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EXACT SOLUTION OF THE REFLEXION PROBLEM IN NON-LINEAR OPTICS

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In non-linear optics one considers media in which the polarisation is not proportional to the electric field. Since the advent of lasers this no longer seems to be a purely academic problem.

Non-linear wave equations have been treated in other disciplines (gas-dynamics, hydraulics) by means of techniques based on the properties of characteristics. It might be advantageous to adapt these methods to non-linear optics.

As an example we treat the reflexion of a plane polarised wave at normal incidence. The dielectric occupies the half space \( x > 0 \) and the Maxwell equations for the transmitted wave reduce to

\[
\frac{\partial E}{\partial x} - \frac{\partial B}{\partial t} = 0 ,
\]

\[
\frac{\partial B}{\partial x} - \frac{\partial D}{\partial t} = 0 .
\]

(1)

We now consider \( D \) to be a given function of \( E \). Defining a function \( c(E) \) by the relation

\[
c^{-2} = \mu \frac{dD}{dE} ,
\]

(2)

we write eqs. (1) in the form

\[
\frac{\partial E}{\partial x} - \frac{\partial B}{\partial t} = 0 ,
\]

\[
\frac{\partial B}{\partial x} - \frac{1}{c^2} \frac{\partial E}{\partial t} = 0 .
\]

(3)

Using the method of characteristics (see, e.g., ref. 1) it is easily shown that the solutions of (3) must satisfy the relations

\[
dE = c \ dB \quad \text{for} \quad \frac{dx}{dt} = c ,
\]

\[
dE = -c \ dB \quad \text{for} \quad \frac{dx}{dt} = -c .
\]

(4)

There exist solutions for which \( dE + c \ dB = 0 \) throughout. These so-called simple waves travel to the right with local speed \( c \). They represent a pure transmitted wave. If the boundary condition is \( E = F(t) \) for \( x = 0 \), this solution is, in implicit form

\[
E = F \left( t - \frac{x}{c(E)} \right) .
\]

It remains to join this solution to the vacuum fields. These can be written in the form

\[
E_{\text{vac}} = f_1 \left( t - \frac{x}{c_0} \right) + f_r \left( t + \frac{x}{c_0} \right) ,
\]

\[
B_{\text{vac}} = -\frac{1}{c_0} f_1 \left( t - \frac{x}{c_0} \right) + \frac{1}{c_0} \left( t + \frac{x}{c_0} \right) ,
\]

(5)

where \( f_1(t) \) and \( f_r(t) \) denote the time dependence of incident and reflected waves. The former function is arbitrary, and the problem is to express \( f_r \) in terms of \( f_1 \).

We take \( \mu \) to be equal in vacuum and dielectric. The joining conditions then simply are

\[
E_{\text{vac}} = E , \quad B_{\text{vac}} = B \quad \text{for} \quad x = 0 .
\]

Using (2) and (4) we find

\[
B_{\text{vac}} = B = -\int_0^{E_{\text{vac}}} \frac{dE}{c} = -\mu \frac{1}{c_0} \int_0^{E_{\text{vac}}} \left( \frac{dD}{dE} \right)^{\frac{1}{2}} dE .
\]

From (5) it is then seen that the solution of our problem is

\[
f_1 - f_r = c_0 \mu \frac{1}{c_0} \int_0^{f_1+f_r} \left( \frac{dD}{dE} \right)^{\frac{1}{2}} dE .
\]

References


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