order to prevent torsional restraint, a hydrostatic bearing was applied at the load introduction point. Force transfer between the hydrostatic bearing at the crown of the arch and the ram at the baseline was achieved through a tension rod, resulting in a directed load.

After reaching approximately 60 % of the ultimate load, the ratio between arch deflection and load increased indicating the onset of non-linear behavior. The arches displayed significant out-of-plane deformations, twist and yielding at specific locations in the arch-rib in the vicinity of the ultimate load indicating that elastic-plastic buckling failure occurred prior to the attainment of the in-plane plastic collapse load.

The comparison of multiple tests on identical arches resulted in nearly identical ultimate loads with a maximum difference of 3.1 % showing high repeatability and a robust testing procedure. The experiments showed that the magnitude of the maximum lateral imperfections has a small influence on the ultimate load which is contrary to observations from earlier finite element studies. In addition, an increase in subtended angle results in a small increase in failure load. The experimental results in the form of load-displacement graphs and load-strain graphs are used to validate a finite element model which is described in a related paper.

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