Integration of dual electromagnetic energy conversions: linear actuation with integrated contactless energy transfer
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Integration of Dual Electromagnetic Energy Conversions
Linear Actuation with Integrated Contactless Energy Transfer
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Summary

Integration of Dual Electromagnetic Energy Conversions
Linear Actuation with Integrated Contactless Energy Transfer

The power supply of long-stroke, linear, moving-coil machines is conventionally obtained through moving cables which have a detrimental effect on the performance and dynamics of the system. To obviate the limitations associated with moving cables, the cables can be replaced with a contactless energy transfer (CET) system. For high-accuracy positioning applications synchronous, permanent magnet machines are favorable as electric machines on account of their excellent servo characteristics. This thesis presents the study on the physical, electromagnetic integration of the CET system in a synchronous, permanent magnet, linear machine. Apart from the focus on the identification and analysis of the physical repercussions associated with the integration a significant deal of attention in this thesis is also paid to the magnetostatic modeling of permanent magnet actuators.

In the first part of the thesis the focus is on the quantitative comparison of four different modeling techniques, namely the harmonic method, the Schwarz-Christoffel method, the boundary element method, and the tooth contour method. This comparison enables a well-founded choice for the most suitable technique for the modeling of the complex electromagnetic problem that arises from the electromagnetic integration of a CET system into a linear machine. Mathematical derivations of the techniques are provided and each technique is applied to the same benchmark topology. The benchmark topology is a synchronous, linear machine with permanent magnets. The results in terms of the flux density distribution in the airgap of the benchmark topology, as obtained by the different modeling methods, are compared to each other and finite element method (FEM) results. Furthermore, from the flux density distribution the force profile of the benchmark topology is calculated by applying different force calculation methods. The force results are compared quantitatively as well.

The second part of the thesis focusses on the actual electromagnetic integration of the CET into the linear machine. The integration is obtained by superimposing two magnetic fields of different amplitudes and frequencies in a single soft-
magnetic body. Electromagnetic cross-coupling effects between the two functionalities are identified by means of electromagnetic analyses. These analyses lead to the formulation of five criteria that ideally should be met for an integration solution to be deemed feasible. These criteria are related to the electromagnetic cross-coupling effects that manifest themselves in terms of mutually induced emf in the CET system due to the machine operation and vice versa, disturbance forces in the machine as a result of CET operation, and the influence of energy transfer capabilities as a function of machine operation. It is shown that most criteria can be met by applying an orthogonal mutual magnetic field distribution between the machine and the CET functionality. Moreover, the manner in which the integration is obtained, enforces specific demands on the soft-magnetic material. Therefore, a brief survey of types of soft-magnetic materials is conducted. The unique material properties of recently developed soft-magnetic composite (SMC) material grades have enabled the electromagnetic integration to become viable. The isotropic SMC allows the superposition of two magnetic fields that differ two orders of magnitude in frequency. Furthermore, three integrated solutions are proposed, i.e. the flat, double sided, and tubular linear topology. All topologies are qualitatively assessed to which extent the criteria are met, and which modeling technique of the first part is best suited for modeling. The assessment shows that the tubular topology is expected to perform the best in comparison to the two others.

The tubular topology applies an orthogonal field orientation, and it is designed to continuously transfer 800 VA of power and have a peak force of 250 N at a maximum speed of 2 m·s⁻¹. As an alternative to computationally expensive 3D, transient, FEM, an analytical model based on multiple harmonic models is presented for modeling and design. Decoupling of the functionalities on account of the orthogonality of the magnetic field orientation and the transformer field being significantly smaller in amplitude than the motor field allows each functionality to be modeled independently by fast 2D harmonic models. Justification of this modeling approach is provided by verification with 3D, FEM simulations. The optimal design is found by a parametric search of physical and geometrical parameters. Expected cross-coupling effects on account of the nonlinearity of the soft-magnetic material are identified through 3D, magnetostatic, FEM simulations. These simulations also show the impact of the assumption of the soft-magnetic material being infinitely permeable in the harmonic model.

A prototype of the integrated tubular topology has been constructed, and practical verification through measurements has been conducted. The measurements have shown the electromagnetic integration of a CET system into a synchronous, permanent magnet, linear motor in a single soft-magnetic body in a manner as presented in this thesis.
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<td>permanent magnet</td>
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Chapter 1

Introduction

1.1 The legacy of electromagnetism

Throughout history, man has frequently stood in awe when he turned his gaze at the sky to behold the electromagnetic phenomena, at the time not yet known as such, that transpired there. Whether it is the dreadful, devastating force of lightning or the unsullied beauty of the aurora polaris, electromagnetic phenomena have undeniably left an indelible imprint on the world view of the day. Unable to comprehend whatever they beheld, man ascribed these celestial spectacles to the creations of deities or even interpreted them as the earthly guises of the gods themselves.

As the Age of Enlightenment budded, so did the understanding of many physical phenomena. Among others, the view on electromagnetism was gradually replaced with a more scientific one, based on reason and logic, rather than an esoteric one. As the understanding of electromagnetism grew and man became able to predict the behavior of electromagnetic phenomena through mathematical models, it soon became apparent that the once feared and misunderstood phenomenon could be turned to their own advantage through technology. Resulting technologies have evolved ever since, and still continue to do so. A profound understanding of electromagnetism has helped to shape the technology driven, and perhaps even more, technology dependent world that we live in today. From that point of view man still marvels at electromagnetic phenomena, but less for its beauty or mystique, but increasingly for the wide variety of technical applications made possible by it.
Chapter 1: Introduction

1.2 Background

The unceasingly increasing demand for electric devices to ease our way of life comes at a price; all the devices have to be powered one way or another. The daunting pursuit of unscrambling Christmas lights that heralds in the Yuletide or the spaghetti wires behind the television set or PC are too familiar, recurring, domestic nuisances to many of us. Although cumbersome when an electric device has to be unplugged and removed or replaced, the tangled up wires do not normally cause the devices to work improperly when adequately shielded from electromagnetic interference. This becomes, however, a completely different story when electrical industrial devices repeatedly move relatively to each other at high velocities. Tangled up wires can be prevented by a sound mechanical mounting. This, however, does not prevent the wires from wearing out; causing downtime in a given production process. Additionally, the use of moving wires can severely limit the freedom of movement in certain applications. Moreover, in the high-precision industry the moving wires introduce undesired dynamic mechanical distortion, varying moving mass, hysteresis effects, and friction.

The conventional way of dealing with the limitations imposed by the use of moving cables is to replace them with slip rings. Carbon contacts, or brushes, are pressed against a copper ring-contact where either can rotate relative to the other as depicted in Fig. 1.1. By widening the slip ring in the axial direction, a 2 DoF (degrees-of-freedom) system with respect to directions of movements is obtained. This means that a simultaneous movement parallel to the axis is also made possible. For the system of Fig. 1.1, with multiple brushes in parallel, this entails that the pitch between the brushes becomes equal to the stroke in the axial direction. The slip ring principle in conjunction with brushes can easily be adapted for translating motion by unrolling the slip rings into flat plates. A variant for translating motion is the overhead line in conjunction with a pantograph that can be found in numerous electrified public transportation systems. In that case, the pantograph is the elongated brush and the overhead wire is equivalent to the
1.2: Background

slip ring. In any case, sufficient contact pressure must be exerted on the brush or pantograph to maintain a good contact. The slip ring solution increases the freedom of movement on one hand, but introduces new drawbacks on the other hand. The requirement of sufficient contact pressure introduces friction between the brushes and the rings. Furthermore, the contact surface between the copper and the brushes is highly resistive and, on top of that, shows nonlinear behavior. Pollution of the rings could obstruct the current flow from the brushes to the ring, and sparking can happen when a circuit break occurs. Also, the brushes are prone to abrasion which leads to downtime for maintenance.

A solution that enables a contactless electric coupling between moving members is obtained by a coupling through a time-varying electromagnetic field. In practice, low to high-power electromagnetic couplings are predominantly realized by magnetic-field couplings (transformers). For lower power ratings electric-field couplings (capacitors) can be applied as an alternative to magnetic-field couplings [51, 66]. A time-varying electromagnetic field distributed throughout space is generated by the stationary member of the electromagnetic link through which the moving member moves. The generated time-varying electromagnetic field is partly linked with the moving part. The energy contained within the linked electromagnetic field is converted into the appropriate form of electric energy. For the same power rating the voltage and frequency rating of a magnetic field coupling is in general significantly lower than that of an electric field on account of the magnetic permeability of free space being five orders of magnitude higher than the permittivity of free space. In literature, many variations of magnetically coupled contactless energy transfer (CET) systems have been proposed of which an overview is provided in chapter 8. CET systems are to a lesser degree susceptible to mechanical friction, abrasion, mechanical distortion, and hysteresis effects. However, additional power electronic converters on the stationary and moving side are necessary to convert the electricity into the desired form.

It is not uncommon for advanced electromechanical systems to comprise several electromechanical actuators. Often multi-DoF or multi-axis motion is realized by a combination of single-DoF actuators. These single-DoF actuators can be fed by separate CET systems. To allow multi-DoF operation some actuators for actuation on one axis are moved in their entirety by a single or multiple actuators that provide actuation on other axes. Hence, the power rating of the CET along the axis that carries additional actuators has to be sufficient as to not only supply the power necessary for actuation along its own axis, but also to provide the power demand of the actuators of the additional axes that are moved by it. Generally, the approach of adding separate CET systems to each axis has a detrimental effect on the force density of the system as a whole. The electric-to-kinetic energy conversion process that occurs in electromechanical actuators and the energy conversion process of the CET through the electromagnetically coupled field are governed by the same physical fundamentals. In both cases the conversion of electric or kinetic energy into, or from, the electromagnetic energy contained within the electromagnetic field form an intermediate step in the overall energy conversion process. The next logical step in improving the force density,
while exploiting the advantages associated with CET systems as power supplies, is the investigation if these closely related processes can be integrated into a single electromagnetic device. The research on the multiple energy conversions for CET and single-DoF actuation in a single device containing permanent magnets, and the modeling of it, is treated in this thesis.

1.3 Research goals and objectives

Transferring energy wirelessly to the moving member of a machine can be obtained through an add-on approach. In the add-on approach a separate CET system that is electromagnetically completely isolated from the actuator is mounted to the actuator. Aspects in regard to add-on solutions are not thoroughly elaborated on in this thesis. Integration of multiple energy conversions in a single device can be approached from two different perspectives, i.e. integration can be considered physical or structural in nature. An integrated CET solution is considered physical in nature when the active (coils and/or permanent magnets) or passive (soft-magnetic materials) magnetic components have a dual functionality. An integrated solution is considered structural in nature where none of the components have a dual functionality, but where the components are placed such that the flux of either functionality influences the electromagnetic behavior of the other.

This thesis aims to research the electromagnetic aspects associated with the integration of the dual energy conversions in a single, linearly moving device in a physical manner rather than a structural one. More in particular, the following aspects in this regard are researched and addressed:

1. Proper choice for an adequate magnetostatic modeling technique for linear energy conversion devices:
   The choice for a particular modeling technique is dictated by both the physical and geometrical nature of the electromagnetic problem at hand. Preferably, the chosen modeling technique is both accurate, fast, and at least able to take into account physical phenomena that dominate the electromagnetic behavior of the problem. For complex electromagnetic problems one has to concede on one, two, or even all of these three aspects. A time consuming, but accurate technique such as the nonlinear finite element method (FEM) is undesirable when numerous model evaluations are required during optimization. Instead, alternative, eligible modeling techniques have to be researched and compared quantitatively. A quantitative comparison enables a well-founded choice for a suitable technique for the modeling of a specific electromagnetic problem. Generally, these modeling techniques require certain assumptions that make them less accurate. However, the loss in accuracy is compensated for by significantly reduced computational efforts. Furthermore, in order for a technique to be applicable, restrictions on the geometry might apply. Hence, the following matters are addressed
1.3: Research goals and objectives

specifically when comparing alternative magnetostatic modeling methods:

- mapping of the physical assumptions required for a modeling technique to be applicable;
- identification of the limitations of a modeling method in terms of geometrical restrictions;
- quantification of the attainable accuracy with respect to the computation time for the different modeling methods.

2. Formulation of requirements with respect to cross-coupling effects associated with integration of CET into a linear actuator:

Integration of CET into a linear actuator in a physical manner results in the superposition of two magnetic fields in one soft-magnetic body. One magnetic field originates from the CET functionality and the other from the actuator functionality. Moreover, each field operates within its own different frequency range. Hence, electromagnetic cross-coupling effects between the functionalities are to be expected. The following issues in regard to the impact on the electromagnetic performance are identified, investigated, and quantified:

- the dependency of the overall electromagnetic performance on the spatial orientations of the flux density component of the CET relative to that of the machine;
- the additional losses in the structure with respect to conventional machines as a result of extra ohmic losses on account of the addition of CET coils and the increased iron losses due to the CET flux density component;
- the superposition of the electromagnetic fields operating in their own frequency domain that both restricts and mutually affects the electromagnetic behavior of both functionalities.

3. Study on topological configurations of integrated solutions and the use of magnetic materials:

Proposed integrated topologies are scrutinized to which extent they are susceptible to cross-coupling effects between the two functionalities. A feasible integrated, topological design must exhibit decoupling between the two functionalities. A good decoupled electromagnetic performance is, among others, determined by the geometrical positioning of active magnetic components and the mutual, spatial magnetic field orientation of the CET and machine. Moreover, the spatial field orientation associated with a specific topology and the range in operating frequency enforce stringent requirements on the type of suitable soft-magnetic material. Material properties such as saturation level, core losses, and the material being isotropic or anisotropic stipulate, and also limit, in a considerable measure the electromagnetic performance of the integrated topology. A comparative assessment with respect to the soft-magnetic material properties is conducted.
4. **Extension to the classical modeling of electromechanical devices:**
   The integration of CET into an actuator requires the classical design philosophy with respect to electrical machines to be revised. A new, problem specific design philosophy is formulated that not only allows the modeling of the actuator, but also incorporates the CET functionality. The increased complexity of the electromagnetic problem as a result of integration calls for a modeling technique that provides an acceptable compromise between calculation time and accuracy for it to be applicable in an optimization procedure. Furthermore, the extended modeling technique preferably enables the identification and quantification of all electromagnetic cross-coupling effects between the CET and actuator.

5. **Realization of a prototype for practical verification:**
   Practical verification on a prototype is not only required to demonstrate the proof-of-principle, but to also quantitatively assess the validity of the design approach and the electromagnetic and thermal modeling. Furthermore, measurements on the prototype permit the actual quantification of cross-coupling effects and additional phenomena that the modeling techniques fail to provide accurate, or even any information on. The effects of manufacturing tolerances that are not considered in the electromagnetic modeling are identified.

### 1.4 Outline of the thesis

This thesis is split into three parts. The first part focuses primarily on the comparative study of magnetostatic modeling techniques and force calculations for electromechanical energy conversion devices. The second part is concentrated on the research questions in regard to the integration of the CET into a linear, permanent magnet actuator. Furthermore, a detailed description of the modeling of the selected prototype and the practical verification is provided in part two. Finally, in the third part conclusions with respect to the first and second part are drawn from the findings in the preceding parts. Also, the scientific contributions of the presented research are provided. Part three concludes with recommendations for further improvement and future research. The first part is aimed to be written in such a manner that it can be read independently of the second part, and the general formulations of the presented modeling techniques are not tailored to a specific application. Apart from providing a comparative analysis between the modeling methods, the emphasis of each individual modeling technique, as presented in the first part, is also on the mathematical derivation from Maxwell’s equations and implementation.
1.4: Outline of the thesis

1.4.1 Part I: EM modeling of energy conversion problems

Chapter 2 begins with a review of the general electromagnetic theory. An overview of analytical and numerical modeling methods for the solving of Maxwell’s equations is given next. Finally, an electromagnetic benchmark problem is presented to which all presented modeling techniques are applied.

The harmonic method (HM), a modeling technique that is based on a Fourier-series description of the electromagnetic field distribution in orthogonally shaped domains, is discussed in chapter 3. The range of application of the HM is addressed. Moreover, it is explained how the electromagnetic boundary conditions are to be applied to obtain the proper electromagnetic field distribution in the domain of interest. The HM is applied to the electromagnetic benchmark problem and the obtained results, in the form of the calculated flux density distribution, are compared to FEM simulations. Also, the effect of increasing the accuracy by taking more harmonics into account on the calculation time is investigated.

Chapter 4 is devoted to the Schwarz-Christoffel (SC) mapping technique. The simplification of an electromagnetic boundary value problem with a polygonal, geometrical configuration of boundaries in a Cartesian or polar coordinate system through the technique of conformal mapping is elaborated on. The complex SC-mapping integral is derived and the procedure for making the electromagnetic benchmark problem fit to be modeled by means of the Schwarz-Christoffel method (SCM) is discussed. The process for the calculation of the flux density distribution in the electromagnetic benchmark problem is provided. More complex boundary value problems require the SC integral to be evaluated numerically. As is the case for the HM, the obtained flux density distribution is compared to FEM simulations. The effect of increasing the accuracy of the method by increasing the number of equivalent point sources on the calculation time is quantified to allow a comparison between the SCM and the HM and the other methods as presented in subsequent chapters of part I.

The numerical solving of the scalar potential of an electromagnetic boundary value problem by means of the boundary element method (BEM) is the topic of chapter 5. The derivation from Maxwell’s equation in integral form into a set of linear equations by means of Green’s identities is addressed. To that end, the boundary of the electromagnetic boundary value problem has to be discretized. The BEM is applied to the geometry of the electromagnetic benchmark problem. The results are once more compared to FEM simulations. To allow a comparison with the other modeling methods the influence on the accuracy and computation time as a function of the discretization of the boundary is mapped.

In chapter 6 the tooth contour method (TCM) is introduced. The TCM is an equivalent magnetic circuit (MEC) based, hybrid, modeling technique that employs the BEM for airgap permeance calculations to improve its accuracy. The classical MEC is explained and the calculation of the airgap permeances by means of the BEM is discussed. The calculated flux linkages of the coils in the electro-
magnetic benchmark problem are compared to FEM simulations. The effects of efforts to increase the accuracy of the method at the expense of an increased calculation time are recorded for comparison with the other methods.

For electromechanical devices forces and torques originating from the electromagnetic field distribution are the quantities of interest. Depending on the form in which the electromagnetic field distribution in a structure is available eventually determines which force calculation should be applied. Three methods of force calculations are presented in chapter 7, i.e. the Lorentz force method, the Maxwell stress tensor method, and the virtual work method. Force calculations based on the field distributions obtained by the different modeling techniques, as described in the previous chapters, on the electromagnetic benchmark problem are determined. The outcome of the force calculations is compared to force calculations from the FEM.

1.4.2 Part II: Linear actuation with integrated contactless energy transfer

Chapter 8 starts with a discussion on the basic principles of electromagnetic energy conversion for CET through capacitively and inductively coupled system, and wave propagation. Next, a literature overview of both fully separated, and physically as well as structurally integrated CET systems in conjunction with electrical machines is provided. The research questions regarding the integration of CET into a linear, permanent magnet actuator are discussed. To that end, five criteria are formulated with respect to the electromagnetic behavior of an integrated solution that, ideally, should be met in order to reach decoupling of the functionalities. Furthermore, a comparative analysis of three different soft-magnetic materials with respect to their respective electromagnetic material properties is conducted. The consequences of the integration of a CET functionality in a machine are quantified by means of combined equations for initial sizing of the CET and the permanent magnet machine. Moreover, three integrated topologies are proposed that are deemed feasible. The feasibility checks are performed by assessing to which extent the three topologies satisfy the formulated criteria.

One of the proposed integrated topologies, namely the tubular topology, is selected for prototyping. An electromagnetic model is thoroughly discussed for the modeling of the integrated, tubular topology in chapter 9. The model exploits the decoupled behavior of the CET and machine in that the CET functionality and machine functionality are modeled separately by means of the HM. Verification and the impact on account of the electromagnetic assumptions, that allow the HM to be applicable, are identified by means of FEM simulations. Furthermore, a steady state thermal equivalent circuit (TEC) model is discussed that is applied to monitor the temperature distribution in the structure during the design process. The design strategy is elaborated on as well. Cross-coupling effects due to the nonlinearity of the soft-magnetic material are identified by means of 3D FEM
simulations.

Practical verification and the proof-of-concept of the integration of the CET into a tubular, permanent magnet motor is demonstrated in chapter 10. Measurements with respect to the electromagnetic behavior are discussed. These include emf, resistance, induction and force measurements, and the quantification and localization of losses. The cross-coupling effects between the functionalities are quantified through stationary and dynamical measurements. Where possible, the verification of the electromagnetic model of chapter 9 that is used for design is provided. Moreover, alterations in design choices that were required during construction, and their impact on the electromagnetic behavior, are explained.

### 1.4.3 Part III: Closing

This thesis concludes with chapter 11. Chapter 11 deals with the conclusions on part I and II. Furthermore, the conclusions and answers to the research questions of section 1.3 are given. Finally, the recommendations for further improvement and future research are addressed.
Part I

Electromagnetic modeling of energy conversion problems
A fast and accurate design method, or modeling technique, is an indispensable and essential part of the design process of any electromagnetic device. In general, a compromise has to be made between the speed and the accuracy of the modeling technique. Opting for a very accurate modeling technique at the expense of a long and time consuming design process might be desirable. However, it might simply not be feasible because of the size or complexity of a device, either physical or geometrical in nature; or because of insufficient computational resources. In any case, making the right assumptions and choosing a proper, less accurate design method can still lead to a satisfactory end result, if one is aware of the shortcomings of the design method. Choosing the proper design method is the most crucial step in the design flow, since it must at least be able to predict the physical phenomena that affect the behavior of the device to be designed the most.

In this chapter a short review of Maxwell’s theory is given first. Furthermore, a brief survey is given of general modeling techniques that are commonly applied to the design of electromechanical devices with their respective advantages and disadvantages in terms of accuracy, applicability, and to which extent the physical phenomena can be taken into account. In the last section a benchmark topology is introduced. In the subsequent chapters the presented techniques that are applied to, explained, and validated by means of the benchmark problem are discussed.
2.1 General electromagnetic theory

This section deals with the general electromagnetic theory that is imperative for a profound understanding of the different modeling techniques that are the topic of later chapters. First, it is described how three different, special cases are obtained from the full Maxwell equations and how they relate to material properties. Next, the magnetic vector and scalar potential are introduced, and it is explained how they can be applied to solve Maxwell’s equations. Maxwell’s equations together with the magnetic vector potential or scalar potential formulation form the foundation from which the discussed modeling tools are derived.

2.1.1 Maxwell’s equations

Maxwell’s equations that govern electromagnetic phenomena are given by [72]

\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \quad \text{Ampère’s law} \quad (2.1a) \]
\[ \nabla \cdot \vec{B} = 0, \quad \text{Gauss’s law for magnetism} \quad (2.1b) \]
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \text{Faraday’s law} \quad (2.1c) \]
\[ \nabla \cdot \vec{D} = \rho_V, \quad \text{Gauss’s law} \quad (2.1d) \]

where \( \vec{H} \) is the magnetic field strength, \( \vec{J} \) is the current density, \( \vec{D} \) is the electric flux density, \( \vec{B} \) is the magnetic flux density, \( \vec{E} \) is the electric field strength, \( \rho_V \) is the free electric volume charge density and \( t \) is the time. These quantities are not independent and are linked through the material properties of the medium which are given by the constitutive relations

\[ \vec{B} = \mu_0 \left( \vec{H} + \vec{M} \right), \quad (2.2a) \]
\[ \vec{D} = \varepsilon_0 \vec{E} + \vec{P}, \quad (2.2b) \]

where \( \mu_0, \varepsilon_0, \vec{M}, \) and \( \vec{P} \) are the permeability and permittivity of free space, the magnetization, and polarization of the material, respectively.

The source terms \( \vec{J}, \vec{M}, \) and \( \vec{P} \) (2.1) and (2.2) can be split into two parts [109]

\[ \vec{J} = \vec{J}_0 + \vec{J}_i, \quad (2.3a) \]
\[ \vec{M} = \vec{M}_0 + \vec{M}_i, \quad (2.3b) \]
\[ \vec{P} = \vec{P}_0 + \vec{P}_i, \quad (2.3c) \]

where the 0-subscript denotes the internal source term of the material that is not dependent on the externally imposed electromagnetic fields (\( \vec{E} \) and \( \vec{H} \)), and
the \(i\)-subscript denotes the term that is induced due to the externally enforced electromagnetic field. The induced source terms are a result of the interaction between the imposed electromagnetic field and the electrically charged particles in the material. The extent to which a material linearly reacts to an externally applied field is given by the electric conductivity, \(\sigma\), magnetic susceptibility, \(\chi_m\), and electric susceptibility, \(\chi_e\), of the material. The induced components of (2.3) can be expressed as

\[
\vec{J}_i = \sigma \vec{E}, \quad (2.4a)
\]

\[
\vec{M}_i = \chi_m \vec{H}, \quad (2.4b)
\]

\[
\vec{P}_i = \varepsilon_0 \chi_e \vec{E}, \quad (2.4c)
\]

With the substitution of (2.4) into (2.3) and by subsequently substituting the obtained result into (2.2), the constitutive relations can be written as

\[
\vec{B} = \mu_0 (1 + \chi_m) \vec{H} + \mu_0 \vec{M}_0 \quad (2.5a)
\]

\[
\vec{D} = \varepsilon_0 (1 + \chi_e) \vec{E} + \vec{P}_0 \quad (2.5b)
\]

where \(\mu_r\) is the relative permeability and \(\varepsilon_r\) is the relative permittivity of the material. Henceforth, the multiplication of the relative permeability with the permeability of free space is simply referred to as the permeability, \(\mu\), of the material, i.e \(\mu = \mu_r \mu_0\). Analogously, the permittivity, \(\varepsilon\), of the material equals \(\varepsilon = \varepsilon_r \varepsilon_0\).

The four partial differential equations of (2.1) together with the constitutive relations of (2.5) mathematically describe the complete electromagnetic behavior of a system. Depending on the frequency, \(f\), at which an electromagnetic device operates and whether or not conductive material is present in the device, (2.1) can be simplified. Electromechanical systems generally operate at relatively low frequencies, typically \(f \ll 1\) MHz. For these frequencies the time dependent displacement current term in (2.1a) can be neglected in the analysis. In this manner, the quasi-static equations are obtained

\[
\nabla \times \vec{H} = \vec{J}, \quad \text{Ampère’s law} \quad (2.6a)
\]

\[
\nabla \cdot \vec{B} = 0, \quad \text{Gauss’s law for magnetism} \quad (2.6b)
\]

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \text{Faraday’s law} \quad (2.6c)
\]

\[
\nabla \cdot \vec{D} = \rho_V. \quad \text{Gauss’s law} \quad (2.6d)
\]

The quasi-static equations are to be applied when eddy currents in conductive media influence the behavior of the device in such a way that they cannot be neglected. Skin and proximity effects in wires [22], core losses in transformers
Chapter 2: Electromagnetic modeling methods

and motors [71] are examples where eddy currents play a dominant role. Even more important are the class of devices of which the working principle is based on the existence of eddy currents in the structure, such as induction machines [40], magnetic braking systems [43] and induction heating [14].

Apart from eddy currents Faraday’s law, (2.6c), causes the induced back electromotive force, emf, when conductors are exposed to time-varying magnetic fields. However, magnetostatic calculations suffice to determine the emf when eddy current effects can be neglected. For this situation Maxwell’s equations can be further reduced to the static equations

\[ \nabla \times \vec{H} = \vec{J}, \quad \text{Ampère’s law} \]
\[ \nabla \cdot \vec{B} = 0, \quad \text{Gauss’s law for magnetism} \]
\[ \nabla \times \vec{E} = 0, \quad \text{Faraday’s law} \]
\[ \nabla \cdot \vec{D} = \rho_V. \quad \text{Gauss’s law} \]

The advantage of using a static equations is that they are now decoupled; meaning that the problem reduces to either a fully magnetostatic problem (only dependent on \( \vec{H} \) and \( \vec{B} \)), or a fully electrostatic one (only dependent on \( \vec{E} \) and \( \vec{D} \)). In practice, a magnetostatic analysis suffices in the majority of the cases for an accurate physical description of electrical machines, such as synchronous and dc-machines. Therefore, the analyses and derivations of different modeling tools in the subsequent chapters hold for the magnetostatic case, unless indicated otherwise.

2.1.2 The magnetic vector potential

The coupled differential equations (2.7a) and (2.7b) can be solved by introducing a vector potential, \( \vec{A} \). Since \( \nabla \cdot \vec{B} = 0 \), the magnetic flux density can be expressed in terms of the vector potential using vector calculus

\[ \vec{B} = \nabla \times \vec{A}. \]

By imposing the Coulomb gauge condition, after substitution of (2.8) into (2.7a) and using (2.5a), the Poisson equation is obtained

\[ \nabla^2 \vec{A} = -\mu_0 \mu_r \vec{J} - \mu_0 \nabla \times \vec{M}_0. \]

The expression for \( \vec{A} \) is dependent on the used coordinate system. The vector potential method is applicable to both 2D and 3D problems. For 2D problems \( \vec{A} \) reduces to a scalar valued quantity in one direction only, i.e. orthogonal to the 2D-plane of interest. Alternatively, the scalar potential method, which is the topic of section 2.1.3, can be applied to find the electromagnetic field solution. The vector potential method is particularly useful for the calculation of the electromagnetic field distribution inside domains with currents and for inductance calculations [26]. The vector potential can only be solved analytically for a handful of problems.
with very simple boundary conditions and linear soft-magnetic materials as shown for 2D geometries in Fig. 2.1. The geometries in Fig. 2.1 are all orthogonal shapes, by which is meant that the edges of the domain are parallel to the axes of the coordinate system and have right angles where they form vertices. For geometrically more complex problems (2.9) can be solved numerically by means of domain discretization of the problem.

### 2.1.3 The magnetic scalar potential

Similar to the vector potential method described in the previous section, vector calculus can be applied to (2.7a) and (2.7b) for current-free regions. In case \( \vec{J} = 0 \, \text{A} \, \text{m}^{-2} \), (2.7a) can be expressed in the scalar potential by applying the vector identity \( \nabla \times (\nabla \phi) = 0 \). Hence, the magnetic field strength is given by

\[
\vec{H} = -\nabla \phi, \tag{2.10}
\]

where \( \phi \) is the magnetic scalar potential in A. By substituting (2.10) into the constitutive relation (2.2a) and the result into (2.7b) the Poisson equation for the magnetic scalar potential is obtained

\[
\nabla^2 \phi = \nabla \cdot \vec{M}_0. \tag{2.11}
\]

The main restriction of the scalar method is already evident from its derivation, i.e. only current-free regions can be considered. This limitation can be overcome by applying the reduced magnetic scalar potential [11]. The method is equally fit to be applied in both 2D and 3D problems. Contrary to the magnetic vector potential approach the solution remains a scalar regardless of the problem being a 2D or 3D one. The same geometrical limitations apply as for the vector potential method. Orthogonal 3D geometries for which (2.11) can be solved analytically are shown in Fig. 2.2. Geometrically complex problems can be solved numerically.
Fig. 2.2: Orthogonal, 3D geometries for which the Poisson equation can be solved analytically: bar (a), cylinder (b), and sphere (c).

2.2 Overview of analytical modeling techniques

As is apparent from the previous section, it is for most practical cases not possible to calculate an analytical expression for the magnetic vector or scalar potential. It is, however, possible to make assumptions or accept limitations that allow the magnetic field distribution inside a structure, or at least a part of it, to be calculated relatively fast within an acceptable accuracy. An overview of commonly used analytical modeling tools, together with a short analysis of what can and cannot be modeled, is given in this section.

2.2.1 Magnetic equivalent circuit method

One of the oldest, and still widely used, modeling techniques is the magnetic equivalent circuit (MEC) method [32, 81]. In this method the magnetic structure of the device is subdivided into a network of simplified flux tubes, known as reluctances or permeances, based on estimated flux paths. The obtained flux tube network together with the sources can be transformed into an equivalent reluctance, or permeance, network for which the flux through the tubes and the scalar potential values at the nodes can be solved by applying Kirchhoff’s laws. It is possible to take saturation effects into account by solving the nonlinear network with an iterative algorithm [49]. The MEC method is less suitable as design tool for electrical machines when high accuracy is required or for problems in free space. The method is described in detail in chapter 6.
2.2: Overview of analytical modeling techniques

2.2.2 The equivalent current method and imaging

The equivalent current method [26] is based on the Poisson equation for the vector potential (2.9). It can be seen that the curl of the magnetization on the right hand side of (2.9) can be expressed as an equivalent current density: \( \mathbf{J}_m = \nabla \times \mathbf{M}_0 \). This implies that the field solution can be obtained by the superposition of the field solution originating from a current density due to a free external source \( \mathbf{J} \) and an equivalent current density originating from a permanent magnet. Superposition can, however, only be applied for linear homogeneous media. A difference in the relative permeability of the permanent magnet and the surrounding linear medium can only be taken into consideration for a limited number of problems where the boundary of the transition between the magnet and the surrounding form parallel straight lines [96]. In other cases a unity relative permeability has to be assumed for the magnets. The equivalent current method is especially suitable for force and torque calculations on permanent magnets and coils in free space [45].

For simple Dirichlet or Neumann boundary conditions the magnetic field distribution between the boundaries can be obtained by applying the method of images [11]. The magnetic field solution can be obtained by the superposition of the field solutions that originate from sources that are geometrically mirrored to the original ones in such a way that the boundary conditions are met. The values of the current of the equivalent, imaged sources have to be adjusted in accordance with the permeability value on either side of the boundary. The field solution due to the source distribution is only valid in the domain containing the non-imaged

Fig. 2.3: A cylindrical permanent magnet in a 2D, axisymmetric coordinate system near a finitely permeable boundary (a), and its equivalent current model (b).
equivalent currents bounded by the boundaries. An example of a cylindrical permanent magnet near a straight boundary is shown in Fig. 2.3. Nonlinearity of soft-magnetic materials cannot be taken into account.

### 2.2.3 The equivalent magnetic charge method and imaging

The equivalent magnetic charge model is similar to the equivalent current model, except that the method is derived from the Poisson equation for the scalar potential (2.11), and only permanent magnet sources can be allowed for. For the magnetic equivalent charge method there are, as the name implies, equivalent magnetic charge source terms. The equivalent magnetic charge term consists of a volume charge density term \( \rho_V = -\nabla \cdot \vec{M}_0 \) and a surface charge density term \( \rho_S = \vec{M}_0 \cdot \vec{n} \), where \( \vec{n} \) is the outward normal vector of the surface of the magnetized body [26]. Because of the divergence free property for magnetic fields of (2.7b), the total amount of positive equivalent magnetic charge and the total amount of negative equivalent magnetic charge in the complete problem must be equal. The applicability and restrictions of the equivalent magnetic charge model are the same as those for the equivalent current method. The technique of imaging is equally applicable, and the solution for the magnetic field is only valid in the domain bounded by the boundaries [42]. For problems with linear material properties a mix of both methods can be applied to incorporate the magnetic flux density field of free current sources through superposition. The geometry of Fig. 2.3 transformed into an equivalent magnetic charge model is shown in Fig. 2.4

![Fig. 2.4: A cylindrical permanent magnet in a 2D, axisymmetric coordinate system near a finitely permeable boundary (a), and its equivalent magnetic charge model (b).](image-url)
2.2: Overview of analytical modeling techniques

2.2.4 Harmonic method

For orthogonal shapes in orthogonal coordinate systems the analytical solution to the Poisson equations for both the vector and scalar potential formulation can be expressed as a Fourier series [31]. When a structure can be divided into adjoining orthogonal domains, the harmonic coefficients of the Fourier series have to be determined by applying the boundary conditions. If all the domains have the same fundamental frequency all linear material properties such as permeability, permittivity and conductivity can be taken into account [11]. For adjoining domains with a different fundamental frequency a special boundary condition has to be applied to find the harmonic coefficients that determine the electromagnetic field distribution in the domains of interest. The harmonic method (HM) is applicable for the full, quasi-static and static formulation of Maxwell’s equations. The HM is fast and accurate due to the electromagnetic field distribution being available in analytical form. However, nonlinearities cannot be allowed for. A thorough discussion of the HM is given in chapter 3 for magnetostatic cases.

2.2.5 Conformal mapping method

Another modeling technique is the conformal mapping method [34]. The method is based on complex functions in the 2D polar or 2D Cartesian domain, which makes it a transformation method. A complex function maps points of one complex plane into another complex plane. Through this complex mapping function the geometry of a problem with complicated boundary conditions, for which (2.9) or (2.11) cannot be solved initially, can be transformed into a model with simple Dirichlet or Neumann boundary conditions for which an analytical expression for the potential or flux density distribution is available. A special case of the conformal mapping method is the topic of chapter 4.

2.2.6 Semi-analytical implementation issues

Although in many cases the magnetic scalar or vector potential can be expressed analytically for a given electromagnetic problem by applying one of the previously described modeling techniques, the evaluation of the potential value is liable to numerical issues when implemented into a computer program. Therefore, the methods can be deemed semi-analytical methods. The numerical issues can be different in nature and are briefly addressed here.

When the boundary configuration requires an infinite number of images for the equivalent current method, magnetic equivalent charge method, or conformal mapping method the expression for the electromagnetic field becomes an infinite series. The practical implementation requires the series to be truncated to a finite series. The number of terms to be evaluated is an unavoidable compromise.
Chapter 2: Electromagnetic modeling methods

between speed and accuracy. The same holds for the HM, where the expression for the potential is an infinite Fourier series.

If the number of domains in the HM become large, it is preferable to calculate the Fourier coefficients through solving the set of equations by matrix inversion. As the number of considered harmonics increase, so does the difference in the order of magnitude between the largest and the smallest element in the matrix. This could cause the matrix to be ill-conditioned. If possible, scaling of the matrix can improve the condition number of the matrix; or else, the number of harmonics have to be reduced further. The same phenomenon could occur, although less common, for the MEC method when flux tubes within one model differ strongly in geometrical size or relative permeability.

If the mapping function for the conformal mapping method has to be determined numerically, it is evident that numerical issues arise. Apart from that, numerical errors are introduced due to the point-to-point property that is inherent to the method, i.e. a finite number of equivalent currents or charges can only be allowed for.

2.3 Overview of numerical modeling techniques

The Poisson equations for the vector and scalar potential can be solved numerically for problems where analytical modeling approaches fail to provide enough information, or cannot solve the problem at all. Numerical methods are very flexible since there are, depending on the method to be applied, hardly any restrictions on the geometrical complexity of the problem, and the electromagnetic field distribution can be calculated throughout the entire domain. All numerical methods are based on the discretization of the domain in order to transform the Poisson equation into a set of linear equations. The discretization is mostly referred to as the mesh. In general, a finer discretization of the domain leads to a more accurate solution of the electromagnetic field distribution compared to a coarse discretization. However, fine discretization results in a computationally more demanding process in terms of simulation time and memory usage, which leaves the applicability of numerical methods to the computational capabilities available to the user. In the following, three of the widely used numerical methods are shortly addressed.
2.3: Overview of numerical modeling techniques

2.3.1 Finite difference method: FDM

The oldest and most intuitive numerical method is the finite difference method (FDM) [26]. The FDM is based on a rectangular discretization of the problem space to solve the Poisson or Laplace equation. The rectangular mesh is realized by means of an orthogonal grid of nodes. The partial differential equations are expressed as the discrete differentiation between adjacent nodes. In this way, the partial differential equation is transformed into a set of linear equations that can be solved in order to find the potential values at the nodes subject to the boundary conditions. Nonlinear material properties can be handled by solving the set of equations through an iterative nonlinear solving algorithm. An illustrative example of a meshed geometry for the FDM is shown in Fig. 2.5a, which depicts a gapped C-core with a permanent magnet. It can be seen that the surrounding space has to be meshed as well, which means that the problem has to be bounded to limit the number of nodes. Non-orthogonal geometrical shapes with slanted or curved edges are more difficult to handle and require a non-orthogonal mesh distribution. For those problems the finite element method is more appropriate.

2.3.2 Finite element method: FEM

At present, the finite element method (FEM) is the most widely used numerical method for the design and analysis of electromechanical devices because of its flexibility and numerical stability [46]. Similar to the FDM the geometry is discretized to solve the Poisson or Laplace equation, but not into nodes but into triangular elements for 2D problems or tetrahedral elements for 3D problems. Other polygonally shaped elements are very well possible, but in practice triangular and tetrahedral ones are most commonly applied, because they allow the meshing of slanted and curved boundaries. To transform the Poisson equation into a set of linear equations, the potential distribution throughout an element as a function of the spatial variables and the potential values at its nodes is firstly

Fig. 2.5: Illustrative examples of discretization of a gapped C-core with a permanent magnet for different numerical methods: FDM (a), FEM (b), and BEM (c).
expressed by an interpolation function. The interpolation function is generally a polynomial that can be of arbitrary order, but in practice a second order polynomial is mostly applied, because it provides a good compromise between accuracy, mesh size, and computation time. Next, the energy functional in terms of the stored energy in the electromagnetic field and the energy of the sources is determined. By minimizing the energy functional with respect to the value of the potential on all nodes, the set of equations is finally obtained. The entries in the system matrix are only dependent on the spatial variables that describe the mesh. The triangulation of the same geometry as for the FDM is illustrated in Fig. 2.5b.

After solving the system the potential values at the nodes are known, and the polynomial interpolation can be applied to calculate the potential values in between nodes. Nonlinearities can be allowed for by implementing an iterative nonlinear solving algorithm such as the Newton-Raphson method. Especially for 2D problems that contain nonlinear soft-magnetic material, the FEM is very suitable. For 3D problems the method soon becomes computationally expensive and time consuming, because of the large number of elements in the domain. As is the case for the FDM, the entire domain has to be discretized as shown in Fig. 2.5b. The boundary condition at infinity can be mimicked by truncation of the domain with a modified boundary condition. This is particularly disadvantageous for open problems in free space where the field slowly diminishes with respect to the distance from the source. For these kind of problems the BEM is a proper alternative.

2.3.3 Boundary element method: BEM

The boundary element method (BEM) [27] is another numerical technique that can be applied to determine the potential distribution in the domain. The derivation of the BEM, in order to obtain the set of linear equations, differs from the derivation of the FDM and the FEM. Where both the FDM and the FEM are derived from the local formulation of the Poisson or Laplace equation; the BEM has a different starting point, and it is derived from the global formulation of the Poisson or Laplace equation. In other words, the method is based on integral equations instead of differential equations. The integral equation only has to be evaluated on the boundaries of the domain and the dimension of the mesh elements is one order lower than that of the problem. In this manner, the number of elements can be reduced significantly compared to the FEM. For unbounded problems or problems in free space the BEM is very suitable, since the domain need not be bounded. An example of the discretization of the gapped C-core with a permanent magnet is shown in Fig. 2.5c.

Similar to the FEM the potential values in between the nodes of the mesh elements is obtained through polynomial interpolation functions. Although the BEM solves a given problem with fewer elements than the FEM does for the same problem, the computational effort is not necessarily proportionally less. Contrary to the FDM
and FEM the system matrix is densely populated, which intensifies the memory usage. Moreover, the material properties inside the domain have to be linear and homogeneous, which makes the BEM unsuitable for problems with nonlinear soft-magnetic material. Hybrid techniques that employ the BEM in linear domains and the FEM in nonlinear ones have been proposed to combine the advantages of both methods [100]. A more detailed discussion about the BEM is given in chapter 5.

2.4 Benchmark topology

In the subsequent chapters three modeling techniques are derived from Maxwell's equations that are considered to be eligible design tools for slotted, permanent magnet, electromechanical devices. The modeling techniques are applied to the same electromechanical test case problem in order to obtain the electromagnetic field distribution within the topology. From the electromagnetic field distribution the force on the translator is calculated. The result of each method is compared to FEM simulations. The sole purpose of the benchmark topology is to enable a comparative, quantitative analysis of the modeling tools.

2.4.1 Geometry and physics of the benchmark topology

The benchmark topology is shown in Fig. 2.6. The benchmark topology is a typical electromechanical problem [47, 59, 88], and it comprises a periodic section of a
Table 2.1: Physical parameters for the benchmark problem

<table>
<thead>
<tr>
<th>variable</th>
<th>value</th>
<th>unit</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{\text{rem}}$</td>
<td>1.3</td>
<td>T</td>
<td>remanence of the permanent magnets</td>
</tr>
<tr>
<td>$\mu_{\text{r,pm}}$</td>
<td>1.05</td>
<td>-</td>
<td>relative permeability of the permanent magnets</td>
</tr>
<tr>
<td>$J_{\text{ph}}$</td>
<td>$4 \cdot 10^6$</td>
<td>A m$^{-2}$</td>
<td>rms phase current density</td>
</tr>
<tr>
<td>$\mu_{\text{r,Fe}}$</td>
<td>$5 \cdot 10^3$</td>
<td>-</td>
<td>relative permeability of the core and back plate</td>
</tr>
</tbody>
</table>

Table 2.2: Geometrical parameters for the benchmark problem

<table>
<thead>
<tr>
<th>variable</th>
<th>value</th>
<th>unit</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_p$</td>
<td>12</td>
<td>mm</td>
<td>pole pitch of the permanent magnets</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>16</td>
<td>mm</td>
<td>pole pitch of the phase coils</td>
</tr>
<tr>
<td>$\alpha_m$</td>
<td>$\frac{2}{3}$</td>
<td>-</td>
<td>permanent magnet width ratio</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>$\frac{9}{16}$</td>
<td>-</td>
<td>phase-slot width ratio</td>
</tr>
<tr>
<td>$h_1$</td>
<td>5</td>
<td>mm</td>
<td>height from the origin of the back iron</td>
</tr>
<tr>
<td>$h_2$</td>
<td>9</td>
<td>mm</td>
<td>height from the origin of the magnet array</td>
</tr>
<tr>
<td>$h_3$</td>
<td>10</td>
<td>mm</td>
<td>height from the origin of the airgap</td>
</tr>
<tr>
<td>$h_4$</td>
<td>20</td>
<td>mm</td>
<td>height from the origin of the phase slots</td>
</tr>
<tr>
<td>$h_5$</td>
<td>25</td>
<td>mm</td>
<td>height from the origin of the translator</td>
</tr>
</tbody>
</table>

three phase, four pole, synchronous, permanent magnet, linear motor with surface mounted permanent magnets and concentrated phase coils. The magnetization direction of the permanent magnets is indicated by arrows. The phase coils are indicated with uppercase A, B and C, where the superscript plus and minus sign give the polarity of the phase current. The numerical values of the physical quantities are tabulated in Table 2.1, and the numerical values of the geometrical variables of Fig. 2.6 are given in Table 2.2. If the geometry of Fig. 2.6 is considered to be the initial position ($\Delta x = 0$ mm) where the phase coils and the core can move relative to the magnets and back iron in the direction of the positive x-axis, then the current densities for the A, B and C phase are respectively given by

\[
J_A = J_{\text{ph}} \sqrt{2} \cos \left( \frac{\Delta x}{\tau_p} \pi + \frac{2}{3} \pi \right),
\]

\[
J_B = J_{\text{ph}} \sqrt{2} \cos \left( \frac{\Delta x}{\tau_p} \pi \right),
\]

\[
J_C = J_{\text{ph}} \sqrt{2} \cos \left( \frac{\Delta x}{\tau_p} \pi - \frac{2}{3} \pi \right),
\]

where $\Delta x$ is the relative displacement in the x-direction.
2.4.2 FEM as a benchmark modeling technique

The FEM is applied as a benchmark modeling technique to which all the other presented modeling methods in the subsequent chapters will be assessed in terms of accuracy. The FEM is applied to find the electromagnetic field distribution in the airgap and the force between the stator and translator. The choice for the FEM as a benchmark technique is dictated by the feature that it requires no assumption or limitation with respect to the geometry and physics of the benchmark problem in order for the method to be applicable. Hence, the errors that are introduced on account of model assumptions are averted. To, furthermore, minimize the error due to the finite element discretization of the benchmark problem and interpolation of the electromagnetic field quantities the minimal mesh element size is chosen to be less than $0.25\, \text{mm}$ to obtain a fine resolution. Since discretization in itself, however fine, remains a source of error, the comparison with the results obtained by the modeling techniques to be presented in the next chapters are not expressed in terms of accuracy, but rather in terms of discrepancy. The FEM simulations are carried out by using the commercially available FEM software package Flux 11.1 by Cedrat. All simulations are carried out with second-order mesh elements.
Chapter 3

The Harmonic Method

In this chapter the harmonic method (HM), or harmonic model, is discussed. The method is applicable in Cartesian, cylindrical or spherical coordinate systems. Here, the general methodology is explained for 2D problems in the $xy$-plane of the Cartesian coordinate system only to demonstrate the procedure for setting up the HM for a given electromechanical problem. The benchmark topology is the given 2D problem by means of which the method is explained. The general procedure can be adapted to other problems in, possibly, different coordinate systems. In part II of this thesis the method is applied to the design of the tubular motor with an integrated coaxial transformer. In this chapter the flux density distribution in the airgap of the benchmark topology is calculated by applying the HM. The obtained results are compared to FEM simulations.

The HM has been successfully applied to a wide range of electromechanical problems. The analytical solution of the static magnetic field distribution of a wire in between two parallel, soft-magnetic plates as presented in [31] can be considered the basis of the method. Over the years the HM, for 2D problems in between two plates, has been extended to incorporate, among others, eddy current phenomena for the analyses of rotating [76] and linear [20, 78] induction motors, eddy current loss analyses [71, 102], and inclusion of permanent magnets in the airgap [12, 117]. The extension of 2D magnetic problems with slotted structures is presented in [30, 67, 114]. Examples of the calculation of the magnetic field for 3D problems can be found in [44, 74].

The HM is based on the analytical solution of the Laplace or Poisson equation for orthogonal domains that is obtained by applying the method of separation of variables. The solution for the potential is only valid in orthogonal domains with homogeneous, linear material properties. This allows the solution for the potential to be expressed as a Fourier series. Furthermore, in this chapter the convention is
Chapter 3: The Harmonic Method

adopted that for 2D problems the trigonometric functions of the Fourier series are dependent on the values on the abscissa only. When an electromagnetic problem is composed of a concatenation of orthogonal domains, the electromagnetic field distribution in each domain can be calculated by applying the proper boundary conditions at the interfaces of the domain transitions.

Apart from the material properties being constant there are some additional limitations to the extent to which the HM can be applied. First, the concatenation of the domains can only occur, by the adopted convention, in the direction parallel to the ordinate. Secondly, all the source terms within the domain or on the boundaries have to be expressible as Fourier series. Finally, adjacent domains with a different fundamental spatial frequency can only be taken into account by setting the tangential magnetic field component zero on both lateral sides, where the lateral sides are parallel to the ordinate. This assumption allows the potential to be expressed as a Fourier series for which the fundamental spatial frequency is determined by the width of the narrower domain. This implies for the benchmark topology that the concatenation of domains can only occur in the $y$-direction when the periodicity is chosen in the $x$-direction. The domain representation of the benchmark topology is shown in Fig. 3.1. Furthermore, it can be seen that the core consists of infinitely permeable material, allowing the spatial frequencies of domains $\Omega^{(3)}$, $\Omega^{(4)}$ and $\Omega^{(5)}$ to be determined by the widths of the domains $l_3$, $l_4$ and $l_5$, respectively. It has to be noted that $l_3 = l_4 = l_5$. For the sake of clarity the domain index is denoted as a superscript $k$ in parentheses.

![Orthogonal domain representation of the benchmark topology with the indication of the type of boundary conditions.](image)
3.1 Fourier series representation of sources

For 2D problems with current sources and/or permanent magnets the magnetic vector potential method can be applied. To solve (2.9) by means of the HM, the terms of the right member describing the source distribution in a domain have to be expressed as Fourier series. If for the problem at hand the source distribution within a domain cannot be expressed as a Fourier series, the HM is unsuitable. However, for most practical electromechanical devices the source terms can be represented by Fourier series. For that purpose, the source terms within a domain may only vary in the direction parallel to the abscissa and they have to be invariant in the other direction. Since only the $xy$-plane of a Cartesian coordinate system is considered, the analysis is simplified. The magnetic vector potential only has a nonzero component in the $z$-direction. Therefore, only currents in the $z$-direction are taken into account and are only dependent on $x$. Furthermore, the magnetization vector only has an $x$ and $y$-component which, also, is only dependent on $x$.

The Fourier series for the internal magnetization vector in the $\Omega^{(k)}$ domain for the 2D Cartesian coordinate system is given by

$$
\vec{M}^{(k)}_0(x) = \vec{M}^{(k)}_o + \sum_{n=1}^{\infty} \vec{M}^{(k)}_{s_n} \sin \left( w^{(k)}_n x \right) + \vec{M}^{(k)}_{c_n} \cos \left( w^{(k)}_n x \right),
$$

(3.1)

where $\vec{M}^{(k)}_o$ is the vector containing the nonharmonic offset values in the $x$ and $y$-direction of the internal magnetization pattern, $\vec{M}^{(k)}_{s_n}$ and $\vec{M}^{(k)}_{c_n}$ are vectors containing the $n$th Fourier coefficients for each direction, and $w^{(k)}_n$ is the spatial frequency. Analogously, a current density source is described by

$$
J_z^{(k)}(x) = J_o^{(k)} + \sum_{n=1}^{\infty} J_{s_n}^{(k)} \sin \left( w^{(k)}_n x \right) + J_{c_n}^{(k)} \cos \left( w^{(k)}_n x \right),
$$

(3.2)

where $J_o^{(k)}$ contains the offset terms, and $J_{s_n}^{(k)}$ and $J_{c_n}^{(k)}$ contain the Fourier coefficients of the sine and cosine terms. The spatial frequency is determined by the length of a domain, $l_k$. For periodic boundary conditions at the lateral sides of a domain the spatial frequency with respect to Fig. 3.1 is defined as

$$
w^{(k)}_n = \frac{2n\pi}{l_k}. 
$$

(3.3)

However, if the domain exhibits boundary conditions on the lateral sides for which the magnetic field tangent to the boundary is zero, e.g. for $k \in \{3, 4, 5\}$ in Fig. 3.1, the spatial frequency is given by

$$
w^{(k)}_n = \frac{n\pi}{l_k}. 
$$

(3.4)

The average source terms values, or offset, and the Fourier coefficients can be found by applying standard Fourier analysis to the source term pattern. The
source patterns for the benchmark problem are shown in Fig. 3.2. The internal magnetization vector in domain 1 only has a $y$-component and the Fourier coefficients with reference to Tables 2.2 and 2.1 are represented by

\[ \vec{M}_o^{(1)} = \vec{0}, \]  \hspace{1cm} \text{(3.5a)}

\[ \vec{M}_s^{(1)} = \frac{B_{\text{rem}} e_y}{\mu_0} \sum_{j=1}^{N_{mp}} \sum_{i=1}^{2} (-1)^{i+j} \cos \left[ \frac{n\pi}{N_{mp}} \left((-1)^i \alpha_m + 2j - 1\right)\right], \]  \hspace{1cm} \text{(3.5b)}

\[ \vec{M}_c^{(1)} = \vec{0}, \]  \hspace{1cm} \text{(3.5c)}

where $\hat{e}_y$ is the unit vector parallel to the ordinate and $N_{mp}$ the number of magnetic poles. For the benchmark topology the number of poles equal $N_{mp} = 4$.

The current density distributions in the phase slot domains equal

\[ J_o^{(k)} = \frac{1}{2} \left(J_L^{(k)} + J_R^{(k)}\right), \]  \hspace{1cm} \text{(3.6a)}

\[ J_s^{(k)} = 0, \]  \hspace{1cm} \text{(3.6b)}

\[ J_c^{(k)} = \frac{2}{n\pi} \left(J_L^{(k)} - J_R^{(k)}\right) \sin \left(\frac{n\pi}{2}\right), \]  \hspace{1cm} \text{(3.6c)}

where

\[ J_L^{(k)} = \begin{cases} -J_C & \text{for } k = 3 \\ -J_A & \text{for } k = 4 \\ -J_B & \text{for } k = 5 \end{cases}, \quad J_R^{(k)} = \begin{cases} J_A & \text{for } k = 3 \\ J_B & \text{for } k = 4 \\ J_C & \text{for } k = 5 \end{cases}, \]  

with respect to (2.12).
3.2 Solution within the orthogonal domain

The solution to the Poisson equation of (2.9) for orthogonal domains can be obtained through the technique of separation of variables. For 2D Cartesian problems the orthogonal domains are rectangles in the $xy$-plane. The magnetic vector potential in the $z$-direction for the $\Omega^{(k)}$ domain is given by

$$A_z^{(k)}(x, y) = A_0^{(k)}(y) + \sum_{n=1}^{\infty} \frac{A_n^{(k)}(y)}{w_n^{(k)}} \sin \left( \frac{w_n^{(k)} x}{w_n^{(k)}} \right) + \frac{A_{cn}^{(k)}(y)}{w_n^{(k)}} \cos \left( \frac{w_n^{(k)} x}{w_n^{(k)}} \right),$$

(3.7)

with

$$A_0^{(k)}(y) = A_0^{(k)} + B_0^{(k)} y - \frac{1}{2} \mu^{(k)} J_o^{(k)} y^2,$$

$$A_n^{(k)}(y) = a_n^{(k)} e^{(y-h_n^{(k)}) w_n^{(k)}} + b_n^{(k)} e^{(h_n^{(k)}-y) w_n^{(k)}} + S_{yn}^{(k)},$$

$$A_{cn}^{(k)}(y) = c_n^{(k)} e^{(y-h_n^{(k)}) w_n^{(k)}} + d_n^{(k)} e^{(h_n^{(k)}-y) w_n^{(k)}} + S_{ycn}^{(k)},$$

where $a_n^{(k)}$, $b_n^{(k)}$, $c_n^{(k)}$, $d_n^{(k)}$, $A_0^{(k)}$, and $B_0^{(k)}$ are constants, all with respect to the $k$th domain and $n$th harmonic, that have to be calculated subject to the boundary conditions. The values of $h_n^{(k)}$ and $h_n^{(k)}$ are the value of the $y$-coordinate of the horizontal bottom and top side of the $k$th rectangle as indicated by $h_1$, $h_2$, $h_3$, and $h_4$ in Fig. 3.1. The source terms $S_{yn}^{(k)}$ and $S_{ycn}^{(k)}$ read

$$S_{yn}^{(k)} = \mu^{(k)} \frac{J_y^{(k)}}{w_n^{(k)}} + \mu_0 M_y^{(k)},$$

$$S_{ycn}^{(k)} = \mu^{(k)} \frac{J_y^{(k)}}{w_n^{(k)}} - \mu_0 M_y^{(k)},$$

where $M_y^{(k)}$ and $M_y^{(k)}$ are the values in the $y$-direction of $M_y^{(k)}$ and $M_y^{(k)}$ in (3.1), respectively. In the absence of any source terms in the interior of the domain these expressions take zero value.

The expression for the flux density components are found through (2.8). The flux density component in the $x$-direction becomes

$$B_x^{(k)}(x, y) = B_0^{(k)}(y) + \sum_{n=1}^{\infty} B_{xyn}^{(k)}(y) \sin \left( \frac{w_n^{(k)} x}{w_n^{(k)}} \right) + B_{xcn}^{(k)}(y) \cos \left( \frac{w_n^{(k)} x}{w_n^{(k)}} \right),$$

(3.8)

where

$$B_0^{(k)}(y) = B_0^{(k)} - \mu^{(k)} J_x^{(k)} y,$$

$$B_{xyn}^{(k)}(y) = a_n^{(k)} e^{(y-h_n^{(k)}) w_n^{(k)}} - b_n^{(k)} e^{(h_n^{(k)}-y) w_n^{(k)}},$$

$$B_{xcn}^{(k)}(y) = c_n^{(k)} e^{(y-h_n^{(k)}) w_n^{(k)}} - d_n^{(k)} e^{(h_n^{(k)}-y) w_n^{(k)}}.$$
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The expression for the flux density component in the \(y\)-direction is equal to

\[
B^{(k)}_y(x, y) = \sum_{n=1}^{\infty} B^{(k)}_{ys_n}(y) \sin \left( w_n^{(k)} x \right) - B^{(k)}_{yc_n}(y) \cos \left( w_n^{(k)} x \right),
\]

(3.9)

where

\[
B^{(k)}_{ys_n}(y) = c_n^{(k)} e^{(y-h_n^{(k)}) w_n^{(k)}} + d_n^{(k)} e^{(h_n^{(k)}-y) w_n^{(k)}} + S^{(k)}_{ys},
\]

\[
B^{(k)}_{yc_n}(y) = a_n^{(k)} e^{(y-h_n^{(k)}) w_n^{(k)}} + b_n^{(k)} e^{(h_n^{(k)}-y) w_n^{(k)}} + S^{(k)}_{yc}.
\]

3.3 Boundary conditions

The coefficients \(a_n^{(k)}, b_n^{(k)}, c_n^{(k)}, d_n^{(k)}, \) and \(B^0_n^{(k)}\) have to be known to determine the flux density distribution in the domains. The solution of these coefficients is obtained by a set of linear equations that are governed by the electromagnetic boundary conditions. The general magnetic boundary conditions on the interface, \(\Gamma\), between a domain \(j\) and a domain \(k\) are given by

\[
\vec{n} \times \vec{H}^{(k)} - \vec{n} \times \vec{H}^{(j)} = \vec{J}_\Gamma^{(k)},
\]

\[
\vec{n} \cdot \vec{B}^{(k)} - \vec{n} \cdot \vec{B}^{(j)} = 0,
\]

(3.10a, b)

where \(\vec{n}\) is the normal vector on the boundary pointing from domain \(j\) in the direction of domain \(k\), and \(\vec{J}_\Gamma\) is a surface current on the boundary \(\Gamma\). It has to be noted that in the previous section the boundary conditions on the lateral sides, parallel to the ordinate, have tacitly been taken into account. As a matter of fact, applying these boundary conditions yields (3.3) and (3.4). The boundary conditions parallel to the abscissa remain to be solved. Four types of boundary conditions are encountered for the harmonic models in this thesis. These are explained in the following. It has to be noted that none of the interfaces between domains carry currents; therefore, \(\vec{J}_\Gamma\) vanishes in (3.10a). Furthermore, it is convenient to express (3.10a) in terms of the flux density and magnetization using (2.5a). Equation (3.10a) then becomes

\[
\vec{n} \times \left( \vec{B}^{(k)} - \mu_0 \vec{M}^{(k)} \right) \mu_r^{(k)} - \vec{n} \times \left( \vec{B}^{(j)} - \mu_0 \vec{M}^{(j)} \right) \mu_r^{(j)} = \vec{0}.
\]

(3.11)

3.3.1 Continuous boundary conditions

For adjacent domains with the same fundamental spatial frequency (3.10) can be applied directly at the interface. In the considered Cartesian coordinate system the normal vector is parallel to the ordinate. If the vector points from the
3.3: Boundary conditions

$j^\text{th}$ domain towards the $k^\text{th}$ domain the continuous boundary conditions for the tangential magnetic field strength are given by

\[ \mu_r^{(k)} \left[ B_x^{(j)}(x, y_T) - \mu_0 M_x^{(j)}(x) \right] = \mu_r^{(j)} \left[ B_x^{(k)}(x, y_T) - \mu_0 M_x^{(k)}(x) \right], \quad (3.12) \]

or equivalently,

\[ \mu_r^{(k)} \left[ B_y^{(j)}(y_T) - \mu_0 M_y^{(j)}(y_T) \right] - \mu_r^{(j)} \left[ B_y^{(k)}(y_T) - \mu_0 M_y^{(k)}(y_T) \right] = 0, \quad (3.13a) \]

\[ \mu_r^{(k)} \left[ B_{x y}^{(j)}(y_T) - \mu_0 M_{x y}^{(j)}(y_T) \right] - \mu_r^{(j)} \left[ B_{x y}^{(k)}(y_T) - \mu_0 M_{x y}^{(k)}(y_T) \right] = 0, \quad (3.13b) \]

\[ \mu_r^{(k)} \left[ B_{x c}^{(j)}(y_T) - \mu_0 M_{x c}^{(j)}(y_T) \right] - \mu_r^{(j)} \left[ B_{x c}^{(k)}(y_T) - \mu_0 M_{x c}^{(k)}(y_T) \right] = 0, \quad (3.13c) \]

where $y_T = h_T^{(k)}$. Similarly, for the magnetic flux density normal to the continuous boundary the sets of equations are equal to

\[ B_{y T}^{(j)}(x, y_T) = B_{y T}^{(k)}(x, y_T), \quad (3.14) \]

or equivalently,

\[ B_{y y T}^{(j)}(y_T) - B_{y y T}^{(k)}(y_T) = 0, \quad (3.15a) \]

\[ B_{y c T}^{(j)}(y_T) - B_{y c T}^{(k)}(y_T) = 0. \quad (3.15b) \]

The number of harmonics have to be the same in the adjacent domains. For the benchmark topology there is only one continuous boundary condition, viz. between domain $\Omega^{(1)}$ and $\Omega^{(2)}$. Hence, (3.13) and (3.15) hold for $j = 1$ and $k = 2$ and the magnetization terms parallel to the abscissa vanish.

3.3.2 Neumann boundary conditions

When the magnetic field components on a boundary are assigned a value, Neumann boundary conditions have to be applied. This type of boundary condition is useful when the problem has one or more symmetry axes. Depending on the nature of the symmetry either the tangential or normal component of the magnetic field, or both for that matter, is set to zero. Neumann boundary conditions also occur when one of the domains on either side of the boundary is infinitely permeable. In this situation the tangential component of the magnetic field strength along the boundary is zero. Hence, the set of equations for domains with an infinitely permeable boundary or odd symmetry (mirrored equivalent currents flow in the same direction) is given by

\[ B_x^{(k)}(x, y_T) - \mu_0 M_x^{(k)}(x) = 0, \quad (3.16) \]

or equivalently,

\[ B_y^{(k)}(y_T) - \mu_0 M_y^{(k)}(y_T) = 0, \quad (3.17a) \]

\[ B_{x y}^{(k)}(y_T) - \mu_0 M_{x y}^{(k)}(y_T) = 0, \quad (3.17b) \]

\[ B_{x c}^{(k)}(y_T) - \mu_0 M_{x c}^{(k)}(y_T) = 0. \quad (3.17c) \]
where \( y_{\Gamma} = h_{3}^{(k)} \) or \( y_{\Gamma} = h_{4}^{(k)} \) depending on whether the boundary condition applies to the bottom or top side of the domain, respectively. In case of even symmetry (mirrored equivalent currents flow in the opposite direction) the Neumann boundary conditions satisfy

\[
B_{y}^{(k)}(x, y_{\Gamma}) = 0,
\]

or equivalently,

\[
B_{y c}^{(k)}(y_{\Gamma}) = 0, \quad B_{y c}^{(k)}(y_{\Gamma}) = 0.
\]

Domains 1, 3, 4 and 5 in Fig. 3.1 of the benchmark topology have an infinitely permeable boundary at \( y_{\Gamma} = h_{1} \) and \( y_{\Gamma} = h_{4} \), respectively, which requires (3.17) to be applied. Furthermore, due to the Neumann boundary conditions on the lateral sides of the slots \( B_{x c}^{(k)} = 0 \) for \( k \in \{3, 4, 5\} \).

### 3.3.3 Compound boundary conditions

Compound boundary conditions appear at the interface of domains with different, fundamental spatial frequencies. They are referred to as compound boundary conditions, because they consist of a combination of a continuous and Neumann boundary condition. This type of boundary condition occurs at the interface \( y_{\Gamma} = h_{3} \) between domain 2 and domains 3, 4, and 5 of the benchmark topology. At the positions along the interface where adjacent domains share a common boundary, the continuous boundary conditions hold. The Neumann boundary condition applies to the wider domain only, where there is no common boundary. First, the tangential field component is considered:

\[
B_{x}^{(j)}(x, y_{\Gamma}) - \mu_{0} M_{x}^{(j)}(x) = \begin{cases} 
\frac{\mu_{0}^{(j)}}{\mu_{0}^{(k)}} \left[ B_{x}^{(k)}(x, y_{\Gamma}) - \mu_{0} M_{x}^{(k)}(x) \right] & \text{for } \delta_{j,k} \leq x \leq \delta_{j,k} + l_{k}, \\
0 & \text{elsewhere},
\end{cases}
\]

where \( \delta_{j,k} \) is the offset in the \( x \)-direction of the \( k \)th domain with respect to the \( j \)th domain. Equation (3.20) is the equality of two Fourier series with different fundamental frequencies. The magnetic field distribution in the \( x \)-direction on the boundary can be expressed as a Fourier series with the same fundamental spatial frequency as the wide domain. To express the equality in terms of Fourier coefficients of the wider domain, \( j \), the formula for calculating the Fourier coefficients of a series is applied to the expressions on both sides of the equals sign. Due to the orthogonality properties of the trigonometric functions the Fourier coefficient in the wider domain can be expressed as a series of the terms in the other domain,
3.3: Boundary conditions

i.e.

\[
B^{(j)}_{x \alpha}(y \Gamma) - \mu_0 M^{(j)}_{x \alpha} = \sum_{k=k_1}^{k_N} \left[ \frac{\mu_c^{(j)}}{\mu_r^{(k)}} B^{(k)}_{x \alpha}(y \Gamma) - \mu_0 M^{(k)}_{x \alpha} \right] \gamma_0^{(k)}(n) \\
+ \sum_{k=k_1}^{k_N} \sum_{m=1}^{\infty} \frac{\mu_c^{(j)}}{\mu_r^{(k)}} B^{(k)}_{x \alpha m}(y \Gamma) - \mu_0 M^{(k)}_{x \alpha m} \gamma_c^{(k)}(n, m),
\]

\[
B^{(j)}_{x c m}(y \Gamma) - \mu_0 M^{(j)}_{x c m} = \sum_{k=k_1}^{k_N} \left[ \frac{\mu_c^{(j)}}{\mu_r^{(k)}} B^{(k)}_{x c m}(y \Gamma) - \mu_0 M^{(k)}_{x c m} \right] \zeta_0^{(k)}(n) \\
+ \sum_{k=k_1}^{k_N} \sum_{m=1}^{\infty} \frac{\mu_c^{(j)}}{\mu_r^{(k)}} B^{(k)}_{x c m}(y \Gamma) - \mu_0 M^{(k)}_{x c m} \zeta_c^{(k)}(n, m),
\]

(3.21)

(3.22)

where \(k_1\) and \(k_N\) denote the index value of the first domain and last domain that share a common boundary with the \(j^{th}\) domain, respectively. The correlation functions indicated by \(\gamma\) and \(\zeta\) are a result of the Fourier coefficient calculations and yield

\[
\gamma_0^{(k)}(n) = \frac{2}{l_j} \int_{\delta_{j,k}}^{\delta_{j,k}+l_k} \sin \left( w_n^{(j)} x \right) \, dx,
\]

(3.23a)

\[
\gamma_c^{(k)}(n, m) = \frac{2}{l_j} \int_{\delta_{j,k}}^{\delta_{j,k}+l_k} \sin \left( w_n^{(j)} x \right) \cos \left( w_m^{(k)} x \right) \, dx,
\]

(3.23b)

\[
\zeta_0^{(k)}(n) = \frac{2}{l_j} \int_{\delta_{j,k}}^{\delta_{j,k}+l_k} \cos \left( w_n^{(j)} x \right) \, dx,
\]

(3.24a)

\[
\zeta_c^{(k)}(n, m) = \frac{2}{l_j} \int_{\delta_{j,k}}^{\delta_{j,k}+l_k} \cos \left( w_n^{(j)} x \right) \cos \left( w_m^{(k)} x \right) \, dx.
\]

(3.24b)

For the normal component of the magnetic field the boundary condition is given by

\[
B^{(j)}_y(x, y \Gamma) = B^{(k)}_y(x, y \Gamma) \quad \text{for} \quad \delta_{j,k} \leq x \leq \delta_{j,k} + l_k.
\]

(3.25)

Similarly, to express the equality in terms of Fourier coefficients of the narrower domain, \(k\), the formula for calculating the Fourier coefficients of a series is applied to the expression on both sides of the equals sign. However, the fundamental spatial frequency is now equal to that of the narrow domain and the procedure is to be repeated for all the narrow domains sharing a boundary with the \(j^{th}\) domain

\[
B^{(j)}_{y \alpha m}(y \Gamma) = \sum_{n=1}^{\infty} B^{(j)}_{y \alpha \alpha m}(y \Gamma) \varepsilon^{(k)}_{n} \varepsilon^{(k)}_{m} - B^{(j)}_{y \alpha \alpha m}(y \Gamma) \varepsilon^{(k)}_{n} \varepsilon^{(k)}_{m},
\]

(3.26)
where the correlation functions, $\epsilon$, are

$$
\epsilon_{s_j}(m, n) = \frac{2}{l_k} \int_0^{l_k} \sin(w_m^{(k)} x) \sin(w_n^{(j)} x) \, dx,
$$

(3.27a)

$$
\epsilon_{c_j}(m, n) = \frac{2}{l_k} \int_0^{l_k} \sin(w_m^{(k)} x) \cos(w_n^{(j)} x) \, dx.
$$

(3.27b)

Applying the compound boundary conditions allows the value of the coefficients of the flux density component on the boundary of domain to be expressed as a series of all the coefficients in the domains that share the same boundary. In this way, the set of equations is obtained that satisfy the boundary conditions at interfaces with unequal width. For the benchmark topology this boundary condition applies for $j = 2$, $k_1 = 3$, and $k_N = 5$ with respect to Fig. 3.1.

### 3.3.4 Dirichlet boundary conditions

The previously discussed types of boundary conditions suffice to obtain the solution in terms of magnetic flux density or magnetic field strength, since all the unknowns describing these fields can be calculated. However, these boundary conditions do not allow $A_o$ for the expression of the vector potential in (3.7) to be calculated. When it is desirable to have the complete expression for the vector potential, e.g. for flux linkage and induction calculations, Dirichlet boundary conditions have to be applied. In that case, the value of the magnetic vector potential is given a value at the boundary where the Dirichlet boundary condition holds. The offset term, $A_o$, can be chosen freely as long as the value of (3.7) throughout the entire domain remains finite. If at least one of the domains extends to infinity, the vector potential is mostly chosen such that

$$
\lim_{y \to \pm \infty} A_z^{(k)}(x, y \Gamma) = 0
$$

(3.28)

is satisfied. The magnetic vector potential expression in the other domains is found by applying a continuous boundary condition for the vector potential expressions at the interfaces of the domains. Hence,

$$
A_z^{(j)}(x, y \Gamma) = A_z^{(k)}(x, y \Gamma).
$$

(3.29)

The set of equations that result from applying the boundary conditions can be transformed into a matrix equation. Through matrix inversion the values of the coefficients can subsequently be calculated. The technique is explained in the next section where the magnetic flux density in the airgap of the benchmark topology is calculated through the HM.
3.4 Harmonic model for the benchmark topology

The boundary condition equations in terms of the coefficients are obtained by substitution of (3.7), (3.8), and (3.9) into the boundary conditions that govern the problem. In order to obtain a finite number of equations, the series for each domain have to be truncated. The number of harmonics that are taken into account for the domains 1 and 2 are indicated by $N_n$ and for domains 3, 4 and 5 by $N_m$.

For domain $\Omega^{(1)}$ the matrix, $M_1$, that operates on its respective coefficients, $a^{(1)}_n$, $b^{(1)}_n$, $c^{(1)}_n$, and $d^{(1)}_n$ is given by

$$M_1 = \begin{bmatrix}
E_{1,2} & -I_n & 0 & 0 \\
0 & 0 & E_{1,2} & -I_n \\
\mu_r^{(2)} I_n & -\mu_r^{(2)} E_{1,2} & 0 & 0 \\
0 & 0 & \mu_r^{(2)} I_n & -\mu_r^{(2)} E_{1,2} \\
I_n & E_{1,2} & 0 & 0 \\
0 & 0 & I_n & E_{1,2} \\
O_1 & O_1 & O_1 & O_1
\end{bmatrix}, \quad (3.30)$$

where $I_n$ is the $N_n \times N_n$ identity matrix and $O_1$ is a zero matrix of size $(2N_n + 6N_m) \times N_n$. $E_{1,2}$ is an $N_n \times N_n$ matrix of which only the diagonal entries are nonzero:

$$E_{1,2(n,n)} = e^{(h_1 - h_2)w_n^{(1)}}. \quad (3.31)$$

The matrix that operates on the coefficients of domain $\Omega^{(2)}$ is

$$M_2 = \begin{bmatrix}
O_2 & O_2 & O_2 & O_2 \\
-\mu_r^{(1)} E_{2,3} & \mu_r^{(1)} I_n & 0 & 0 \\
0 & 0 & -\mu_r^{(1)} E_{2,3} & \mu_r^{(1)} I_n \\
-\mu_r^{(3)} I_n & -\mu_r^{(3)} E_{2,3} & 0 & 0 \\
0 & 0 & \mu_r^{(3)} I_n & -\mu_r^{(3)} E_{2,3} \\
-\epsilon_c & -\epsilon_s E_{2,3} & \epsilon_s & \epsilon_s E_{2,3} \\
O_3 & O_3 & O_3 & O_3
\end{bmatrix}, \quad (3.32)$$

where $O_2$ and $O_3$ are zero matrices of size $2N_n \times N_n$ and $3N_m \times N_n$, respectively. Only the entries on the diagonal of $E_{2,3}$, with size $N_n \times N_n$, are nonzero and are given by

$$E_{2,3(n,n)} = e^{(h_2 - h_3)w_n^{(1)}}. \quad (3.33)$$
The correlation matrices, \( \epsilon_s \) and \( \epsilon_c \), both have size \( 3N_m \times N_n \). Each correlation matrix consists of three separate correlation matrices of size \( N_m \times N_n \) that apply to the common boundary of domain 2 with that of each individual slot region of domains 3, 4, or 5

\[
\epsilon_s = \begin{bmatrix} \epsilon_{s,2,3} & \epsilon_{s,2,4} & \epsilon_{s,2,5} \end{bmatrix}^T, \\
\epsilon_c = \begin{bmatrix} \epsilon_{c,2,3} & \epsilon_{c,2,4} & \epsilon_{c,2,5} \end{bmatrix}^T
\]

(3.34a)

(3.34b)

where the entries of the individual matrices for \( k \in \{3, 4, 5\} \) are given by

\[
\epsilon_{s,2,k(m,n),} = \frac{2l^2 \sin \left[ \frac{w_n}{(2) \delta_{2,k}} \right]}{\pi \left[ m^2 R^2 - 4n^2 R_k^2 \right]} - (-1)^m \sin \left[ \frac{w_n}{(2) \left( \delta_{2,k} + l \right)} \right],
\]

(3.35a)

\[
\epsilon_{c,2,k(m,n),} = \frac{2l^2 \cos \left[ \frac{w_n}{(2) \delta_{2,k}} \right]}{\pi \left[ m^2 R^2 - 4n^2 R_k^2 \right]} - (-1)^m \cos \left[ \frac{w_n}{(2) \left( \delta_{2,k} + l \right)} \right].
\]

(3.35b)

Finally, the matrix that operates on the coefficients of the slots is

\[
M_3 = \begin{bmatrix}
O_4 & O_4 \\
-\mu_r^{(2) \gamma_c E_{3,4}} & -\mu_r^{(2) \gamma_c E_{3,4}} \\
-\mu_r^{(2) \zeta_c E_{3,4}} & -\mu_r^{(2) \zeta_c E_{3,4}} \\
-E_{3,4} & -I_{3m} \\
I_{3m} & -E_{3,4}
\end{bmatrix}
\]

(3.36)

where \( I_{3m} \) is the identity matrix of size \( 3N_m \times 3N_m \), \( O_4 \) is a \( 6N_n \times N_m \) zero matrix, and \( E_{3,4} \) is a \( 3N_m \times 3N_m \) matrix of which only the entries on the main diagonal are nonzero. The values on the diagonal are given by

\[
E_{3,4} = \begin{bmatrix}
e_3 & 0 & 0 \\
0 & e_4 & 0 \\
0 & 0 & e_5
\end{bmatrix}
\]

(3.37)

with

\[
e_{k(m,n),} = e^{(k_3-h_4)w_m} \quad \text{for} \quad k \in \{3, 4, 5\}.
\]

(3.38)

The correlation matrices, \( \gamma_c \) and \( \zeta_c \), with size \( N_n \times 3N_m \) are, again, composed of smaller matrices through

\[
\gamma_c = \begin{bmatrix} \gamma_{c,2,3} & \gamma_{c,2,4} & \gamma_{c,2,5} \end{bmatrix},
\]

(3.39a)

\[
\zeta_c = \begin{bmatrix} \zeta_{c,2,3} & \zeta_{c,2,4} & \zeta_{c,2,5} \end{bmatrix},
\]

(3.39b)

where the entries of the individual matrices for \( k \in \{3, 4, 5\} \) are given by

\[
\zeta_{c,2,k(m,n),} = \frac{4n^2 \left[ \sin \left[ \frac{w_n}{(2) \delta_{2,k}} \right] - (-1)^m \sin \left[ \frac{w_n}{(2) \left( \delta_{2,k} + l \right)} \right] \right]}{\pi \left[ m^2 R^2 - 4n^2 R_k^2 \right]},
\]

(3.40a)

\[
\gamma_{c,2,k(m,n),} = \frac{4n^2 \left[ \cos \left[ \frac{w_n}{(2) \delta_{2,k}} \right] - \cos \left[ \frac{w_n}{(2) \left( \delta_{2,k} + l \right)} \right] \right]}{\pi \left[ m^2 R^2 - 4n^2 R_k^2 \right]},
\]

(3.40b)
Next, the source vector of the matrix equation is given by

\[ \vec{V}_{sys} = \begin{bmatrix} \vec{O}_1 & -\vec{S}_{yc} & -\vec{S}_{ys} & \vec{B}_{\gamma_0} & \vec{B}_{\zeta_0} & \vec{J}_v & \vec{O}_2 \end{bmatrix}^T, \] (3.41)

where \( \vec{O}_1 \) and \( \vec{O}_2 \) are row vectors of length \( 4N_n \) and \( 3N_m \), respectively, which contain zeros only. The elements in row vectors \( \vec{S}_{yc} \) and \( \vec{S}_{ys} \) correspond to the source term expressions, \( S \), of (3.7) for \( k = 1 \). The current vector \( \vec{J}_v \) with length \( 3N_m \) is composed of smaller vectors with length \( N_m \) of which the elements of the individual vectors are given by (3.6c). Therefore,

\[ \vec{J}_v = \mu_r^{(2)} \begin{bmatrix} \vec{J}_{(3)}^{(3)} & \vec{J}_{(4)}^{(4)} & \vec{J}_{(5)}^{(5)} \end{bmatrix}^T. \] (3.42)

The transpose of the vectors \( \vec{B}_{\gamma_0} \) and \( \vec{B}_{\zeta_0} \) in the source vector are obtained through

\[ \begin{align*}
\vec{B}_{\gamma_0}^T &= \mu_r^{(2)} \gamma_0 \vec{B}_s, \quad (3.43a) \\
\vec{B}_{\zeta_0}^T &= \mu_r^{(2)} \zeta_0 \vec{B}_s, \quad (3.43b)
\end{align*} \]

where the entries for \( k \in \{3, 4, 5\} \) of the correlation matrices are

\[ \begin{align*}
\gamma_{0(n,k-2)} &= \cos \left[ \frac{w_n^{(2)} \delta_{2,k}}{n\pi} \right] - \cos \left[ \frac{w_n^{(2)} (\delta_{2,k} + l_k)}{n\pi} \right], \quad (3.44a) \\
\zeta_{0(n,k-2)} &= \sin \left[ \frac{w_n^{(2)} (\delta_{2,k} + l_k)}{n\pi} \right] - \sin \left[ \frac{w_n^{(2)} \delta_{2,k}}{n\pi} \right], \quad (3.44b)
\end{align*} \]

and \( \vec{B}_s \) is the vector containing the nonharmonic terms inside each slot given by (3.6a). So,

\[ \vec{B}_s = \mu_r^{(3)} (h_4 - h_3) \begin{bmatrix} J_\delta^{(3)} & J_\delta^{(4)} & J_\delta^{(5)} \end{bmatrix}^T. \] (3.45)

The system matrix consists of the matrices that operate on all coefficients. Hence, the system matrix becomes

\[ \mathbf{M}_{sys} = \begin{bmatrix} \mathbf{M}_1 & \mathbf{M}_2 & \mathbf{M}_3 \end{bmatrix}. \] (3.46)

With the system matrix known the matrix equation is finally obtained

\[ \mathbf{M}_{sys} \vec{q} = \vec{V}_{sys}, \] (3.47)

where \( \vec{q} \) is the column vector containing the coefficients that are obtained once the system is solved. For the benchmark topology the coefficients within \( \vec{q} \) are arranged in the following way

\[ \vec{q} = \begin{bmatrix} \vec{a}^{(1)} & \vec{b}^{(1)} & \vec{c}^{(1)} & \vec{d}^{(1)} & \vec{a}^{(2)} & \vec{b}^{(2)} & \vec{c}^{(2)} & \vec{d}^{(2)} & \vec{C} & \vec{D} \end{bmatrix}^T, \] (3.48)
Chapter 3: The Harmonic Method

where

\[ \vec{C} = \begin{bmatrix} \vec{c}^{(3)} & \vec{c}^{(4)} & \vec{c}^{(5)} \end{bmatrix}, \]

\[ \vec{D} = \begin{bmatrix} \vec{d}^{(3)} & \vec{d}^{(4)} & \vec{d}^{(5)} \end{bmatrix}. \]

By substitution of the results of (3.48) into (3.8) and (3.9) for domain \( \Omega^{(2)} \) the flux density components along a horizontal line in the airgap can be calculated. The flux density components are calculated in the middle of the airgap at \( y = \frac{1}{2} (h_2 + h_3) \). The results for \( N_n = 100 \) and \( N_m = 38 \) compared to FEM simulations are shown in Figs. 3.3 and 3.4 for the \( x \) and \( y \)-component, respectively. It can be seen that a good correspondence between the HM and the FEM is obtained. The discrepancy, \( \epsilon_\delta \), between the rms-values of the flux density components
ponents compared to the FEM in Figs. 3.3 and 3.4 are 0.26% and 0.45% for the $x$ and $y$-component, respectively. The time required for the evaluation of the coefficients and the field quantities for one relative position of the translator with respect to the stator is equal to 62 ms.

The accuracy and calculation time both depend on the number of harmonics that are taken into account during the simulation. The rms-value of the flux density components obtained by the HM as a function of the number of harmonics compared to the FEM results are shown in Fig. 3.5. The discrepancy between the rms-values of the magnetic flux density components for the HM and FEM are shown in Fig. 3.6a, and the corresponding calculation time with respect to the number of harmonics taken into consideration is shown in Fig. 3.6b. It has to be noted that the number of harmonics, $N_n$, on the abscissa apply to domains $\Omega^{(1)}$ and $\Omega^{(2)}$. The choice for the number of harmonics in the slot domains is determined through $N_m = [2N_n l_m t_e^{-1}]$ to minimize the error [29]. The discrepancy for both rms-values of the $x$ and the $y$-component with respect to the FEM solution is $\epsilon_x < 1\%$ for $N_n \geq 11$ with an accompanying calculation time of $t = 2.2$ ms. A further increase in the number of harmonics for $N_n \geq 20$ at the expense of an increased calculation time has a negligible effect on the values of the flux density components. Figure 3.6b shows that the calculation time is approximately quadratically proportional with the number of harmonics taken into account.

The discrepancy is caused by the fact that the relative permeability of domain $\Omega^{(1)}$ is considered to apply to the entire domain, i.e. $\mu_r^{(1)} = 1.05$. Hence, the different permeability value of the air in between the permanent magnets is not allowed for. Obviously, this error increases as the value of the relative permeability of the permanent magnets increases, and it cannot be reduced by increasing the number of harmonics.

3.5 Additional remarks and numerical issues

A useful feature of the HM is that the spatial frequency spectrum is available once the coefficients are determined. The amplitude for each harmonic of the sine and cosine terms are known from which the amplitude and phase spectra can be calculated. The amplitude spectra of the flux density components, as depicted in Figs. 3.3 and 3.4, are shown in Fig. 3.7 for the first twenty harmonics.

The method can be adapted to different 2D problems with different formulations in a polar [50] or axisymmetric [114] coordinate systems. Changing from the Cartesian coordinate system to another does not change the structure of the matrix equation or the correlation functions for a problem with the same configuration of orthogonal subdomains. Only the expressions for $B_{y_0}$, $B_{x_0}$, $B_{y_1}$, $B_{x_1}$, and $B_{y_2}$ are dependent on the coordinate system to be applied. Instead of an exponential function with base $e$ they contain Bessel functions for the axisymmet-
Chapter 3: The Harmonic Method

ric coordinate system, or exponential functions with a radius dependent base for the polar coordinate system. For 3D problems it can be more convenient to solve the Poisson equation for the scalar potential. With the scalar potential method, however, it is not possible to take current sources into account. Moreover, the Fourier representation of the potential for 3D problems contains a double series that is dependent on two coordinates.

Truncation of the series is not only required to obtain a finite number of equations, but also to guarantee numerical stability. In domains where the source term distribution has a discontinuous behavior in the direction of the abscissa, the truncated series exhibits oscillations at the discontinuity, known as Gibb’s phenomenon [35]. Moreover, the system matrix has to be well-conditioned to acquire a sound numerical solution. The number of harmonics can be increased,
3.6 Conclusions

The HM is an accurate and fast method for the calculation of the magnetic field distribution when the geometry of the problem is divisible into orthogonal subdomains. The method is equally well applicable to geometries that can be represented as a composition of orthogonal domains in Cartesian, cylindrical, or spherical coordinate systems in both 2D and 3D. The analytical expressions of the amplitudes of the harmonics are coordinate system dependent. Having the
expressions for the field distribution in each domain available in analytical form allows fast and accurate post processing. The modular, systematic approach of decomposing the electromagnetic problem into rectangular subdomains opens up the possibility of forming the system matrix automatically through a computer program.

The HM is applicable under the physical assumption that the material properties have to be constant throughout the entire subdomain, or they have to be assumed constant. Furthermore, the lateral boundary conditions that are parallel to the ordinate have to be of the Neumann or periodic type and the spatial source variation within a subdomain may only be dependent on the coordinate of the abscissa. For the benchmark problem the assumption of a constant permeability in the subregion containing the permanent magnets has resulted in an error in the magnetic field calculation in the airgap. The restrictions as a result of the assumption of the boundary conditions on the lateral boundaries has prevented finitely permeable material for the core and end-effects of the mover of the benchmark problem from being modeled.

Comparison of the rms-values of the flux density components in the $x$ and $y$-direction obtained with the HM and FEM simulations have shown an attainable discrepancy of $\epsilon_d < 1\%$ with a corresponding calculation time of $t = 2.2\text{ ms}$. 
Chapter 4

The Schwarz-Christoffel Method

The Schwarz-Christoffel (SC) mapping is a special case of conformal mapping, which was introduced in section 2.2.5. The SC mapping is applicable to polygonally shaped domains where the Laplace equation holds [11]. The domain enclosed by the polygonal boundary, described in the complex plane, is transferred into a simpler domain for which the boundary value problem can be solved analytically. The transformation from one domain into another is obtained by the complex mapping function. The sources in the polygonal regions have to be represented by equivalent point sources first. The positions of the point sources in the polygonal domain are then mapped through the mapping function to the corresponding positions in the other. In the simplified, mapped domain the potential distribution is calculated, and the potential distribution in the polygonal domain is finally found through the mapping function. This procedure and the derivation of the SC mapping function are discussed in this chapter. The benchmark topology is analyzed with the Schwarz-Christoffel method (SCM) and the results are compared to FEM simulations.

The SCM has been known for many years and applied to a variety of electromagnetic problems which involve the Laplace equation to be solved. Publication regarding electromagnetic problems dealt with the SCM for wave guide and shielding problems [4, 28, 110]. The major limitation preventing the method to be used for electromechanical problems is caused by the fact that the mapping function can only be analytically evaluated for polygons with fewer than four vertices, which are of little practical avail. However, the application of the SCM for electromechanical problems has gained more attention in recent years. In [69] the slotted structure of a machine is significantly reduced to a structure with only one slot that is infinitely deep to come to an analytical mapping. In [70] the SCM is used to calculate the exact permeance between two aligned teeth. However, the resulting equation has to be solved numerically and misalignment cannot be
allowed for. For more complex problems with more than four vertices the mapping can be solved numerically by using the freely available Schwarz-Christoffel toolbox for Matlab\(^1\) by T. A. Driscoll [19]. The toolbox is applied in [59, 79, 80] for force calculations on slotted structures. How to analytically obtain the mapping function for a polygon, therefore, is not elaborated on in this thesis. The technique is thoroughly described in [11].

4.1 Schwarz-Christoffel mapping equation

A complex function can be considered an extension to a real function in the sense that in lieu of mapping real numbers from one number line to another; complex numbers in a complex plane are mapped into another complex plane. Although it can, the complex function or mapping itself does not necessarily contain the imaginary unit, \(j\), but rather operates on numbers in the complex plane. The function operates on real or complex numbers alike and the result of the mapping can be real or complex regardless of the argument being real or complex. A map is said to be conformal when the angle between intersecting curves is preserved under the mapping. In other words, when two curves intersect in one domain at a certain angle at a given point, the mapped curves intersect at the corresponding mapped point in the other domain at the same angle. For the map to be conformal the mapping function has to be analytical, which entails that the Cauchy-Riemann equations have to be satisfied, and its derivative is nonzero. Solutions to the Laplace equation automatically satisfy the Cauchy-Riemann equations [34], which implies that the solutions of a potential problem can be conformally mapped to another domain via the mapping function while preserving the orthogonality between potential and flux lines.

To come to the SC equation a mapping, \(g\), is considered that maps the grid from the complex \(w\)-domain to a grid in the \(z\)-domain by way of \(z = g(w)\), where \(\{w, z\} \in \mathbb{C}\). The derivative of the mapping plays an important role in the derivation of the final SC equation. The derivative of a complex function is obtained in the same way as for real functions and, indeed, all the same rules for differentiation apply. The derivative of the mapping gives the gradient of the mapping, where the real and imaginary part are the vector components in the direction of the real axis and imaginary axis, respectively. The argument of the derivative is the angle between the real axis and the tangent. The graphical interpretation is shown in Fig. 4.1 on a curve for the mapping \(z = g(w)\) at position \(z_p = x_p + y_p j\). For the SCM a mapping with the following derivative is considered

\[
g'(w) = \frac{dg(w)}{dw} = \frac{dz}{dw} = C(w - b)^\alpha, \tag{4.1}
\]

where \(\{b, \alpha\} \in \mathbb{R}\) and \(\{w, C\} \in \mathbb{C}\). The expression for the argument of the

\(^1\)available from http://www.math.udel.edu/~driscoll/SC/
4.1: Schwarz-Christoffel mapping equation

The derivative of the map is calculated for three distinct situations. The first situation is described by \( \{ w \in \mathbb{R} : w > b \} \) for which the argument is expressed as

\[
\arg (g'(w)) = \arg (C) + \alpha \arg (w - b) = \arg (C),
\]

(4.2)

in case \( \{ w \in \mathbb{R} : w < b \} \) the expression for the argument is

\[
\arg (g'(w)) = \arg (C) + \alpha \arg (w - b) = \arg (C) + \alpha \pi,
\]

(4.3)

and finally for \( \{ w \in \mathbb{R} : w = b \} \)

\[
g'(w) = 0.
\]

(4.4)

From the results of (4.2), (4.3), and (4.4) it can be concluded that if the real axis in the \( w \)-domain is travelled over from \(-\infty \) to \( \infty \) that the argument of the derivative of the map is constant until the point \( w = b \) is reached, where the argument jumps by the value \( \alpha \pi \), and then remains constant at the new value. This means that in the \( z \)-domain at the position \( z_b = g(b) \) the slope of the line suddenly changes by the angle \( \alpha \pi \). Hence, the mapping with a derivative given by (4.1) maps the real axis of the \( w \)-domain into a line with a vertex at position \( z_b = g(b) \), where the relative angular displacement between the lines on either side of the vertex equals \( \alpha \pi \). Furthermore, it can be concluded that the multiplication factor, \( C \), in (4.1) is not only a scaling factor, but it also causes an additional rotation of the line in the \( z \)-domain. Finally, it is noteworthy to observe that the derivative of the mapping is zero at \( w = b \), which results in the mapping to be nonconformal at that point. It is easily shown that additional vertices can be
obtained for a line in the $z$-domain by extending (4.1) to
\[
g'(w) = C \prod_{n=1}^{N_v} (w - b_n)^{\alpha_n},
\]
for which the value of the argument is expressed as
\[
\arg (g'(w)) = \arg (C) + \sum_{n=1}^{N_v} \alpha_n \arg (w - b_n),
\]
where $N_v$ is the total number of vertices. A graphical representation of the foregoing is depicted in Fig. 4.2. If the real axis in the $w$-domain is to be mapped on the boundary of a closed polygon the beginning and the end point of the polygonal path coincide. Summation of all the angular displacements at their respective vertices, evaluated in the counterclockwise direction of the polygon, has to equal $-2\pi$:
\[
\sum_{n=1}^{N_v} \alpha_n = -2.
\]
This means that the final angle is defined once all the other $N_v - 1$ vertices are defined. Therefore, the product of (4.5) has to be carried out for $N_v - 1$ terms only. Furthermore, it is more convenient to express the angular displacements at the vertices in terms of the interior angle as demonstratively indicated by $a_2\pi$ in Fig. 4.2 for the second vertex: $\alpha_n = a_n - 1$. Finally, to obtain the mapping function (4.5) has to be integrated
\[
g(w) = A + C \int_{w_0}^{w} \frac{d\zeta}{\prod_{n=1}^{N_v-1} (\zeta - b_n)^{\alpha_n - 1}},
\]
where $A$ is a complex valued integration constant. Equation (4.8) maps the upper half plane of the complex $w$-domain ($\text{Im } w \geq 0$) into the interior domain of a polygon in the complex $z$-domain with $N_v$ vertices located at $z_1, \ldots, z_{N_v}$ with their
4.2 Calculation of the magnetic field through the SC mapping

The first step in applying the SCM is the representation of the problem space into a polygon in the complex $z$-domain, where all the source terms in the domain have to be replaced with equivalent point sources. The polygon can be a closed one or an open one with vertices located at infinity. In any case, the transformation of the sources into point sources is required in order to be able to map their corresponding positions to the $w$-domain by means of the mapping function. The domain, together with the point source representation of the benchmark topology in the $z$-domain, is shown in Fig. 4.3. It can be seen that only the polygonally shaped airgap region is considered, where the soft-magnetic regions are assumed to be infinitely permeable. Furthermore, the permanent magnets and phase currents are replaced by line currents, where the equivalent line currents of the magnets are situated on the lateral sides of the magnets. The value of the equivalent line current for the permanent magnets is calculated by

$$I_m = -\frac{(h_3 - h_2) B_{\text{rem}}}{\mu_0 N_m},$$

(4.9)
where $B_{\text{rem}}$ is the remanence of the permanent magnets, $h_3 - h_2$ the magnet height and $N_n$ the number of equivalent line currents on one of the lateral sides of the magnet. The line currents of the phase coils are given by

$$I_{eq} = \frac{I_{ph}}{N_p},$$  (4.10)

where $I_{ph}$ is the phase current and $N_p$ the number of equivalent line currents.

Next, the mapping of the polygonally shaped airgap domain has to be determined. Due to the periodic boundary conditions on the sides of the domain it is more convenient to apply a mapping that maps the interior of a rectangle in the $w$-domain, instead of the upper half plane in the $w$-domain, into the polygonal airgap domain in the $z$-domain. This can be obtained by consecutively applying two different mappings. The toolbox, however, can directly determine the mapping from a rectangle into the polygon. To that end, the complex numbers that describe the vertices and the corresponding interior angles of the polygon in counterclockwise direction have to be provided as vectors to the toolbox. The exact procedure can be found in the manual of the toolbox [19]. Once the mapping is obtained the positions of the point sources in the $w$-domain can be calculated by means of the inverse mapping

$$\vec{w}_{ps} = g^{-1}(\vec{z}_{ps}),$$  (4.11)

where $g^{-1}$ is the inverse mapping and $\vec{z}_{ps}$ denotes the vector containing the positions of the point sources in the $z$-domain. The locations of the point sources in the rectangular $w$-domain are depicted in Fig. 4.4. Since the map is conformal, the value of the potential that can be calculated at a point in the $w$-domain is equal to the potential of the corresponding mapped point in the $z$-domain. Hence, the potential distribution on a set of points in the $z$-domain are determined by calculating the values of the potential at the corresponding mapped positions in the $w$-domain due to the point sources distribution in the $w$-domain. The flux density distribution in either domain, on the other hand, is determined by a coordinate system dependent gradient of the potential. The flux density in the $z$-domain can be calculated if the flux density distribution in the $w$-domain is known via [34]

$$B(z) = \frac{B(w)}{g'(w)}.$$  (4.12)

where $g'(w)$ is the conjugate of the derivative of the mapping. It has to be noted that $B \in \mathbb{C}$, where the real and imaginary part are the flux density components in the direction of the positive real and positive imaginary axis, respectively.

The real and imaginary part of the flux density distribution in the $w$-domain at a point $w = u + vj$ due to the $k$th line current at $w = \psi_k + \nu_kj$ in a rectangle with
4.2: Calculation of the magnetic field through the SC mapping

periodic boundary conditions, width $l$, and height $h$ are given by

$$
\text{Re} \left[ B_k (u,v) \right] = \frac{\mu_0 I}{4h} \sum_{n=-\infty}^{\infty} \frac{\sin \left[ \frac{\pi}{h} (v + \nu_k) \right]}{\cosh \left[ \frac{\pi}{h} (u - \psi_k + nl) \right] - \cos \left[ \frac{\pi}{h} (v + \nu_k) \right]} + \frac{\mu_0 I}{4h} \sum_{n=-\infty}^{\infty} \frac{\sin \left[ \frac{\pi}{h} (v - \nu_k) \right]}{\cosh \left[ \frac{\pi}{h} (u - \psi_k + nl) \right] - \cos \left[ \frac{\pi}{h} (v - \nu_k) \right]},
$$

(4.13a)

$$
\text{Im} \left[ B_k (u,v) \right] = \frac{\mu_0 I}{4h} \sum_{n=-\infty}^{\infty} \frac{\sinh \left[ \frac{\pi}{h} (u - \psi_k) \right]}{\cosh \left[ \frac{\pi}{h} (u - \psi_k + nl) \right] - \cos \left[ \frac{\pi}{h} (v + \nu_k) \right]} - \frac{\mu_0 I}{4h} \sum_{n=-\infty}^{\infty} \frac{\sinh \left[ \frac{\pi}{h} (u - \psi_k) \right]}{\cosh \left[ \frac{\pi}{h} (u - \psi_k + nl) \right] - \cos \left[ \frac{\pi}{h} (v - \nu_k) \right]},
$$

(4.13b)

where $I$ is the value of the line current. The complex flux density due to all line currents is obtained by superposition of the flux density due to all individual line currents

$$
B(u,v) = \sum_{k=1}^{K} \text{Re} \left[ B_k (u,v) \right] + j \text{Im} \left[ B_k (u,v) \right],
$$

(4.14)

where $K$ is the total number of line currents.
Chapter 4: The Schwarz-Christoffel Method

4.3 SCM applied to the benchmark topology

The method is applied to determine the flux density along a horizontal path in the middle of the airgap at $y = \frac{1}{2}(h_2 + h_3)$ as indicated by the dashed line in Fig. 4.3. The flux density is calculated on the mapped path in the $w$-domain, as indicated by the dashed line in Fig. 4.4, due to the line current distribution in the $w$-domain by applying (4.13) and (4.14). Finally, (4.12) is applied to find the flux density distribution in the $z$-domain. The results in comparison to FEM simulations are shown in Fig. 4.5 and Fig. 4.6 for the $x$ and $y$-component, respectively. It can be seen that the result are in good correspondence with respect to the FEM solution. The simulation was carried out with $N_p = N_m = 50$ and $n \in \{-1, 0, 1\}$ in (4.13). The discrepancy, $\epsilon_d$, between the rms-values of the flux...
4.3: SCM applied to the benchmark topology

Fig. 4.7: The rms-values of the flux density components obtained by the SCM as a function of the number of equivalent point sources compared to FEM.

Fig. 4.8: Discrepancy with FEM (a), and the calculation time (b) of the SCM as a function of the number of equivalent point sources for one fixed position.

density components compared to the FEM in Figs. 4.5 and 4.6 are 2.3% and 4.1% for the $x$ and $y$-component, respectively. The time required for the calculation of the field quantities for one relative position of the translator with respect to the stator is equal to 1.09 s. It has to be noted that this time does not include the time for the calculation of the mapping itself. The calculation of the mapping is not affected by the number of relative positions of the translator with respect to the stator, for which the magnetic field is to be evaluated, since it has to be calculated only once prior to the field computations. The additional calculation time for the mapping is equal to 1.3 s.

The rms-values of the flux density components obtained by the SCM as a function
of the number of equivalent point sources compared to those obtained by FEM are depicted in Fig. 4.7. The discrepancy with respect to the FEM results and the calculation time for one position for different numbers of equivalent point sources after the mapping has been calculated are shown in Fig. 4.8.

The simulations for Figs 4.7 and 4.8 have been conducted with the number of equivalent point sources for the phase coils, $N_p$, and the permanent magnets, $N_m$, set equal, i.e. $N_m = N_p$. For $N_m = 17$ the discrepancy between the rms-values of the $x$ and the $y$-component with respect to the FEM solution is $\epsilon_5 < 3\%$ with a calculation time per position of $t = 0.63s$. An additional percentage point in the discrepancy can be gained by increasing the number of point sources to $N_m = 100$ at the expense of a significantly higher computation time. The computation time is proportional to the number of equivalent point sources. The calculation time does not only include the evaluation of (4.13), but also includes the inverse mapping of the location of the point sources and the path along which the flux density components are evaluated in the $w$-domain, the conjugate of the derivative of the map, and the calculation of the complex flux density values in the original $z$-domain via (4.12).

Assuming a unity relative permeability instead of 1.05 for the permanent magnet in order for the field calculation to be carried out by means of the equivalent current method introduces an error. This error manifests itself as the asymptotic behavior of the discrepancy for increasing $N_m$ in Fig. 4.7a. An additional error is introduced on account of numerical inaccuracies associated with the calculation of the mapping.

### 4.4 Additional remarks and numerical issues

The calculation time of the mapping function is only dependent on the number of vertices of the polygon. The SCM provides an alternative design tool for electromagnetic problems for which the geometry of the problem is not necessarily divisible into orthogonal domains. The SCM is only applicable in the 2D complex plane. Hence, only 2D problems in the Cartesian or polar coordinate system can be handled.

In the benchmark example ideal Neumann boundary conditions were applied, since the soft-magnetic material was assumed to be infinitely permeable. It is possible, however, to take the influence of linear, finitely permeable, soft-magnetic material into account. To that end, the expressions of (4.13) have to be modified to obtain the flux density distribution in the airgap due to the non-ideal boundary condition [31]. The field inside the soft-magnetic material cannot be calculated, since the mapping only applies to the interior of the domain (airgap region) and not the exterior. Nonlinear soft-magnetic materials cannot be taken into account.

Aside from the physical limitations and issues there are also some numerical issues
that require prudence to be called for when using the SC toolbox. Solving (4.8) numerically by means of the toolbox is a computationally demanding task. The calculation time is approximately proportional to the number of vertices of the polygon to the power of three [18]. Depending on the problem convergence is not always guaranteed. Furthermore, conformality is not guaranteed for elongated regions due to numerical accuracy [18]. An elongated region is a region with a large or small height-to-width ratio. Another effect known as crowding can occur as well [19]. Crowding is visible in Fig. 4.4 for the line currents of the phase coils. It can be seen that they tend to accumulate in the \( w \)-domain for line currents that are located deeper in the phase slot in the \( z \)-domain. In case of severe crowding, e.g. very deep and thin slots, the locations after mapping might become unreliable and could introduce errors on account of limitations with respect to the precision of the binary representation of numbers [6].

4.5 Conclusions

It has been shown how to obtain the magnetic field distribution in the airgap of the benchmark problem by means of the SCM. The SCM is able to determine the electromagnetic field distribution in domains with a geometrical configuration of non-orthogonal, polygonally shaped boundaries. The applicability of the SCM is restricted to problems in the 2D Cartesian or polar coordinate system.

Physically, the problem has to be represented by an equivalent current model that can be mapped through the complex mapping into a simpler domain where the boundary value problem can be solved. The material inside the polygonal domain has to be linear and homogeneous.

Only one domain can be taken into account and elongated polygonal domains have to be avoided to ensure numerical stability. The flux density distribution in the airgap of the benchmark topology has been determined, and the discrepancy with FEM results in terms of percentage are in the order of 3% within a calculation time of \( t = 0.63 \) s for a single relative position of the benchmark topology. The discrepancy can be reduced further to 2% at the cost of a three times higher computation time. The calculation time of the flux density distribution at a given position is linearly dependent on the number of equivalent currents in the problem. The additional time required for the calculation of the mapping is approximately proportional to the cube of the number of vertices of the polygon and is equal to \( 1.3 \) s for the benchmark topology.
In this chapter the boundary element method (BEM) for the numerical solving of the Laplace equation for linear, 2D, magnetostatic scalar potential problem with mixed boundary conditions is addressed. The BEM transforms the Laplace equation into a set of linear equations through discretization of the boundaries of the domain. The field distribution on the domain boundaries is known once the set of linear equations is solved. The potential problem need not necessarily be a magnetic scalar potential one. The method can be applied to any linear problem governed by the Laplace equation for scalar potentials, e.g. electrostatics, steady state heat conduction, fluid flow et cetera. The problem specific formulation can be obtained by replacing the magnetostatic quantities, $\varphi$ and $H$, by the problem specific ones. First, a brief mathematical review of the BEM is given on how to obtain the set of linear equations from the generic Laplace equation [27]. The resulting formulation has been implemented in Matlab. The implemented code is then applied to a boundary value problem based on the benchmark topology. The results are compared with FEM simulations for verification.

5.1 Conventions

Before going into detail about the mathematical analysis of the BEM, it is convenient to establish conventions regarding notation. In general, the BEM can have two formulations, i.e. the interior or the exterior formulation. The distinction is depicted in Fig. 5.1, in which in each figure two identical domains with the same boundary shape are shown. A problem is classified an interior one if the domain of interest ($\Omega$) is enclosed by the boundary of the domain ($\Gamma$) as shown in Fig. 5.1a. Conversely, a BEM problem is regarded an exterior one if the domain of interest lies outside the boundary and extends to infinity as shown in
Chapter 5: The Boundary Element Method

Fig. 5.1: Convention of the directions of the vectors for an interior (a), and an exterior (b) BEM formulation.

Fig. 5.1b. The outward normal vector, \( \vec{n} \), and the direction of integration, \( d\vec{\Gamma} \), on the boundary are indicated for both situations. Furthermore, the location of a point in the coordinate system is defined by a space vector, \( \vec{x} = x\hat{e}_x + y\hat{e}_y \), where \( \hat{e}_x \) and \( \hat{e}_y \) are the unit vectors in the \( x \) and \( y \)-direction of the coordinate system, respectively.

5.2 Laplace equation in integral form

The magnetic scalar potential distribution, \( \varphi \), in a source-free domain, \( \Omega \), is governed by the Laplace equation

\[
\nabla^2 \varphi (\vec{x}) = 0 \quad \forall \Omega, \tag{5.1}
\]

and the magnetic field strength value, \( H \), in the direction of vector \( \vec{v} = v_x\hat{e}_x + v_y\hat{e}_y \), is defined as

\[
H (\vec{x}) = \nabla \varphi (\vec{x}) \cdot \vec{v} \quad \forall \Omega. \tag{5.2}
\]

In order to solve (5.1) by means of the BEM, it has be rewritten into the integral form by applying the method of weighted residuals. In the method of weighted residuals the solution for the scalar potential is approximated by a polynomial. The polynomial approximation, \( \varphi_p \), leads to an error compared to the exact expression for \( \varphi \). The error, which is also known as the residual, is expressed as

\[
\epsilon_{\text{res}} = \nabla^2 \varphi_p - \nabla^2 \varphi = \nabla^2 \varphi_p. \tag{5.3}
\]
The residual, $\epsilon_{\text{res}}$, is multiplied with a test function (weighted) and then the integral over the domain has to be equal to zero

$$\int_{\Omega} \epsilon_{\text{res}} w \, d\Omega = \int_{\Omega} \left( \nabla^2 \varphi_p \right) w \, d\Omega = 0, \tag{5.4}$$

where $\Omega$ is the interior of the domain and $w$ is the test function. In this manner, the residual is minimized over the entire domain. Depending on the preferred method, e.g. collocation method, least squares, Galerkin’s method, or method of moments, different weighting functions can be applied [27]. In the remainder of the chapter the subscript $p$ of the approximated scalar potential is omitted.

### 5.3 Representation formula

The integral formulation of the Laplace equation of (5.4) still contains domain integrals. In order to eliminate the domain integrals, Green’s second identity is applied which leads to

$$\int_{\Omega} \left( w \nabla^2 \varphi - \varphi \nabla^2 w \right) \, d\Omega = \oint_{\Gamma} \left( w H - \varphi (\nabla w \cdot \vec{n}) \right) \, d\Gamma, \tag{5.5}$$

where $\Gamma$ is the boundary enclosing the domain of $\Omega$ and $\vec{n}$ the outward normal vector on the boundary. The left member of the domain integral on the left hand side of the equals sign vanishes by the substitution of (5.4) into (5.5). The subsequent substitution of (5.2) then yields

$$- \int_{\Omega} (\varphi \nabla^2 w) \, d\Omega = \oint_{\Gamma} (wH - \varphi (\nabla w \cdot \vec{n})) \, d\Gamma. \tag{5.6}$$

The final domain integral can be eliminated by choosing the Dirac delta function as solution to the Poisson equation for the test function, i.e.

$$\nabla^2 w = -\delta \left( \vec{x}, \vec{\xi} \right), \tag{5.7}$$

where $\vec{\xi} = \xi_x \hat{e}_x + \xi_y \hat{e}_y$ is the load point. The load point is the position vector where the delta function takes the nonzero value. The sifting property of the Dirac delta function ensures that the remaining domain integral reduces to the value of the potential at the load point position $\vec{\xi}$:

$$\varphi \left( \vec{\xi} \right) = \oint_{\Gamma} (wH - \varphi (\nabla w \cdot \vec{n})) \, d\Gamma. \tag{5.8}$$

Finally, the differential equation of (5.7) has to be solved to obtain the expression for $w$ and its gradient. The expressions for $w$ and $\nabla w$ are dependent on the number of dimension of the problem and are referred to as the fundamental solutions.
For 2D problems the fundamental solutions are

\[ u(\vec{x},\vec{\xi}) = w = -\frac{1}{2\pi} \ln |\vec{x} - \vec{\xi}|, \]  
\[ q(\vec{x},\vec{\xi}) = \nabla w \cdot \vec{n} = -\frac{(\vec{x} - \vec{\xi}) \cdot \vec{n}}{2\pi |\vec{x} - \vec{\xi}|^2}, \]  

(5.9a)

(5.9b)

The fundamental solutions, \( u \) and \( q \), describe the potential and field strength distribution in free space due to a point source located at the load point. With (5.9) equation (5.8) becomes the so-called representation formula

\[ \varphi(\vec{\xi}) = \oint_{\Gamma} H(\vec{x}) u(\vec{x},\vec{\xi}) - \varphi(\vec{x}) q(\vec{x},\vec{\xi}) \, d\Gamma. \]  

(5.10)

The representation formula gives the value of the potential at position \( \vec{\xi} \) in the domain, \( \Omega \), once the potential, \( \varphi \), and field strength distribution, \( H \), on the boundary, \( \Gamma \), are known.

### 5.4 Boundary integral equation

To solve the potential and field strength distribution on the boundary, the load point in (5.10) has to be located on the boundary. On the boundary, however, the sifting property of the delta function is not defined, and the boundary has to be modified in order to be able to evaluate the integral of (5.10). The boundary modification is shown in Fig. 5.2. In the vicinity of the load point the boundary by-passes \( \vec{\xi} \) through a circle with radius \( \rho \). The integral of the first term of (5.10) can be split into two parts, and a limiting process is applied for the improper integrals

\[ \int_{\Gamma} H u \, d\Gamma = -\frac{1}{2\pi} \lim_{\rho \to 0} \int_{\Gamma'} H \ln |\vec{x} - \vec{\xi}| \, d\Gamma - \frac{1}{2\pi} \lim_{\rho \to 0} \int_{\Gamma'} H \ln |\vec{x} - \vec{\xi}| \, d\Gamma. \]  

(5.11)
5.5: Discretization of the boundary integral equation

The left integral of (5.11) can be evaluated normally. The right integral of (5.11) is weakly singular and, therefore, reduces to zero. The same approach is applied to the right member of (5.10)

$$\int_{\Gamma} \varphi q d\Gamma = -\frac{1}{2\pi} \lim_{\rho \to 0} \int_{\Gamma'} \frac{\varphi (\mathbf{x} - \mathbf{\xi}) \cdot \mathbf{n}}{|\mathbf{x} - \mathbf{\xi}|^2} d\Gamma' - \frac{1}{2\pi} \lim_{\rho \to 0} \int_{\Gamma} \frac{\varphi (\mathbf{x} - \mathbf{\xi}) \cdot \mathbf{n}}{|\mathbf{x} - \mathbf{\xi}|^2} d\Gamma.$$  

(5.12)

The left integral of (5.12) can also be evaluated normally. The right integral of (5.12) is strongly singular and has to be evaluated in the following way

$$\lim_{\rho \to 0} \int_{\Gamma} \varphi \rho \frac{(\mathbf{x} - \mathbf{\xi}) \cdot \mathbf{n}}{|\mathbf{x} - \mathbf{\xi}|^2} d\Gamma = \lim_{\theta \to 0} \int_{\Gamma} \varphi \frac{\rho}{\rho^2} \rho d\theta = \alpha \varphi (\mathbf{\xi}),$$  

(5.13)

where $\alpha$ is the angle between the lines on either side of $\mathbf{\xi}$. Substitution of the results of (5.11) and (5.13) into the representation formula of (5.10) gives the boundary integral equation

$$c (\mathbf{\xi}) \varphi (\mathbf{\xi}) + \oint_{\Gamma} \varphi (\mathbf{x}) q (\mathbf{x}, \mathbf{\xi}) d\Gamma = \oint_{\Gamma} H (\mathbf{x}) u (\mathbf{x}, \mathbf{\xi}) d\Gamma,$$  

(5.14)

where

$$c (\mathbf{\xi}) = \begin{cases} 
1 - \frac{\alpha}{2\pi} & \text{for } \mathbf{\xi} \in \Gamma \\
1 & \text{for } \mathbf{\xi} \in \Omega \\
0 & \text{for } \mathbf{\xi} /\in \Omega, \mathbf{\xi} /\in \Gamma.
\end{cases}$$  

(5.15)

The boundary integral equation has to be solved to find the field distribution on the boundary subject to the imposed boundary conditions. For most practical geometries (5.14) cannot be evaluated analytically, but requires the geometry to be discretized in order to solve (5.14) numerically. Since the boundary integral equation has to be evaluated along the boundary ($\Gamma$), only the boundary of the domain of interest has to be subdivided into boundary elements.

5.5 Discretization of the boundary integral equation

An example of the subdivision of the boundary of a domain, or mesh, is illustrated in Fig. 5.3. The dots in Fig. 5.3 represent the end points of each straight mesh element. The potential and field strength distribution along each individual mesh element is given by the order of the polynomial approximation of $\varphi_p$ in (5.3). For an $n^{th}$ order polynomial approximation $n + 1$ points on each mesh element have
to be evaluated. The values of the potential and field strength distribution along each element can be found by interpolation through so-called shape functions. In this thesis only constant, or zero-order, elements are considered, which yield the simplest BEM formulation. For zero-order elements the values of the potential and field strength distribution along each element are constant. The point where the actual values of the potential and field strength are calculated is situated in the middle of the element. Higher order elements in general give more accurate results for the same mesh density. However, higher order elements lead to a more complex formulation [115] and a larger set of equations, causing the computational efforts to increase accordingly. For each given problem a compromise between the order and the total number of mesh elements has to be reached. In practice, second order elements are most commonly applied. For the calculation of the permeance values of the TCM, to which the BEM is applied in this thesis, the simple zero-order approximation proves to provide a good compromise between accuracy and mesh size. Additional reading on the use of shape functions for higher order mesh elements can be found in [83].

The discretized version of the boundary integral equation of (5.14) for constant mesh elements at a load point located on the $i^{\text{th}}$ mesh element is given by

$$
\frac{1}{2} \phi (\vec{\xi}_i) + \sum_{j=1}^{N} \left[ \varphi_j \int_{\Gamma_j} q (\vec{x}, \vec{\xi}_i) \, d\Gamma \right] = \sum_{j=1}^{N} \left[ H_j \int_{\Gamma_j} u (\vec{x}, \vec{\xi}_i) \, d\Gamma \right], \quad (5.16)
$$

where $j$ is the index number of the mesh elements and $N$ the total number of mesh elements. Equation (5.16) can be reduced to a set of linear equations by applying the collocation method. In the collocation method the load point is placed in the center of the first mesh element, $i = 1$, and (5.16) is evaluated for all $N$ mesh elements. This yields the first linear equation. The graphical representation is shown in Fig. 5.4 for the contribution of the $j^{\text{th}}$ element to the potential value at the load point on the $i^{\text{th}}$ element. This procedure is sequentially repeated with the load point placed in the center of the other elements to obtain a matrix equation of the form

$$
\mathbf{F} \vec{\phi} = \mathbf{G} \vec{H}, \quad (5.17)
$$

where $\vec{\phi}$ and $\vec{H}$ are column vectors of length $N$ that give the potential and field
5.5: Discretization of the boundary integral equation

strength on the mesh elements, and $F$ and $G$ are $N \times N$ matrices that are only dependent on the geometry.

The entries of $F$ and $G$ are given by the integrals of (5.16) and they can be calculated analytically or numerically. To obtain the analytical expression for the entries of $F$ and $G$, Fig. 5.4 is considered. Figure 5.4 shows the $j^{th}$ mesh element with the load point, $\vec{\xi}_i$, located at the center of the element. The entries of the $F$ and $G$ matrix with respect to the $j^{th}$ mesh element in Fig. 5.4 are given by

$$F_{i,j} = \begin{cases} \frac{\theta_1 - \theta_2}{2\pi} & \text{for } i \neq j \\ \frac{1}{2} & \text{for } i = j \end{cases}$$

(5.18a)

$$G_{i,j} = \begin{cases} \frac{L}{2\pi} \left[ \ln \left( \frac{\cos(\theta)}{\ell} \right) + 1 \right] \tan(\theta) - \theta \bigg| \theta = \theta_1 & \text{for } i \neq j \\ \frac{L}{2\pi} \left( 1 + \ln \left( \frac{2}{\ell} \right) \right) & \text{for } i = j \end{cases}$$

(5.18b)

where

$$\vec{\xi}_i = \frac{1}{2} (\vec{x}_{i+1} + \vec{x}_i), \quad L = |\vec{x}_{i+1} - \vec{x}_i|,$$

$$\vec{\Gamma} = \frac{\vec{x}_{j+1} - \vec{x}_j}{|\vec{x}_{j+1} - \vec{x}_j|}, \quad \vec{n} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \vec{\Gamma},$$

$$\ell = \left( \vec{x}_j - \vec{\xi}_i \right) \cdot \vec{n}, \quad \theta_0 = \arctan2 \left( \frac{n_y}{n_x} \right),$$

$$\theta_1 = \arctan \left( \frac{1}{\ell} (\vec{x}_j - \vec{\xi}_i) \cdot \vec{\Gamma} \right), \quad \theta_2 = \arctan \left( \frac{1}{\ell} (\vec{x}_{j+1} - \vec{\xi}_i) \cdot \vec{\Gamma} \right),$$

in which $\arctan2$ is the four quadrant inverse tangent function.
5.6 Mixed boundary value problems

Since (5.17) contains both the potential values on each mesh element in $\vec{\varphi}$ as well as the field strength values on each element in $\vec{H}$, Dirichlet problems or mixed boundary problems can be dealt with relatively easy. In either case, (5.17) has to be transformed into a standard matrix equation

$$A \vec{y} = \vec{b}, \quad (5.19)$$

where $A$ is an $N \times N$ matrix, $\vec{b}$ a column vector containing all known boundary values of either the potential or the field strength on the mesh elements, and $\vec{y}$ is the column vector to be solved. For full Dirichlet problems the potential vector, $\vec{\varphi}$, only contains known values and the field strength vector, $\vec{H}$, contains all unknowns. Therefore, $A$, $\vec{y}$, and $\vec{b}$ in (5.19) are equal to $G$, $\vec{H}$, and the matrix multiplication of $F$ and $\vec{\varphi}$, respectively. Solving (5.19) by matrix inversion, therefore, directly yields the value of the field strength distribution in the direction of the outward normal along the boundary of the domain. For mixed boundary problems with a mixture of known potentials or field strengths on the boundary (5.17) has to be rearranged prior to applying (5.19). The values to be calculated have to be on one side of the equals sign, whereas all the imposed values have to appear on the other side. This is obtained by swapping the columns of $F$ and $G$ after sign inversion. The rearrangement is illustrated by means of an example. Assume the following matrix equation

$$
\begin{bmatrix}
F_{11} & F_{12} & F_{13} & F_{14} \\
F_{21} & F_{22} & F_{23} & F_{24} \\
F_{31} & F_{32} & F_{33} & F_{34} \\
F_{41} & F_{42} & F_{43} & F_{44}
\end{bmatrix}
\begin{bmatrix}
\varphi_{1k} \\
\varphi_{2u} \\
\varphi_{3k} \\
\varphi_{4u}
\end{bmatrix}
= 
\begin{bmatrix}
G_{11} & G_{12} & G_{13} & G_{14} \\
G_{21} & G_{22} & G_{23} & G_{24} \\
G_{31} & G_{32} & G_{33} & G_{34} \\
G_{41} & G_{42} & G_{43} & G_{44}
\end{bmatrix}
\begin{bmatrix}
H_{1u} \\
H_{2k} \\
H_{3u} \\
H_{4k}
\end{bmatrix},
$$

where the subscript $k$ and $u$ denote whether the value for the potential or field strength on the mesh element is known or unknown, respectively. After rearrangement and sign inversion of the columns the following matrix equation is obtained

$$
\begin{bmatrix}
-G_{11} & -G_{12} & -G_{13} & -G_{14} \\
-G_{21} & -G_{22} & -G_{23} & -G_{24} \\
-G_{31} & -G_{32} & -G_{33} & -G_{34} \\
-G_{41} & -G_{42} & -G_{43} & -G_{44}
\end{bmatrix}
\begin{bmatrix}
\vec{y}
\end{bmatrix}
= 
\begin{bmatrix}
-F_{11} & -F_{12} & -F_{13} & -F_{14} \\
-F_{21} & -F_{22} & -F_{23} & -F_{24} \\
-F_{31} & -F_{32} & -F_{33} & -F_{34} \\
-F_{41} & -F_{42} & -F_{43} & -F_{44}
\end{bmatrix}
\begin{bmatrix}
\vec{b}
\end{bmatrix}.
$$

All the unknown values are now contained within $\vec{y}$, and $\vec{b}$ contains the numeric values as a result of the matrix multiplication on the right-hand side of the equals sign. Once $\vec{y}$ is solved, the potential and field strength vectors are obtained by the inverse rearrangement.
5.7 Multiple domains and periodic boundary conditions

So far, only single domains were considered where either the potential or the field strength on the boundary is imposed. More practical problems, however, involve multiple domains with different material properties. The BEM for single domains with source free interior regions can easily be extended to problems with multiple source free interior regions. Figure 5.5a shows a multiple domain problem with two domains with different material properties. The multiple domain problem can be decomposed into two single domain problems as shown in Fig. 5.5b. The two resulting subdomains can be meshed independently except for the edges that coincide at the interface of the material transition. At the interface edges the meshes must be identical in the sense that coinciding elements have the same length and orientation. The interface edges for domains 1 and 2 in Fig. 5.5b are formed by mesh elements with indices \(j_1^{(1)}\) to \(j_M^{(1)}\) and \(j_1^{(2)}\) to \(j_M^{(2)}\), respectively. Each interface mesh, therefore, consists of \(M\) elements. Since neither the potential nor field strength is known on the interface mesh, additional boundary conditions are required to solve (5.17)

\[
\begin{align*}
\varphi_{j_m^{(1)}} - \varphi_{j_M^{(2)} - m + 1} &= 0 & \text{for } m \in \{1, \ldots, M\}, \\
\mu_1 H_{j_m^{(1)}} + \mu_2 H_{j_M^{(2)} - m + 1} &= \rho_S & \text{for } m \in \{1, \ldots, M\},
\end{align*}
\]

where \(\mu_1\) and \(\mu_2\) are the permeabilities of regions 1 and 2, respectively, and \(\rho_S\) is the magnetic surface charge density on the interface. For an electrostatic problem \(\mu_1, \mu_2, \text{ and } s\) would, respectively, represent the permittivity of region 1, the permittivity of region 2, and the electric surface charge density on the boundary. The most straightforward way of incorporating the boundary conditions is by...
appending the matrices of (5.17) with boundary conditions matrices as follows
\[
\begin{bmatrix}
F^{(1)} & 0 \\
0 & F^{(2)} \\
Φ^{(1)} & Φ^{(2)} \\
0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
φ^{(1)} \\
φ^{(2)} \\
\end{bmatrix}
= \begin{bmatrix}
G^{(1)} & 0 \\
0 & G^{(2)} \\
Ψ^{(1)} & Ψ^{(2)} \\
0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
H^{(1)} \\
H^{(2)} \\
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
S \\
\end{bmatrix},
\]
(5.21)

where the entries of the boundary matrices, Φ^{(1)}, Φ^{(2)}, Ψ^{(1)}, Ψ^{(2)}, and the source vector, \( \vec{S} \), in regard to Fig. 5.5 are given by
\[
\begin{align*}
Φ^{(1)}_{m,j} &= 1 \\
Φ^{(2)}_{m,j,M+1} &= -1 \\
Ψ^{(1)}_{m,j} &= µ_1 \\
Ψ^{(2)}_{m,j,M+1} &= µ_2 \\
S_m &= ρ_S
\end{align*}
\]
for \( m \in \{1, \ldots, M\} \).
(5.22)

Rearrangement of the columns for mixed boundary value problems has to be applied to come to the matrix equation of (5.19). The same approach can be applied for periodic boundary conditions on the edges of a domain. The edges are now on the same domain, but the foregoing still applies. In this case, the domain index, indicated by the superscript number in parentheses, is the same, as are the material properties coefficients, µ. Moreover, for periodical boundaries \( ρ_S = 0 \text{ A m}^{-1} \) in (5.20b).

### 5.8 Field solution in the interior of the domain

The representation formula of (5.10) can be applied to find the potential distribution at a load point located in the interior of a domain, \( \vec{ξ}_Ω \), when the field distribution on the boundary is known. First, the representation formula is discretized
\[
ψ_Ω \left( \vec{ξ}_Ω \right) = \sum_{j=1}^{N} H_j \int_{Γ_j} u \left( \vec{x}, \vec{ξ}_Ω \right) dΓ - \sum_{j=1}^{N} φ_j \int_{Γ_j} q \left( \vec{x}, \vec{ξ}_Ω \right) dΓ,
\]
(5.23)
where \( j \) is the index of the mesh elements. The discretized representation formula can be expressed as a matrix operation
\[
ψ_Ω \left( \vec{ξ}_Ω \right) = \vec{G}_p \cdot \vec{H} - \vec{F}_p \cdot \vec{φ},
\]
(5.24)
where \( \vec{φ} \) and \( \vec{H} \) are given by the solutions to (5.17). As the load point, \( \vec{ξ}_Ω \), always lies within the interior of the domain, the integrals of (5.23) are definite.
Therefore, the entries for $\vec{F}_p$ and $\vec{G}_p$ are given by the respective expression for $i \neq j$ in (5.18).

The field strength in the interior domain at the load point is obtained by taking the gradient of (5.23)

$$\vec{H}_\Omega (\vec{\xi}_\Omega) = \left[ \sum_{j=1}^{N} H_j \int_{\Gamma_j} \frac{\partial u (\vec{x}, \vec{\xi}_\Omega)}{\partial x} d\Gamma - \sum_{j=1}^{N} \varphi_j \int_{\Gamma_j} \frac{\partial q (\vec{x}, \vec{\xi}_\Omega)}{\partial x} d\Gamma \right] \hat{e}_x$$

$$+ \left[ \sum_{j=1}^{N} H_j \int_{\Gamma_j} \frac{\partial u (\vec{x}, \vec{\xi}_\Omega)}{\partial y} d\Gamma - \sum_{j=1}^{N} \varphi_j \int_{\Gamma_j} \frac{\partial q (\vec{x}, \vec{\xi}_\Omega)}{\partial y} d\Gamma \right] \hat{e}_y. \quad (5.25)$$

The integrals in (5.25) can be evaluated analytically. The matrix representation of (5.25) is given by

$$\vec{H}_\Omega (\vec{\xi}_\Omega) = \vec{G}_f \cdot \vec{H} - \vec{F}_f \cdot \vec{\varphi}, \quad (5.26)$$

where $\vec{F}_f$ and $\vec{G}_f$ are $2 \times N$ matrices containing the values of the integrals of (5.25). The entries for $\vec{F}_f$ and $\vec{G}_f$ with respect to Fig. 5.4 can be expressed as

$$F_{f,\omega,j} = \frac{n_x}{4\pi \ell} \left( \sin [2(\theta_0 + \theta_2)] - \sin [2(\theta_0 + \theta_1)] \right)$$

$$+ \frac{n_y}{4\pi \ell} \left( \cos [2(\theta_0 + \theta_1)] - \cos [2(\theta_0 + \theta_2)] \right), \quad (5.27a)$$

$$G_{f,\omega,j} = \frac{n_y}{4\pi \ell} \left( \sin [2(\theta_0 + \theta_1)] - \sin [2(\theta_0 + \theta_2)] \right)$$

$$+ \frac{n_x}{4\pi \ell} \left( \cos [2(\theta_0 + \theta_1)] - \cos [2(\theta_0 + \theta_2)] \right), \quad (5.27b)$$

$$F_{f,\theta,j} = \frac{1}{2\pi} \left( \ln [\cos (\theta_1)] - \ln [\cos (\theta_2)] \right) \sin (\theta_0)$$

$$+ \frac{1}{2\pi} (\theta_1 - \theta_2) \cos (\theta_0), \quad (5.27c)$$

$$G_{f,\theta,j} = \frac{1}{2\pi} \left( \ln [\cos (\theta_2)] - \ln [\cos (\theta_1)] \right) \cos (\theta_0)$$

$$+ \frac{1}{2\pi} (\theta_1 - \theta_2) \sin (\theta_0). \quad (5.27d)$$

### 5.9 The BEM applied to the benchmark topology

The BEM has been implemented in Matlab by means of which a magnetostatic problem is solved. The magnetic flux density components are obtained by $\vec{B} = \mu \vec{H}$. The geometry of the benchmark topology of Fig. 2.6 is considered. The boundary value problem to be solved is schematically shown in Fig. 5.6, where bold lines indicate a Dirichlet boundary condition with an imposed potential value of zero, the
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Fig. 5.6: Magnetostatic BEM model for the benchmark topology.

Fig. 5.7: Boundary element mesh of the benchmark topology.

dashed lines indicate a periodical boundary conditions, and solid lines represent continuous boundary conditions at the material transition of different regions. The problem is divided into five regions indicated by the superscript number between parentheses. For the benchmark problem the relative permeability of region \( \Omega^{(1)} \) is equal to \( \mu_r^{(1)} = 1 \) and the relative permeabilities of the remaining regions are equal to \( \mu_r^{(2)} = \mu_r^{(3)} = \mu_r^{(4)} = \mu_r^{(5)} = \mu_{r_{pm}} = 3 \). This particular value for \( \mu_{r_{pm}} \) is deliberately chosen to show that its value does not affect the discrepancy as opposed to the HM and SCM. Furthermore, positive and negative unity magnetic surface charge densities are assigned to the top edges of the regions \( \Omega^{(2)} \) to \( \Omega^{(5)} \), as depicted in Fig. 5.6. It has to be noted that the problem is physically different from the benchmark problem in chapters 3 and 4. All Dirichlet boundaries in Fig. 5.6 have an assigned potential of \( \phi = 0 \) A. However, for the problem to be identical to the ones in chapters 3 and 4 the Dirichlet boundary would still apply for infinite permeable boundaries, but the potential values for the top and bottom one would be different. The actual difference in potential value between the two boundaries is not known in advance and, therefore, cannot be imposed.

Before the boundary integral equation for a problem can be solved, the boundary has to be meshed. The applied mesh is shown in Fig. 5.7. It can be seen that...
in the vicinity of the vertices of the domain the mesh is finer. In the corners the magnetic potential exhibits high gradients, therefore, the mesh is denser in the corners to obtain a more accurate solution. The finer meshing of the vertices is obtained by geometrical progression of the mesh element lengths from the center of an edge to its vertices. Regions $\Omega^{(1)}$, and $\Omega^{(2)}$ to $\Omega^{(5)}$ each contain 342 and 36 mesh elements, respectively. Hence, the total number of mesh elements equals $N_{\text{BEM}} = 486$. The magnetic field distribution along a line in the middle of the airgap at $y = \frac{1}{2} (h_2 + h_3)$ with respect to Fig. 2.6 is calculated and compared to FEM simulations. The results for the calculated magnetic scalar potential, flux density component in the $x$-direction and $y$-direction along the line are shown in Figs. 5.8, 5.9, and 5.10, respectively. It can be seen that the results are in good agreement with the FEM simulations. The discrepancy between the rms-value of the potential obtained by the BEM and the FEM of Fig. 5.8 is 0.18% and the calculation time is equal to 160 ms.

Results in terms of the rms-values of the flux density and the accompanying simulation time, both as function of the number of mesh elements, are shown in Fig. 5.11 and Fig. 5.12, respectively. The flux density values are compared to FEM simulations in Fig. 5.11. The discrepancy between the FEM and BEM results is $\epsilon_4 < 1\%$ for both the $x$ and $y$-component of the flux density for mesh sizes for which $N_{\text{BEM}} \geq 300$. The corresponding calculation time is 87 ms. It has to be noted that from the number of mesh elements with respect to accuracy in itself only superficial conclusions can be drawn. The actual mesh distribution is of importance as well. An adequate mesh distribution based on the understanding of the physical nature of the problem can yield more accurate results with fewer mesh elements, and therefore reduced calculation time, than a uniformly distributed mesh with more elements in which the physical nature of the problem is unheeded.

Since the BEM does not impose any geometrical restrictions on the domains, it yields accurate results. The only physical restriction is the material to be linear and homogeneous. Obviously, a more accurate field solution is obtained when the mesh is finer at the expense of a longer calculation time. The BEM matrices ($\mathbf{F}$ and $\mathbf{G}$) are densely populated compared to the FEM. As a consequence, the BEM has a more intense use of memory. Furthermore, nonlinear material properties can be taken into account, but they require domain discretization which negates the advantage of reducing the computational effort due to boundary discretization. The BEM can be extended to solve the Poisson equation. However, this requires domain discretization of the regions with the source term when its integral cannot be evaluated analytically. It has to be noted that numerical domain integration does not increase the number of elements in the matrices, it is only required to determine the values of the source vector. Furthermore, the BEM can also be formulated for solving the vector potential, which is especially beneficial when domains with current sources have to be dealt with. These additional features have not been implemented, because they are not required for the permeance value calculation to which the BEM is applied in chapter 6.
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Fig. 5.8: Magnetostatic scalar potential distribution for the benchmark topology calculated with the BEM and the FEM.

Fig. 5.9: Magnetic flux density distribution in the $x$-direction for the benchmark topology calculated with the BEM and the FEM.

Fig. 5.10: Magnetic flux density distribution in the $y$-direction for the benchmark topology calculated with the BEM and the FEM.
5.10 Conclusions

The BEM, as formulated in the foregoing, is an accurate and flexible tool to numerically calculate the field quantities for linear scalar potential problems that are governed by the Laplace equation. The complexity of the geometry does not limit the applicability of the method, i.e. non-orthogonal, polygonal shapes can be allowed for without increasing complexity or losing accuracy.

Physical restrictions are the requirement of the material properties within a domain being linear and homogeneous, and the inability of taking domains with current source into account for the BEM formulation as presented in this chapter. However, the latter restriction does not apply in general for the BEM, and it can...
be ameliorated by a vector potential formulation.

To enhance the overall accuracy of the BEM, higher order polynomial approximations can be applied instead of zero-order ones at the expense of a more complex mathematical formulation. With higher order elements the same accuracy can be obtained with fewer elements. Only the boundary of the domain is discretized in order to calculate the field quantities, which reduces the computational effort compared to numerical methods that require domain discretization such as the FDM and the FEM. The BEM has successfully been implemented and applied to the geometry of the benchmark topology with different physical properties than the benchmark topology of chapters 3 and 4. The obtained discrepancy, depending on the number of mesh elements, compared to FEM simulations is within 1\% for a mesh not exceeding 300 elements with an accompanying calculation time of 87 ms. Contrary to the HM and the SCM the accuracy is not dependent on the value of the permeability of the permanent magnet.
The tooth contour method (TCM) is a hybrid modeling techniques that combines the MEC method with another accurate modeling method that is applied to adequately determine the permeance values of the flux tubes in the airgap [62]. The soft-magnetic parts of the model are transformed into an equivalent permeance network using the classical MEC approach [81]. The airgap permeances, however, are not calculated classically by discretization of the airgap into flux tubes of simplified shape for which the permeance values can be expressed analytically. Instead, the calculation of the permeance values is based on the scalar potential and flux distribution on the boundary of a subproblem of the airgap. Within the subproblem only the local potential and flux distribution are considered and, therefore, the subproblem can be relatively small in size. Depending on the geometry of the subproblem the user has freedom of choice regarding the model to be applied, i.e. either numerical or analytical if possible [41, 86]. Since the airgap permeance calculation is based on an accurately determined electromagnetic field distribution in the airgap, and not on the geometry of simplified flux tube shapes, the overall accuracy of the electromagnetic field distribution inside the complete structure is improved. In this manner, a more accurate MEC model is created that can, if desired, allow for nonlinear effects.

In this chapter the TCM is discussed. The classical MEC is briefly explained and the focus is on the calculation of the airgap permeances by means of the BEM [52, 53]. The method is applied to the benchmark topology and the results in terms of flux linkages are compared to FEM simulations.
6.1 Equivalent network components

Each MEC consists of a network of passive components, the permeances or reluctances, and active components which are the sources of magnetomotive force (mmf). To transform the magnetic structure into a MEC, the structure is divided into flux tubes. The division of the structure into flux tubes is based on an estimated flux line distribution inside the structure. While the flux traverses from one side of a flux tube to the other, it is assumed that no leakage occurs at the longitudinal faces of the tube. Furthermore, the flux is perpendicular to the planes where the flux enters and leaves the flux tube. At these transversal planes the magnetic scalar potential, mmf, value throughout the surface is equal. A flux tube together with the flux flow and equipotential transversal faces is schematically shown in Fig. 6.1. Crucial in the method is the calculation of the proper value of the corresponding permeance or reluctance of the flux tube in the equivalent network. The value of the reluctance in general is given by

\[ R = \frac{\Delta F}{\phi}, \]  

(6.1)

where \( R \) is the reluctance, \( \Delta F \) the mmf-drop over the reluctance, and \( \phi \) the flux through it. The permeance value can simply be calculated by \( \mathcal{P} = R^{-1} \). Since the mmf-drop and the flux through a reluctance are only known once the complete field distribution is known, it is more convenient to assume a uniform flux distribution throughout the whole flux tube. In that case, the permeance value of the flux tube can be expressed as a function of its geometrical dimensions.

The geometry dependent expression for the passive and active components of the equivalent network can be derived by considering the simple magnetic circuit of Fig. 6.2. The main flux path is assumed to be as indicated by the dashed line in Fig. 6.2a, and the flux is assumed to be uniformly distributed. The magnetic sources are a coil with \( N \) turns around the leg indicated by subscript 1 and a
永久磁铁与内部磁化

永久磁铁与内部磁化，$M_0$，组成腿，其子标为3。磁化方向由箭头指示。根据Fig. 6.2a沿磁通路径应用安培定律

$$\oint \vec{H} \cdot d\vec{l} = \sum_{n=1}^{4} H_n l_n = N_t I, \quad (6.2)$$

其中$N_t$是匝数，$I$是通过线圈的电流。将(2.5a)代入(6.2)并使用磁通与磁通密度的关系（在所有情况下磁通都是垂直于面积）

$$\phi = BA$$

(6.2)变为

$$\phi \sum_{n=1}^{4} \frac{l_n}{\mu_0 \mu_r A_n} = N_t I + M_0 \mu_r l_3 \Rightarrow \phi \sum_{n=1}^{4} \mathcal{R}_n = \mathcal{F}_c + \mathcal{F}_m, \quad (6.3)$$

其中$\mu_r$和$A_n$分别为第$n$个磁通管的相对磁导率和横截面积，相应的mmf源项分别为$\mathcal{F}_c$和$\mathcal{F}_m$。注意文献中永久磁体的mmf源项通常表示为$\mathcal{F}_m = -H_c I$，其中$H_c$是硬磁性材料的矫顽力。Fig. 6.1中的任意形状磁通管的磁通函数值的一般表达式为

$$\mathcal{R} = \int \frac{1}{\mu_0 \mu_r(l) A(l)} dl. \quad (6.4)$$

Fig. 6.2: A simple magnetic circuit: the physical representation (a), and its MEC representation (b).
6.2 Airgap permeance calculations via the classical approach

The airgap in electromechanical devices is a necessity that is required to allow relative movement from the moving part with respect to the stationary part. The airgap abruptly interrupts the magnetic circuit and causes the flux lines to disperse in the air due to the low permeability of the air compared to the permeability of the soft-magnetic material. This effect is known as flux fringing and is shown in Fig. 6.3 for the benchmark topology. Furthermore, it can be seen from Fig. 6.3 that not all flux lines cross the airgap and link the moving part with the stationary part. This effect is referred to as flux leakage.

A MEC representation in flux tubes of the airgap of the benchmark topology is depicted in Fig. 6.4. All the flux tubes have a constant cross-section and, therefore, the flux lines inside the tubes flow parallel to the lateral sides. The constant cross-section of the tubes ensures (6.4) to be applicable [118]. The flux tubes that allow the flux to cross the airgap from the stator to the translator are shown in Fig. 6.4a. It can be seen that the fringing is represented by curved tubes. The leakage tubes, which have both transversal planes on either the stator or translator, are shown in Fig. 6.4b. The division of the soft-magnetic core in flux tubes is straightforward and (6.4) can be applied without difficulty, since the flux lines inside the core flow more or less uniformly, as can be seen from Fig. 6.3.

It is obvious from the comparison between the actual flux distribution in Fig. 6.3 and the simplified flux tube representation in Fig. 6.4 that an error is introduced in the calculation of the electromagnetic field distribution. If the inclusion of the relative movement is of interest, e.g. for force calculations, the situation might occur that for different relative positions different networks with a different number of components are required. Moreover, the leakage and fringing flux are very sensitive to variations in the geometry, which could complicate a geometrical optimization routine or render the outcome of one unreliable. These errors and
6.3 Airgap permeances calculations via the potential distribution

Since the longitudinal faces of a flux tube are assumed to be without leakage, the actual geometrical shape of the flux tube comprises the space confined in between two adjacent flux lines of the actual flux distribution. The transversal faces of the tube are the faces where the two adjacent flux lines are connected to the boundary of a material transition. The paths of the flux lines are a priori not known and even if they were known, (6.4) can generally not be evaluated in that case. However, if the potential distribution along the transversal faces of a tube is known, the flux through the transversal planes can be calculated. To accurately calculate the permeance the actual potential distribution is not of interest, but only the potential difference, or mmf drop, between the transversal planes. The potential drop can artificially be imposed, and (6.1) has to be evaluated after the flux is calculated. The transversal planes are assumed to be equipotential planes for each flux tube.

uncertainties, nevertheless, can be reduced if more information could be obtained of how the flow actually would be, based on (6.1) rather than (6.4).

6.3 Airgap permeances calculations via the potential distribution

Fig. 6.5: Division of the contours of the airgap in the benchmark topology into 42 line segments.
The generic procedure for the correct calculation of the airgap permeances is illustrated by means of an example. The airgap of the benchmark topology is considered. The first step in the procedure is to divide the boundaries, or contours, of the airgap into smaller line segments as portrayed in Fig. 6.5 for the benchmark problem. These line segments eventually form the equipotential transversal planes of the flux tube. The total number of indexed line segments equals in this case 42, but a different number is possible for the same problem depending on the desired extent of discretization. In the second step the line segments that form a noncontinuous or nonperiodic boundary have to be assigned a constant potential in order to calculate the potential and flux distribution within the domain, from which the permeance value of the flux tube between the line segments can be calculated. To find the permeance between the $k$th and $m$th line segment the potential of the $k$th segment is set to one, and all the other noncontinuous or nonperiodic boundaries are assigned the value zero. This particular, element-wise, potential distribution along the boundaries of the airgap can mathematically be expressed as

$$\varphi_n = \delta_{nk},$$  \hspace{1cm} (6.5)

where $\varphi_n$ is the assigned scalar potential value of the line segment with index number $n$, $\delta_{nk}$ the Kronecker delta, and $k$ the line segment with the assigned unity potential value. It has to be noted that additional boundary conditions, dictated by the nature of the problem, or simply to reduce the size of the model, can be taken into account. The periodicity is allowed for by requiring $\varphi_{10} = \varphi_{23}$. Line segments 11 to 22 are continuous boundaries. Next, the Laplace equation (2.11) subject to the boundary conditions of (6.5) has to be solved to obtain the potential and flux distribution within the domain. The flux that leaves the domain through the $m$th line segment has to be determined. The flux through the $m$th line segment can only originate from the imposed potential at the $k$th line segment, because the potentials on all other line segments are equal to zero. Therefore, the flux through the flux tube between line segments $k$ and $m$ equals the calculated flux through line segments $m$. Finally, with the potential drop and flux distribution known (6.1) can be evaluated. Through the convenient choice of the unity scalar potential drop over the $k$th line segment with respect to the others, the permeance value of the flux tubes between the $k$th and $m$th segment is equal to the total amount of flux entering the $m$th line segment

$$P_{k,m} = \iint_{S_m} \vec{B}_{m,k} \cdot \vec{n} \, dA,$$ \hspace{1cm} (6.6)

where $P_{k,m}$ is the permeance value of the flux tube between line segments $k$ and $m$, $S_m$ the surface of the $m$th line segment, $\vec{B}_{m,k}$ the flux density vector on the surface $S_m$ originating from a uniform potential distribution on $k$, and $\vec{n}$ the outward normal vector to $S_m$.

In principle, any technique, such as the ones described in the previous chapters if applicable, can be applied to solve the potential and flux distribution and a
proper choice is at the user’s discretion. Analytical tools are preferable for solving the boundary value problem, because of their speed and accuracy. However, for more complex airgap shapes analytical techniques may not provide a solution, which forces one to resort to numerical techniques. Based on the foregoing, it has to be noted that for the calculation of the permeances, only the potential and flux distribution on the boundary of the domain are of interest. It is, therefore, unnecessary to calculate the complete potential distribution in the interior of the domain when the solution on the boundary of the domain suffices. The BEM is a flexible and accurate tool for linear problems where the solution on the boundary only is of the essence. Furthermore, the BEM provides the solution for both the potential and flux distribution on the boundary, once the set of equations is solved. How to apply the BEM for the calculation of the potential and flux distribution is the topic of the next section.

6.4 Airgap permeances calculations via the BEM

In chapter 5 the BEM has thoroughly been discussed and has successfully been applied to a scalar potential problem for the benchmark topology. For the calculation of the airgap permeances a similar problem is faced with different initial boundary values. In fact, for the same mesh the $G$ and $F$ matrices are identical to those in chapter 5 for the benchmark problem, since their entries only depend on the geometry. The source vectors, however, are different due the different source distribution on the boundary in accordance with section 6.3. Furthermore, no charge density is present on any of the boundaries for the calculation of the permeances. Solving the new boundary value problem is analogous with the procedure presented in chapter 5.

Once the field distribution on the boundary is found, (6.6) has to be evaluated to determine the permeance value. For the zero-order BEM formulation (6.6) reduces to

$$P_{k,m} = -\mu_0 \mu_r \frac{d_z}{N_m} \sum_{n} H_{mn} l_{mn}, \quad (6.7)$$

where $d_z$ is the depth of the domain, $H_{mn}$ is the field strength on the $n^{th}$ boundary element of the $m^{th}$ line segment as a result of solving (5.17), $l_{mn}$ is the length of the $n^{th}$ boundary element of the $m^{th}$ line segment, and $N_m$ the number of boundary elements the $m^{th}$ line segment is divided into. It is observed that for a single BEM simulation, (6.7) can be evaluated for all permeance values between the $k^{th}$ line segment and all the other line segments.

The procedure of solving the boundary value problem and evaluation of (6.7) has to be repeated to obtain all the values of the flux tubes in the airgap and also to obtain the displacement dependency. A straightforward implementation may, therefore, lead to a very slow model due to the considerable number of
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required BEM simulations. However, the computational effort can be reduced if the following issues are considered.

Since the permeances as a function of relative displacement are smooth functions, it is not necessary to individually calculate the permeances for each new position. It is computationally less demanding to calculate the position dependent permeances in advance for a few fixed positions before the MEC network is solved. Through interpolation the values of the permeances can be found when the MEC matrix is formed. Furthermore, permeances between line segments can exhibit symmetry or can even be identical, but for a translation with respect to the relative displacement. In that case, the calculation of one displacement dependent permeance profile suffices. The translation or symmetry can afterwards be taken into account at the interpolation stage. Moreover, not all flux tubes contribute significantly to the electromagnetic behavior of the problem. The permeance values of these tubes can be assigned zero value. As mentioned earlier, the entries of the $G$ and $F$ matrices are solely determined by the geometry. If no relative displacement occurs between boundary elements, their respective entries in the matrices remain equal. Therefore, for the displacement dependent permeance calculation only the entries for which the corresponding elements are displaced need to be recalculated. Meaning that the entries for all stationary boundary elements with respect to other stationary elements are the same if the mesh on the stationary boundaries does not change with displacement. The same holds for the line segments on the moving parts.

6.5 Solving the MEC network

In order to find the potential and flux distribution inside the equivalent network, the network matrix has to be formed and the matrix equation solved. The network is governed by Kirchhoff’s laws. In general, a network can either be solved by applying Kirchhoff’s voltage or current law. Although, Kirchhoff’s voltage law yields the fewest number of equations, it can only be applied to flat 2D networks. Therefore, Kirchhoff’s current law is applied, because it is more flexible. First, the nodes in the network are numbered. For each node Kirchhoff’s current law is evaluated to obtain a set of linear equations. The matrix equation is given by

$$
\mathbf{P} \vec{u} = \vec{\phi},
$$

(6.8)

where $\mathbf{P}$ is the admittance matrix and $\vec{u}$ and $\vec{\phi}$ are the potential and flux vectors, respectively. The admittance matrix consists of four smaller matrices

$$
\mathbf{P} = \begin{bmatrix}
\mathbf{Y} & -\mathbf{S}^T \\
\mathbf{S} & \mathbf{0}
\end{bmatrix},
$$

(6.9)
where $Y$ is a square, symmetrical, $N \times N$ matrix containing all the permeance values between the nodes. The nondiagonal entries of $Y$ are given by

$$Y_{n,m} = Y_{m,n} = P_{n,m} \quad \text{for } n \neq m,$$

(6.10)

where $P_{n,m}$ is the value of the permeance between nodes with indices $n$ and $m$. The values on the diagonal are found through

$$Y_{n,n} = -\sum_{m=1}^{N} P_{n,m},$$

(6.11)

where $N$ is the total number of nodes in the network. If the element between two nodes is a source the value for $Y_{n,m} = 0$. $S$ is the super-node matrix that allows mmf sources to be modeled with Kirchhoff’s current law. The number of rows of $S$ is determined by the number of mmf sources. If the network contains $N_{\text{mmf}}$ mmf sources, $S$ has size $N_{\text{mmf}} \times N$. For the $k^{\text{th}}$ mmf source between nodes $n$ and $m$ the entries on the $k^{\text{th}}$ row of the super-node matrix, $S$, become

$$S_{k,q} = \begin{cases} 
-1 & \text{for } q = n, \\
1 & \text{for } q = m, \\
0 & \text{elsewhere.}
\end{cases}$$

(6.12)

In the absence of any flux sources in the network the flux vector in (6.8) is given by

$$\vec{\phi} = \begin{bmatrix} \vec{0} & \vec{F} \end{bmatrix}^T,$$

(6.13)

where $\vec{0}$ is the zero row vector of length $N$, and $\vec{F}$ the row vector with $N_{\text{mmf}}$ elements. The $k^{\text{th}}$ element of $\vec{F}$ is given by the value of the $k^{\text{th}}$ mmf source. The potential vector, $\vec{u}$, is obtained by matrix inversion. The potential vector gives the potential value at the nodes of the network. The flux through a permeance between nodes $n$ and $m$ is simply found by

$$\phi_{nm} = (u_n - u_m) P_{n,m}.$$ 

(6.14)

### 6.6 The TCM applied to the benchmark topology

The TCM is applied to the benchmark topology of Fig. 2.6. The MEC is shown in Fig. 6.6. The soft-magnetic and permanent magnet flux tubes are shown in black and the nodes are numbered. The values of the soft-magnetic and permanent magnet flux tubes can be calculated by applying (6.4) for tubes of rectangular or trapezoidal shape. The airgap permeances are not shown for the sake of clarity, but they connect the open nodes on the magnets and the teeth as illustratively indicated by the gray permeance between nodes 6 and 26. The calculated displacement dependent permeance profile for three flux tubes that connect the magnets
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Fig. 6.6: Magnetic equivalent circuit for the benchmark topology.

with the teeth are depicted in Fig. 6.7. The value of $\mathcal{P}_{3,22}$ and $\mathcal{P}_{3,23}$ can respectively be obtained by mirroring $\mathcal{P}_{3,13}$ and $\mathcal{P}_{3,14}$ with respect to the ordinate. The permeances between the other magnets and teeth are obtained by translation of the curves in Fig. 6.7. It has to be noted that the displacement, $\delta_x$, is not the displacement of the translator relative to the stator, but the misalignment of a magnet with respect to a tooth. At the aligned position, $\delta_x = 0$ mm, the center of the magnet is directly located underneath the center of a tooth. The leakage permeances in between the magnets and in between the slots have been taken into account as well, but their profiles are not shown. The source terms with respect to (2.12), (6.3), and Tables 2.1 and 2.2 are given by

$$ F_m = \frac{B_{\text{rem}}}{\mu_0 \mu_{\text{rel}}} (h_2 - h_1), \quad (6.15a) $$

$$ F_A = \frac{\alpha_{\mu_0} \tau_0}{4} J_A (h_4 - h_3), \quad (6.15b) $$

$$ F_B = \frac{\alpha_{\mu_0} \tau_0}{4} J_B (h_4 - h_3), \quad (6.15c) $$

$$ F_C = \frac{\alpha_{\mu_0} \tau_0}{4} J_C (h_4 - h_3), \quad (6.15d) $$

After the matrix equation has been formed and solved, the potential distribution on the nodes and the flux through the flux tubes is known. Contrary to the HM, SCM, and BEM the flux density distribution in the airgap cannot be calculated even with the flux distribution known. Although the value of each flux tube is known, no information is available on its geometry when calculated through the BEM. Hence, the flux density distribution within a tube cannot be retrieved. Instead, the flux linkages of the phase coils are calculated. The flux linkages of phase coils A, B and C are found by calculating the flux through the mmf-sources between nodes 17 and 18, nodes 28 and 29, and nodes 39 and 40, respectively.
The results for the flux linkages as a function of the relative displacement between the mover and stator compared to FEM simulations are shown in Fig. 6.8. A reasonably good agreement between the results is obtained for the shape and amplitude of the wave forms. The discrepancies, $\epsilon_\delta$, between the rms-values of the flux linkages obtained by the FEM and the TCM are 3.9%, 4.5%, and 3.7% for phase A, B, and C, respectively.

The effect of increasing the number mesh elements in the BEM for the calculation for the airgap permeances on the discrepancy between the flux linkages obtained by FEM and TCM is visualized in Fig. 6.9a. By increasing the number of mesh elements in the BEM the airgap permeance calculations become more accurate. Furthermore, a denser mesh will result in smoother permeances as a function of misalignment. The flux linkage value of phase B converges toward $\lambda = 230\mu$Wb for $N_{\text{BEM}} \geq 300$. Hence, increasing $N_{\text{BEM}}$ will not result in an improved accuracy.
Fig. 6.9: Absolute (a) and relative (b) comparison of the rms-values of the flux linkage of phase B obtained by the TCM as a function of the number of BEM elements compared to FEM.

as can be seen by the discrepancy curve of Fig. 6.9b. Evidently, the calculation time of the airgap permeance is equivalent to Fig. 5.12. However, multiple BEM simulations are required for the calculation of the different airgap permeances. Therefore, for a fixed value of $N_{BEM}$ the calculation time of all the permeance values for one relative position of the stator with respect to the translator is an integer multiple of the calculation time corresponding to the value of $N_{BEM}$ in Fig. 5.12.

6.7 Additional remarks

There is a phase shift visible between the flux linkage in Fig. 6.8 obtained by the FEM and the TCM. This phase shift originates from the assumption that flux lines enter the top side of the permanent magnet perpendicularly due to the assumed equipotential nature of the transversal faces of the flux tubes. Especially for the flux of the phase coils this assumption is erroneous. As a result, the armature reaction flux cannot be determined adequately, since its flux lines do not necessarily cross the surface of the magnet perpendicularly. This is evident from the calculation of the discrepancy in case there is no current in the phase coils ($J_{ph} = 0 \text{ A m}^{-2}$). In that case, the phase shift and armature field are zero and the discrepancies decrease to 2.2%, 2.9%, and 2.1%, respectively. However, by comparing the rms-values of the flux linkages errors that are solely ascribed to the phase shift are not reflected in the values of the discrepancy. Therefore, increases in the discrepancy are only on account of the armature reaction field being erroneously calculated.

The calculation time can be split in two parts, i.e. the calculation time of the displacement dependent permeance calculation by means of the BEM and the
time for the solving of the MEC. The calculation time of a permeance profile equals 3.3 s and the solving of the MEC circuit, including the interpolation of the permeance values, is equal to 92 ms for each step. The calculation time of the permeance profiles as depicted in Fig. 6.7 depends on the number of profiles, the number of desired data points for a profile, and the number of mesh elements in the BEM model. The accuracy with which the permeance profile can be determined is not solely dependent on the number of mesh elements, but also on the mesh distribution. In general, a denser mesh is required on the boundary where the scalar potential distribution is concentrated. The calculation of the permeance profiles of Fig. 6.7 has been conducted for a BEM model with 488 mesh elements and with a spatial interval between the data points on the abscissa of 0.5 mm. Evidently, the overall accuracy and calculation time are also affected by the choice of the layout of the MEC model.

The MEC method hinges on the assumptions of flux being uniformly distributed within the flux tubes. A more accurate field solution, therefore, can only be obtained by a finer division into flux tubes of the problem [2]. Due to the uniform flux distribution local saturation effects are difficult to model. Furthermore, for nonlinear problems with a large number of nonlinear permeances convergence problems might arise for the iterative algorithm. An additional error is caused by the fact that the material transition coincides with transversal equipotential plane of a flux tube. Therefore, the flux is always perpendicular to the planes of the material transition, even when the permeability values of the two materials are in the same order of magnitude. This effect is especially apparent for machines with surface mounted magnets. In this case, a finer division of the MEC will not ameliorate the accuracy of the method. The method is better fit for problems with iron boundaries on both sides of the airgap, such as reluctance, flux switching, and interior permanent magnet machines. The major drawback of the TCM is the computationally demanding BEM method repeatedly being applied for the airgap permeance calculation. A faster nonlinear TCM model is obtained when the MEC is applied in conjunction with the HM for the permeance calculations. However, restrictions are imposed on the shape of the airgap of the problem as elucidated in chapter 3. In other cases, it might be faster to directly implement the nonlinear problem in the FEM.

6.8 Conclusions

The TCM is a MEC based hybrid modeling technique for which the accuracy is increased by means of an alternative calculation of the airgap permeances based on scalar potential calculations. In this chapter the BEM for scalar potential formulation has been applied to determine the airgap permeances instead of calculations based on estimated, geometrically simplified flux tube shapes in the airgap. The technique does not impose any limitations in terms of the geometry in order to be applicable. The TCM provides information on the field distribution
inside the soft-magnetic material. Therefore, the technique can be extended to incorporate nonlinear materials. In that case, the matrix equation of the network has to be solved iteratively by means of a solving algorithm for nonlinear equations. If applicable, the number of additional simulations for the calculation of the airgap permeances can be minimized by taking symmetry or repetitiveness of the problem into account. Errors in the field distribution calculation are introduced due to the equipotential nature of the transversal faces of the flux tubes and, furthermore, are dependent on the fineness of the MEC network and mesh distribution of the BEM model. The equipotential nature of the transversal faces makes the technique less useful in case permanent magnets form the boundary on one side of the airgap. For topologies with soft-magnetic material on both side of the airgap the TCM is well suited.

The TCM has been applied to the benchmark problem. The flux linkages of the phase coils have been calculated and compared with FEM results. The discrepancy between the rms-value of the flux linkage profiles obtained by the TCM and the FEM are in the range of 3.7% to 4.5%. The total calculation time of the permeance profiles is equal to 3.3s and the solving of the MEC matrix takes 92ms for a single relative displacement between the translator and the stator. Only a global field distribution is available for the TCM; unlike the HM, SCM, or BEM, by means of which the local electromagnetic field distribution within the domain can accurately be determined.
Chapter 7

Force calculations

In the foregoing chapters it is shown how to obtain the magnetic flux density field distribution or flux linkages by means of different modeling techniques. For electromechanical applications the quantity of merit is the force or torque exerted on the moving part of a device. The actual conversion from electromagnetic energy to mechanical energy, originating from the forces acting on the electrically charged particles due to the electromagnetic field, can be calculated in different ways. This chapter deals with commonly applied methods for force calculations from the results of the calculated magnetic field. Three methods are discussed in brief, i.e. Lorentz force method, Maxwell stress tensor (MST) method and virtual work (VW) method. The applied modeling method for the determination of the magnetic field stipulates which method is applicable. For each modeling technique presented in the previous chapters the force on the benchmark topology is determined by the method most suited for the modeling method in question. Furthermore, the effect of increasing the accuracy in regard to flux density on the force profile calculations is addressed. Finally, the presented modeling methods of the previous chapters are compared.

7.1 Lorentz force method

When the exact, local magnetic flux density distribution within a current carrying conductor is known, the Lorentz force method can be applied for the calculation of the forces acting on the conductors [26]. An electrically charged particle moving through an electromagnetic field experiences the Lorentz force according to

\[ \mathbf{F} = q \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right), \]

(7.1)
where \( \vec{F} \) is the force vector, \( q \) the charge, and \( \vec{v} \) the velocity vector of the particle. For magnetostatic problems the electric field component, \( \vec{E} \), vanishes. A current carrying conductor can be considered as a flow of a charge density distribution contained within the confines of the conductor. For the magnetostatic case the volume force density vector is given by

\[
\vec{f}_V = \vec{J} \times \vec{B}, \tag{7.2}
\]

where the volume force density vector in \( \text{N m}^{-3} \) is denoted by \( \vec{f}_V \) and \( \vec{J} \) is the current density vector in \( \text{A m}^{-2} \). The force on the conductor is found by integration of (7.2) over the volume, \( V_c \), of the conductor

\[
\vec{F} = \oint_{V_c} \vec{J} \times \vec{B} \, dV. \tag{7.3}
\]

The Lorentz force method is particularly useful for open problems without iron boundaries, or for problems with iron boundaries that can be represented as such by imaging. It is also applicable to calculate the forces on permanent magnets. However, the permanent magnets have to be expressed as an equivalent current density distribution in order to evaluate (7.3) over the volume of the equivalent current density distribution. This implies that a unity relative permeability for the magnets has to be assumed.

### 7.2 Maxwell stress tensor method

The MST method is a more general method for the calculation of the forces acting on a body when the exact, local field distribution of the problem is known. The MST equation for magnetostatics is obtained by substitution of Ampère’s law for the magnetostatic case, (2.7a), into the Lorentz volume force density equation of (7.2) [42]

\[
\vec{f}_V = -\frac{1}{\mu_0} (\nabla \times \vec{B}) \times \vec{B}. \tag{7.4}
\]

By applying the proper vector identity (7.4) can be written as

\[
\vec{f}_V = \frac{1}{\mu_0} \left[ (\vec{B} \cdot \nabla) \vec{B} - \frac{1}{2} \nabla (|\vec{B}|^2) \right]. \tag{7.5}
\]

The result of (7.5) can more conveniently be expressed in tensor form by

\[
\vec{f}_V = \frac{1}{\mu_0} \nabla \cdot \mathbb{T}, \tag{7.6}
\]

where \( \mathbb{T} \) is the \( 3 \times 3 \), second order MST of which the entries are given by

\[
T_{i,j} = B_i B_j - \frac{1}{2} |\vec{B}|^2 \delta_{ij}, \tag{7.7}
\]
with $\delta_{ij}$ as the Kronecker delta function and the indices, $i$ and $j$, denote the directions parallel to the axes of the coordinate system. The total force on a body can be found by integration over the volume of the body, similar to (7.3). By applying the divergence theorem the volume integral reduces to a surface integral

$$
\vec{F} = \frac{1}{\mu_0} \oint_S \mathbf{T} \cdot \vec{n} \, dS, \quad (7.8)
$$

where $S$ is the surface enclosing the body and $\vec{n}$ the normal vector of $S$. The outcome of the integral is independent of the integration path of the surface enclosing the body. Hence, the surface of integration enclosing the body can be chosen arbitrarily, allowing the evaluation of (7.8) to be simplified when choosing the surface appropriately.

### 7.3 Virtual work method

An alternative way of force calculations is provided by the method of virtual work (VW). The VW method is based on the electromagnetic energy difference of the complete problem due to a relative movement of components [24]. The calculation of the electromagnetic energy in the system does not necessarily require the field distribution to be known locally, as is required by the Lorentz and MST method. This makes the VW method more generally applicable.

The VW method is based on the energy balance between the electric, electromagnetic and kinetic energy in a lossless system:

$$
dW_{el} = dW_{em} + dW_k \Rightarrow 
\dot{i}d\lambda = dW_{em} + F \, dx, \quad (7.9)
$$

where $W$ is the work and the subscripts $el$, $em$ and $k$ denote the electric, electromagnetic and kinetic work, respectively. The current, infinitesimal flux linkage, force and infinitesimal displacement are denoted by $i$, $d\lambda$, $F$ and $dx$, respectively. The electromagnetic field is considered to be dependent on the mutually independent variables $\lambda$ and $x$: $W_{em}(\lambda, x)$. The differential of the electromagnetic energy function is then given by

$$
dW_{em} = \frac{\partial W_{em}}{\partial \lambda} \left. \frac{\partial \lambda}{dx} \right|_{x=\text{constant}} + \frac{\partial W_{em}}{\partial x} \left. \frac{\partial x}{\partial \lambda} \right|_{\lambda=\text{constant}}, \quad (7.10)
$$

For constant $\lambda$ the infinitesimal flux linkage vanishes in (7.9) and (7.10), i.e. $\partial \lambda = 0$. Hence,

$$
F = - \left. \frac{dW_{em}}{dx} \right|_{\lambda=\text{constant}}. \quad (7.11)
$$
Chapter 7: Force calculations

For most practical electromechanical devices it is hard to guarantee a constant flux linkage when moving from one position to another. It is more convenient to keep the current constant instead of the flux linkage as the position changes. To that end, the electromagnetic co-energy, $W_{\text{em}}'$, is introduced. The relation between energy and co-energy is given by

$$W_{\text{em}} = i\lambda - W_{\text{em}}'. \quad (7.12)$$

Substitution of (7.12) into (7.9) and subsequently applying the product rule for differentiation yields the energy balance in terms of the co-energy

$$\lambda di = -dW_{\text{em}}' + Fdx. \quad (7.13)$$

Analogous to the analysis for the energy balance the expression for the force in terms of the co-energy becomes

$$F = \frac{dW_{\text{em}}'}{dx} \bigg|_{i=\text{constant}}. \quad (7.14)$$

The electromagnetic energy and co-energy in the system are respectively given by

$$W_{\text{em}} = \int_V \left[ \int_0^B \vec{H} \cdot d\vec{B} \right] dV, \quad (7.15a)$$

$$W_{\text{em}}' = \int_V \left[ \int_{H_c}^H \vec{B} \cdot d\vec{H} \right] dV, \quad (7.15b)$$

where $V$ is the volume of the problem, and $H_c$ the coercivity of the material. For linear material with constant permeability these expressions reduce to

$$W_{\text{em}} = \frac{1}{2\mu_0\mu_r} \int_V |\vec{B}|^2 dV, \quad (7.16a)$$

$$W_{\text{em}}' = \frac{\mu_0\mu_r}{2} \int_V |\vec{H}|^2 dV. \quad (7.16b)$$

It has to be observed that for linear materials the expressions for the energy and co-energy are equal: $W_{\text{em}} = W_{\text{em}}'$. To numerically find the force at position $x_F$, the energy or co-energy of the problem has to be calculated at positions $x_F \pm \Delta x$. The force in the direction of displacement is then respectively found by evaluating

$$F(x_F) = \frac{W_{\text{em}}(x_F - \Delta x) - W_{\text{em}}(x_F + \Delta x)}{2\Delta x}, \quad (7.17a)$$

$$F(x_F) = \frac{W_{\text{em}}'(x_F + \Delta x) - W_{\text{em}}'(x_F - \Delta x)}{2\Delta x}, \quad (7.17b)$$

while the current or flux is kept constant for both positions, respectively.
7.4 Forces on the benchmark topology

In the previous chapters the magnetic field distributions of the benchmark topology were determined in different ways. For the HM, SCM, and TCM the force profiles as a function of displacement on the benchmark topology are calculated. The results are compared to FEM simulations.

7.4.1 Force calculation for the HM

The MST method is applied to the magnetic field solution obtained by the HM. The mover has to be enclosed by a surface over which (7.8) can be evaluated. For 2D problems the surface reduces to a closed contour enclosing the mover. The chosen contour is a rectangle with the edges parallel to the ordinate located at \( x = 0 \) and \( x = l_2 \) in regard to Fig. 3.1. The edges parallel to the abscissa are located at \( y = h \) and \( y \to \infty \), where \( h_2 \leq h \leq h_3 \). On account of the periodic boundaries parallel to the ordinate and the flux density approaching zero at \( y \to \infty \), it suffices to evaluate the MST integral along the edge at \( y = h \) parallel to the abscissa in the airgap. The outward normal of that line is parallel to the ordinate (\( \vec{n} = -\hat{e}_y \)). With this, the MST integral for the force vector component in the direction of displacement with respect to (3.8) and (3.9) becomes

\[
F_x = -\frac{d}{\mu_0} \int_0 \frac{N_{mp} \tau_p}{\mu_0} B_y^{(2)}(x, h) B_y^{(2)}(x, h) \, dx. \tag{7.18}
\]

The integral of (7.18) can be evaluated analytically

\[
F_x = \frac{dN_{mp} \tau_p}{\mu_0} \sum_{n=1}^{\infty} \left[ \frac{b_n^{(2)} c_n^{(2)} - a_n^{(2)} d_n^{(2)}}{e^{(h_2 - h_3) w_n^{(2)}}} \right]. \tag{7.19}
\]

It can be seen that the expression of (7.19) is indeed independent of \( h \). The force calculations also could have been found by means of the VW method. However, the VW method requires surface integrals to be evaluated twice for all domains for each position. Therefore, the VW method is computationally more expensive than the MST method.

7.4.2 Force calculation for the SCM

When the magnetic field is calculated through the SCM by means of equivalent point currents, it seems obvious to calculate the force by means of the Lorentz force method. However, due to the singularities in the magnetic field that originate from the point sources themselves, the Lorentz force cannot be calculated directly. To find the force on a point current the field distribution due to all
other point currents, including the imaged ones, has to be calculated excluding the field distribution due to the point source where the force is calculated on. Each force calculation, therefore, requires a new field calculation. To also avoid domain integration the MST method is applied in lieu of the VW method for the calculation of the force profile of the benchmark topology. The procedure for the MST, as discussed for the HM, still applies. Equation (7.18), however, has to be evaluated numerically. The integrands of (7.18) at height $y = h$ are obtained as described in chapter 4.

### 7.4.3 Force calculation for the TCM

The field distribution obtained by the TCM is only globally known. Therefore, the VW method is applied for the force calculations. The energy or co-energy in a flux tube is obtained by applying (7.16) to the volume of the flux tube. Since the flux inside the flux tube is considered to be uniformly distributed, the energy and co-energy in the $n^{th}$ tube are given by

$$W_{em,n} = \frac{1}{2} \phi_n^2 R_n,$$

(7.20a)

$$W'_{em,n} = \frac{1}{2} (\Delta F_n)^2 P_n,$$

(7.20b)

where $\phi_n$ and $\Delta F_n$ denote the flux through and mmf-drop over the $n^{th}$ flux tube with reluctance or permeance value $R_n$ or $P_n$, respectively. Since the problem is linear, either equation can be applied for the calculation of the co-energy in a flux tube. The energy of the complete system is found by summation of the co-energy stored in all permeances of the MEC

$$W'_{em} = \sum_n W'_{em,n},$$

(7.21)

### 7.5 Force results for different methods

The results for the different force profile calculations compared to FEM simulations are shown in Fig. 7.1. From Fig. 7.1 it can be concluded that all methods provide a good estimate of the average thrust force. It can, however, clearly be observed that the shapes of the profiles differ for each modeling technique. The force profiles for the HM and the SCM are in good agreement with the FEM simulation, whereas the discrepancy of the force profile of the TCM in terms of amplitude is more apparent. Furthermore, a slight phase shift of the TCM force profile compared to the others is visible. The phase shift is caused by the error in the magnetic field distribution of the phase coils for the reasons discussed in section 6.6. The overall error being larger for the TCM is explained by the fact
that the force calculation is based on a coarse global magnetic field distribution instead of a detailed local one, as is the case for the HM and the SCM. A very fine flux tube discretization allows the MST method to be used for MEC as presented in [1]. However, the flux tube discretization inclines to adopt the appearance of a fine FDM or even FEM mesh yielding a large set of equations. Furthermore, the assumption of a unidirectional flux flow within the tubes and the assumed equipotential planes at the transversal faces of the flux tubes remain a source of error.

In the previous chapters the gain in accuracy of the calculated magnetic field quantities at the expense of an increased calculation time has been mapped. Next, the influence of increasing the accuracy of the magnetic field quantities on the force calculations is addressed. For each method the average thrust force, $\bar{F}$, and the peak-to-peak force ripple, $\Delta F$, are calculated. The force results for the HM, SCM, and TCM are shown in Figs. 7.2, 7.3, and 7.4, respectively. For comparison, the number of harmonics, $N_n$, for the HM, the number of equivalent point sources, $N_m$, for the SCM, and the number of mesh elements, $N_{BEM}$, for the permeance calculations for the TCM are determined for which the relative discrepancy in the average thrust force and the force ripple is within $\pm 5\%$ or $\pm 10\%$ of the force results obtained by the FEM. The ranges which correspond to the $\pm 5\%$ or $\pm 10\%$ discrepancy with respect to the average force and force ripple are indicated by the gray curves in Figs. 7.2, 7.3, and 7.4.

For the HM the discrepancy with the FEM is within a $\pm 5\%$ margin when the number of harmonics are $N_n \geq 3$ and $N_n \geq 22$ for the average thrust force and force ripple, respectively. Their respective calculation times equal $t \geq 1.3\text{ ms}$ and $t \geq 7.7\text{ ms}$. The SCM requires $N_m \geq 6$ for the number of equivalent point sources with an accompanying simulation time of $t \geq 0.48\text{ s}$ to obtain an average thrust force that is within the $\pm 5\%$ margin. For the force ripple calculation the $\pm 5\%$
margin is not reached for the number of point sources considered. A margin of ±10% on the discrepancy of the force ripple is attained for $N_m \geq 60$ with $t \geq 1.2$ s. The calculation time for the mapping function ($t = 1.3$ s) has been excluded. Finally, for the TCM the number of BEM elements equal $N_{BEM} \geq 164$ to satisfy the ±5% margin criterion on the average force, whereas a margin of even ±10% for the force ripple is not met altogether. However, increasing the number of BEM elements further than $N_{BEM} \geq 500$ will not improve the margin on the force ripple and it is converged to $\Delta F = 12.8$ N which corresponds to a discrepancy of 16.1%. The calculation time of the permeance profiles equals $t = 1.3$ s for $N_{BEM} = 164$, and $t = 3.3$ s for $N_{BEM} = 500$.

The performance of the modeling techniques in terms of calculation time and discrepancy with FEM are summarized in Table 7.1. The defining quantity of each method is indicated in bold in Table 7.1. The defining quantity is the discrepancy

---

**Fig. 7.2:** The average thrust force (a), and the force ripple (b) as a function of the number of harmonics of the HM.

**Fig. 7.3:** The average thrust force (a), and the force ripple (b) as a function of the number of equivalent point sources of the SCM.
that requires the most computational effort to converge to its minimal value. For the HM, SCM, and TCM the defining quantity proves to be the margin on the discrepancy of the force ripple calculation. Since the BEM formulation as considered in chapter 5 is only being applied for permeance calculations, the calculation of forces would have no physical meaning as mentioned in section 5.9. Hence, no force calculations have been conducted on the field quantities of the BEM results. The defining quantity of the BEM is, therefore, the flux density component in the \( y \)-direction. The other values in the columns of Table 7.1 correspond to the calculation associated with the computational effort required to obtain the value of the defining quantity. It has to be observed that the calculation time is split into two parts; the preparatory calculation time indicated by ‘set-up’ in Table 7.1, and the calculation time per relative position of the translator with respect to the stator of the benchmark topology indicated by ‘step’ in Table 7.1. Only the SCM and the TCM require preparatory calculations that contribute significantly to the total calculation time. From Table 7.1 it is concluded that the HM performs the best both in terms of discrepancy on the field and force calculation, and calculation time.

### 7.6 Conclusions

Several modeling techniques have been presented in part I of this thesis. Furthermore, methods for force calculations from the calculated magnetic field distribution have been addressed. The methods have been explained and verified by means of magnetic field and force calculations on an electromagnetic benchmark problem. The modeling of the same benchmark problem by means of the different techniques enables a comparative analysis between the methods. The results of the methods have been compared to FEM simulations on the bench-
Chapter 7: Force calculations

Table 7.1: Performances of modeling methods in terms of calculation time and discrepancy with respect to field quantities and forces.

<table>
<thead>
<tr>
<th>$\epsilon_\delta$ [%]</th>
<th>HM</th>
<th>SCM</th>
<th>BEM</th>
<th>TCM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_x$</td>
<td>0.2</td>
<td>2.2</td>
<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td>$B_y$</td>
<td>0.2</td>
<td>1.6</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda_B$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.4</td>
</tr>
<tr>
<td>$\bar{F}$</td>
<td>1.9</td>
<td>2.0</td>
<td>-</td>
<td>0.3</td>
</tr>
<tr>
<td>$\Delta F$</td>
<td>5.0</td>
<td>10.0</td>
<td>-</td>
<td>16.1</td>
</tr>
<tr>
<td>$t$ [s] set-up</td>
<td>0.0</td>
<td>1.3</td>
<td>0.0</td>
<td>3.3</td>
</tr>
<tr>
<td>$t$ [s] step</td>
<td>$7.7 \cdot 10^{-3}$</td>
<td>1.2</td>
<td>$8.7 \cdot 10^{-2}$</td>
<td>$9.2 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

The HM has proven to be the fastest and most accurate method for determining the flux density distribution inside the airgap and the force profile of the benchmark problem. The TCM performs the worst in terms of magnetic field calculations, force calculations, and simulation time. The SCM is faster and more accurate than the TCM, but significantly slower and more inaccurate than the HM. Furthermore, the major drawback of the SCM and TCM are the requirement of additional, preparatory simulations to determine the mapping function and the permeance profiles, respectively. No force calculations have been conducted on the field result obtained by BEM, because it has only been implemented to solve the Laplace equation for scalar potential problems that cannot handle current regions (coils).
Part II

Linear actuation with integrated contactless energy transfer
Chapter 8

Contactless energy transfer in electrical machines

The concept of CET through inductive coupling in conjunction with electrical machines has been known and researched for many years. A patent [36] was filled as early as 1917 in which a mechanical rectifying device is presented, composed of a gapped three-phase rotating transformer in conjunction with a synchronous machine with a wound rotor. The use of a CET solution obviates the limitations inherent to the use of slip rings and brushes, such as wear, friction, reliability, speed, and sparking. The use of transformers requires the electricity to be available in ac form. The operating frequency of transformers is predominantly stipulated by the size of the transformer, the applied soft-magnetic core material, and the generated heat loss.

In this chapter the basic principles of electromagnetic CET systems are discussed, and an overview is given of existing technologies of CET systems in general and in particular for electrical machines. A distinction is made between add-on solutions and integrated ones. Add-on solutions are considered systems where the individual magnetic flux paths and components of the transformer and the machine are fully separated, whereas integrated solutions are system where the flux paths of the transformer and machine overlap or coincide, or where some components have a dual functionality. Next, an analysis is conducted on which desired electromagnetic requirements ought to be met in order for an integrated solution to be deemed feasible. Also the limitations in range of application, dictated by the choice for different soft-magnetic materials, are addressed based on saturation levels and losses. Then, the impact of the integration of CET, in terms of force density as a function of the power to be transferred contactlessly, is quantified by means of simplified motor, transformer, and thermal equations. Finally, three integrated topologies for a linear, permanent magnet, synchronous machine with
8.1 Electromagnetic contactless energy transfer principles

Contactless energy transfer is made possible by the interaction between the electric and electromagnetic field quantities as given by Maxwell’s equations in (2.1). The reciprocity of Maxwell’s equations allows the flow of energy in a system to be bi-directional, under the condition that the electric and field quantities are time-varying. The interaction of the electric quantities and the field quantities originates from the forces exerted on the electric charges present in matter when exposed to an electromagnetic field as given by the Lorentz force of (7.3). Conversely, an external force exerted on a charge, which causes it to move, gives rise to an electromagnetic field. The principles and power transfer capabilities for CET through a capacitive or inductive link are discussed in the following. Furthermore, CET by means of wave propagation is shortly addressed.

8.1.1 Capacitive energy transfer

Electric energy can be wirelessly transferred through the electric field by means of a capacitive coupling [66, 68, 111]. The principles of capacitive energy transfer is based on electrostatic induction as shown in Fig. 8.1a. The accumulated charges in the primary plates of the capacitors repel the charges of equal polarity in the opposing plates on the secondary side. This effect allows the energy to

Fig. 8.1: Capacitive energy transfer by means of the electric field: physical representation (a), and basic resonant network implementation (b).
be transferred through the resulting electric field without mechanical connection. The coupling only works for time-varying currents, because a direct current (dc) would stop flowing as soon as the capacitor is fully charged. A time-varying current changes the polarity of the plates through successive charging and discharged of the plates. An apparent time-varying current, known as the displacement current, passes through the capacitor as a consequence of the effect of electrostatic induction. It can be seen from Fig. 8.1a that not all field lines cross the spacing between the primary and secondary plates. These leakage fields, indicated in gray in Fig. 8.1a, cause the load current on the secondary side to be lower than the source current on the primary side when the energy flow is from the primary to the secondary.

The capacitances of the link form a series-impedance that gives rise to a voltage drop over the wireless link when power is transferred. To compensate for this voltage drop a resonant series inductor, $L_{\text{res}}$, is applied of which the value is tuned to the operating frequency of the system via

$$\omega^2 = \frac{1}{L_{\text{res}}C_{\text{eq}}}$$

where $C_{\text{eq}}$ is the total equivalent capacitance of the link. The basic network configuration is shown in Fig. 8.1b for which $C_{\text{eq}} = \frac{1}{2}C$. The leakage capacitances, $C_{\sigma}$, are shown in gray. The maximum amount of power that can be transferred can be determined by considering the impedance of a capacitor. For sinusoidal currents the magnitude of the voltage drop over a capacitor can be expressed in terms of its impedance or in terms of the amplitude of the sinusoidally varying electric field distribution, $E$. For a parallel plate capacitor the voltage is given by

$$V_C = \frac{I_C}{2\pi f C} = \frac{Ed}{\sqrt{2}}, \quad (8.1)$$

where $V_C$, and $I_C$ are the rms-values of the voltage over and the current through one capacitor, respectively, and $d$ the distance between the plates in m. It has been assumed that the electric field is uniformly distributed and confined to the volume in between the plates. For this simplified situation the capacitance of a parallel plate capacitor is approximated by

$$C \approx \frac{\varepsilon_0 \varepsilon_r A}{d}, \quad (8.2)$$

where $A$ is the surface area of the capacitor plates in m$^2$. The electric field may not exceed the value for which electric discharge occurs through the dielectric disposed in between the capacitor plates. Substitution of (8.2) into (8.1) yields the maximum rms current through the capacitive link. The maximum apparent power transfer is obtained by $S_{\text{cap}} = V_p I_C$ which gives

$$S_{\text{cap}} = \varepsilon_0 \varepsilon_r \pi f A E V_p \sqrt{2}, \quad (8.3)$$

where the value of $E$ is determined by the electric field value at which electric discharge through the dielectric occurs. For a given dielectric the maximum allowable electric field is determined by the breakdown voltage. Therefore, according to (8.3) the current through the capacitors can only be increased further by increasing the operating frequency, relative permittivity of the dielectric, or surface area
Chapter 8: Contactless energy transfer in electrical machines

Fig. 8.2: Inductive energy transfer by means of the magnetic field: physical representation (a), and basic resonant network implementation (b).

8.1.2 Inductive energy transfer

Electric energy can also be transferred contactlessly through the magnetic field by means of an inductive coupling or transformers. The principle of inductive energy transfer is based on Faraday’s law of induction of \((2.1c)\), which states that a time-varying magnetic field induces a time-varying emf in a conductor enclosing the magnetic field. The principle is graphically shown in Fig. 8.2a. The time-varying magnetic flux originating from the primary coil is coupled with the secondary coil. In accordance with Lenz’s law, the coupled flux induces an emf in the secondary coil of which the polarity is such as to oppose the magnetic field when a current as a result of the induced emf is allowed to flow. Leakage fields are indicated in gray in Fig. 8.2a. Leakages cause a voltage drop on the secondary side when the direction of the energy flow is from the primary to the secondary. The ratio between the primary and secondary voltage is proportional to the winding ratio of the primary and secondary coils. In this manner, any desired voltage level can be
obtained on the secondary side. A similar voltage/current transformation is not possible for capacitively coupled CET systems. However, the value of the series inductance, or impedance, could be tuned to exploit the voltage drop over the link to obtain a lower voltage on the secondary side. In this case, the ratio between the primary and secondary voltage is dependent on the secondary current.

The basic network representation is shown in Fig. 8.2b, where the actual secondary impedances, $L_{\sigma s}$ and $R_{\text{load}}$, are referred to the primary side. The referred impedances relate to the actual impedances via $L'_{\sigma s} = L_{\sigma s}N_p^2N_s^{-2}$ and $R'_{\text{load}} = R_{\text{load}}N_p^2N_s^{-2}$, where $N_p$ and $N_s$ are the number of turns of the primary and secondary coil, respectively. The voltage drop over the leakage inductances can be compensated by applying a resonant series capacitor, $C_{\text{res}}$, that is tuned to the operating frequency via $\omega^2 = L_{\sigma}^{-1}C_{\text{res}}^{-1}$. $L_{\sigma}$ is the total leakage inductance of the series connection of the two leakage inductances $L_{\sigma p}$ and $L'_{\sigma s}$. Alternatively, a resonant parallel capacitor, instead of a series one, can be applied as well. A portion of the current on the primary side is not transferred to the secondary, but flows through the magnetization inductance, $L_m$. This so-called magnetization current is required to maintain the magnetic field. A core of soft-magnetic material can be applied to increase flux density levels in the transformer coils and so boost the power density of the system.

A similar analysis as presented in section 8.1.1 for the maximum amount of transferrable power can be conducted for transformers. To that end, the flux inside the core is assumed to be uniformly distributed, and the core material is assumed to be linear. The relation between the primary voltage and the magnetization flux density in the core is determined by Faraday’s law through

$$v_p = -AN_p \frac{dB}{dt}, \quad (8.4)$$

where $B$ is the flux density in the core, $A$ the area of the cross-section of the core through which the transformer flux flows, and $v_p$ the periodic primary voltage. For a purely sinusoidal primary voltage with an rms-value of $V_p$ and frequency $f$ in Hz, (8.4) reduces to

$$V_p = \pi f A \hat{B} N_p \sqrt{2}, \quad (8.5)$$

where $\hat{B}$ is the amplitude of the sinusoidally varying flux density. The apparent power is simply given by $S_{\text{ind}} = V_p I_p$ for a single-phase transformer. With this, (8.5), and (2.5a) the apparent power for sinusoidal electric waveforms can be expressed as

$$S_{\text{ind}} = \mu_0 \mu_r \pi f A \hat{H} N_p I_p \sqrt{2}. \quad (8.6)$$

Equations (8.3) and (8.6) show some parallels. In both cases the power rating can be enhanced by increasing the frequency or the cross-section penetrated by the electric or magnetic field. Although different in nature, the maximum field values are determined by the material properties of the dielectric or soft-magnetic
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Fig. 8.3: Energy transfer by means of electromagnetic wave propagation.

core material. Where the electric field is limited by the breakdown voltage of the
dielectric for capacitive links, the magnetic field level is limited by the saturation
level of the soft-magnetic material, when applied. It has to be observed that
by equating (8.3) and (8.6) the primary voltage of the capacitive link has to be
much higher at given $A$ and $f$ than the primary voltage of an inductive link
on account of $\mu_0\varepsilon_0^{-1} \approx 10^5 \text{ V/m}$. As is the case for dielectrics, the use of soft-
magnetic material gives rise to additional losses which increase with the operating
frequency as is discussed in more detail in section 8.4.2. The value for $I_p$ is
determined by the thermal properties of the transformer and the environment it
is placed in. The choice for the number of primary turns, $N_p$, is determined by
the maximum values of the magnetization current and the magnetization flux.
The magnetization flux as a function of magnetization current can be estimated
by a MEC model for topologies with soft-magnetic cores. Once the permeance
value seen by the magnetization flux is known, the number of primary turns can
be expressed as

$$N_p = \frac{\hat{B}A}{\hat{i}_{\text{mag}}P_{\text{mag}}},$$

(8.7)

where $\hat{i}_{\text{mag}}$ is the peak value of the magnetization current and $P_{\text{mag}}$ is the perme-
ance value seen by the magnetization flux. An overview of different implementa-
tions of inductive CET systems for moving applications is provided in section 8.2.

8.1.3 Wave propagation

Capacitive or inductive CET systems apply either the electric or magnetic field
to contactlessly transfer energy. Instead of either using the magnetic or electric
field for CET, they can both by applied in an electromagnetic wave to transfer
power as shown in Fig. 8.3. The apparent power density contained within an
electromagnetic wave and the direction in which the wave propagates are given
by the Poynting vector

$$\vec{S}_{\text{wav}} = \vec{E} \times \vec{H}.$$

(8.8)
Theoretically, electromagnetic waves can transfer energy from a transmitter (primary) to a receiver (secondary) over large distances in the order of kilometers. For the waves to propagate over these distances the frequency has to be typically in the high kilohertz to low gigahertz range ($5 \leq \log_{10} f \leq 10$). The disadvantage of applying electromagnetic wave propagation is the difficulty with which the propagation direction can be focused. Waves diffuse through space according to radiation patterns that are closely related to the antenna configuration [5, 16]. Therefore, only a fraction of the energy content of the electromagnetic waves is actually transferred to the receiver as depicted by the black wave in Fig. 8.3. As the distance between the sender and the receiver increases, the amount of received energy decreases even further. Obstacles in the path of propagation could reduce the amount of power received by the secondary side due to heat dissipation in the obstacle or reflection of the waves. If an obstacle in the space through which the electromagnetic waves propagate - including stray waves - is an animate one, the levels of the electric and magnetic wave-component have to be sufficiently low as to not cause any damage to the living tissue. This restricts the power density level of the electromagnetic waves in accordance with IEC norms. Another disadvantage is the use of such high frequencies for high power. Clearly, higher frequencies cause the switching losses in the power electronic circuit to increase.

Rectennas have been proposed [73] to scavenge energy from devices that emit radio waves for powering small, low power, electronic devices. CET systems have been proposed that apply self-resonant coils in the near-field to wirelessly transfer power over a couple of meters [60, 91, 101]. Distances between the transmitter and receiver up to $2.4\text{ m}$ with a power transfer of $60\text{ W}$ operating at a frequency of $f \approx 10\text{ MHz}$ have been realized [60]. The efficiency for this particular system monotonically decreases with the distance between the transmitter and receiver. When the primary and secondary are in close proximity, an efficiency of $\eta > 0.8$ can be obtained. As the distance between transmitter and receiver increases and approaches $2.4\text{ m}$ the efficiency drops to $\eta \approx 0.2$. In [101] the efficiency is kept more or less constant for small distances by applying an adaptive frequency control to keep the system in resonance with varying distances between transmitter and receiver.

### 8.2 Inductive contactless energy transfer solutions

Inductively coupled coils provide an alternative to conventional wired or slipping connections in order to circumvent the limitations associated with these approaches. These include limited freedom of movement, wear, undesired mechanical dynamics, and added mass. High efficiency and high power density are the most important requirements of an inductive coupling. Power electronic converters are required to shape the voltage and current waveforms in order for them to be fit for inductive power transfer. Additional converters contribute to the losses and add to increased complexity of the system. However, for specific appli-
cations the elimination of limitations, inherent to conventional connections, might outweigh the concessions on the efficiency, power density, and complexity when applying inductive couplings. The electromagnetic configuration of the CET system is highly dependent on the application in which it is to be applied. The type of relative movement of the inductively coupled coils can be rotational or translational. Although rotational and translation CET systems obey the same basic physical principles, the implication on the electromagnetic behavior can be very different. For rotational movement the transformer can generally be compact and the coils occupy a comparable volume of space. For translational, relative movement one of the coils can occupy a considerably larger volume than the other. The discrepancy in occupied volume is dependent on the stroke and the chosen electromagnetic configuration. The repercussions of the volume of one of the coils being larger than the other manifest themselves in terms of a reduced magnetic coupling and higher ohmic losses compared to rotational transformers with a similar configuration.

8.2.1 Add-on CET systems

Supplying a moving body with electric energy is a situation that is frequently faced in complex, electromechanical applications. The electric energy can be used for different purposes. The electric energy can be employed to power the phase coils of a moving coil actuator that produces the movement, for the power supply of additional electric devices on the moving body, or a combination of both. Supplying the electric energy wirelessly can be realized by adding moving transformers to the design. For a contactless power supply to moving phase coils the application of a separate moving transformer does not impose any restriction on the actuator type. However, the use of the additional component, being the transformer, can lead to a significantly bulkier design.

Some basic moving transformer topologies for translational and rotational motions are depicted in Fig. 8.4 [84, 103]. All the topologies shown in Fig. 8.4 have only one degree-of-freedom (DoF), meaning that the relative mechanical displacement is restricted to be in one direction only. The direction of movement is indicated by the relative, mechanical speed: \( v_m \) for translational motion and \( \omega_m \) for rotational motion. By comparison, the magnetic coupling for the topology of Fig. 8.4a is significantly lower than for the topology of Fig. 8.4b, because the ratio between the flux penetrating the surface area of the secondary coil with respect to that penetrating the area of the primary coil is smaller for the translational topology than for the rotational one. Furthermore, the magnetic coupling is dependent on the stroke for the translational CET system, i.e. as the stroke increases the magnetic coupling reduces for a constant surface area of the secondary coil. Moreover, the length of the wire is significantly longer for the primary coil than for the secondary in case of a translational system giving rise to increased voltage drops and losses.
The magnetic coupling can be increased by applying a soft-magnetic core. Depending on the relative permeability of the soft-magnetic material to be applied, its geometrical configuration, and airgaps being present in the core, the flux inside the core can be significantly higher than outside the core. Therefore, the ratio of the amount of linked flux with respect to the total flux generated by the primary coil can be increased significantly. The inclusion of soft-magnetic cores in the topologies of Figs. 8.4a and 8.4b is depicted in Figs. 8.4c and 8.4d, respectively. Although the magnetic coupling increases when a soft-magnetic core is applied, the leakage inductance is hardly affected by it. When the same current is passed through the primary coils of Figs. 8.4a and 8.4c, the amount of magnetic flux through the primary that has no overlap with the core is scarcely affected. Hence, the values of the leakage inductance are in the same order of magnitude for topologies with the same coil configuration whether a core is being applied or not. The gain in magnetic coupling is solely ascribed to the increased magnetization inductance which is the result of the enhanced flux in the core.

Variants of, and extensions to, the basic topologies of Fig. 8.4 for stationary and
moving systems have been researched over the years. In [21] a rotational transformer with a soft-magnetic, gapped core is proposed to allow an unrestricted movement of a robot arm around its parallel axis. Contrary to the topology of Fig. 8.4d, where the cross-section of the airgap is parallel to the rotational axis, the airgap is perpendicular to the rotational axis. Furthermore, the primary and secondary are separated by the airgap leading to a so-called adjacent coil configuration instead of the coaxial coil configuration of Fig. 8.4d. A similar topology with adjacent coil configuration is presented in [61] where the tolerance on the misalignment of the rotational axis of the secondary with respect to the primary axis is investigated. A comparative study between an adjacent and coaxial coil configuration is presented in [82]. A three phase rotational transformer in conjunction with an induction machine is elaborated on in [98]. The presence of an airgap in the core for rotational transformers is inevitable to guide the leads of the coils out of the core.

Translational, cored topologies are described in [7, 8, 48, 63, 89, 90, 104, 105]. In Fig. 8.5 a gapless core with an elongated, flexible primary winding to decrease the leakage inductance is shown [7]. Due to the own weight of the forward and return conductors of the flexible primary coil, either is pulled toward the other by gravity. The conductors are pushed apart by the leg of the core around which the primary coil is wrapped. In this manner, the cross-section of the primary coil that has no overlap with the core is significantly reduced, leading to a low leakage inductance. However, the primary coil is exposed to friction, abrasion, and fatigue. Furthermore, the primary coil can only be mounted in one direction with respect to the surface of the earth for the conductors to be pulled together again by gravity after the core passes.

An E-shaped and S-shaped, semi-open core, as depicted in Figs 8.6a and 8.6b respectively, with an elongated primary coil is applied in [48]. The E-shaped core is obtained by connecting two U-shaped cores of which the indentations are in the same direction. The S-shaped core is obtained by flipping over one of the U-shaped cores in such a way that the two indentations point in the opposite direction. The open sides of the core allow a stiff mechanical support of the primary coil. A variation of this approach can be found in [99]. A similar construction with a coaxial coil configuration (Fig. 8.6c) is proposed in [89], where the core is a hollow, open cylinder.

A different approach is given in [63] where the core is elongated instead of the primary winding (Fig. 8.7). The primary and secondary are wound around a gapless, rectangular core with a circular cross-section. The magnetic coupling of the core becomes position dependent due to the leakage flux. Further away from the primary the flux in the core gradually decreases due to the leakage flux, which results in a decreasing coupling when the secondary coil moves away from the primary.

In [90] the coupling of a single-sided, soft-magnetic, E-core CET system is increased by adding an additional, orthogonally orientated, secondary coil around
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Fig. 8.5: Gapless CET solutions with a flexible primary to reduce the leakage inductance.

Fig. 8.6: Open cored or gapped CET solutions that allow a stiff support of the primary coil for an E-core (a), S-core (b), and coaxial (c) configuration.

Fig. 8.7: CET solutions with an elongated stationary soft-magnetic core.
the same leg as the normal secondary so as to link the leakage flux perpendicular to the coupled flux as well. This approach allows for a more constant coupling as a function of transversal displacements or perturbations. Consequently, the need for an accurate alignment of the primary with respect to the secondary becomes less stringent. This principle is shown in Fig. 8.8.

In [104] and [105] the influence on the electromagnetic behavior due to the inclusion of soft-magnetic material at several strategic places for different coil configurations on the position dependency of the magnetic coupling is researched. Furthermore, [104] applies a segmented primary, where an active switching algorithm of the primary coils is required. An example for a topology with soft-magnetic material on the primary side only is shown in Fig. 8.9.

Multi-DoF, low power, CET system with flat primary and secondary coils for domestic and office use have been proposed by [75] and [107]. An industrial planar, 2 DoF, CET system for powers up to 35 kW is investigated in [39]. Both are depicted in Figs 8.10a and b, respectively. It has to be noted that the core of the mover in Fig. 8.10b has been cut and set transparent for the sake of clarity.

It has to be noted that, depending on the chosen topology, an airgap in the core of translational transformers can be omitted. However, the mechanical stiffness of the elongated components limits the maximum stroke. A low stiffness can cause the components to bend under gravitational or magnetic forces, which could cause the stationary and moving parts to graze during motion. Adding a mechanical support enhances the stiffness makes the use of airgaps inevitable at the expense of a reduced magnetic coupling.

### 8.2.2 Integrated approaches

The same electromagnetic fundamentals govern the working principles of transformers and electrical machines. This is apparent from the use of equivalent transformer models for the analysis of the electric behavior of machines in the classical approach [13, 113]. Moreover, the components machines and transformer consists of are generally the same, i.e. coils and a soft-magnetic structure. Possible permanent magnets can be replaced by equivalent coils in the transformer model. It seems, therefore, obvious to integrate a transformer into the design of a machine to exchange electrical power between the stationary and moving members of the machine.

In [87] the transformer of a dc-dc converter is integrated in the structure of a three phase machine to supply a 14 V or 42 V dc-bus for automotive applications as shown in Fig. 8.11. The stationary phase coils of the machine simultaneously act as the primary of a transformer. Additional stationary coils, which form the secondary coils of the transformer, are placed in the slots of the phase coils. Therefore, the teeth of the motor also form the transformer core. Although, the transformer is fully integrated in the machine, all its components are on the stator.
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Fig. 8.8: E-core solution of a CET system with an additional orthogonal secondary winding to also link the leakage flux from the primary.

Fig. 8.9: CET solutions with a segmented primary with a core on the primary side only.

Fig. 8.10: 2D, planar CET solutions for domestic or office (a), and industrial (b) applications.
Hence, no electric energy with respect to the CET is exchanged between the stator and the rotor. The transformer is integrated by adding additional secondary coils to an existing machine design wherein the integration of a transformer has not been anticipated.

Two topologies for a synchronous, permanent magnet, rotational machine with an integrated CET are proposed in [93] and [94]. In the first topology the three-phase primary coils and three-phase motor coils are separated and placed in the slots of the stator. The three-phase secondary coils are placed on the rotor behind the permanent magnets. The three-phase, motor windings are wound according to the fractional slot winding principle, whereas the three-phase primary coil-set of the CET on the stator is wound in accordance with the integer slot winding principle. To ensure electromagnetic decoupling and allow decoupled control of the functionalities the fields originating from the CET and motor may not share any common time-harmonics. This solution requires the use of two power electronic converters, i.e. one for each functionality. Another approach is presented where only one converter is required to simultaneously control both functionalities. To that end, the second topology is introduced that employs one three-phase coil-set on the stator that is simultaneously used as the three-phase primary of the CET and the phase coils of the motor. The three-phase secondary coil-set behind the permanent magnets is replaced with single-phase coils that are aligned with the $d$-axis of the rotor. Alignment of the single-phase secondary with the $d$-axis ensures the CET to operate on the $d$-axis in order to obtain electromagnetic decoupling between the functionalities. In both cases the magnetic field of the CET has to cross the airgap including the permanent magnets. This will introduce additional eddy current losses in the permanent magnets. Furthermore, the magnetic field of the CET and motor are confined within the same plane, which causes the soft-magnetic material to saturate easier as a result of the superimposed magnetic fields. The schematic geometrical representation for the variant with the single-phase secondary is shown in Fig. 8.12.

Integrated approaches for linear machines have been proposed in [25] and [77]. In [25] the integrated design can only operate in either motor or CET mode, but not in both modes simultaneously. The topology presented in [25] is used as a transformer to charge a battery of a tram at standstill and applied as motor for linear propulsion for driving the tram. The topology of [77] in Fig. 8.13 applies an additional elongated coil around the U-shaped stator of a transversal flux machine. The secondary I-shaped core and the pick-up coil are separated from the translator and dragged along by it. The major drawback of this design is the position dependency of the magnetic coupling. Since the secondary core and the translator are separated, the leakage inductance of the transformer changes as the relative position of the translator with respect to the stator changes. The leakage flux passes through the core of the translator, the amount of which is position dependent due to the change in the magnetic circuit as a result of the slotted structure.

The integrated multi-DoF topology of Fig. 8.14 is presented in [17]. The CET
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Fig. 8.11: Integrated CET solution with separated secondary coils on the stator and motor coils that simultaneously operate as primary coils.

Fig. 8.12: Integrated CET solution with combined three-phase, primary CET and motor coils on the stator with permanent magnets and single-phase secondary CET coils on the rotor.
Fig. 8.13: Integrated CET solution in a linear transversal flux machine with separated primary and secondary coils.

Fig. 8.14: Integrated, planar, multi-DoF CET solution with a secondary coil on the mover and stationary coils that can either operate as primary coils or motor coils.
is integrated in a planar, multi DoF actuator. The coils that allow a magnet array to levitate and move are intended to be applied as primary coils for a coreless transformer when the secondary pick-up coil is positioned over them. The secondary pick-up coil is mounted to the side of the levitated magnet array. An active switching algorithm for the primary coils is applied. CET during the movement of the levitated magnet array has been proven with separate coils for the transformer and actuator. However, the dual function of the coils has never been demonstrated.

The same problems with respect to leakage inductance caused by elongated transformer components are encountered when integrating CET into linear machines compared to integrations into rotary machines as discussed for separated CET systems in section 8.2.1. In rotary machines the magnetic fields of both the motor and CET are confined to the mechanical structure, whereas the magnetic fields in linear topologies extend beyond the space where there is overlap between the stator and translator. A thorough electromagnetic analysis is to be conducted when integrating CET into machines in order to prevent undesired cross-coupling and saturation effects from occurring in the design. The design considerations to minimize these effects are the topic of the next section.

8.3 Design considerations and specifications

In order for an integrated design to work properly the impact on some key electromagnetic phenomena have to be taken into consideration when integrating a CET system into an electrical machine. Five criteria are proposed that an integrated design should ideally satisfy:

1. No undesired emf is induced in the transformer coils due to the time-varying magnetic field of the motor and vice versa;
2. The magnetic coupling of the transformer is independent with position;
3. The magnetic coupling of the transformer is independent of the machine phase current;
4. The force or torque is independent of the transferred power of the CET;
5. Integration must not lead to overheating of the system.

The formulation of these five criteria is the result of qualitative analyses on the expected interaction between the functionalities on the electromagnetic level as presented in this section. In the following these criteria are individually addressed and elaborated on in separate subsections.
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8.3.1 Mutually induced emf

Integration of CET into an electrical machine can electromagnetically be considered as the superposition of two, time-varying, magnetic fields; one originating from the coils of the transformer and the other from the coils and permanent magnets of the machine. Both magnetic fields can induce an emf in both the machine and the transformer coils. When an emf is induced in the phase coils of the machine due to the magnetic field of the transformer, it is desirable that the induced emf does not produce a disturbance force or torque in the machine. Conversely, the magnetic field of the machine must not induce an emf in the transformer coils that cause a disturbance in the output voltage of the transformer. Moreover, if the emfs that are induced in the transformer coils cause currents in the electric CET circuit, these currents would cause a damping force on the mover. The same current could even damage the electronic circuitry of the CET. For these reasons it is imperative to establish electromagnetic decoupling between the transformer and machine.

Electromagnetic decoupling can be obtained by a proper spatial orientation of machine coils with respect to the magnetic field of the transformer coils and vice versa. Figure 8.15 shows three possible orientations of the coils in which decoupling between the coils is ensured. The concept of null-flux coils is applied in Fig. 8.15a. One of the coils, which could either be a machine or transformer coil, is wound in an eight-shape pattern. In this manner, the flux from the normally wound coil (bold line in Fig. 8.15a) does not induce an emf in the null-flux coil because the flux seen by the left half is equal to the one seen by the right half of the null-flux coil, but in opposite direction. Hence, the total flux and induced emf equal zero. Conversely, the fluxes originating from both halves of the null-flux coil do not induce an emf in the other coil. Electromagnetic decoupling is guaranteed as long as the relative displacement of the coils is in the vertical direction in accordance with the arrow in Fig. 8.15a. Variations on the null-flux principle for a CET system are presented in [10], where the sections of the different null-flux
coils have two half, four quarter, or spiral sections.

When a single-phase transformer is applied within a three-phase electrical machine the property of the sum of the motor phase currents being zero can be exploited. In Fig. 8.15b the transformer coil is wound such that the flux of the three-phase motor is fully encompassed. The total net flux seen by the transformer coil equals zero and, therefore, no emf is induced in the transformer coil. The magnetic flux of the transformer coil does induce an emf in each phase coil of the motor. Ideally, the induced emfs in the motor coils are equal and cancel when a wye-connection is applied. A delta connection of the motor coils would have the same effect as a short-circuited secondary transformer coil. To ensure decoupling the relative coil positions have to remain fixed. Furthermore, it has to be noted that the material may not saturate, since saturation may cause an imbalance in the three-phase flux distribution that could result in mutually induced emfs. The approach of Fig. 8.15b could for instance be applied when the primary of the transformer and the motor coils are on the stator of the machine.

By spatially orienting the coils orthogonally with respect to their relative flux orientation an alternative manner of decoupling can be acquired. This coil configuration is schematically shown in Fig. 8.15c. The flux of one coil is tangential to the surface of the other, ensuring that no emf is induced. Independent rotation around the dotted center line of Fig. 8.15c, possibly combined with a translational movement parallel to the dotted center line, does not induce an emf due to the symmetry of the field, nor does a relative translational movement of the coils solely restricted to the direction parallel to the plane to which a coil surface is confined. These directions are indicated with arrows.

The approaches of Figs. 8.15a and 8.15b can be applied when the magnetic fields of the transformer and machine are in-plane, i.e. both fields are confined to the same 2D plane. When the magnetic fields are confined to two planes that have an orthogonal relative orientation, the approach of Fig. 8.15c has to be applied.

8.3.2 Position independent magnetic coupling

When the open circuit test is performed on the secondary side of the integrated transformer, it is desirable to have a stable and constant secondary voltage that does not fluctuate with position. Fluctuation in the secondary, open circuit voltage can be compensated by controlling the amplitude or frequency of the primary voltage which results in a fluctuation of the amplitude and/or frequency of the flux in the core. However, it is more elegant to minimize the secondary voltage fluctuation by a sound electromagnetic design of the transformer as to eliminate the need of voltage or frequency control. Voltage fluctuations are caused by the position dependency of the magnetic coupling. The position dependent behavior of the magnetic coupling can be ascribed to two effects; being saturation and change in the magnetic flux path. The effects are illustrated by means of a general, qualitative analysis of the magnetic field distribution of the integrated topology of
Fig. 8.16: Integration of CET into a reluctance actuator where the magnetic 
coupling of the transformer is position dependent.

To demonstrate the effect of the change in the flux path the core material of the stator and the mover as a whole is assumed to be not saturated. In that case, the magnetization flux is predominantly determined by the permeance values of the left- and right-hand side airgap. A relative movement of the mover core in the $y$-direction results in a change of the airgap volume, which in turn affects the permeance value of the gaps. The change in the airgap permeances gives rise to an alteration in the magnetization flux. As a consequence the ratio between the transformer leakage flux and magnetization flux will result in a modified magnetic coupling. On the other hand, a change in position of the mover causes the magnetic loading, or magnetic flux density in the core, of the reluctance actuator to change. If, for sake of argument, the flux in the core is assumed to be uniformly distributed, the permeability of the nonlinear, soft-magnetic core material is a function of the flux density inside. The magnetization flux of the transformer is affected by the change in permeability on account of the magnetic loading of the actuator, which reverberates in alterations of the magnetic coupling and secondary voltage. In practice, the effect of change in flux path and saturation occur simultaneously and the position dependent magnetic coupling cannot solely be attributed to either effect.
8.3.3 Influence of the machine current on the magnetic coupling

The electric loading of the machine should not influence the magnetic coupling of the transformer in order to maintain a constant secondary voltage. Variation in the phase current of the machine can lead to voltage fluctuations on the secondary side of the transformer if the magnetic coupling is sensitive to the amplitude of the machine current. A change in the phase current gives rise to a change of the flux in the core which leads to a change in the spatial permeability distribution throughout the volume of the soft-magnetic core. The effect is exactly the same as the position dependent saturation effect as described in the previous subsection, with the distinction that the change in the permeability distribution originates from a magnetic field perturbation due to the machine phase-current rather than the relative position of the mover with respect to the stator.

In the topology of Fig. 8.16 the effect is observed by comparing the total flux, which is the vector sum of the transformer and machine flux, in the two side-legs of the core parts. The transformer and machine flux have an in-plane orientation, which means that both fluxes are confined to the same 2D plane. The machine flux revolves in the counterclockwise direction around the left-hand side of the machine coil and in the clockwise direction around the right-hand side machine coil. The transformer flux rotates in the counterclockwise direction only. This entails that the resulting total flux is amplified in one leg and attenuated in the other. Consequently, the operating point in the $BH$-curve of each leg is different and dependent on the amplitude of the total flux. Obviously, the magnetic coupling is affected by the change in operating point of the $BH$-curve as a function of the machine current. However, if the airgap is the dominant permeance in the magnetic circuit, the change in magnetic coupling is negligible if the working point on the $BH$-curve is below the knee-point.

8.3.4 Influence of the transferred power on the force or torque

In the previous subsection the influence of the machine current on the secondary voltage is discussed. A similar effect occurs when the transformer field influences the force or torque of the machine. In principle the magnetization flux of the transformer is not dependent on the secondary transformer current when a fixed primary voltage is applied and, therefore, it can be considered constant. However, the leakage flux of both the primary and secondary increase as the primary and secondary current increase. If the leakage flux passes through the core, it could affect the local, total flux and cause local saturation. In turn the local saturation may lead to a change in the force or torque profile.

The amplification and attenuation of the total flux in the side-legs of the topology of Fig. 8.16 cause an undesired pulsating disturbance torque on the mover. The
pulsation is caused by the sinusoidal behavior of the transformer flux. The disturbance torque is independent of the secondary current, since it is caused by the magnetization field. If the amplitude of the transformer field is low with respect to the motor field the impact of the disturbance torque is negligible. However, the situation is different when the leakage flux is considered. The leakage flux of the transformer for the topology of Fig. 8.16 is concentrated in the vicinity of the airgap in between the primary and secondary coils and is predominantly parallel to the $x$-axis. The horizontal leakage flux component might cause the center leg of the mover to saturate and in that way influence the force on the mover. Therefore, the leakage flux is dependent on the power to be transferred and, consequently, so is the force.

The cross-coupling effects of the electric loading of the machine on the magnetic coupling, and the influence of the transferred power on the force or torque can be reduced significantly by applying a mutual, orthogonal orientation of the fluxes of the transformer and machine in lieu of an in-plane orientation. This is indicated in Fig. 8.17, where the effect of the in-plane and orthogonal flux orientation on the total resulting magnetic field is compared. For both the in-plane and orthogonal orientation the machine field vectors, $\vec{B}_m$, have the same length as do the transformer field vectors, $\vec{B}_t$. The total field vector, $\vec{B}$, is obtained by the vector sum of the transformer and motor field vector. The in-plane situation of Fig. 8.17a is considered where both $\vec{B}_m$ and $\vec{B}_t$ are confined to the $xy$-plane. Hence, the total field vector is also confined to the $xy$-plane. For fixed lengths of $\vec{B}_m$ and $\vec{B}_t$ the arrowhead of the total field vector has to lie on the dashed circle depending on the angle, $\theta$, between the machine and the transformer field vectors. For a fixed relative position of the mover with respect to the stator and a fixed machine current, $\theta$ is fixed and so is $\vec{B}_m$. However, the amplitude of $\vec{B}_t$ is time dependent. If the length of $\vec{B}_t$ in Fig. 8.17a is the maximum amplitude of the transformer field vector, then the dotted straight line in the dashed circle is the locus along which the arrowhead of the total field vector moves as a function of time for fixed
8.3: Design considerations and specifications

Fig. 8.18: Variation in the total field vector length, normalized to the machine field vector length, as a function of the angle and ratio in length between the machine and transformer field vector.

$\vec{B}_m$ and $\theta$. The length of the total field vector can be expressed as

$$|\vec{B}| = \sqrt{|\vec{B}_m|^2 + |\vec{B}_t|^2 \cos^2 (\omega t) + 2|\vec{B}_m||\vec{B}_t|\cos (\omega t) \cos (\theta)}, \quad (8.9)$$

where $t$ is the time and $\omega$ the radial frequency of the transformer field. The minimum and maximum length of the total field vector can be calculated by determining the minima and maxima of (8.9) through differentiation with respect to time. The variation in length of the total field vector, $\Delta |\vec{B}|$, is the difference between its maximum and minimum value. The normalized variation in the total field vector length with respect to the machine field vector length is shown in Fig. 8.18 for different relative lengths of the transformer field vector with respect to the machine field vector. It can be concluded that for all ratios in length the variation is minimal when $\theta = \frac{\pi}{2}$ rad and maximal when $\theta = 0$ rad or $\theta = \pi$ rad. Furthermore, the variation becomes larger as the amplitude of the transformer field vector increases. Especially for transformer field vectors with increased lengths a larger reduction in variation is observed when the field vectors have a perpendicular instead of a parallel orientation. By enforcing a transformer field vector that is always orthogonally oriented with respect to the machine field vectors the lowest possible variation in the total field vector is guaranteed regardless of the orientation of the machine vector inside the $xy$-plane. This orthogonal orientation is depicted in Fig. 8.17b, where the locus along which the arrowhead of the total field vector moves is indicated by the dotted line. Hence, the fields predominantly being parallel in the topology of Fig. 8.16 is the worst possible relative orientation imaginable in regard to the minimization of the mutual influences of the machine and transformer currents on the power transfer capabilities and force, respectively.

For the orthogonal orientation the sweep on the $BH$-curve for one cycle of the transformer field is significantly smaller, which results in a smaller variation of
the permeability distribution of the soft-magnetic material. Alternatively, for the same sweep in the BH-curve either the machine or transformer field can have a larger amplitude when orthogonal orientation of the fields is applied. Hence, the orthogonal field orientation permits more lenient restrictions on the magnetic field levels when integrating CET into an electrical machine. On the other hand, an in-plane field orientation allows coils to simultaneously be used as transformer and machine coils.

8.3.5 Thermal aspects with respect to integration

Evidently, the addition of extra coils into the structure of a machine, to allow CET, introduces additional heat sources that are the result of the ohmic resistance of these coils. Generally, the frequency at which the transformer operates is significantly higher than the frequency of the phase currents of the motor. The high frequency of the transformer field results in additional iron losses of the soft-magnetic core. Moreover, the skin-effect in the transformer coils and the proximity effect in both the transformer and machine coils are additional sources of heat. When applied in a permanent magnet machine the electric conductivity of the magnets has to be taken into consideration as well when the high-frequency transformer flux penetrates them. Eddy currents are induced into magnets due to the time-varying field in combination with the nonzero conductivity of the permanent magnets. The transformer flux that penetrates the magnets can be the magnetization flux, the leakage flux, or both depending on the chosen topological configuration. When the magnetization flux penetrates the permanent magnets the losses are constant and independent of the transferred power. Otherwise, the eddy current losses in the magnets become dependent on the transferred power. In case of the topology of Fig. 8.16 the eddy current losses in the permanent magnet originate from the leakage flux only, since the magnetization flux bypasses the permanent magnet through the side-legs. In any case, the integration will eventually result in a compromise between force or torque density and the power transfer capabilities. Therefore, adequate thermal modeling is an essential part during the design process of an integrated solution.

8.4 Soft-magnetic materials and losses

The soft-magnetic core is an essential part for both transformers and machines. By adding soft-magnetic material the force density for motors and the power transfer capabilities of transformers are greatly improved compared to their coreless counterparts. In both cases, the core acts as a flux conduit and greatly enhances the flux density level of the magnetic field. However, the inclusion of the soft-magnetic material introduces nonlinear behavior and gives rise to additional losses in the form of heat or acoustic energy due to magnetostriction when a time-varying magnetic field is applied [106]. The core or iron losses are classically decomposed into
three components: eddy current losses, hysteresis losses, and excess losses [9]. The eddy current losses originate from the nonzero electric conductivity of the core material. When a time-varying magnetic field penetrates the core, eddy currents are induced that contribute to the generated heat due to the Joule effect. The hysteresis losses are ascribed to the domain wall movement of the magnetic domains which continuously change shape due to the applied time-varying magnetic field. This change in shape is accompanied with very short, local eddy current surges that lead to losses due to the Joule effect. The excess loss is attributed to the remaining energy loss associated with the interaction and motion of the domain walls when the contribution of the short eddy current surges to the losses is ignored. All three classical losses always contribute, to a greater or lesser extent, to the total loss depending on the choice of the soft-magnetic material. Acoustic energy is generated due to time-varying, internal mechanical stresses in the core on account of the magnetic field. In general, the acoustic losses are negligible compared to the Joule losses. Three types of soft-magnetic materials are compared in this section, viz. electric steel, ferrite, and soft-magnetic composites (SMC). The choice of the proper magnetic material is predominantly determined by the saturation level and the frequency dependent loss characteristic of the material. The approximate ranges of application in terms of frequency and maximum flux density levels are shown in Fig. 8.19 for the three considered materials. The step-like shape of the curves in Fig. 8.19 is ascribed to the material grade of the materials as prescribed by their respective manufacturers [15, 23, 37, 112]. The frequency range could be extended by operating the material at higher frequencies and at lower flux density levels to keep the losses within acceptable values.
Fig. 8.20: Eddy current reduction by lamination of electric steel: eddy current flow in laminated (a), and nonlaminated electric steel (b).

8.4.1 Materials

Electric steel can sustain flux density levels as high as $B = 2.3 \text{T}$ with a maximum relative permeability of $\mu_r = 1.5 \cdot 10^4$ when applying a cobalt-iron alloy [112]. For more conventional silicon-iron alloys the maximum flux density level is around $1.6 \leq B \leq 1.8 \text{T}$. Due to the high electric conductivity of the electric steel, typically in the range of $1.0 \cdot 10^6 \leq \sigma \leq 5.0 \cdot 10^6 \text{S m}^{-1}$, the frequency range is limited to near-dc frequencies for bulk material [15]. However, the frequency range can significantly be extended by applying laminations. Thin sheets of electric steel that are electrically isolated from each other are stacked to reduce the eddy current losses in the steel. The insulating layers in between the laminations prevent the eddy currents from flowing in the direction perpendicular to the plane of the laminations over the full height of the core as portrayed in Fig. 8.20. The induced eddy currents are now confined to the lamination sheet whereby the cross-section, through which the horizontally orientated eddy currents mainly flow in regard to Fig. 8.20, is reduced compared to the nonlaminated situation. The reduced cross-section leads to an increased electric resistance and as a result the amplitude of the eddy currents decrease. Thinner laminations result in a further reduction of the core losses. Typical lamination thicknesses vary between $0.10 \text{mm}$ to $0.65 \text{mm}$ for high and low frequency applications, respectively [15]. The lamination of steel causes the steel to become anisotropic both electrically and magnetically. Isotropy is already present in the laminations on account of the rolling direction during the manufacturing process. Evidently, the electric anisotropy is caused by the insulating layers. Similarly, the inclusion of the isolating layers introduces tiny airgaps in the direction perpendicular to the laminations, which significantly reduces the permeability of the stack in that direction. On top of that, the permeability in the perpendicular direction is already low due to the anisotropy of a single lamination. Hence, laminated electric steel can only be applied in application where the flux is confined to 2D planes. This enforces an in-plane orientation of the transformer flux with respect to the machine flux when integrating CET into a machine. The resulting variation in the total magnetic field vector for in-plane
orientation in combination with the high electric conductivity of the steel restricts
the maximum frequency of the transformer flux in order to limit the core losses.

From Fig. 8.19 it can be seen that the frequency range for which ferrite is applica-
able is superior to both steel and SMC. The large frequency range is ascribed to the
low electric conductivity of $1.0 \leq \sigma \leq 1.0 \cdot 10^1 \text{S m}^{-1}$ for different ferrite grades.
Furthermore, ferrite has good isotropic behavior which allows 3D magnetic field
orientation. However, the flux density level is poor with respect to steel and SMC.

The temperature dependent flux density level is maximum around $B = 0.5 \text{T}$ for
lower frequencies and $B = 0.3 \text{T}$ for high frequencies. The maximum relative
permeability of ferrite is approximately around $\mu_r = 3.0 \cdot 10^3$ for material grades
for frequencies lower than $1 \text{MHz}$ and $6.0 \cdot 10^2 \leq \mu_r \leq 1.5 \cdot 10^3$ for higher frequen-
cies [23]. The low saturation levels for ferrite cause the material to be unfit as
the soft-magnetic core material for an integrated solution. The concession on the
length of the machine field vector can be reduced to nearly zero by applying an
orthogonal flux orientation in combination with a high-frequency, low amplitude,
transformer field for ferrite cores, due to the low core losses at high frequencies.
Nevertheless, the reduction in magnetic loading as a result of the low saturation
level causes the force density to deteriorate significantly.

A similar technique of reducing the cross-section through which the eddy currents
flow in order to diminish the core losses is also applied in soft-magnetic composite
materials (SMC). The distinction with respect to the laminated approach is that
iron powder with a small particle encapsulated by a thin insulating layer is applied
to further reduce the cross-section. The particle size is typically less than $100 \mu m$
[37]. The iron particles are electrically isolated from each other so that the in-
duced eddy currents are confined to the volume of the particle. The final shape of
the soft-magnetic material is obtained by compacting the powder under high pres-
sure in a mold. The mechanical strength and magnetic properties are obtained by
heat treatment of the compacted powder. The structure of the SMC and the eddy
current flow in the particles due to a magnetic field that is directed perpendicular
to the plane of the paper is shown in Fig. 8.21. The random, granular structure of the iron particles of the SMC ensures 3D isotropic electromagnetic behavior of the material. Not all iron particles are perfectly insulated from each other, causing the material to exhibit some bulk electric conductivity. It can be seen from Fig. 8.21 that small voids are present in between the particles. These voids act as an airgap that is distributed throughout the material. This distributed airgap lowers both the relative permeability, the maximum saturation level and the electric conductivity of the core material. Generally, the most suitable material grade for a specific application has to be based on a compromise between a low conductivity and high permeability. The maximum attainable relative permeability equals $\mu_r = 8.5 \cdot 10^2$ with an electric conductivity of $\sigma = 1.4 \cdot 10^4 \text{S m}^{-1}$ at a maximum flux density level of $B = 1.6 \text{T}$. Alternatively, for material grades with low electric conductive the maximum flux density, relative permeability, and electric conductivity equal $B = 1.4 \text{T}$, $\mu_r = 2.9 \cdot 10^2$, and $\sigma = 1.3 \cdot 10^2 \text{S m}^{-1}$, respectively [37].

The SMC material can be considered a compromise between steel and ferrite in terms of its electromagnetic material properties. This unique peculiarity makes it very useful to be applied in an integrated solution. First, in spite of the low relative permeability the maximum allowable flux density level is sufficiently high to allow a good magnetic loading of the machine. Secondly, the isotropic behavior together with the maximum flux density level of the SMC enables an orthogonal field orientation for decoupling of the functionalities and minimal reduction in the magnetic loading of the machine. Lastly, the eddy current losses of especially the high-frequency, transformer field can be kept low due to the low electric conductivity. An overview of the electromagnetic material properties of electric steel, ferrite, and SMC is given in Table 8.1.

### Table 8.1: Electromagnetic material properties for electric steel, ferrite, and SMC

<table>
<thead>
<tr>
<th>Material</th>
<th>$B_{\text{max}}$ [$\text{T}$]</th>
<th>$\mu_{\text{rmax}}$ [-]</th>
<th>$\sigma$ [Sm$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>electric steel</td>
<td>1.6 – 2.3</td>
<td>$1.0 \cdot 10^3 – 1.5 \cdot 10^3$</td>
<td>$1.0 \cdot 10^6 – 5.0 \cdot 10^6$</td>
</tr>
<tr>
<td>ferrite</td>
<td>0.3 – 0.5</td>
<td>$6.0 \cdot 10^2 – 3.0 \cdot 10^3$</td>
<td>$1.0 \cdot 10^6 – 1.0 \cdot 10^1$</td>
</tr>
<tr>
<td>SMC</td>
<td>1.4 – 1.6</td>
<td>$2.9 \cdot 10^2 – 8.5 \cdot 10^2$</td>
<td>$1.3 \cdot 10^2 – 1.4 \cdot 10^4$</td>
</tr>
</tbody>
</table>

8.4.2 Core losses

Core losses are the heat dissipation in the soft-magnetic material under the influence of a time-varying magnetic field. The eddy current or Joule losses can directly be taken into account in the magnetic field solution by solving, where possible, the quasi-static Maxwell equations of (2.6) for soft-magnetic bulk materials where the electric conductivity is homogeneously distributed throughout the material. However, the Joule effect due to electric conductivity contributes only
partly to the total losses of a soft-magnetic material. Their contribution is linked to the shape of the (minor) hysteresis loops of the $BH$-curve of the material and the behavior of the material on a microscopic scale. The losses of a soft-magnetic material can be quantified by applying an empirically derived equation for the mass loss density as function of the flux density level and the frequency. The mass loss density for sinusoidal excitation are given by Steinmetz’s equation \[108\]

$$dP = c_1 \dot{B}^{c_3} f^{c_3},$$

or equivalently when decomposed into the three separate losses [9]

$$dP = c_{\text{eddy}} \dot{B}^2 f^2 + c_{\text{hys}} \dot{B}^{\alpha} f + c_{\text{exc}} \dot{B}^{\gamma} f^2,$$

where $dP$ are the losses per unit mass and $c$ are constants that have to be determined empirically. Despite the fact that the equations of (8.10) are only valid for sinusoidally time-varying magnetic fields of amplitude $\dot{B}$, they are generally applied, possibly in modified form, to calculate the core losses for applications with nonsinusoidal, time-varying, magnetic fields [85, 92, 95]. For nonsinusoidally varying fields (8.10b) is expressed in a more general form as

$$dP = \frac{c'_{\text{eddy}}}{T} \int_0^T \left[ \frac{B(t)}{T} \right]^2 dt + \frac{c'_{\text{hys}}}{T} \dot{B}^{\alpha} f + \frac{c'_{\text{exc}}}{T} \int_0^T \left[ \frac{B(t)}{T} \right]^2 dt,$$

where the period is given by $T = f^{-1}$ and the primed coefficients of (8.11) indicate that the loss coefficients are numerically different from those of (8.10b) for nonsinusoidal flux densities. When applying (8.11) the Steinmetz coefficients, as supplied by the manufacturer of the soft-magnetic material, have to be corrected before (8.11) can be applied. The total core losses are found by integration of $dP$ over the volume of the core geometry.

### 8.5 Initial sizing of integrated topologies

In the previous section the focus was solely on the electromagnetic implications and losses of a machine with an integrated CET. These electromagnetic implications, logically, have repercussion on the geometrical sizing of an integrated topology with respect to the sizing of a machine without CET. In this section a quantitative analysis is provided by means of simplified machine, transformer, and thermal equations that can be applied for the initial sizing of a given topology. These basic equations are based on the desired force or torque density and power transfer capabilities.
Chapter 8: Contactless energy transfer in electrical machines

Fig. 8.22: Geometrical reduction of the geometry of the machine structure for the purpose of initial sizing.

Initial sizing of permanent magnet machines

For the initial sizing of brushless, permanent magnet machines without an integrated CET only the airgap is considered and, in the case of slotted structures, the structure is reduced to a slotless one where the current is concentrated in current sheets as described in [33] and depicted in Fig. 8.22. Next, both the flux density level distribution in the airgap, or magnetic loading, and the Ampère-turns value, or electric loading, in the current sheets are assumed constant at their respective rectified average and rms-values. Applying the Lorentz force equation of (7.2) to the reduced geometry of Fig. 8.22 yields an estimate of the force density

\[ f_V = \frac{Q B_m A_g}{V_m}, \tag{8.12} \]

where \( f_V \) is the volume force density in \( \text{N m}^{-3} \), \( B_m \) is the magnetic loading in \( \text{T} \), \( V_m \) the volume of the machine, and \( Q \) the electric loading per unit length in \( \text{Am}^{-1} \), and \( A_g = d_z l_g \). For rotary machines the expression of (8.12) has to be multiplied by the arm of the conductors to obtain the volume torque density. The electric loading per unit length is given by

\[ Q = \frac{N_{cpp} n_{ph} N_{ph} I_{ph}}{l_g} = \frac{N_{cpp} n_{ph} J_{ph} A_{ph}}{l_g}, \tag{8.13} \]

where \( N_{cpp} \) is the number of coils per phase, \( n_{ph} \) is the number of phases, \( N_{ph} \) is the total number of conductors per phase coil, \( I_{ph} \) the rms-value of the currents in the conductors, \( J_{ph} \) the rms-value of the current densities in the conductors, and \( A_{ph} \) the total surface area of the phase-coil conductors. Expressing (8.12) in terms of the phase current density makes the volume force density independent of the number turns of the phase coils. Equation (8.12) is generally valid for all types of brushless, permanent magnet machines [33], it can, however, be expressed in terms of the geometrical parameters of the problem specific machine topology to estimated the initial sizes.
Initial sizing of transformers

In section 8.1.2 the equation for the maximum apparent power transfer for sinusoidally operated, inductively coupled systems has already been derived. The result of (8.6) can be expressed in terms of the winding area of the conductors of the primary and the secondary, $A_w$, and the current density, $J_t$, through the transformer coils by substitution of $N_p I_B = J_t A_w$. The apparent power rating of the transformer as a function of the amplitude of the magnetic flux density, $\hat{B}$, instead of the magnetic field, $\hat{H}$, is then given by

$$S_t = \pi f A_c \hat{B}_t J_t A_w \sqrt{2}. \quad (8.14)$$

The general expression of (8.14) can be expressed in the specific topological variables to obtain the initial sizing of the transformer. It is noteworthy to observe that the apparent power is independent of the number of turns when expressed in the current density. Therefore, (8.14) is valid for all transformers with the same geometry, regardless of the turn ratio as long as the filling factor of the coils remains constant. The apparent power rating and primary voltage of (8.4) are both linearly dependent on the amplitude of the flux density, $\hat{B}$, and the frequency, $f$. The power rating and primary voltage are not affected by a change in $\hat{B}$ or $f$ as long as their product is kept constant; however, the core losses are. Depending on the empirically determined coefficients of (8.10) the core losses can be reduced by choosing the proper value of $\hat{B}$ and $f$. Hence, for a given geometry the core losses can be minimized without compromising the voltage or power rating of the transformer by tuning the flux density and frequency.

Estimation of the temperature

The sizing equations of (8.12) and (8.14) are subject to the maximum temperature rise in the topology. The temperature rise is caused due to all the losses in the structure. The total core losses can be estimated by (8.10). The ohmic losses in a coil are given by

$$P = \frac{l A J^2}{\sigma c_f}, \quad (8.15)$$

with $\sigma$ as the conductivity of the coil material, $c_f$ the copper filling factor, and $l$ and $A$ are the length and cross-section of the coil, respectively. For short heat flux paths the temperature inside the structure can be estimated by assuming an infinite thermal conductivity of the structure when the convection coefficient on the outer surface is sufficiently low. By sufficiently low is meant that the equivalent thermal resistance of the convection is dominant. In that case, the temperature distribution under steady state conditions throughout the structure
is constant and can be calculated by Newton’s convection law [3]
\[
T = T_a + \frac{P_{\text{loss}}}{h_{\text{con}} A_o} \Delta T,
\] (8.16)
where \(T_a\) is the ambient temperature in K, \(h_{\text{con}}\) the thermal convection coefficient, and \(A_o\) the outer surface area of the structure where the heat is exchanged through convection. Again, \(P_{\text{loss}}\) and \(A_o\) can be expressed in terms of the specific geometrical parameters of the topology to determine the initial sizes of the structure in conjunction with (8.12) and (8.14). Conversely, the compromise between the power transfer capabilities and force density can quickly be approximated for a fixed integrated topology.
8.5.1 Illustrative example for initial sizing

The impact of the integration of CET into a permanent magnet machine on the volume force density is demonstrated by means of the integrated, tubular topology of Fig. 8.23. Only a periodical section of the geometry is shown in Fig. 8.23 and the following analysis applies to the periodic section only. Slits are present in the teeth of a slotted tubular motor to fit the secondary windings into the mover core. The direction of flow of the currents is such that an orthogonal field orientation is obtained. The primary and secondary current of the transformer flow in the \( z \)-direction of the coordinate system and the three-phase currents flow in the \( \theta \)-direction. The important dimensions for initial sizing of the topology of Fig. 8.23 are tabulated in Table 8.2. The geometrical variables of (8.12) and (8.14) can be expressed in those provided in Table 8.2.

The performance of an electromechanical device in general is determined by the maximum losses, which in turn are determined by the maximum allowable temperature in the structure. The maximum losses can be approximated with (8.16) and are equal to 15 W for a maximum temperature rise of \( \Delta T = 95 \) K for the topology of Fig. 8.23. It has to be noted that only the convection with a convection coefficient of \( h_{\text{con}} = 15 \text{ Wm}^{-2} \text{K}^{-1} \) on the outer diameter of the core is considered. In the analysis the total losses, \( P_{\text{loss}} \), are composed of the total ohmic losses of all phase, primary and secondary coils according to (8.15) and the core losses. For simplicity, only the eddy current component of (8.10b) due to the transformer field is taken into account for the core loss calculation, since it is considered to be dominant. The total loss equation is then given by

\[
P_{\text{loss}} = c_{\text{eddy}} B_t f^2 + \sum_k \frac{l_k A_k J_k^2}{\sigma c_{f_k}},
\]

where \( k \) is the index for a coil. The current density levels of both the transformer current and phase current are eliminated from the total loss equation by substituting (8.12) and (8.14) into (8.17). In this manner, the volume force density can be expressed in terms of the flux density levels of the motor and the CET, the apparent power rating of the CET, and the total power loss, \( P_{\text{loss}} \). Next, the flux density level of the motor is eliminated from (8.17) by substitution of

\[
B_m = \begin{cases} 
\sqrt{B_{\text{max}}^2 - B_t^2} & \text{for an orthogonal field orientation,} \\
B_{\text{max}} - \hat{B}_t & \text{for an in-plane field orientation,}
\end{cases}
\]

where \( B_{\text{max}} \) is the maximum flux density level which is given by the saturation level of the soft-magnetic material. The result for the force density as a function of varying apparent power ratings of the CET and varying flux density levels of the CET when \( P_{\text{loss}} = 15 \text{ Wm}^{-2} \text{K}^{-1} \) is shown in Fig. 8.24a, where it has to be noted that all quantities are normalized with respect to their maximum values and the frequency of the transformer field is fixed to \( f = 5 \text{ kHz} \). The maximum flux density level equals \( B_{\text{max}} = 1.5 \text{ T} \). The physical parameters are tabulated in Table 8.3.
The maximum volume force density and apparent power rating of the transformer equal \( f_{V \text{max}} = 264 \text{kNm}^{-3} \) and \( S_{\text{t max}} = 340 \text{VA} \), respectively. The normalized values allow a more general, comparative analysis of the impact of the CET on the motor performance, since the actual values are problem specific. It is observed from Fig. 8.24a that the apparent power rating and the value of the transformer field are restricted by the zero level-line for the force density, \( f_{V} = 0 \text{Nm}^{-3} \). Beyond the zero level-line the losses due to the transformer only are greater than the maximum allowable losses for those combinations of the apparent power and flux density levels. The obvious compromise between the machine force density and the apparent power is illustrated by Fig. 8.24a, i.e. as the apparent power increases the force density diminishes for a fixed flux density level of the transformer field. However, there is a path visible for which the decline in force density with increasing apparent power is less severe. This path is indicated by the bold,
dashed line in Fig. 8.24a. It is, therefore, desirable that a design of an integrated solution is operated on, or at least in the vicinity of, that line. However, it cannot be sufficiently emphasized that the graph is based on some greatly simplified equations that provide an initial estimate of the performance of a permanent magnet machine with integrated CET. For a more accurate behavior extensive electromagnetic and thermal modeling is required as discussed in chapter 9.

Had an in-plane field orientation been applied for the same topology, then the performance would have looked like the one depicted in Fig. 8.24b. For the calculation of Fig. 8.24b the total loss and all geometrical values are kept the same with respect to those for the calculation of Fig. 8.24a, except that the magnetic loading in (8.18) has been obtained for an in-plane field orientation instead of an orthogonal one. It can then be seen that for the same apparent power and flux density level of the transformer a lower force density is obtained. Obviously, the fixed coil configuration cannot generate an in-plane field orientation and, therefore, Fig. 8.24b is only exemplary. However, it once more underlines the benefit of an orthogonal field orientation over an in-plane one.

8.6 Electromagnetically integrated designs

Three designs of machines with an electromagnetically integrated CET are proposed in this section. The working principles of each design are elaborated on, and the extent to which each design satisfies the five criteria of section 8.3 in regard to the electromagnetic behavior are analyzed. Based on this analysis the most suitable topology of the three is selected for further modeling and design for the manufacture of a prototype.

8.6.1 Flat, synchronous, permanent magnet, linear machine

In Fig. 8.25 a periodic cross-section of a flat, synchronous, permanent magnet, slotted, linear machine with integrated CET is portrayed in a cutaway view, where the periodicity is in the direction of the $x$-axis. The integrated flat solution can be considered as the combination of a flat, linear machine and a gapped, double C-core transformer [58]. The magnetic field orientation of the transformer and machine flux in the soft-magnetic core parts is schematically shown in Fig. 8.26. It can be seen that electromagnetic decoupling of the machine and transformer coils is obtained by an orthogonal field orientation. Hence, the soft-magnetic core material has to be isotropic which prohibits laminated steel to be applied.

The magnetic coupling of the transformer is independent with position from a geometrical point of view, since no geometrical variation in the permeance path of the transformer flux occurs as the relative position of the translator with respect to the stator changes. However, fluctuation in the magnetic coupling with position
Fig. 8.25: Schematic cutaway representation of a periodic section of the integrated, flat topology.

Fig. 8.26: Orthogonal magnetic field orientation of the fluxes in the core for the integrated, flat topology: transformer flux (■) and machine flux (■) in the $xz$-plane (a), and the $yz$-plane (b).
might occur due to the position dependent machine field distribution which affects the permeability distribution throughout the soft-magnetic core in accordance with the phenomenon described in section 8.3.2. The same phenomenon causes the magnetic coupling of the transformer to be dependent on the motor phase-current. Yet, under normal operating conditions the effect of the fluctuation in the magnetic coupling is small, because the magnetic flux density level in the core is predominantly determined by the magnetic field of the permanent magnets. It can be seen from Fig. 8.26 that the flux line distribution in the teeth of the translator core is less concentrated, meaning that the flux density level of the transformer in the teeth is low. Therefore, the effect of the magnetization flux of the transformer will not significantly contribute to the local saturation in the teeth and will not introduce significant disturbance forces.

Apart from the criteria of section 8.3 there are some additional aspects that require further discussion. It is observed from Fig. 8.26b that the field orientation is not fully orthogonal, since the machine and transformer flux both have a component in the z-direction in the teeth of the translator core. Although the z-component does not induce an emf in the phase coils due to symmetry of the transformer field, it has to be taken into account when the flux density of the transformer is to be increased in order to prevent local saturation effects from occurring. Generally, the flux density of the transformer field is small in the teeth and, consequently, the component in the z-direction is even smaller. The permanent magnet array is of the quasi-Halbach type, since a back iron in between the magnet array and the primary coil would provide a low reluctance, lossy, leakage path for the transformer flux and thereby decrease the magnetic coupling. To focus the magnetic field of the permanent magnets in the airgap of the machine while maintaining a good magnetic coupling, a quasi-Halbach magnet array is applied. However, the leakage flux, of especially the primary coil, will induce eddy currents in the permanent magnets which leads to additional losses. Finally, the legs, around which the secondary coils are wound, establish an additional path for the machine flux. The machine flux through the secondary legs magnetically pre-biases the soft-magnetic material there, which affects the magnetization flux of the transformer.

8.6.2 Double sided, linear, flux switching machine

Another integrated solution is shown in Fig. 8.27. The integrated topology consists of a double sided, linear, flux switching machine with a U-shaped stator core around which the primary coil of the transformer is wound, and the secondary coil encloses the full translator core. The direction of magnetization of the permanent magnets is parallel to the x-axis. It is apparent that the magnetic coupling of this topology is position dependent, since the magnetization path alters as the translator moves with respect to the stator. The flux switching machine topology is based on the topology for a rotary flux switching motor presented in [116]. This particular design of the flux switching machine with multi-toothed poles is
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Fig. 8.27: Schematic cutaway representation of the integrated, double sided, linear, flux switching topology.

Fig. 8.28: In-plane magnetic field orientation of the fluxes in the core for the integrated, flux switching topology: transformer flux (■) and machine flux (■) in the xy-plane (a), and the yz-plane (b).
chosen to minimize the total airgap reluctance seen by the magnetization flux and, also, to minimize the fluctuation in the permeance path due to geometrical changes with position. By applying a double sided structure in conjunction with a U-shaped stator, a U-I core transformer configuration is realized.

Decoupling between the machine and transformer coils is obtained by applying the null-flux principle for the phase coils. Two phase coils, belonging to the same phase, that oppose each other with respect to the \(xz\)-symmetry plane of Fig. 8.27 are wound in opposite direction as to prevent an emf from being induced due to the transformer field. The distinction with the null-flux principle, as depicted in Fig. 8.15a, is that opposing phase coils are not in the same plane; however, the principle applies nonetheless on grounds of symmetry.

The field distribution of the transformer and motor flux is shown in Fig. 8.28. It can clearly be seen that the topology exhibits an in-plane field orientation. This poses the major drawback of this topology, since the teeth saturate under normal machine operation. Clearly, the saturation of the teeth affects the magnetization flux density distribution and vice versa. Therefore, serious cross-coupling effects between machine and CET operation are to be expected. Moreover, it can be seen in Fig. 8.28b that the magnetization flux influences the machine field differently on either side of the symmetry plane. The machine field in a leg on one side is always enhanced by the transformer field whereas the other is attenuated by it.

Due to the in-plane field orientation the translator can be constructed of a laminated steel stack. The stator has to be of isotropic material when constructed out of one part. Alternatively, a combination of two perpendicularly orientated stacks can be applied, i.e. a U-shaped stack, stacked in the direction of the \(x\)-axis, to guide the transformer flux and a stack with the same staking direction as the translator stack for the stator teeth. The U-shaped transformer stack can also be replaced by SMC or ferrite depending on the flux density level to reduce the core losses.

The major advantage of the topology of Fig. 8.27 is that hardly any field from the phase or transformer coils penetrates the permanent magnets. Consequently, the eddy current losses in the permanent magnets will be low for this configuration. In comparison, it can be concluded that the integrated, flat solution is more suitable since more cross-coupling effects in regard to the criteria of section 8.3 are averted by applying an orthogonally, decoupled magnetic field configuration.
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Fig. 8.29: Schematic cutaway representation of a periodic section of the integrated, tubular topology, where the return path of the primary (not shown) closes as a concentrated bundle of strands outside of the outer radius of the core.

8.6.3 Tubular, synchronous, permanent magnet, linear machine

A periodic section of a tubular, synchronous, permanent magnet, slotted, linear motor with integrated CET is depicted in Fig. 8.29. The integrated, tubular topology combines the high force density properties of the tubular synchronous machine (TSM) with the high magnetic coupling characteristics of a coaxial transformer. Moreover, the transformer is gapless, which leads to an increased magnetic coupling. Electromagnetic decoupling of the transformer and phase coils is obtained by an orthogonal spatial orientation of both the coils and magnetic fields and, therefore, only isotropic soft-magnetic core material is applicable for the core.

The field orientation is shown in Fig. 8.30. Since the flux path of the magnetization flux of the transformer does not change with displacement, the magnetic coupling is not dependent on the relative position of the translator with respect to the stator from a geometrical point of view. As is the case for the integrated, flat topology, a possible position dependency of the magnetic coupling is introduced by the position dependency of the permeability distribution due to the flux density distribution of the machine. The effect as discussed in section 8.3.3 makes the coupling also dependent on the phase current. However, in comparison with the integrated, flat topology the variation in the angle between the transformer and machine field vectors is significantly less. This is already apparent from the comparison of the number of planes of symmetry along which orthogonality of the
fields is guaranteed in Figs. 8.26b and 8.30b. The flat topology has only one plane of symmetry formed by the $xz$-plane in Fig. 8.26b, whereas the number of planes of symmetry for the tubular topology are equal to the number of secondary coils. Due to the irregularities in the core geometry at the inner radius, as a consequence of the slits in which the secondaries are disposed, the orthogonality of the magnetic fields is lost there. Because the flux density levels for both transformer and machine inside the core decrease inversely proportionally with the radius, the saturation effect will commence at the inner rim and will gradually expand toward the outer rim as the effect increases. For the flat topology the flux density level is more or less constant throughout the teeth and, therefore, the saturation effect will occur over the entire tooth volume. The effect is that the flat core saturates as a whole and that for the tubular core the material at the inner rim can be saturated whereas the material on the outer rim is not. As local saturation in the core is only affected by the magnetization flux, no disturbance force is introduced that is dependent on the amount of power being transferred. Conversely, the dependency of the magnetic coupling on the phase current is less significant compared to the flat topology, because of the flux density being inversely proportional with the radius.

The core being gapless is beneficial for the magnetic coupling, however, it significantly limits the stroke of the translator due to the mechanical stiffness of the stator. The return path of the primary, which is not shown in Fig. 8.29, has to close beyond the outer radius of the topology. The advantage of a coaxial transformer is the secondary leakage field being zero. The leakage flux from the primary is located in between the primary and the secondary coil. This leakage field penetrates the permanent magnets and causes the eddy current losses there
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to be dependent on the power to be transferred.

8.6.4 Modeling of integrated topologies

Each of the three integrated topologies have their own geometrical particulars that to a large extent dictate which of the presented modeling techniques of part I is best suited for analysis and design. All integrated geometries have a 3D field distribution which rules out the SCM to be applied to any of the presented integrated solutions. On account of the geometrical complexity of the proposed topologies full, 3D modeling can only be obtained by applying the BEM or the TCM.

By decomposing the 3D problems into two 2D problems more flexibility in the choice for the modeling method is obtained. For the flat, integrated topology this yields two subproblems with cross-sectional views similar to those depicted in Fig. 8.26. The quasi-Halbach configuration of the magnet array gives rise to errors when applying the TCM due to the assumptions of a solely perpendicular field component on the edges of the magnets for permeance calculations. The SCM is unable to take the relative permeability of the permanent magnets into account. Therefore, the motor part in the $xz$-plane can best be modeled by the HM and the CET part in the $yz$-plane can either be modeled by the HM or the SCM. Generally, the HM will have a shorter computation time than the SCM for a comparable accuracy.

For the integrated, flux switching topology the use of the HM or the SCM is ruled out for the subproblem in the $xy$-plane. The problem exhibits no periodicity. Even by adopting a periodicity over the full length of the translator the HM proves to be of no avail. The large ratio between the airgap width and the slot width gives rise to numerical issues for the HM. The polygon that describes the domain for the SCM contains too many vertices that lead to unacceptably long computation time. The TCM on the other hand is very well suited for the subproblem in the $xy$-plane, since the airgap does not contain any permanent magnets and both sides are made of soft-magnetic material. Only a small portion of the airgap has to be considered for the airgap permeance calculation. Moreover, nonlinearity of the core can be allowed for. For the modeling of the subproblem in the $yz$-plane the HM is the best option, because it results in the lowest computation time.

The tubular topology can be decomposed in a problem in $rz$-plane and $r\theta$-plane. Problems in the $rz$-plane can in generally not be solved by the SCM. Again, the quasi-Halbach array renders the TCM unfit; leaving the HM is the best option for problems in the $rz$-plane. The subproblem in the $r\theta$-plane can best be solved by the HM. The HM allows the relative permeability of the magnet array to be taken into account and is, moreover, the fastest modeling technique.
Table 8.4: Performance of the three presented integrated topologies with respect to the first 4 criteria of section 8.3 and field orientation

<table>
<thead>
<tr>
<th></th>
<th>flat</th>
<th>flux switching</th>
<th>tubular</th>
</tr>
</thead>
<tbody>
<tr>
<td>decoupled motor and transformer emf</td>
<td>++</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>position independent coupling</td>
<td>+</td>
<td>--</td>
<td>++</td>
</tr>
<tr>
<td>phase current independent coupling</td>
<td>+</td>
<td>--</td>
<td>+</td>
</tr>
<tr>
<td>force independent of transferred power</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>field orientation</td>
<td>orthogonal</td>
<td>in-plane</td>
<td>orthogonal</td>
</tr>
</tbody>
</table>

8.6.5 Remarks for the proposed topologies

For all three proposed topologies electromagnetic decoupling of the transformer and phase coils is obtained. Based on the analyses for the three proposed topologies it becomes clear that for none of the proposed integrated topologies a really position independent magnetic coupling can be realized. The contribution of the position dependent coupling originating from geometrical variations can be eliminated as shown for the flat and tubular topology. When superimposing two time-varying magnetic fields, whether orthogonally orientated or not, interaction between them can in general not be averted. Therefore, a completely position independent magnetic coupling, a phase current independent coupling, and disturbances on the force or torque caused by the transformer loading are always present when integrating CET into a machine. However, by utilizing an orthogonal orientation these cross-coupling effects can be minimized. The cross-coupling effects are significantly less for both the flat and tubular topology with respect to the flux switching topology. The tubular topology is expected to perform better than the flat topology based on the analysis with respect to the increased magnetic coupling as a result of the gapless core, a ‘more orthogonal’ field orientation, and the impact of saturation effects being more localized due to the cylindrical structure. The results of the performance analysis based on the five criteria of the three proposed topologies are tabulated in Table 8.4.
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8.7 Conclusions

The electromagnetic principles of capacitive and inductive CET systems have been addressed and compared. It has been shown that capacitive CET systems have to be operated at high voltage levels and frequencies to obtain the maximum power density. Through inductive coupled CET systems higher power densities are obtained with lower voltage levels and frequencies as compared to capacitive systems on account of the permittivity of free space being five orders of magnitude smaller than the permeability of free space.

The similarities and dissimilarities for rotating and translating inductively coupled CET system in terms of their electromagnetic behavior have been addressed. Translating CET system suffer from higher leakage flux and ohmic losses, especially as the stroke increases, compared to rotating systems, due to the elongated primary. The electromagnetic performance of the CET systems for both types of movement can significantly be improved by applying soft-magnetic material. An overview of different, existing CET system has been given both for add-on and integrated topologies.

Five important electromagnetic criteria have been proposed and explained in detail, which preferably should be met when integrating CET into a synchronous permanent magnet machine. The criteria describe the possible cross-coupling effects that can occur when integrating CET into a machine. It has been described how cross-coupling phenomena can be tackled by exploiting mutual orientation of the transformer and machine coils and the orientation of the vectors of the magnetic fields of the transformer and machine. Not all cross-coupling effects can be eliminated due to the superposition of two time-varying magnetic fields, which cause local fluctuations in the working point on the \( BH \)-curve. These fluctuations affect the machine performance by introducing disturbance torques and forces and the transformer performance by causing a varying magnetic coupling. However, it has been shown that cross-coupling can be minimized by applying an orthogonal orientation of the transformer coils with respect to machine coils and an orthogonal mutual orientation of the magnetic fields. However, an orthogonal field orientation requires the soft-magnetic material to be isotropic.

The applicability of different soft-magnetic materials for integrated topologies has been discussed. Based on the saturation levels, isotropic properties, and frequency dependent iron losses laminated steel, soft-magnetic composite (SMC) material, and ferrite have been compared. Laminated steel is an inappropriate material due to the high iron losses at low frequency and anisotropic electromagnetic behavior that prevents and orthogonal field orientation to be applied, whereas isotropic ferrite is unsuitable due to the low saturation level. SMC is viable as core material for integrated topologies as a result of the saturation level, isotropic electromagnetic behavior, and low electric conductivity.

The impact of the integration of a transformer into the structure of a machine
has been quantitatively estimated by means of simplified, initial sizing equations for machines, transformers, and thermal structures. The reduction in force density as a function of the apparent power to be transferred has been analyzed for orthogonal and in-plane magnetic field orientations. It has been shown that for a fixed topology, wherein the total losses were kept constant, an increased apparent power rating of the transformer generally lowers the force density of the machine. However, the reduction of the force density can be minimized by applying the proper value of the flux density level of the transformer field. Moreover, for an orthogonal field orientation this reduction is less severe compared to the same topology which utilizes an in-plane orientation. Hence, an orthogonal field orientation not only diminishes cross-coupling effects, but also leads to a higher force density level and, thereby, is the preferred magnetic field configuration.

Finally, three topologies for linear permanent magnet machines with integrated CET have been proposed that are eligible for prototyping, viz. the flat, flux switching, and the tubular solution. The three topologies have qualitatively been assessed to which extent the five criteria are satisfied. Also, the modeling methods by means of which the electromagnetic behavior of each of the proposed topologies can most accurately be determined have been addressed. Only the flat and tubular topology exploit the favorable orthogonal field orientation. Furthermore, the magnetic coupling is highly position dependent for the flux switching topology because of the change in geometry due to movement. The tubular topology has been selected for further electromagnetic analysis and prototyping.
Chapter 9

Design of the integrated, tubular topology

The integrated, tubular topology of Fig. 8.29 is selected to be modeled and designed for prototyping. This chapter discusses the electromagnetic and thermal modeling of the integrated tubular topology. Due to the mutual, orthogonal, electromagnetic field distribution of the coaxial transformer for the CET with respect to the tubular synchronous machine (TSM) field two separate harmonic models are derived to quantify the electromagnetic behavior. It is demonstrated in this chapter by way of FEM simulations that this assumption holds. The magnetic loading of the TSM and force profile are obtained by means of an axisymmetric, magnetostatic, harmonic model. The eddy current losses induced in the permanent magnets due to the leakage field of the CET are calculated through a quasi-static harmonic model in the polar coordinate system. Furthermore, the losses in the complete structure are determined, and a steady state thermal equivalent model (TEC) is developed to monitor the temperature distribution in the structure. FEM simulations on the full topology are conducted to not only check the validity of the presented modeling strategy, but also to gain insight in the electromagnetic implications associated with integration of CET into a TSM.

The physical nature and geometrical complexity of the integrated topology greatly restrict the choice of the appropriate electromagnetic modeling method. Preferably, the electromagnetic model should be able to provide the full, 3D, time-varying, electromagnetic field distribution throughout the structure, including eddy current effects and the nonlinearity of the soft-magnetic core material. Only a transient, nonlinear, 3D FEM model provides an accurate electromagnetic field distribution for complex structures that concurrently contain permanent magnets, time-varying currents, nonlinear soft-magnetic and conductive materials. Transient, 3D FEM models are demanding with respect to computational resources,
which lead to time expensive models that are unsuitable as design tool. In this chapter an alternative model is proposed that exploits the decoupling of the CET and TSM functionalities that allows the electromagnetic phenomena of the CET and TSM to be modeled separately; each in their own frequency domain. To minimize the computational effort, both the model for the CET and TSM are harmonic models. The individual models and the overall design procedure are thoroughly treated in the following sections.

9.1 Harmonic model for force calculations

The components which actively contribute to the production of the thrust force on the integrated topology are shown in Fig. 9.1a, where a periodical section of the integrated, tubular topology is depicted. The nonmagnetic shaft is not an active part in an electromagnetic sense from a magnetostatic point of view, it is essential nonetheless to provide mechanical stability to the permanent magnet array. The geometry exhibits two types of periodicity, viz. an actual, physical
9.1: Harmonic model for force calculations

periodicity in the circumferential direction, depending on the number of spatial segmentations of the secondary CET coils, \( N_{\text{cir}} \), and an assumed periodicity in the axial direction. The assumption of the axial periodicity is adopted to make the problem fit for force profile calculations by means of the HM. So, end-effects due to the finite length of the topology along the axial direction are neglected. Although the CET coils do not actively contribute to the force production along the axial direction of the actuator, the slits in which the secondary coils are placed cause a disturbance in the 2D, magnetic flux distribution of the TSM. Hence, when calculating the force profile by means of a 2D, axisymmetric model a reduction factor, \( \beta \), is introduced to account for the force reduction due to the presence of secondary slits in the soft-magnetic core. The 2D, axisymmetric, magnetostatic, harmonic, model and the 2D harmonic model in the polar coordinate system for the calculation of the force reduction factor are addressed in this section. The force results are compared to FEM simulations for model verification.

9.1.1 Axisymmetric motor model

The parameterized, axisymmetric geometry for the harmonic model for the force profile calculation is shown in Fig. 9.1b. The motor type is a permanent magnet, three-phase TSM, where the translator consists of a soft-magnetic core with three slots with concentrated phase-windings and teeth with pole shoes. The stator has two pole pairs in a quasi-Halbach, permanent magnet array configuration mounted on a nonmagnetic shaft.

The harmonic model is divided into nine domains and the soft-magnetic core is assumed to have an infinite permeability. Hence, the nonlinearity of the core and its effects on the performance are neglected in the model. To increase the numerical stability of the method, the width in the axial direction of the domains \( \Omega_k^{(5)} \) is set equal to the width of domains \( \Omega_k^{(4)} \) for \( k \in \{A, B, C\} \). In other words, the axial width of the tooth-tip is assumed to apply for the entire slot domain. The actual width of the phase coil domains is indicated in Fig. 9.1b by the hatched surfaces. The commutation of the phase-coil currents is given by (2.12). To maintain the original electrical loading, the rms phase current density value in (2.12), \( J_{\text{ph}} \), has to be replaced with \( J_{\text{ph}}\alpha_s^{-1} \). Where \( \alpha_s \) is the slot-opening width ratio as depicted in Fig. 9.1b. The current densities in the slot domains are independent of \( z \). This entails that there is no harmonic content in the spatial Fourier series description of the current sources, i.e. \( J_{s_n}^{(5)} = J_{c_n}^{(5)} = 0 \, \text{A m}^{-2} \forall n > 0 \) in (3.2). The quasi-Halbach configuration of the permanent magnets in domain \( \Omega^{(2)} \) is described by the superposition of two magnetization patterns: one square wave shape in the radial direction and one in the axial direction in accordance with (3.1).
Magnetic field solution in the domain

The axisymmetric problem of Fig. 9.1 closely resembles the benchmark problem of section 3.4. The axisymmetric problem has additional domains between the phase current domains and the airgap, and the magnetization pattern is different due to the quasi-Halbach array. Other than that, the same correlation functions still apply and, indeed, the formulation of the matrix equation for the calculation of the coefficients is obtained in exactly the same manner as described in section 3.4. However, since the problem is in the axisymmetric coordinate system instead of a Cartesian one, the magnetic field solution is different. For a magnetostatic problem in an axisymmetric coordinate system, the general expression for the magnetic field solution is given by

\[
A^{(k)}_0(r, z) = A^{(k)}_0(r) + \sum_{n=1}^{\infty} A^{(k)}_{wn}(r) \sin \left( w_n(z) \right) + \frac{A^{(k)}_{nc}(r)}{w_n} \cos \left( w_n(z) \right), \quad (9.1)
\]

with

\[
A^{(k)}_0(r) = A^{(k)}_0 r^{-1} + \frac{1}{2} B^{(k)}_0 r - \frac{1}{4} \mu^{(k)} J^{(k)}_0 r^2,
\]

\[
A^{(k)}_{wn}(r) = a^{(k)}_n I_1(w_n r) + b^{(k)}_n K_1(w_n r) + S^{(k)}_{rwn}(r),
\]

\[
A^{(k)}_{nc}(r) = c^{(k)}_n I_1(w_n r) + d^{(k)}_n K_1(w_n r) + S^{(k)}_{rnc}(r).
\]

The expression for the radial flux density field component is given by

\[
B^{(k)}_r(r, z) = \sum_{n=1}^{\infty} B^{(k)}_{rwn}(r) \sin \left( w_n(z) \right) - B^{(k)}_{rcn}(r) \cos \left( w_n(z) \right), \quad (9.2)
\]

where

\[
B^{(k)}_{rwn}(r) = a^{(k)}_n I_1(w_n r) + b^{(k)}_n K_1(w_n r) + S^{(k)}_{rwn}(r),
\]

\[
B^{(k)}_{rcn}(r) = a^{(k)}_n I_1(w_n r) + b^{(k)}_n K_1(w_n r) + S^{(k)}_{rcn}(r),
\]

and the axial flux density component becomes

\[
B^{(k)}_z(r, z) = B^{(k)}_0(r) + \sum_{n=1}^{\infty} B^{(k)}_{zwn}(r) \sin \left( w_n(z) \right) + B^{(k)}_{zcn}(r) \cos \left( w_n(z) \right), \quad (9.3)
\]

in which

\[
B^{(k)}_0(r) = B^{(k)}_0 - \mu^{(k)} J^{(k)}_0 r,
\]

\[
B^{(k)}_{rwn}(r) = a^{(k)}_n I_0(w_n r) - b^{(k)}_n K_0(w_n r) - S^{(k)}_{rwn}(r),
\]

\[
B^{(k)}_{rcn}(r) = a^{(k)}_n I_0(w_n r) - b^{(k)}_n K_0(w_n r) - S^{(k)}_{rcn}(r).
\]

\( I_n \) and \( K_n \) in (9.1), (9.2) and (9.3) denote the \( n^{th} \) order, modified Bessel functions of the first and second kind, respectively. The expressions for the source terms
equal

\[ S_{zn}^{(k)}(r) = \left( \mu_{zn}^{(k)} - \mu_0 M_{zn}^{(k)} \right) Q_{zn}(r), \]
\[ S_{rn}^{(k)}(r) = \left( \mu_{rn}^{(k)} + \mu_0 M_{rn}^{(k)} \right) Q_{rn}(r), \]
\[ S_{rcn}^{(k)}(r) = \left( \mu_{rcn}^{(k)} + \mu_0 M_{rcn}^{(k)} \right) Q_{rcn}(r), \]
\[ S_{rscn}^{(k)}(r) = \left( \mu_{rscn}^{(k)} - \mu_0 M_{rscn}^{(k)} \right) Q_{rscn}(r), \]

in which

\[ Q_{zn}^{(k)}(r) = K_1(w_n^{(k)} r) \int_{w_n^{(k)} \rho_b^{(k)}}^{w_n^{(k)} r} r' I_1(r') \, dr' - I_1(w_n^{(k)} r) \int_{w_n^{(k)} \rho_b^{(k)}}^{w_n^{(k)} r} r' K_1(r') \, dr', \]
\[ Q_{rn}^{(k)}(r) = K_0(w_n^{(k)} r) \int_{w_n^{(k)} \rho_b^{(k)}}^{w_n^{(k)} \rho_b^{(k)}} r' I_1(r') \, dr' + I_0(w_n^{(k)} r) \int_{w_n^{(k)} \rho_b^{(k)}}^{w_n^{(k)} \rho_b^{(k)}} r' K_1(r') \, dr'. \]

These integrals have no analytical solution and have to be evaluated numerically.

In the lower limits of the integrals the variable \( \rho_b^{(k)} \) appears. Analogous to \( h^{(k)} \) in section 3.2, \( \rho_b^{(k)} \) is the radial position of the inner radius of the \( k \)th domain.

**Estimation of the flux density in the teeth**

After solving the matrix equation the coefficients are known, and by that the flux density distribution in the airgap. Due to the assumed infinite permeability of the core, the spatial flux distribution inside cannot be calculated with the Fourier technique. However, the average flux density in the narrow part of a tooth \( (\rho_6 \leq r \leq \rho_7) \) can be estimated by calculating the amount of flux that penetrates the dashed line between point \( p_1 \) and \( p_2 \) in the radial direction as depicted in Fig. 9.2. The flux is given by the difference of the magnetic vector potential values at the end-points of the line. For an assumed uniform flux density distribution inside the \( k \)th tooth, its value at \( r = \rho_6 \) is calculated via

\[ B_{\text{tan}k} = \frac{A_y^{(4)}(\rho_6, \delta_k) - A_y^{(4)}(\rho_6, \delta_k - w_t)}{(1 - \alpha_p \alpha_s) \tau_c}, \quad (9.4) \]

where

\[ w_t = (1 - \alpha_p \alpha_s) \tau_c. \]
Chapter 9: Design of the integrated, tubular topology

In practice, the core consists of nonlinear, soft-magnetic material. As long as local saturation effects, especially on the pole shoes, are small and the flux density component on the contour of the tooth predominantly has a normal orientation with respect to the contour, (9.4) gives a good approximation of the average flux density level in the tooth. The value of the flux density level in the teeth can be applied to estimate the core losses and to determine the proper value of the phase-coil width coefficient, \( \alpha_p \), as to avoid saturation of the teeth.

**Force profile calculation**

The force on the mover is calculated by adapting the MST integral of (7.18) to the axisymmetric problem. The radius of the cylindrical surface parallel to the \( z \)-axis in the airgap, along which the MST is evaluated, is denoted by \( \rho_g \) for which \( \rho_4 < \rho_g < \rho_5 \). The MST integral becomes

\[
F_z = -\frac{\beta N_{ax}}{\mu_0} N_{mp} \frac{2\pi}{\tau_p} \int_0^{\frac{2\pi}{\tau_p}} \int_0^{\rho_5} B_z^{(2)}(\rho_g, z) B_z^{(2)}(\rho_g, z) \rho_g d\theta d\rho_g.
\]  
(9.5)

Evaluating (9.5) analytically results in an expression independent of the radius of the surface of integration

\[
F_z = \frac{\beta N_{ax} N_{mp} \tau_p}{\mu_0} \frac{1}{\tau_p} \sum_{n=1}^{\infty} \frac{1}{n} \left[ b_n^{(2)} c_n^{(2)} - a_n^{(2)} d_n^{(2)} \right],
\]  
(9.6)

where \( \beta \) is the force reduction factor and \( N_{mp} \) is the number of poles per periodic, axial segment. \( N_{ax} \) denotes the number of \( 4\tau_p \)-wide periodic, axial, segments the full topology consists of. For the topology of Fig. 9.1 the number of poles per periodic, axial segment equal \( N_{mp} = 4 \). It has to be noticed that in the absence of secondary slits the force reduction factor reduces to \( \beta = 1 \). In the next subsection the calculation of the force reduction factor by means of the HM is explained.
9.1.2 Force reduction factor calculation

The reduction in force is a result of the reduced airgap reluctance as a consequence of the presence of the secondary slits. The reduction in the airgap reluctance causes the energy contained within the airgap to be lower when the mmf-drop over the gap is constant. Equations (7.11) and (7.14) show that the relative decrease in force must equal the relative reduction in energy. From the substitution of (6.1) into (7.20) it follows that the energy contained in the airgap is proportional to the amount of flux through it. Hence, the force reduction factor is calculated using

\[ \beta = \frac{\phi_{\text{eff}}}{\phi_{\text{max}}}, \]  

(9.7)

where \( \phi_{\text{max}} \) is the amount of flux crossing the airgap in the absence of slits, and \( \phi_{\text{eff}} \) is the amount of flux crossing the airgap in case slits are present.

The same procedure as described in section 6.3 for airgap permeance calculations is applied for the calculation of the amount of flux crossing the airgap. This approach is valid because the actual, absolute flux values are not of interest, but only the relative decrease of it. To that end, Fig. 9.3 is considered in which \( \rho_{\text{eff}} \) is the effective airgap length. Figure 9.3 shows a 2D, scalar potential model of the airgap shape in the polar coordinate system consisting of two orthogonal domains of unequal width. A unity mmf-drop is imposed over the airgap. The expression for the scalar potential distribution in the \( k^{\text{th}} \) domain of the gap equals

\[ \varphi^{(k)}(r, \theta) = \varphi_{0}^{(k)} + B_{o}^{(k)} \ln r + \sum_{n=1}^{\infty} \left[ a_{n}^{(k)} \left( \frac{r}{\rho_{t}} \right)^{w_{n}^{(k)}} + b_{n}^{(k)} \left( \frac{r}{\rho_{b}} \right)^{w_{n}^{(k)}} \right] \cos(w_{n}^{(k)} \theta), \]

(9.8)
where

\[ w_{n}^{(1)} = nN_{\text{cir}}, \quad \rho_{b}^{(1)} = \rho_{5} - \rho_{\text{eff}}, \quad \rho_{t}^{(1)} = \rho_{5}, \]

\[ w_{n}^{(2)} = \frac{nN_{\text{cir}}}{2\alpha_{t}}, \quad \rho_{b}^{(2)} = \rho_{5}, \quad \rho_{t}^{(2)} = \rho_{6}. \]

After solving the linear set of equations by applying the boundary conditions as discussed in section 3.3, the flux that crosses the airgap is calculated by

\[ \phi_{\text{eff}} = -\mu_{0} \int_{N_{\text{cir}}}^{N_{\text{cir}}} \frac{\partial \varphi^{(1)}(r, \theta)}{\partial r} r \, d\theta, \]

\[ = -\frac{2\pi \mu_{0}}{N_{\text{cir}}} B_{o}^{(1)}. \] (9.9)

The value of the maximum flux in case there are no secondary slits is given by

\[ \phi_{\text{max}} = \frac{2\pi \mu_{0}}{N_{\text{cir}} \ln \left( \frac{\rho_{5}}{\rho_{5} - \rho_{\text{eff}}} \right)}. \] (9.10)

The force reduction factor of (9.7) becomes

\[ \beta = -B_{o}^{(1)} \ln \left( \frac{\rho_{5}}{\rho_{5} - \rho_{\text{eff}}} \right). \] (9.11)

The force reduction factor as derived in the aforementioned is accurate in case the flux in the airgap is solely radially oriented. A closer inspection of (9.6) reveals that (9.11) does not fully account for the complete reduction in force. Changes in the axial component influence the force reduction factor as well. Furthermore, due to the quasi-Halbach array there is no distinctly assignable radial airgap length, since the flux is distributed over the entire inner area. To compensate for these effects, the value of the length of the effective, radial airgap \( \rho_{\text{eff}} \) is to be chosen such as to minimize the error with respect to force calculations obtained by preparatory, magnetostatic, 3D FEM simulations. The 3D FEM simulations are to be conducted before the actual design process is performed. It suffices to conduct a parameter search for two values of \( \alpha_{t} \) and two values of \( \rho_{5} \) in the 3D FEM model to tune \( \rho_{\text{eff}} \).

### 9.1.3 Model verification through the 3D FEM model

Verification of the proposed analytical method is required for validation of assumptions and to quantify the accuracy of the design tool. For that purpose, the results that are obtained by means of the motor model are compared to magnetostatic, 3D FEM simulations. Both the FEM and the HM calculations are
9.1: Harmonic model for force calculations

Table 9.1: Geometrical parameters for verification of the 2D harmonic models

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>topology 1</td>
<td>topology 2</td>
</tr>
<tr>
<td>$\tau_c$</td>
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</tr>
<tr>
<td>$\tau_p$</td>
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<tr>
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<td>0.5</td>
</tr>
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</tr>
<tr>
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<td>40</td>
</tr>
<tr>
<td>$\rho_8$</td>
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</tr>
<tr>
<td>$N_{\text{cir}}$</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

The FEM verification has been executed for three different sets of geometrical parameters with respect to Fig. 9.1. The geometrical values are tabulated in Table 9.1. Furthermore, the value of the secondary slit ratio, $\alpha_t$, has been varied as well to verify the validity of the model for the force reduction factor calculation. It has to be observed that the radial heights of all domains were kept constant during the simulations except for domain $\Omega^{(1)}$. Hence, the simulations show the impact on the force profile for varying shaft radii. The rms current density in the phase coils equals $J_{\text{ph}} = 5 \text{ A mm}^{-2}$, and the remanence of the permanent magnets is set to $B_{\text{rem}} = 1.3 \text{ T}$. The value of the effective airgap length has been found to be equal to $\rho_{\text{eff}} = 2.5 \text{ mm}$. The results for the force profile obtained by the harmonic force model and magnetostatic, 3D FEM simulations are shown in Fig. 9.4, where the lines and markers represent the data from the harmonic and FEM model, respectively. It can be seen from Fig. 9.4 that the results from the harmonic model match the results from the FEM well; both in terms of force ripple amplitude and average value of the thrust force. The error between HM and FEM results is within 2.5% for the cases considered.
9.2 The harmonic model for eddy current calculations

The active components of the integrated CET are shown in Fig. 9.5a. The primary transformer coil is disposed inside the hollow shaft over the full length of the stator. The secondary transformer coil consists of a series connection of a total of $N_{cir}$ coils that are equidistantly distributed over the circumference of the core. The return path of the primary coil closes beyond the outer confines of the translator. The return path of the primary is not shown in Fig. 9.5a. The shaft provides the mechanical support for the primary coil. Where the shaft in the magnetostatic motor model can be neglected in the modeling, it must be taken into account in the quasi-static transformer model when it is made up of conductive material. The leakage flux of the CET that penetrates the shaft gives rise to eddy currents in it that contribute to the losses. The same phenomenon occurs in the conductive permanent magnet array. To quantify the amount of power that is dissipated in the shaft and permanent magnet array, a 2D, quasi-static, harmonic model in the polar coordinate system is formulated. Since the power losses are calculated by means of a 2D model, the effects on the eddy current distribution inside the shaft and permanent magnet array due to the slot-openings in between the pole shoes are neglected. The results from the quasi-static, harmonic model are compared to 2D and 3D, steady state, ac, FEM simulations for validation.
9.2: The harmonic model for eddy current calculations

9.2.1 Transformer model in the polar coordinate system

The parameterized geometry for the harmonic model in the polar coordinate system for the eddy current calculations is shown in Fig. 9.5b. The model has six domains, of which domains $\Omega^{(1)}$ and $\Omega^{(5)}$ consist of air. Domains $\Omega^{(2)}$ and $\Omega^{(6)}$ are the primary and secondary coil-region, respectively. It has to be noted that only the coil side in the hollow shaft of the primary coil is considered in the model. Likewise, only the coil side of the secondary coil in the airgap of the structure is considered. These assumptions are enforced by the ideal boundary condition that forms the outer confine of the model to make it fit for modeling with the HM. All the field and source information beyond that boundary cannot be taken into consideration. The shaft and the magnet array domains are represented by domains $\Omega^{(3)}$ and $\Omega^{(4)}$, respectively. Both domains have a nonzero conductivity. The total current in the entire domain must add up to zero. Hence, the total amount of imposed current flowing in the positive $z$-direction within domain $\Omega^{(2)}$ must equal the amount of current flowing in the negative $z$-direction within domain $\Omega^{(6)}$, i.e. $J_z^{(2)} = -J_z^{(6)}$. So, the contribution of the magnetization current in the primary coil-side to the eddy current distribution is neglected. The currents through the coil sides of the CET are determined by the contin-
uous, apparent power rating and the voltage rating of the CET. As is the case
for the current source distributions in the coil regions of the magnetostatic motor
model of section 9.1.1, the currents in all coil regions are spatially invariant, hence
\[ J_{s_n}^{(k)} = J_{c_n}^{(k)} = 0 \text{ A m}^{-2} \forall n > 0, k \in \{2, 6\}. \]

**Magnetic field solution in the domain**

For quasi-static problems, which are governed by (2.6), the time dependent, mag-
netic vector potential is expressed by the diffusion equation

\[ \nabla^2 \vec{A} - \mu \sigma \frac{\partial \vec{A}}{\partial t} = -\mu \vec{J} - \mu_0 \nabla \times \vec{M}_0, \tag{9.12} \]

where \( \sigma \) is the conductivity of the medium. The nonspatial, independent variable,
being time, can be treated as an additional spatial variable to apply the method of
separation of variables to solve (9.12). In a polar coordinate system the magnetic
vector potential is then written as [11]

\[ \vec{A}(r, \theta, t) = \vec{A}(r, \theta) T(t), \tag{9.13} \]

where \( \vec{A} \) is the magnetic vector potential distribution that only depends on the
geometry of the problem, and \( T(t) \) is the time function. For purely sinusoidal
time-varying sources the time function becomes \( T(t) = e^{j\omega t} \). Successive substitu-
tion of the time function into (9.13) and the result into the diffusion equation of
(9.12) yields the inhomogeneous Helmholtz equation

\[ \nabla^2 \vec{A} - j\omega \mu \sigma \vec{A} = -\mu \vec{J} - \mu_0 \nabla \times \vec{M}_0, \tag{9.14} \]

where \( \vec{J} \) and \( \vec{M}_0 \) denote the amplitude of the sinusoidally varying current density
and magnetization. It can be seen that the Helmholtz equation is time invariant.
This means that the expression for \( \vec{A} \) provides the amplitude of the sinusoidally
time-varying, magnetic vector potential field. For media with zero conductivity
(9.14) reduces to (2.9). It is observed that the magnetization term, \( \vec{M}_0 \), is included
in (9.14) for the sake of completeness and can be deemed artificial, because it only
takes a nonzero value for \( \omega = 0 \text{ rad s}^{-1} \).

The geometry of Fig. 9.5b has an axis of even symmetry at \( \theta = 0 \) rad, which means
that the Fourier series description of the field distribution only contains cosine-
terms. The expression for the vector potential in the \( k^{th} \) domain of Fig. 9.5b reads

\[ A_z^{(k)}(r, \theta) = A_0^{(k)}(r) + \sum_{n=1}^{\infty} \frac{A_0^{(k)}}{w_n^{(k)}} \cos \left( w_n^{(k)} \theta \right), \tag{9.15} \]
The amplitudes of the radial and circumferential flux density components are

\[ \mathbf{B}_{\nu}^{(k)}(r) = a_n^{(k)} \mathcal{I}_{n}^{(k)}(\nu^{(k)} r) + b_n^{(k)} \mathcal{K}^{(k)}_{w_n^{(k)}}(\nu^{(k)} r), \quad \text{for } k \notin \{3, 4\} \]
\[ \mathbf{B}_{\nu}^{(k)}(r) = a_n^{(k)} \mathcal{I}_{n}^{(k)}(\nu^{(k)} r) + b_n^{(k)} \mathcal{K}^{(k)}_{w_n^{(k)}}(\nu^{(k)} r), \quad \text{for } k \in \{3, 4\} \]

where

\[ \nu^{(k)} = \sqrt{j \omega \mu^{(k)} c^{(k)}}. \]

The amplitudes of the radial and circumferential flux density components are respectively expressed as

\[ \hat{B}_{r}^{(k)}(r, \theta) = \sum_{n=1}^{\infty} B_{r, s_n^{(k)}}(r) \sin \left( \frac{\nu^{(k)} r}{n} \right), \quad (9.16) \]
\[ \hat{B}_{\theta}^{(k)}(r, \theta) = B_{0}^{(k)}(r) + \sum_{n=1}^{\infty} B_{\theta, c_{n}^{(k)}}(r) \cos \left( \frac{\nu^{(k)} r}{n} \right), \quad (9.17) \]

in which

\[ B_{r, s_{n}^{(k)}}^{(k)}(r) = \begin{cases} a_n^{(k)} \left( \frac{r}{\rho_{n}^{(k)}} \right)^{w_{n}^{(k)}-1} + b_n^{(k)} \left( \frac{r}{\rho_{n}^{(k)}} \right)^{-w_{n}^{(k)}-1}, & \text{for } k \notin \{3, 4\} \\ a_n^{(k)} \mathcal{I}_{n}^{(k)}(\nu^{(k)} r) + b_n^{(k)} \mathcal{K}^{(k)}_{w_n^{(k)}}(\nu^{(k)} r), & \text{for } k \in \{3, 4\} \end{cases} \]
\[ B_{\theta, c_{n}^{(k)}}^{(k)}(r) = \begin{cases} a_n^{(k)} \left( \frac{r}{\rho_{n}^{(k)}} \right)^{w_{n}^{(k)}-1} - b_n^{(k)} \left( \frac{r}{\rho_{n}^{(k)}} \right)^{-w_{n}^{(k)}-1}, & \text{for } k \notin \{3, 4\} \\ a_n^{(k)} \mathcal{I}_{n}^{(k)}(\nu^{(k)} r) + \mathcal{I}_{n}^{(k)}(\nu^{(k)} r) - \mathcal{K}^{(k)}_{w_n^{(k)}}(\nu^{(k)} r), & \text{for } k \in \{3, 4\} \end{cases} \]
\[ B_{0}^{(k)}(r) = \begin{cases} a_n^{(k)} \mathcal{I}_{1}^{(k)}(\nu^{(k)} r) - B_{0}^{(k)} r^{-1}, & \text{for } k \notin \{3, 4\} \\ a_n^{(k)} \mathcal{I}_{1}^{(k)}(\nu^{(k)} r) - b_n^{(k)} \mathcal{K}_{1}^{(k)}(\nu^{(k)} r), & \text{for } k \in \{3, 4\} \end{cases} \]

The field description for the conductive domains contains an additional unknown for each domain that cannot be solved by only applying the boundary conditions.
as presented in section 3.3. An additional boundary condition is required to completely solve the set of equations. The value of the additional unknown is found by requiring that the net eddy current, \( J_e^{(k)} \), in a conductive domain equals zero. The eddy current in a conductive region is given by

\[
J_e^{(k)} = \sigma^{(k)} \frac{\partial A^{(k)}}{\partial t}.
\]  

(9.18)

Hence,

\[
\int_{\rho_b^{(k)}}^{\rho_i^{(k)}} \int_{-\pi N_{cir}}^{\pi N_{cir}} J_e^{(k)} r d\theta dr = 0 \quad \Rightarrow \quad \int_{\rho_b^{(k)}}^{\rho_i^{(k)}} A_0^{(k)}(r) r dr = 0.
\]  

(9.19)

Equation (9.19) results in the relation

\[
b_0^{(k)} = -\frac{X_I^{(k)}}{X_K^{(k)}} \rho_0^{(k)},
\]  

(9.20)

where

\[
X_I^{(k)} = \int_{\rho_b^{(k)}}^{\rho_i^{(k)}} I_0(\nu^{(k)} r) r dr, \quad \text{and} \quad X_K^{(k)} = \int_{\rho_b^{(k)}}^{\rho_i^{(k)}} K_0(\nu^{(k)} r) r dr.
\]

These integrals have to be evaluated numerically. Equation (9.20) completes the set of equations that allows all the unknowns to be solved. The field quantities have complex values. The physical quantities are obtained by taking the real part of their respective complex values.

**Calculation of the eddy current losses**

When the eddy current distribution within a domain is known, the power density distribution in the domain is calculated through

\[
dP_e^{(k)} = \frac{J^{(k)} \bar{J}^{(k)}}{2\sigma^{(k)}},
\]  

(9.21)

where \( \bar{J}^{(k)} \) denotes the complex conjugate of the eddy current distribution in the \( k^{th} \) domain. The losses in the domain per meter depth are found by integration
9.2: The harmonic model for eddy current calculations

The harmonic model for eddy current calculations is based on the assumption that the eddy current density is proportional to the time derivative of the magnetic field. This allows for the calculation of eddy current losses using an integral equation over the surface of the domain.

of (9.21) over the surface of the domain

\[
P_e^{(k)} = \frac{N_{\text{cir}}}{2\sigma^{(k)}} \int_{\rho_h^{(k)}}^{\rho_i^{(k)}} \int_{-\pi}^{\pi} j^{(k)} f^{(k)} \rho \, \mathrm{d}\rho \, \mathrm{d}\theta,
\]

\[
= \pi \omega^2 \sigma^{(k)} \left[ \rho_i^{(k)} \left| A_0^{(k)} \right|^2 \right. \rho \left. \right] \mathrm{d}r + \frac{1}{2} \sum_{n=1}^{\infty} \rho_{c_n}^{(k)} \left| A_{c_n}^{(k)} \right|^2 \rho \mathrm{d}r.
\]

(9.22)

Neither integral in (9.22) can be solved analytically and, therefore, both of them have to be computed numerically. The total eddy current losses are obtained by multiplying (9.22) with the depth of the domain. The losses in the part of the shaft and permanent magnet array that is not enclosed by the soft-magnetic core of the translator is determined by a 1D harmonic model. The 1D model has the same division into domains as in Fig. 9.5, except that the model only has five domains wherein domain \(\Omega^{(5)}\) extends to infinity. Hence, \(A_{c_n}^{(k)} = B_{c_n}^{(k)} = B_{\theta c_n}^{(k)} = 0 \forall k\) for the 1D model and the depth is equal to the stroke of the machine. In the 1D model the contribution of the eddy current losses due to the return path of the primary is neglected.

9.2.2 Model verification through the FEM

The eddy current loss calculations of the harmonic model in the polar coordinate system as a function of \(\alpha_t\) for the three different topologies of Table 9.1 are compared to both 2D and 3D FEM simulations in Fig. 9.6. All simulations are
carried out under identical conditions. The frequency and the current density in the secondary coil are set to \( f = 10 \text{ kHz} \) and \( J_{\text{sec}} = 5 \text{ A mm}^{-2} \), respectively, and the depth of the 2D models equals \( 4\tau_p \). The results of the harmonic model are in excellent agreement with the results of the 2D FEM simulations. The results of the harmonic model with respect to 3D FEM simulations are overestimated by maximally 5.3% in regard to Fig. 9.6. The relative discrepancy between the models increases with larger values for \( \alpha_s, \alpha_p \), since the influence of the enhanced width of the slot-openings will become more significant. However, the results of Fig. 9.6 justify the use of the 2D harmonic model for the values of \( \alpha_s, \alpha_p \leq 0.4 \) for estimation of the eddy current losses in the conductive shaft and permanent magnet array due to the leakage flux of the CET under nominal load conditions.

Furthermore, the eddy current losses due to the additional magnetization current cannot be allowed for due to the inability of the HM to handle problems for which the total net current in the domain does not add up to zero. Nevertheless, the magnetization current is chosen to be less than 10% of the nominal load current, and the contribution to the losses due to the magnetization flux lies in the same order of magnitude.

It has to be noted that the quasi-Halbach magnetization pattern is obtained by an assembly of individual magnets with different magnetization directions. It is assumed that the eddy currents in the magnet array assembly as a whole are not obstructed by the transitions between magnets with a different magnetization orientation. In other words, possible insulating layers in the form of an adhesive or air in between magnets are neglected. In any case, the presence of insulating layers is ill-defined and among others depends on the production process of the quasi-Halbach array. An adhesive being applied between the magnets or the choice for the coating of the permanent magnets will affect the eddy losses due to the leakage field of the transformer. Insulating layers will have a positive effect and reduce the eddy losses by obstructing the eddy current flow in the axial direction through segmentation. Therefore, the eddy current loss results as calculated in this section apply to the worst case scenario and provide a conservative estimate of the losses.

### 9.3 Electric transformer equations

The number of primary turns are a key parameter of the transformer since they determine the magnetization current and the amount of flux inside the soft-magnetic core. Equation (8.5) can be applied to determine the amount of flux in the core for a given geometry, primary voltage rating, and operating frequency. However, the flux density level in the core is not constant, but has an inversely proportional relation with respect to the radius for a coaxial, circular transformer. The transformer being circular and the T-shaped cross-section of the teeth of the core of
the integrated topology require (8.5) to be modified to
\[ V_p = 3\sqrt{2}\pi \rho c f N_p N_{ax} \hat{B} \left( 1 - \alpha_p \right) \ln \left[ \frac{\rho_7}{\rho_6} \right] + \ln \left[ \frac{\rho_8}{\rho_7} \right], \] (9.23)
where \( \rho \) is the radial position inside the core where the peak flux density equals \( \hat{B} \). The circumferentially oriented flux distribution is assumed to be uniform and the fringing of flux into the pole shoes is neglected. This approach is valid for linear, soft-magnetic core material. The use of nonlinear, soft-magnetic material only affects the spatial flux distribution as a consequence of the permeability distribution, while maintaining the total amount of flux within the core. In other words, saturation due to the superimposed TSM and CET field at the inner rim of the core enforces a redistribution of the transformer flux toward the outer rim of the core to keep the amount of flux constant for a fixed frequency and primary voltage. The redistribution of the transformer flux reverberates in an alteration of the magnetization current.

The magnetization current of the transformer is determined by the self-inductance of the primary coil as stipulated by (8.7). The calculation of the primary self-inductance of the permeance value seen by the magnetization flux is given by
\[ L_p = N_p^2 \mathcal{P}_{mag}. \] (9.24)
Adapting (6.4) to the problem specific geometry yields the magnetization permeance of a ring of height \( h \), with inner radius \( r_i \), and outer radius \( r_o \)
\[ \mathcal{P}_{mag} = \frac{\mu_0}{2\pi} \int_{r_i}^{r_o} \int_0^h \frac{\mu_r(r,z)}{r} \, dz \, dr. \] (9.25)

The T-shaped cross-section of a tooth above a secondary slit can be thought of as a combination of two coaxial, toroidal cores. For linear core material with constant \( \mu_r \) and a uniform flux distribution the integral of (9.25) reduces to \( \mu_r \ln \left( \frac{r_o r_i^{-1}}{r} \right) \).

However, for nonlinear material the spatial distribution of the relative permeability inside the core is determined by the magnetic flux density distribution in the teeth. Since the amplitude of the flux density of the CET is chosen to be significantly lower than the flux density of the TSM, the permeability distribution is only determined by the flux density distribution of the TSM for isotropic, soft-magnetic material. Unfortunately, neither the TSM field distribution in the core, nor the CET field in the core can be determined accurately by means of the harmonic models of sections 9.1 and 9.2.

To still obtain an approximation of the magnitude of the magnetization current, the result of (9.4) is considered. Equation (9.4) provides an estimate of the radial component of the flux density value of the TSM field at radial position \( r = \rho_6 \).

Assuming that there is only a uniform, radial flux density component of the TSM field in the teeth, the flux density distribution elsewhere in a tooth is then expressed as
\[ B(r) = \frac{\rho_6}{r} B_{tsm}. \] (9.26)
Table 9.2: Verification of the calculation of the magnetization permeance seen by the primary coil

<table>
<thead>
<tr>
<th></th>
<th>analytical</th>
<th>2D FEM</th>
<th>3D FEM</th>
</tr>
</thead>
<tbody>
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<td>$\mathcal{P}_\text{mag} , [\mu\text{H}]$</td>
<td>0.88</td>
<td>0.90</td>
<td>0.95</td>
</tr>
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</table>

The relative permeability as a function of the flux density level can be determined from the $BH$-curve of the soft-magnetic material. Substitution of (9.26) in the obtained relative permeability function results in an estimate of the relative permeability distribution inside a tooth. The relative permeability distribution is subsequently substituted into (9.25) and then evaluated numerically. In the back iron of the core in between adjacent teeth ($\rho_7 \leq r \leq \rho_8$) the flux is predominantly axially orientated and, therefore, constant. There, (9.26) becomes

$$B_{bi,k}(r) = \frac{2(1 - \alpha_p) \rho_0 \tau_c}{3(\rho_8^2 - \rho_6^2)} \left( B_{tsm,k+1} - B_{tsm,k} \right),$$  \hspace{1cm} (9.27)

where $B_{tsm,k}$ and $B_{tsm,k+1}$ denote the flux density values at $r = \rho_6$ in the $k^{th}$ and $(k + 1)^{th}$ tooth on both sides of the back iron, respectively.

The calculation of the self-inductance of the primary in the above-mentioned is a coarse approximation due to the lack of accurate information on the field distribution in the core and the disregard of the nonlinearity of the soft-magnetic material in the axisymmetric motor model. To make a conservative estimate of the magnetization current, it is assumed in the calculation that the CET flux in the core is confined to the part of the core for which $\rho_6 \leq r \leq \rho_8$. Again, the contribution of the flux in the pole shoes to the self-inductance is neglected, which leads to a lower value of the self-inductance compared to the actual one.

The results for the magnetization permeance for the geometry of topology 2 of Table 9.1 obtained by means of the analytical method, 2D, and 3D nonlinear FEM simulations are compared in Table 9.2. The value for the magnetization permeance for the 2D FEM simulation is obtained by evaluating (9.25) over the same core surface as the analytical model, i.e. $\rho_6 \leq r \leq \rho_8$.

When the self-inductance is known, the rms-value of the magnetization current is finally obtained by

$$I_{\text{mag}} = \frac{V_p}{\omega L_p},$$  \hspace{1cm} (9.28)

where the primary leakage inductance is assumed to be significantly smaller than the magnetization inductance. By neglecting the contribution of the flux in the pole shoes, in the air of the phase-coil slots, and the flux in the end-teeth of the full topology to the total magnetization flux, the actual magnetization current will be smaller than calculated by means of (9.28). In this way, a safety margin is
9.4: Estimation of the core losses

To monitor the temperature rise of the integrated topology during the design process, all the losses in the system have to be known. The core losses in the soft-magnetic material are composed of a low- and high-frequency component due to the TSM and CET functionality, respectively. The losses of each component are calculated separately by applying the Steinmetz equation of (8.10). Each component is calculated under the assumption that the other component is absent. Next, the losses of the individual components are superimposed. In order to be able to apply (8.10), the Steinmetz coefficients have to be determined in advance. The coefficients are calculated by curve fitting of (8.10a) to measurements of the core losses on ring samples of the isotropic SMC material for varying flux densities and frequencies.

As is the case for the approximation of the magnetization current, the spatial flux density distribution in the core is required for the calculation of the core losses. Obviously, the calculation of the flux density distribution in the core for the TSM as discussed in section 9.3 is also applicable to the calculation of the core losses. The value of the magnetic flux density inside the core of the CET is inversely proportional with the radius. Hence, (9.26) also applies to the flux density distribution of the CET throughout the entire core when assuming a uniform field distribution in the circumferential direction.

The core losses in a single, T-shaped core-section are calculated by integration of (8.10a) over the volume of the section after substitution of (9.26) and/or (9.27) into (8.10a). The core losses due to the high-frequency, CET field in the volume of the entire core become

$$P_{Fe_cet} = \frac{6\pi c_1 r_c \rho_m \hat{B}_{cet}^2}{2 - c_2} \int_N \left[(1 - \alpha_t)(1 - \alpha_s \alpha_p) \left(\rho_6^{2 - c_2} - \rho_5^{2 - c_2}\right)ight]$$

$$+ \left(1 - \alpha_p\right)\left(\rho_7^{2 - c_2} - \rho_6^{2 - c_2}\right) + \left(\rho_8^{2 - c_2} - \rho_7^{2 - c_2}\right),$$

(9.29)

where \(\rho_m\) is the mass density of the core material and \(\hat{B}_{cet}\) the value of the amplitude of the sinusoidally varying CET flux density at position \(r = \rho_6\). Equation (9.29) is only valid in case \(c_2 \neq 2\). In case \(c_2 = 2\), (8.10b) has to be applied in lieu of (8.10a).

The core losses in a single T-shaped tooth due to the low frequency, TSM field are obtained in a similar way. However, the frequency of the magnetic field of the TSM in the core is dependent on the relative speed, \(v\), of the translator with respect to the stator, and the pole pitch, i.e. \(f = \frac{1}{2} v T_p^{-1}\). It has to be noted that
by directly applying Steinmetz’s equation, higher harmonic content in the TSM field is neglected. The core losses due to the first harmonic of the TSM field equal

\[
P_{\text{Fe \_tsm}} = \frac{6\pi c_1 \tau_c \rho_m \rho_b^2 v c_3 \hat{B}_{\text{tsm}} N_{\text{ax}}}{(2 - c_2)(2\tau_p)c_3} \left[ (1 - \alpha_t)(1 - \alpha_s \alpha_p) \left( \rho_b^{2-c_2} - \rho_5^{2-c_2} \right) 
+ (1 - \alpha_p) \left( \rho_7^{2-c_2} - \rho_6^{2-c_2} \right) \right] + 3\pi c_1 \tau_c \rho_m \hat{B}_{\text{bi}} c_2 N_{\text{ax}} \left( \rho_8^2 - \rho_7^2 \right) \left[ \frac{v}{2\tau_p} \right]^{c_3},
\]

(9.30)

where \( \hat{B}_{\text{tsm}} \) and \( \hat{B}_{\text{bi}} \) are the maximum values of the flux density in the teeth and back iron.

Different numerical results for the Steinmetz coefficients, \( c_1, c_2, \) and \( c_3 \) are obtained for the low-frequency TSM losses and the high-frequency CET losses because of the discrepancy in the frequencies and field levels. The core losses of the TSM function are highly dependent on the motion profile of the TSM. When the motion profile is known in advance, (9.30) can be evaluated with an effective speed value. Contrary to the core losses associated with the TSM function, the core losses due to the magnetization flux of the transformer are constant, regardless of the amount of power being transferred. Since the same approximation for the spatial, magnetic flux density distribution in the core is applied as described in section 9.3, the same uncertainties in regard to accuracy are introduced. To make a conservative estimate of the core losses, the diminished magnetic flux density level in the core above the pole shoes due to the fringing of the CET flux into the pole shoes is neglected in (9.29). In lieu, the magnetic flux density is considered to not fringe at all into the pole shoes, but to maintain constant over the full circumference at its maximum level for \( \rho_6 \leq r \leq \rho_8 \). Moreover, the field distribution in the pole shoes is set equal to the field of a fully closed ring with the inner radius of \( r_i = \rho_5 \) and the outer radius of \( r_o = \rho_6 \). This particular approach leads to an overestimation of the core losses due to the high-frequency CET flux. For the losses due to the TSM function a safety margin is introduced by evaluating (9.30) at the maximum relative speed of the translator. Finally, it has to be observed that the core losses due to the CET will be significantly higher than the ones due to the TSM as a result of the higher frequency.

### 9.5 Thermal modeling of the integrated topology

When the power loss density distribution due to the electromagnetic field distribution in the topology is quantified, the Poisson equation for steady state heat conduction has to be solved to obtain the temperature distribution within the structure. In this section the temperature distribution within the structure is determined by means of the thermal equivalent circuit (TEC) method. The method is explained and the results are compared to 3D, steady state, thermal FEM simulations.
9.5.1 Three-dimensional thermal equivalent network

The heat equation for steady state thermal problems with temperature independent medium properties is similar to the Poisson equation for the magnetic scalar potential of (2.11), and is given by

\[ \nabla^2 T = -\frac{q}{\kappa}, \]  

(9.31)

where \( T \) is the temperature, \( q \) the volume power density, and \( \kappa \) the thermal conductivity of the medium [38]. Due to the analogy of (9.31) with (2.11), the temperature distribution can be calculated by means of a TEC model. The TEC can be considered the thermal counterpart of the MEC method of chapter 6, where the scalar magnetic potential, \( \varphi \), and magnetic flux, \( \phi \), in the MEC are equivalent to the temperature, \( T \), and heat flux, \( \phi_{th} = -\kappa \nabla T \), respectively. Furthermore, (6.4) also applies for the calculations of the thermal permeances, where \( \mu_0 \mu_r \) is replaced with \( \kappa \).

To obtain a TEC network the geometry of the problem has to be discretized into flux tubes. The integrated topology is divided into cylindrically shaped flux tubes with a 3D heat flow. The 3D heat flow in the flux tubes is allowed for by decomposing the direction of the flow into permeances with heat flows parallel to the axes of the cylindrical coordinate system as shown in Fig. 9.7. The values of the six thermal permeances in a cylindrically shaped flux tube with respects to its dimensions, as indicated in Fig. 9.7, are obtained through

\[ P_\theta = \frac{2\kappa l_z}{\theta} \ln \left[ \frac{r_o}{r_i} \right], \]  

(9.32a)

\[ P_z = \frac{\kappa \theta}{l_z} (r_o^2 - r_i^2), \]  

(9.32b)

\[ P_{r_o} = \frac{\kappa \theta l_z}{\ln \left[ \frac{2r_o + r_i}{r_i + r_o} \right]}, \]  

(9.32c)

\[ P_{r_i} = \frac{\kappa \theta l_z}{\ln \left[ \frac{r_o + r_i}{2r_i} \right]}. \]  

(9.32d)

Possible power dissipation in a flux tube is taken into account by the current source that is connected between the ground node and the center node of the flux tube. The amount of dissipated power in W in the flux tubes is denoted by \( Q \). The heat dissipation is assumed to occur uniformly over the volume, \( V \), of the tubes. Hence, \( Q \) is calculated as \( Q = qV \), which expressed in the geometrical dimensions becomes

\[ Q = \frac{1}{2} q \theta l_z \left( r_o^2 - r_i^2 \right). \]  

(9.33)

The proper values of \( q \) follow from the eddy current and core loss calculations of sections 9.2 and 9.4, respectively. The remaining copper losses of all coils
are obtained by applying (8.15). The volume power densities in the core, shaft, permanent magnet array, and coil regions are obtained by dividing the obtained losses by the volume of their respective regions.

The heat is removed from the system to the ambient by convection and heat radiation at the outer faces of the cylindrical flux tubes that form the confines of the structure. Removal of heat by means of convection due to airflow in the airgap is neglected. The linearized radiation coefficient and the convection coefficient are mimicked in the TEC by an equivalent convection permeance, $P_{\text{con}}$, between the nodes on the outer faces and the ground node of the TEC. The value of the equivalent convection permeance is calculated by means of

$$P_{\text{con}} = h_{\text{con}}A, \quad (9.34)$$

where $A$ is the area of the outer faces of a flux tube at the transition between the structure and the ambient. Hence, each node on the outer face of a flux tube is connected to the ground node in the TEC through a different, equivalent convection permeance. To obtain the actual temperature in the structure, the potential at the ground node has to be set equal to the ambient temperature.

The divisions of the topology with $N_{\text{ax}} = 1$ into cylindrical flux tubes is depicted in Fig. 9.8, where $s$ is the stroke of the translator. Furthermore, the equivalent convection permeance for one outer face of a flux tube is shown. The model of Fig. 9.8 consists of nine different volume regions with different material properties and power loss densities. The circumferential periodicity is exploited in the TEC model by only modeling $N_{\text{cir}}^{-1}$ of the structure with adiabatic boundary conditions (zero convection at the boundary) at $\theta = 0 \text{rad}$ and $\theta = 2\pi N_{\text{cir}}^{-1} \text{rad}$.
9.5: Thermal modeling of the integrated topology

For model verification the TEC model of Fig. 9.8 is applied to the topologies of Table 9.1 with $\alpha_t = 0.2$. The TEC network is formulated and solved as explained in section 6.5. The simulations are executed with a convection coefficient of $h_{\text{con}} = 12.5 \text{ W m}^{-2} \text{ K}^{-1}$, a stroke of $s = 0.2 \text{ m}$, an ambient temperature of $T_a = 298 \text{ K} (25^\circ \text{C})$, and thermal conductivities and power loss values in the volume regions as tabulated in Table 9.3. Multiply valued cells contain the values for regions that have an internal and external part as indicated by ‘in’ and ‘ex’ in Fig. 9.8. The thermal conductivities of the volume regions are determined by the material properties of the regions. Furthermore, the outer faces of the cylindrical elements that are parallel to the $\theta z$-plane of the coordinate system are assigned adiabatic boundary conditions. For the sole purpose of model verification the values of the power loss, $Q$, are chosen arbitrarily, and are not based on previously described calculation methods of the core, eddy current, or copper losses.

Fig. 9.8: Division into cylindrical, heat flux tube elements of the integrated topology with nine types of volume regions.

Table 9.3: Losses and thermal conductivity per volume region

<table>
<thead>
<tr>
<th>phase</th>
<th>prim</th>
<th>sec</th>
<th>pm (in/ex)</th>
<th>shaft (in/ex)</th>
<th>core</th>
<th>air</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$ [W m K$^{-1}$]</td>
<td>1</td>
<td>1</td>
<td>5/5</td>
<td>16.3/16.3</td>
<td>17</td>
<td>0.026</td>
</tr>
<tr>
<td>$Q$ [W]</td>
<td>3</td>
<td>2</td>
<td>1/3</td>
<td>0.5/1.5</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

9.5.2 Three-dimensional, thermal, FEM verification
Table 9.4: Verification of the TEC model with steady state, 3D, thermal FEM

<table>
<thead>
<tr>
<th>Topology 1</th>
<th>Topology 2</th>
<th>Topology 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEC</td>
<td>FEM</td>
<td>TEC</td>
</tr>
<tr>
<td>$T_{\text{max}} \text{[°C]}$</td>
<td>89.7</td>
<td>87.1</td>
</tr>
<tr>
<td>$T_{\text{mag}} \text{[°C]}$</td>
<td>79.8</td>
<td>78.8</td>
</tr>
</tbody>
</table>

Generally, the maximum allowable temperature inside the structure is determined by the thermal class of the wire insulation of the coils and the temperature dependent demagnetization curve of the permanent magnets. To prevent the permanent magnets from demagnetizing the temperature must not exceed the value at which demagnetization occurs. Therefore, the overall maximum temperature in the structures of the three different topologies and the maximum temperature in the permanent magnet array obtained by the TEC model are compared to steady state, 3D, FEM simulations. The results are listed in Table 9.4. Table 9.4 shows that the TEC model is within 3.0% accurate compared to the 3D, steady state, thermal, FEM model.

9.6 Material properties

Material properties are the quantities that eventually determine the performance and geometrical sizes of the electromagnetic system. Up to now, only the general procedures for the calculating of physical quantities have been addressed and been validated by means of FEM simulations. However, validation of the methods and quantification of the performance require the numerical values of the different material properties to be known. Validations of the methods in the previous sections have been carried out by tacitly taking into account the material properties of the actual materials to be applied in the integrated topology. In this section an overview is provided of the materials and the values of material properties constants the integrated solution consists of.

9.6.1 SMC core

Isotropic, soft-magnetic material for the core permits the superposition of two orthogonally oriented magnetic fields. Apart from being isotropic, the core material for the integrated tubular solution has to exhibit a high saturation level and low core losses over the full range of the operating frequency. Material that satisfies these requirements is soft-magnetic composite (SMC) material. The specific SMC material grade is Somaloy 5P130i. This material has recently been developed by
Fig. 9.9: Material properties of SMC Somaloy 5P130i: $BH$-curve at 50 Hz (a), and core losses (b).

Table 9.5: Steinmetz coefficients for low- and high-frequency core losses of Somaloy 5P130i

<table>
<thead>
<tr>
<th></th>
<th>$1.0 \cdot 10^1 \leq f \leq 1.0 \cdot 10^2$</th>
<th>$5.0 \cdot 10^3 \leq f \leq 1.0 \cdot 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>6.356$\cdot 10^{-2}$</td>
<td>3.740$\cdot 10^{-3}$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>1.711</td>
<td>2.113</td>
</tr>
<tr>
<td>$c_3$</td>
<td>1.037</td>
<td>1.476</td>
</tr>
</tbody>
</table>

Höganäs AB, Sweden. In fact, the combination of the high saturation level and the low conductivity make the efficient integration of CET into the TSM possible. The $BH$-curve, and the core losses at different frequencies and flux density levels of Somaloy 5P130i are depicted in Fig. 9.9. The data of Fig. 9.9 are obtained by measurements on ring samples with a Brockhaus MPG 200 hystograph.

The $BH$-curve of Fig. 9.9a is applied in section 9.3 for the primary inductance calculation. The data of the core losses of Fig. 9.9a are curve fitted to the Steinmetz equation for the calculation of the Steinmetz coefficients. The core losses of the TSM and the CET occur in different domains of the curves of Fig. 9.9b. Therefore, two sets of Steinmetz coefficients are curve fitted to the data of the core losses. Separated fits are more accurate in the local domains of operation of each functionality. The Steinmetz coefficients for the TSM core losses at low frequencies and the CET core losses at high frequencies are given in Table 9.5. The bulk thermal conductivity of the SMC material equals $\kappa = 17 \text{ W m}^{-1} \text{ K}^{-1}$.
9.6.2 Other materials

Rare-earth magnets are applied in the permanent magnet array. The selected material grade is Vacuumschmelze GmbH. The permanent magnets consist of a sintered Neodymium-Iron-Boron alloy (NdFeB). The physical properties of the alloy are presented in Table 9.6.

The shaft consists of stainless steel grade 316. This particular grade of stainless steel is selected due to its nearly nonmagnetic behavior. The physical properties of stainless steel 316 are tabulated in Table 9.7.

All the coils are made of copper. Copper as a bulk material has a high thermal conductivity (around $\kappa = 390 \text{ W m}^{-1} \text{ K}^{-1}$). Only in the direction parallel to the current flow the thermal conductivity reaches these high values. Each turn of the coil is electrically insulated from the others. Furthermore, the coil as a whole is electrically insulated from the core material by additional layers of insulation between the coil and the core. Moreover, air-filled cavities are present in between the windings. The layers of insulation, small contact surfaces, and the air-filled cavities obstruct a steady thermal flow in the directions perpendicular to the current flow. Therefore, the bulk thermal conductivity of the volume of all coils is set to $\kappa = 1 \text{ W m}^{-1} \text{ K}^{-1}$ [97]. The electric conductivity of the copper for the calculation of the ohmic losses in the coils through (8.15) is set to $\sigma = 4.46 \cdot 10^7 \text{ S m}^{-1}$. The conductivity of copper takes this value when the material has a temperature of $T = 383 \text{ K} (110^\circ \text{C})$. 

Table 9.6: Physical material properties of NdFeB-permanent magnets grade VA-CODYM 655 HR

<table>
<thead>
<tr>
<th>$B_{\text{rem}}$ [T]</th>
<th>$\mu_r$ [-]</th>
<th>$\sigma$ [MS m$^{-1}$]</th>
<th>$\kappa$ [W m$^{-1}$ K$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>typical value</td>
<td>1.28</td>
<td>1.05</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Table 9.7: Physical material properties of stainless steel grade 316

<table>
<thead>
<tr>
<th>$\mu_r$ [-]</th>
<th>$\sigma$ [MS m$^{-1}$]</th>
<th>$\kappa$ [W m$^{-1}$ K$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>typical value</td>
<td>$\leq 1.02$</td>
<td>1.35</td>
</tr>
</tbody>
</table>
9.7 Design specifications

Before the actual design process can be conducted the design specifications have to be formulated. The design specifications are determined by physical or geometrical requirements or restrictions. In this section the design specifications are given for the integrated tubular topology.

The most important and defining physical quantities are irrefutably the maximum thrust force and the amount of continuous, apparent power transfer. For the integrated topology the maximum thrust force must at least equal \( F_z \geq 250 \, \text{N} \) at a maximum relative speed of \( v = 2 \, \text{m s}^{-1} \), and the required amount of transferrable apparent power of the CET equals \( S_{\text{cet}} = 800 \, \text{VA} \). The mechanical peak power output of the TSM occurs at maximum force demand at maximum speed. Hence, the peak power demand of the TSM is \( P_{\text{mech}} = F_z v = 500 \, \text{W} \). The continuous power demand of the TSM is determined by the motion profile. More than ample power is transferred to fulfill the continuous power demand of the TSM. The surplus power can be applied to wirelessly supply additional electric apparatus on the translator. To guarantee a sound mechanical integrity of the SMC the thickness of the back iron of the translator and the pole shoes are set to \( \rho_8 - \rho_7 = 5 \, \text{mm} \), and \( \rho_6 - \rho_5 = 4 \, \text{mm} \), respectively. Furthermore, the core loss characteristics of the SMC stipulate a feasible range of the operating frequency of the CET of \( 5 \leq f \leq 10 \, \text{kHz} \) at flux density levels of \( \hat{B}_{\text{cet}} \leq 150 \, \text{mT} \). Finally, the thickness of the hollow, stainless steel shaft, \( \rho_3 - \rho_2 \), is preferably kept as small as possible to minimize the eddy losses without compromising the mechanical stiffness in the radial direction. Mechanical structural analysis have resulted in a minimal thickness of \( \rho_3 - \rho_2 = 2 \, \text{mm} \).

Both the force and apparent power transfer rating have to be met within a fixed volume. The volume is defined by the outer radius of the core, \( \rho_8 \), the pole pitch, \( \tau_p \), and the number of axial, periodic segments, \( N_{\text{ax}} \). These three geometrical parameters are assigned fixed values, i.e. \( \rho_8 = 50 \, \text{mm} \), \( \tau_p = 12 \, \text{mm} \), and \( N_{\text{ax}} = 3 \). The volume of the stator is determined by the stroke, \( s \), and the translator length. The stroke is fixed to \( s = 500 \, \text{mm} \). The outer radius, \( \rho_8 \), is determined by the size of available molds for compaction of the SMC disks.

The remaining physical design specifications are determined by the material properties of the used materials and the environment in which the machine operates. A summary for all the design specification is provided in Table 9.8. The remaining geometrical parameters can be varied freely to obtain a geometry that satisfies the design constraints of Table 9.8.
### Table 9.8: Design specification for the integrated, tubular topology

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum thrust force</td>
<td>$F_z$</td>
<td>$\geq 250$</td>
<td>N</td>
</tr>
<tr>
<td>continuous apparent power transfer</td>
<td>$S_{cet}$</td>
<td>800</td>
<td>VA</td>
</tr>
<tr>
<td>maximum speed</td>
<td>$v$</td>
<td>2.0</td>
<td>m s$^{-1}$</td>
</tr>
<tr>
<td>number of axial segments</td>
<td>$N_{ax}$</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>stroke</td>
<td>$s$</td>
<td>500</td>
<td>mm</td>
</tr>
<tr>
<td>outer core radius</td>
<td>$\rho_4$</td>
<td>50.0</td>
<td>mm</td>
</tr>
<tr>
<td>pole pitch</td>
<td>$\tau_p$</td>
<td>12.0</td>
<td>mm</td>
</tr>
<tr>
<td>phase-coil pitch</td>
<td>$\tau_c$</td>
<td>16.0</td>
<td>mm</td>
</tr>
<tr>
<td>shaft thickness</td>
<td>$\rho_3 - \rho_2$</td>
<td>2.0</td>
<td>mm</td>
</tr>
<tr>
<td>airgap length</td>
<td>$\rho_5 - \rho_4$</td>
<td>1.0</td>
<td>mm</td>
</tr>
<tr>
<td>thickness of pole shoes</td>
<td>$\rho_6 - \rho_5$</td>
<td>4.0</td>
<td>mm</td>
</tr>
<tr>
<td>thickness of back iron</td>
<td>$\rho_8 - \rho_7$</td>
<td>5.0</td>
<td>mm</td>
</tr>
<tr>
<td>frequency range of the CET field</td>
<td>$f$</td>
<td>5-10</td>
<td>kHz</td>
</tr>
<tr>
<td>transformer winding ratio</td>
<td>$N_p N_s^{-1}$</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>amplitude of the secondary voltage</td>
<td>$\hat{V}_s$</td>
<td>200</td>
<td>V</td>
</tr>
<tr>
<td>maximum flux density of the TSM</td>
<td>$\hat{B}_{tms}$</td>
<td>1.3</td>
<td>T</td>
</tr>
<tr>
<td>maximum flux density of the CET</td>
<td>$\hat{B}_{cet}$</td>
<td>150</td>
<td>mT</td>
</tr>
<tr>
<td>maximum temperature</td>
<td>$T_{max}$</td>
<td>110</td>
<td>°C</td>
</tr>
<tr>
<td>maximum permanent magnet temperature</td>
<td>$T_{mag}$</td>
<td>90</td>
<td>°C</td>
</tr>
<tr>
<td>ambient temperature</td>
<td>$T_0$</td>
<td>25</td>
<td>°C</td>
</tr>
</tbody>
</table>

### 9.8 Design by parametric search

The design procedure is performed by means of a parametric search subject to the design specifications. The major advantage of a parametric search over a gradient based optimization routine is the ability to monitor the behavior of the objective function over the entire design space. Furthermore, the global optimum can be identified within the design space, whereas gradient based optimization algorithms only converge to a local optimum near the initial guess of the optimal set of variables. Therefore, the choice for the initial guess has to be well founded. However, a parametric search requires a significantly higher number of model evaluations than a gradient based optimization. Hence, a fast model is preferable to map the objective as function of its arguments with a sufficient resolution within a reasonable amount of time. From part I it has been concluded that the HM provides a good compromise between speed and accuracy and, therefore, is well-suited to be applied in a parametric search routine.

To find the global optimum of the objective for the integrated, tubular topology
the geometrical and physical parameters that are not assigned fixed values in Table 9.8 are free to be varied within their feasible range. However, some of the parameters exhibit mutual dependencies. The dependencies of parameters, and how it is dealt with in the procedure for the parametric search, are addressed. Identification of the influence of some geometrical and physical parameters on the performance of the integrated solution is investigated first, before the procedure and results of the parametric search are presented. The outcome of the search for the topology with the highest force is provided at the end of this section.

9.8.1 Identification of individual parameters on the performance

Before the actual design process is executed the influence of some key geometrical parameters on the overall performance of the integrated topology is examined by varying them independently. In the following analyses topology 2 of Table 9.1 is selected to identify the influence of the key parameters on the force and the combined eddy current losses in the shaft and permanent magnet array due to the leakage flux of the CET. Only the eddy current losses in the stator that is enclosed by the mover are considered. The leakage field is calculated under maximum, nominal power transfer conditions of the CET. Under maximum power transfer conditions the leakage field between the primary and secondary coils is maximum and, therefore, leads to the highest eddy losses.

The results in terms of force and eddy losses are normalized, with respect to their respective maxima. Normalization allows a better quantitative analysis than analysis by comparison of absolute values. Moreover, through normalization of the force of the TSM the presented results become independent of the value of the electric loading and number of axial segments. Normalization of the eddy losses results in independency of the CET-coil current densities, operating frequency, and number of axial segments. Consequently, the presented results are valid for all current densities and frequency ranges. The absolute results can be obtained by scaling of the normalized results.

First, the influence of the Halbach magnet ratio, $\alpha_m$, and the slot-opening ratio, $\alpha_s$, on the normalized thrust and cogging force of the machine is examined, while all the other parameters are kept constant in accordance with Table 9.1 for topology 2. Variation of $\alpha_m$ and $\alpha_s$ only affects the magnetic loading of the TSM. The electric loading of the TSM is kept constant during the search since $\alpha_p$ has been fixed, i.e. the total current through the phase coils is equal for all values of $\alpha_m$ and $\alpha_s$. Since the 2D transformer model does not allow for variations of parameters that affect the geometry in the $z$-direction, no changes on the eddy current losses are observed when $\alpha_m$ and $\alpha_s$ are varied. The normalized thrust and cogging force are shown in Fig. 9.10. It can be seen that the maximum trust force is obtained for geometries with a large width of the slot-opening (lower values of $\alpha_s$). The lowest cogging is obtained when the value of the Halbach magnet ratio
is approximately $\alpha_m \approx 0.3$ or $\alpha_m \approx 0.6$. The electric loading can be increased without compromising the cogging force by increasing the phase-slot width (increase of $\alpha_p$) as long as the width of the slot-opening is kept constant, i.e. $\alpha_s \alpha_p$ is constant. This will, however, lead to a higher flux density level in the teeth.

The influence of the relative circumferential slit-width ratio, $\alpha_t$, and the number of circumferential segments, $N_{\text{cir}}$, and by that the number of secondary slits, on the force reduction factor and eddy current losses is shown in Fig. 9.11. It is evident from Fig. 9.11 that an increase in the number of circumferential segments is beneficial for both the force reduction factor and the eddy losses. By distributing the secondary coil winding over more slits for a fixed value of $\alpha_t$, the increase in the effective, radial airgap length becomes less significant, which eventually results in a higher thrust force. Furthermore, the value of the coefficients $a_n^{(k)}$ and $b_{n}^{(k)}$ in (9.15) become smaller for $n > 0$, when the secondary current is more evenly spread over the circumference of the mover. This leads to lower eddy losses. However, a further increase in the number of slits raises implementation issues due the resulting small width of the individual slits.

The effects of varying the ratio of the stator radius with respect to the outer radius of the translator, $\rho_4 \rho_5^{-1}$, and magnet height, $\rho_4 - \rho_3$, are depicted in Fig. 9.12. The maximum force is obtained when the stator to translator radius is around $\rho_4 \rho_5^{-1} \approx 0.4$. An increase of the magnet height leads to a higher magnetic loading and, consequently, to a higher thrust force. An increase in eddy current losses is observed for larger magnet heights and stator radii. Both effects are caused by the increase in volume of the conductive shaft and PM material. The volume of the conductive regions increases with the square of the radii, whereas the flux
Fig. 9.11: Force reduction factor (a), and total normalized eddy current losses (b) for varying relative slit-width ratio, $\alpha_t$, and the number of circumferential segments $N_{cir}$.

Fig. 9.12: Normalized thrust force (a), and total normalized eddy current losses (b) for varying ratio of the stator to the translator radius, $\rho_4/\rho_8$, and permanent magnet height, $\rho_4 - \rho_3$. 
density level is inversely proportional with the radius. Hence, an energy efficient topology is expected to have the lowest amount of volume of conductive materials.

Finally, the influence of the eddy losses as function of the operating frequency, \( f \), and the current density of the CET coils, \( J_{\text{cet}} \), is shown in Fig. 9.13. An increase in current density leads to a higher AC leakage flux through the conductive regions that increases the losses. It is evident that a higher frequency increases the losses as well. For a fixed geometry the normalized thrust force is not affected by variations in frequency and current density when thermal issues are neglected.

9.8.2 Design procedure and optimal design

In addition to the parameters of Table 9.8 some parameters are fixed as well based on the findings of the previous subsection. Modifications in the axial direction, i.e. variations in \( \alpha_m \), \( \alpha_p \), and \( \alpha_s \) have no influence on the eddy current losses results of the 2D, polar, transformer model. Nevertheless, it has been shown in section 9.2.2 that for small slot-openings the transformer model provides accurate results. Therefore, \( \alpha_m = 0.33 \) and \( \alpha_p \alpha_s = 0.29 \), since these values lead to the lowest cogging force with the highest accompanying thrust force as can be derived from Fig. 9.10. The number of circumferential segments are fixed to \( N_{\text{cir}} = 8 \). It can be seen from Fig. 9.11b that the level lines become nearly vertical for \( N_{\text{cir}} \geq 8 \), which entails that a further increase in the number of circumferential segments does not lead to a significant eddy current loss reduction. To keep the eddy current losses minimal in the shaft, its radial thickness, \( \rho_3 - \rho_2 \), is preferably as small as possible without compromising the mechanical stiffness in the radial direction. So, the thickness of the shaft is fixed to \( \rho_3 - \rho_2 = 2 \text{ mm} \).
There are dependencies between the physical and certain geometrical parameters. During the parametric search either the geometrical or physical parameters can be varied, and the values of the dependent parameters are calculated. Here, the physical parameters, $\hat{B}_t$, $f$, $J_p$, $J_{ph}$, and $J_s$ are varied during the search and the dependent geometrical parameters, $\alpha_t$, $\rho_1$, and $N_p$ are adjusted accordingly. Finally, the flow chart as depicted in Fig. 9.14 is obtained for one cycle of the parametric search routine.

First, the values of the independent parameters for a particular step are loaded. Next, the dependent parameter-values of the primary voltage, primary current, and secondary current are calculated. The magnetization current, which may not be higher than 10% of the nominal primary current, is taken into consideration by a multiplication factor of 1.1. Then, a preliminary value of the number of minimal primary turns is determined through (9.23). The definitive value of the number of primary turns is obtained by rounding up the preliminary value to an integer multiple of the number of circumferential segments. The rounding up of the number of turns ensures a symmetrical field distribution by preventing an imbalance in the number of conductors per secondary slit. Furthermore, the magnetization inductance is increased in that manner, which leads to a lower magnetization current. With the number of primary and secondary turns known, the values of $\alpha_t$ and $\rho_1$ are calculated next, and checked whether they are feasible. If so, the routine continues and the axisymmetric motor model and the force reduction model are evaluated. Subsequently, the flux density level in the teeth and the magnetization current of the primary are determined via (9.4) and (9.28), respectively.

What follows is a second feasibility check. When passed, all the losses are calculated by successively evaluating the quasi-static, harmonic, transformer model in the polar coordinate system for the eddy current losses, (8.15) for all coils, (9.29) for the core losses due to CET operation, and (9.30) for the core losses due to TSM operation. Based on the losses calculation the temperature distribution throughout the structure is calculated by means of the 3D TEC model. Lastly, the final feasibility check with respect to temperature is executed. When the final feasibility check is passed, all the input and output parameters are saved and the routine is repeated for the next set of input parameters. When a topology fails to pass a feasibility check, the current cycle is aborted and the next parametric step is initiated.

The parametric search has been conducted to determine the feasible topology with the highest thrust force. The values of the geometrical parameters for the optimal design are listed in Table 9.9. The physical quantities, as calculated by the combined models, are tabulated in Table 9.10. It has to be observed that there is little discrepancy between the maximum temperature of the permanent magnet array and the overall maximum. This is caused due to the additional sources of heat on the stator. Apparently, the maximum allowable magnet temperature determines the thermal loading of the structure. Additional heat sources are absent in a conventional TSM where the thermal loading is predominantly determined by the phase-coil temperature.
load independent parameters

\[ V_p = \frac{N_p V_s}{N_s \sqrt{2}} \]

\[ I_p = 1.1 \frac{S_{sc}}{V_p}, \quad I_s = \frac{S_{sc}}{V_s} \]

calculation of \( N'_p \) for \( \rho = \rho_6 \) via (9.23)

\[ N_p = N_{cir} \left\lceil \frac{N'_p}{N_{cir}} \right\rceil \]

\[ A_p = \frac{N_p I_p}{J_p} \Rightarrow \rho_1 = \sqrt{\rho_2^2 - \frac{A_p}{\pi}} \]
\[ A_s = \frac{N_s I_s}{J_s} \Rightarrow \alpha_t = \frac{A_s}{\pi(\rho_2^2 - \rho_2^5)} \]

\[ \text{[\(\rho_1 \in \mathbb{R}\) \& \(\alpha_t \leq 0.4\)]} \]

2D motor model

calculation of \( \beta \)

Fig. 9.14: Flow chart for the parametric search procedure for optimization.
Table 9.9: Optimal geometry for the integrated, tubular topology obtained by parametric search

<table>
<thead>
<tr>
<th>description</th>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Halbach magnet width ratio</td>
<td>$\alpha_m$</td>
<td>0.33</td>
<td>-</td>
</tr>
<tr>
<td>phase-slot width ratio</td>
<td>$\alpha_p$</td>
<td>0.62</td>
<td>-</td>
</tr>
<tr>
<td>slot-opening width ratio</td>
<td>$\alpha_s$</td>
<td>0.46</td>
<td>-</td>
</tr>
<tr>
<td>relative secondary slit-width ratio</td>
<td>$\alpha_t$</td>
<td>0.23</td>
<td>-</td>
</tr>
<tr>
<td>inner primary radius</td>
<td>$\rho_1$</td>
<td>8.6</td>
<td>mm</td>
</tr>
<tr>
<td>outer primary radius</td>
<td>$\rho_2$</td>
<td>14.0</td>
<td>mm</td>
</tr>
<tr>
<td>outer shaft radius</td>
<td>$\rho_3$</td>
<td>16.0</td>
<td>mm</td>
</tr>
<tr>
<td>outer magnet array radius</td>
<td>$\rho_4$</td>
<td>20.0</td>
<td>mm</td>
</tr>
<tr>
<td>outer airgap radius</td>
<td>$\rho_5$</td>
<td>21.0</td>
<td>mm</td>
</tr>
<tr>
<td>outer pole shoe radius</td>
<td>$\rho_6$</td>
<td>25.0</td>
<td>mm</td>
</tr>
<tr>
<td>outer slot radius</td>
<td>$\rho_7$</td>
<td>45.0</td>
<td>mm</td>
</tr>
<tr>
<td>outer core radius</td>
<td>$\rho_8$</td>
<td>50.0</td>
<td>mm</td>
</tr>
<tr>
<td>end-tooth width</td>
<td>$l_{end}$</td>
<td>3.7</td>
<td>mm</td>
</tr>
<tr>
<td>number of circumferential segments</td>
<td>$N_{cir}$</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>number of axial segments</td>
<td>$N_{ax}$</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>axial length of the translator</td>
<td>$l_{tra}$</td>
<td>151.4</td>
<td>mm</td>
</tr>
</tbody>
</table>

Table 9.10: Values of physical parameters for the optimal design of the integrated, tubular topology

<table>
<thead>
<tr>
<th>description</th>
<th>parameter</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximum thrust force</td>
<td>$F_z$</td>
<td>330</td>
<td>N</td>
</tr>
<tr>
<td>frequency of the CET field</td>
<td>$f$</td>
<td>5.0</td>
<td>kHz</td>
</tr>
<tr>
<td>max. TSM flux density at $r = \rho_6$</td>
<td>$B_{tsm}$</td>
<td>0.93</td>
<td>T</td>
</tr>
<tr>
<td>max. CET flux density at $r = \rho_6$</td>
<td>$B_{cet}$</td>
<td>0.1</td>
<td>T</td>
</tr>
<tr>
<td>rms phase current density</td>
<td>$J_{ph}$</td>
<td>2.2</td>
<td>A mm$^{-2}$</td>
</tr>
<tr>
<td>rms primary current density</td>
<td>$J_p$</td>
<td>1.0</td>
<td>A mm$^{-2}$</td>
</tr>
<tr>
<td>rms secondary current density</td>
<td>$J_s$</td>
<td>2.4</td>
<td>A mm$^{-2}$</td>
</tr>
<tr>
<td>total power loss due to TSM operation</td>
<td>$P_{tsm}$</td>
<td>24.1</td>
<td>W</td>
</tr>
<tr>
<td>total power loss due to CET operation</td>
<td>$P_{cet}$</td>
<td>50.9</td>
<td>W</td>
</tr>
<tr>
<td>maximum temperature</td>
<td>$T_{max}$</td>
<td>89.9</td>
<td>°C</td>
</tr>
<tr>
<td>maximum magnet temperature</td>
<td>$T_{mag}$</td>
<td>88.1</td>
<td>°C</td>
</tr>
</tbody>
</table>
Finally, the width of the end-teeth has to be determined to minimize the additional end-cogging force component as a result of the finite length of the mover. The outer sides of the end-teeth are flat and their shape is obtained by extruding the outer surface of the end-teeth by $l_{\text{end}}$ as depicted in Fig. 9.15. The optimal width is obtained by a parametric search of the end-tooth width in a 2D, nonlinear, magnetostatic FEM model in which the finite length of the mover is taken into consideration. The optimum width of the end-teeth is found to be equal to $l_{\text{end}} = 3.7\text{mm}$.

9.9 Additional finite element analyses and model verification

So far, only the individual modeling methods for force, magnetization current, and different loss calculations have been addressed and been validated by means of separate FEM simulations with identical model assumptions in regard to material properties and electromagnetic boundary conditions. Furthermore, the design procedure of the parametric search has been discussed and applied to determine the optimal values of geometrical and physical quantities for the structure with the highest thrust force subject to the design specifications. However, the validity of the use of separate electromagnetic models for the steps of the design procedure has implicitly been assumed, and the impact of the core material being nonlinear has been neglected in the force calculations due to the limitations inherent to the use of the HM. Moreover, the effect of the alteration in the permeability distribution throughout the core on the force on account of the additional CET flux density component has been assumed to be negligible. Though, the plausibility of this assumption is corroborated by the results of Fig. 8.18, an explicit model verification for nonlinear material has not yet been provided. In this section the
9.9: Additional finite element analyses and model verification

9.9.1 Magnetic field distribution in the nonlinear core

The change in the relative permeability distribution in terms of percentage within the SMC core under the influence of the circumferentially orientated CET flux is investigated through a 3D, magnetostatic, FEM model. To that end, the discrepancy of the spatial distribution of the relative permeability for the situations in which there is no CET flux ($\vec{B}_{\text{cet}} = 0$ T), and for which the amplitude of the circumferential CET flux density component equals $\vec{B}_{\text{cet}} = 0.1$ T is calculated. The results are visualized in Fig. 9.16, where Fig. 9.16a and Fig. 9.16b depict the discrepancy on a cross-section through the center of a pole shoe, and a cross-section through the center of a secondary slit, respectively. Both cross-sections are parallel to the $rz$-plane. It is noteworthy to observe that for both cross-sections the relative permeability decreases ($\Delta \mu_r < 0$) in certain areas and increases ($\Delta \mu_r > 0$) in others. As is to be expected, the largest decrease in permeability occurs directly above the secondary slit due to the fringing of both the TSM and CET flux and, in addition, both fields having maximum values due to the inversely proportional

Fig. 9.16: Relative change of the permeability distribution in the nonlinear core as a result of the increase of the circumferential CET flux component from $\vec{B}_{\text{cet}} = 0$ T to $\vec{B}_{\text{cet}} = 0.1$ T in the cross-section through the center of a pole shoe (a), and the center of the secondary slit (b).
Fig. 9.17: Amplitude of the circumferential flux density component, $\hat{B}_\theta$, in the core in the cross-section through the center of the pole shoe for $B_{cet} = 0$ T (a), and $B_{cet} = 0.1$ T (c), and in the cross-section through the center of the secondary slit for $B_{cet} = 0$ T (b), and $B_{cet} = 0.1$ T (d).

The relationship with respect to the radius. The patches of increased permeability may be unexpected at first, but it is caused by the initially increasing slope of the $BH$-curve for lower values of the magnetic field strength, before it reaches its peak value and finally decreases monotonically with increasing magnetic field strength. Apparently, for the areas of increased permeability the length of the vector of the total magnetic field, that is obtained by the vector sum of the CET and TSM field components, is still smaller than the value for which the relative permeability reaches its maximum.

The influence of the redistribution of the relative permeability on the circumferen-
ential CET flux density component is investigated next. Furthermore, the decoupling of the magnetic fields of the different functionalities is demonstrated. In Fig. 9.17 the amplitude of the circumferential component of the flux density, $\hat{B}$, is shown on the same cross-sections as Fig. 9.16 for $\hat{B}_{\text{cet}} = 0$ T and $\hat{B}_{\text{cet}} = 0.1$ T. In neither Fig. 9.17a nor Fig. 9.17b a circumferential component is present in the total magnetic flux density field. Since the cross-section through the center of the secondary slit also forms a plane of symmetry for the secondary coil, Fig. 9.17b proves that no emf can be induced due to a time-varying TSM flux; and both functionalities are indeed decoupled. It can be seen from Figs. 9.17c and d that the purely $r^{-1}$ relationship of the CET flux is perturbed. The perturbation is caused by the pre-biasing effect of the TSM flux that causes the relative permeability distribution within the core. This effect is clearly visible in Fig. 9.17c, where the value of the circumferential field component at $r = \rho_6$ is higher in the two center-teeth than in the two outer half-teeth. Furthermore, the fringing of the CET flux is apparent from comparing the values of the magnetic flux density of Figs. 9.17c and d at the same radial positions. The same amount of flux of Fig. 9.17c is, as it were, squeezed through the smaller core cross-section of Fig. 9.17d, resulting in higher flux density values. Furthermore, it can be seen that the flux density at $r = \rho_6$ is slightly lower than 0.1 T in Fig. 9.17c and slightly higher in Fig. 9.17d due to the fringing effect.

To numerically express the change in relative permeability, rather than visually, the change in the average, relative permeability over the surfaces of the cross-sections is evaluated for varying $\hat{B}_{\text{cet}}$. The results are shown in Fig. 9.18. From Fig. 9.18 it is concluded that the average permeability in the cross-section through the center of the slit is lower than in the cross-section through the pole shoe. The discrepancy is ascribed to the lower part of the teeth at $r = \rho_6$ being saturated due to the TSM flux component. That the lower average, relative permeability is only ascribed to the saturation of the TSM flux is clear from the average,
relative permeability still increasing initially with increasing magnetization flux density, $\hat{B}_{\text{cet}}$. From the results of Fig. 9.18 it might be concluded that operation at $\hat{B}_{\text{cet}} \approx 0.5 \, \text{T}$ will lead to the highest magnetization induction, since the average, relative permeability is at its maximum. That this statement is erroneous is obvious from (9.25). Equation (9.25) shows that no judgment can be made with respect to the magnetization inductance based on the calculation of the average, relative permeability, since the radius dependent integrand causes the result of the integral to be dependent on the actual spatial distribution of the permeability in the core. Furthermore, the maximum value of the average, relative permeability is only valid for the particular values of the relative position and phase current density of the TSM for which the curves of Fig. 9.18 are obtained.

### 9.9.2 Cross-coupling effects between functionalities

The alteration in the relative permeability distribution and the redistribution of the circumferential CET flux component reverberates in cross-coupling effects between the motor and transformer functionality. First, the dependency of the flux through the secondary coil on the relative displacement of the translator with respect to the stator is investigated for different levels of electric loading of the TSM. The amount of flux that is linked with the secondary coils is calculated by integration of the circumferential flux density component that perpendicularly penetrates the cross-section through the center of the slit. This calculation is conducted for different relative displacements and phase current densities. The results are displayed in Fig. 9.19.

The average value of the flux linkage initially increases with increased electric loading and decreases for $J_{\text{ph}} = 4 \, \text{A mm}^{-2}$. The average flux, or flux linkage per secondary turn, for $J_{\text{ph}} = 0 \, \text{A mm}^{-2}$ equals $\bar{\phi}_0 = 133 \, \mu\text{Wb}$. By choosing $\bar{\phi}_0$ as a
reference value, the maximum relative variation in the flux linkage is calculated by
\[
\Delta \phi = \frac{\phi_{\text{max}} - \phi_{\text{min}}}{\bar{\phi}_0} \cdot 100\%.
\]
(9.35)

where \(\phi_{\text{max}}\) is the maximum value of the flux linkage that occurs at \(\Delta z = -2.5 \text{ mm}\) and \(J_{\text{ph}} = 3 \text{ A mm}^{-2}\), and \(\phi_{\text{min}}\) is the minimum value of the flux linkage that occurs at \(\Delta z = \pm 6 \text{ mm}\) and \(J_{\text{ph}} = 0 \text{ A mm}^{-2}\) in Fig. 9.19. For Fig. 9.19 the maximum relative variation in the flux linkage as function of the relative displacement and electric loading of the TSM equals \(\Delta \phi = 3.2\%\). If the phase current density is limited to the value of Table 9.10, i.e. \(J_{\text{ph}} = 2.2 \text{ A mm}^{-2}\), the variation will be lower, on account of \(\phi_{\text{max}}\) being lower.

In addition to the change in average value of the flux linkage under different electrical loading of the TSM, there is an apparent phase shift discernible in Fig. 9.19. Both the phase shift and the change in average value of the flux linkage are attributed to the spatial reordering of the areas in the core where the relative permeability increases and the areas where it decreases.

Next, the cross-coupling effect of the magnitude of the magnetization flux density on the force profile of the TSM is investigated. The force profiles of the TSM for different values of the magnetization flux density, \(B_{\text{cet}}\), of the CET are portrayed in Fig. 9.20. Initially the force profile is not significantly influenced by an increase of the magnetization flux density in the core. Although the force reduces with increasing magnetization flux density, the force reduction becomes more significant for \(B_{\text{cet}} > 0.4 \text{ T}\). However, even for \(B_{\text{cet}} = 1.0 \text{ T}\) the relative reduction in the average thrust force is only 0.9%. That increasing the magnetization flux has little impact on the force profile is explained by analysis of the flux distribution of the CET with respect to that of the TSM. The shape of the force profile is predominantly determined by the magnetic field of the TSM in the airgap. The
field distribution in the airgap is particularly sensitive to saturation of the core on the edges of the pole shoes. If these edges are already saturated in the absence of any CET flux, the contribution of the CET flux, when it is present, has negligible impact on the local permeability distribution at the edges of the pole shoes. Moreover, the portion of the flux of the CET that fringes into the pole shoes mainly flows in the center of the pole shoes and not at the edges of the bulges. It has to be noticed that the force profile oscillates between the values that correspond with the force profile for $\hat{B}_{cet} = 0 \, \text{T}$ and the amplitude of $\hat{B}_{cet}$. Due to the orthogonal orientation of the TSM and CET fluxes a positive and negative sway of $\hat{B}_{cet}$ yield the same force reduction. Therefore, the force profile oscillates as a whole with double the frequency of the operating frequency of the CET.

The average force of the profiles of Fig. 9.20 is significantly lower than the value for the average thrust force, $F_z$, of Table 9.10. The discrepancy between the values is a result of the assumption of the core being linear, infinitely permeable, and the neglect of end-effects in order for the HM to be applicable. Under these assumptions the HM is accurate, as can be seen from Fig. 9.21, where the HM results are compared between different FEM simulations for the optimal topology. First, the force profile calculated with HM is compared to a linear, periodic, 3D, FEM model of the TSM without CET. Next, the results are shown of the same FEM model in which only the core material has been changed from linear material to SMC Somaloy 5P130i. Already, a decrease of 9.3% is observed in the calculated average thrust force. Furthermore, the peak-to-peak force ripple for the HM equals 3.9% of the average thrust force, whereas the force ripple for the nonlinear, periodic, 3D, FEM model is found to be 7.8% of the average thrust force. Finally,
9.10: Comparison with an add-on solution

The cross-coupling effects associated with integration, as discussed in the previous sections, enforce concessions on both the force density of the TSM and power transfer density of the coaxial transformer. These concessions are mainly attributed to thermal limitations. To quantify the increase in volume as a result of integration the two functionalities have also been design separately to compare to overall volume of the separated solution with the integrated one.

Table 9.11: Comparison of the results obtained by different models in terms of average force, ripple force, and calculation time.

<table>
<thead>
<tr>
<th>modeling method</th>
<th>average force [N]</th>
<th>force ripple [%]</th>
<th>calc. time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HM</td>
<td>330</td>
<td>3.9</td>
<td>3</td>
</tr>
<tr>
<td>FEM (per. lin.)</td>
<td>331</td>
<td>3.6</td>
<td>642</td>
</tr>
<tr>
<td>FEM (per. nl.)</td>
<td>298</td>
<td>7.8</td>
<td>1885</td>
</tr>
<tr>
<td>FEM (full)</td>
<td>300</td>
<td>13.6</td>
<td>28026</td>
</tr>
</tbody>
</table>

The force profile obtained by the full FEM model that includes the nonlinearity of the core, end-effects, and the circumferential flux density component of the CET of $\hat{B}_{\text{cet}} = 0.1\,\text{T}$ is portrayed. It can be seen that the force ripple increases even more to 13.6% on account of the end-effects. However, the average thrust force remains the same with respect to the nonlinear, periodic, FEM model. The low maximal permeability of the SMC is the reason for the large error with respect to the average thrust force. The accuracy of the model for the force calculation can be increased by replacing the axisymmetric HM with a nonlinear, axisymmetric, FEM model at the expense of significantly increased computational efforts.

The results of the force profiles for 25 different positions of the translator with respect to the stator of Fig. 9.21 are tabulated in Table 9.11 to indicate how the loss in accuracy is compensated for by a significant reduction in calculation time. Table 9.11 underlines how impractical the more accurate FEM models would be for design routines, which require numerous model evaluations, on account of the large calculation time. In comparison to the HM, the design process would approximately take 214, 628, and 9342 times longer to finish for the periodic linear, periodic nonlinear, and full FEM models, respectively, for the same number of model evaluations. It, furthermore, has to be noted that the calculation times as provided in Table 9.11 for the FEM models only hold for magnetostatic force calculations in which eddy current losses are not taken into account. Also incorporating the eddy current loss calculation of the CET in the FEM models would result in even more computational expensive transient, nonlinear, 3D, FEM simulation.
Separate tubular motor design

For the design of the separated TSM the model of section 9.1 is applied where the force reduction factor is equal to $\beta = 1$. Furthermore, all geometrical parameters that affect the sizes of the TSM parallel to the axial direction are fixed to the optimal values of the integrated topology as listed in Table 9.9. Also, the radial height of the permanent magnets, back iron, and tooth-tips are fixed to the values of Table 9.9. Hence, only the outer radius, $\rho_8$, of the translator and the radial height of the phase coils, $\rho_7 - \rho_6$, are varied in the parametric search. Equation (8.16) is applied to estimate the temperature distribution in the TSM. The maximum allowable temperature is set to the maximum value of the temperature of the permanent magnets for the integrated solution, namely $T = 90^\circ$C. The result of the parametric search is depicted in Fig. 9.22a. The black dot indicates the minimal outer radius for which the average force is equal to the optimal value of the integrated topology of Table 9.10, i.e. $F_z = 330$ N. The smallest volume is obtained for the topology with an outer radius of $\rho_8 = 37.1$ mm. For this value the total volume occupied by the translator equals $V_{t\text{sm}} = 6.55 \cdot 10^{-4}$ m$^3$. 

Fig. 9.22: Average force for the separate TSM as a function of $\rho_8$ and $\rho_6$ (a), and the maximum temperature of the separate coaxial transformer as a function of inner, $\rho_i$, and outer radius, $\rho_o$, of the core (b).
Separate coaxial transformer design

By separating the sliding, coaxial, transformer from the motor, the operating frequency of the CET can be increased, and as a consequence its volume reduced, when a ferrite core is applied. The coaxial transformer is chosen to be optimized for the smallest core volume rather than highest efficiency. This entails that the peak flux density in the core is chosen at its maximal value at which no saturation occurs. Furthermore, the axial length of the transformer is chosen to be equal to the axial length of the translator, by which it is mechanically supported, to keep the primary leakage inductance as low as possible. The ferrite grade is 3C93 manufactured by Ferroxcube [23]. For this material the maximum peak flux density level equals \( \dot{B}_{\text{cet}} = 0.3 \, \text{T} \). The operating frequency is chosen to be \( f = 25 \, \text{kHz} \) and the voltage levels are as given in Table 9.8. The first step in the design is the calculation of the number of turns through a modified version of (9.23) for a given geometry of the core. Next, the magnetization inductance and current are determined by means of (9.24) and (9.28), respectively. The current density through the coils is stipulated by the nominal load and the ohmic losses are given by (8.15). The core losses are found by modification of (9.29) to the expression for a single, closed ring. The temperature distribution is monitored through (8.16). The core geometry is varied by varying the inner \( (\rho_i) \) and outer radius \( (\rho_o) \) of the ferrite ring-core. The temperature in the core is shown in Fig. 9.22b for different values of \( \rho_i \) and \( \rho_o \). If the temperature inside the transformer is not allowed to exceed \( T = 100 \, \text{°C} \), then the dot indicates the point for the transformer with the smallest outer radius and, therefore, smallest volume. The smallest outer radius equals \( \rho_o = 7.8 \, \text{mm} \) which corresponds to the volume occupied by the moving secondary that equals \( V_{\text{cet}} = 2.89 \cdot 10^{-5} \, \text{m}^3 \).

Comparison of the integrated and add-on solution

The volume occupied by the translator of the integrated topology equals \( V_{\text{int}} = 1.19 \cdot 10^{-3} \, \text{m}^3 \), whereas the total volume of the moving parts of the add-on solution is \( V_{\text{add}} = V_{\text{tsm}} + V_{\text{cet}} = 6.84 \cdot 10^{-4} \, \text{m}^3 \). The integrated solution takes up 74% more space in terms of volume than the add-on solution. However, it has to be noted that by only considering the volume of the individual components, and adding them to obtain the total volume occupied by add-on system, does not necessarily represent the actual volume being occupied. The manner in which the mechanical connection is realized between the motor and the CET can result in a significantly larger volume as a whole than the sum of the individual parts. For example, sufficient clearance between the transformer and motor parts is required to ensure the amount of heat removal from both parts as assumed in the foregoing paragraphs. To make a fairer comparison more information on the structural configuration of the add-on system has to be known as well. Moreover, the reduction in the overall volume is mainly attributed to the use of ferrite, which allows for frequencies to be applied that are significantly higher than 5 kHz. Hence, no concessions have to be made on account of a soft-magnetic material that is
suited for both functionalities. Furthermore, the add-on solution does not suffer from eddy current losses in the permanent magnets due to the leakage flux of the transformer, which also contribute the reduction in volume as a result of less demanding requirements with respect to heat removal from the structure.

9.11 Conclusion and remarks

A general design procedure for the integrated, tubular topology has been presented in this chapter. An alternative to the computationally expensive, transient, nonlinear, 3D FEM as modeling tool has been proposed to identify the electromagnetic behavior of the integrated topology. The decoupling of the TSM and CET functionality has been exploited in the alternative model by modeling each functionality separately by means of the HM; each in its own frequency domain. Furthermore, the temperature distribution in the structure has been determined by means of a steady state, TEC model. All the individual steps in the general design procedure have been discussed in detail and, where possible, model verification of the individual modeling steps by means of FEM simulations have been carried out. The model verifications have been executed under identical electromagnetic assumptions as adopted for the individual models. The model has been applied to identify the implications of the integration of CET into a TSM. Furthermore, the model has been applied in a parametric search routine to find the optimal geometry in terms of the highest average thrust force. Additional 3D FEM simulations have been conducted to map the electromagnetic behavior of the integrated, tubular topology due to the nonlinear soft-magnetic material. The repercussions of the assumption of the core being infinitely permeable have been addressed.

It has been demonstrated that the force prediction by means of the axisymmetric HM in conjunction with the force reduction model in the polar coordinate system provides an accurate estimation of the force profile when the core material is assumed to be linear. The tuning of the force reduction model requires preparatory FEM simulations. The obtained results are accurate within 2.5%. However, when the calculated force profile is compared to the FEM simulation in which the $BH$-curve of the core material and the end-effects are taken into consideration, the error in the average thrust force is 9.3%. The discrepancy is attributed to the low relative permeability of the SMC Somaloy 5P130i. Furthermore, the force ripple increases from 3.9% to 13.6% due to the combined effect of the finite length of the translator and local saturation on the edges of the pole shoes due to the nonlinearity of the soft-magnetic material. The accuracy of the force calculation can be improved by substituting the axisymmetric HM with a nonlinear, periodic, 2D FEM model at the expense of a significantly increased computation time.

The eddy current losses in the permanent magnets and stainless steel shaft due to the high-frequency, leakage flux of the CET have been determined by means of a quasi-static HM in the polar coordinate system. In the model the slot-openings
between the pole shoes have been neglected. It has been shown that for small slot-openings the calculated eddy current losses are accurate within 5.3% compared to steady state, ac, 3D FEM simulations.

The integration of CET into the TSM is made possible by the material properties of the SMC Somaloy 5P130i. The core losses associated with the TSM operation and CET operation of the material have been determined by means of the Steinmetz equation. The field distributions in the core on account of the TSM or CET cannot be determined accurately by means of the HM. The field distribution inside the soft-magnetic material due to the TSM has been estimated based on the mean flux penetrating the teeth of the core. This mean flux has been derived from the magnetic field distribution in the airgap of the axisymmetric HM. The field distribution in the core due to the CET is based on basic transformer equations. The Steinmetz coefficients have been calculated separately for the core losses due to the TSM and CET. Only the first harmonic of the time-varying magnetic flux densities for both the TSM and the CET has been taken into account. A TEC model has been implemented to monitor the temperature distribution in the structure during the design subject to all the losses in the system. The accuracy of the TEC model is within 3.0%.

3D FEM simulation have shown that the functionalities of the TSM and CET are indeed decoupled and that cross-coupling effects between the functionalities are only caused by the nonlinearity of the soft-magnetic core material. The magnitude of the total magnetic flux density vector, which is the vector sum of the TSM and CET flux density vectors, determines the working point on the BH-curve of the SMC material. Changes in the CET field, or TSM field, or both give rise to changes in the working point and manifest themselves as variations in the magnet coupling of the CET and a force reduction of the TSM. A maximum relative variation of 3.2% in the flux linkage of the secondary coil is observed for varying relative displacement of the translator with respect to the stator and varying electric loading of the TSM. A force reduction of maximally 0.9% is observed when the magnitude of the CET flux density component is increased from 0 T to 1.0 T. These numbers show that the integrated, tubular topology solution is feasible and that it exhibits good decoupled electromagnetic behavior. It also shows that the approach of separated modeling of the two functionalities is valid, since the cross-coupling effects are low. The described modeling method has been published in [54, 55, 57].

Finally, a comparison in terms of occupied volume has been made between the integrated solution and an add-on one. It has been shown that the integrated solution occupies 74% more space than the add-on one as a consequence of the use of ferrite instead of SMC and the absence of eddy losses in the permanent magnets. However, the mechanical structure that connects the CET and the TSM has not been considered and may lead to a significant increase in the overall occupied volume of the add-on solution.
In chapter 9 model validation has been conducted by means of magnetostatic FEM simulations. The FEM results have demonstrated the plausibility of the proof-of-concept of the electromagnetic integration of CET in a permanent magnet, synchronous, linear motor. However, the FEM validations in itself are based on the assumption of operation under magnetostatic conditions as a result of the inability to fully incorporate the actual operating conditions, wherein two superimposed magnetic fields operate with different frequencies that differ two orders of magnitude in value. Therefore, to demonstrate the proof-of-concept and to properly quantify the electromagnetic behavior of the tubular topology measurements on a prototype are inescapable. Additionally, measurements permit the validation of the presented modeling method, and the accuracy of the modeling method can be quantified. Finally, measurements provide a decisive answer to the quantification of physical phenomena that the chosen modeling methods fail to provide any or incomplete information on.

The electromagnetic behavior and experimental verification of the integrated tubular topology is mapped in this chapter through measurements on a prototype. First, the synthesis and a system overview of the integrated tubular topology are presented. Next, the measurements with respect to the electromagnetic behavior are discussed. These include emf, resistance, induction, and force measurements, quantification and localization of copper and core losses, and the derivation of the equivalent transformer model for different relative positions of the mover under varying electric loading of the motor. Lastly, a comparative assessment of the practical findings to the theoretically obtained ones is conducted to evaluate the validity and quantify the accuracy of the design method, respectively, where possible.
10.1 Synthesis of the integrated tubular topology

In this section the synthesis of the tubular integrated topology is shortly addressed. An exploded view of a shortened tubular topology is schematically shown in Fig. 10.1. The stator consists of a hollow stainless tube onto which the permanent magnets are adhered. The axially magnetized permanent magnets consist of fully closed rings. The radial field-component of the quasi-Halbach magnets is realized by segmentation of the annular magnets into eight equal parts. Each segment has a parallel magnetization pattern in the direction parallel to the central axis of each segment. Inside the hollow shaft the primary coil is disposed. The actual construction of the assembled stator is shown in Fig. 10.2a.

The soft-magnetic core of the translator consists of compacted SMC disks as shown in Fig. 10.2b. The segmentation of the core into disks is necessary be-
10.1: Synthesis of the integrated tubular topology

cause the complete soft-magnetic core cannot be realized in one compaction step. Segmentation is also necessary to allow the phase coils to be placed in the slots during constructing. The phase coils consist of 119 turns each, and they are wound orthocyclically to increase the copper filling factor of the slots. Each disk has four holes on each side of the outer rim to allow the terminals of the phase coils to be guided out of the core. These holes are present in the outer rim of the disks to compensate for tolerances in parallelism of the flat sides of the disks that are inherent to the compaction process during manufacturing. The holes allow rotation of the disks by an integer multiple of ninety mechanical degrees to compensate for the repetitiveness of the deviations in the parallelism of the flat sides. It has to be noted that both the center and end-disk have been manufactured by the same mold. The proper value of the width of the end-disks, $l_{\text{end}}$, is obtained by placing a plate in the original mold for the center-disks. Furthermore, it is observed that the choice for constructing the end-disks in this manner is a reason for the cogging force to be high. A better cogging force reduction could have been obtained by varying the width over the circumference of the disk (skewing). The core comprises eight center-disks and two end-disks that enclose nine phase coils.

After the core and phase coils have been merged into one component, the secondary transformer coil is wound around the core. The secondary coil consists of eight coils, with seven turns each, connected in series, i.e. one coil in each secondary slit. This amounts to a total of 56 turns for the secondary coil in its entirety. Flat wire is used for the secondary coil to maximize the copper filling factor of the slits. The assembly of the translator is held in place by two plastic end-plates that are connected through four stainless steel studs.

The primary transformer coil also consists of 56 turns. Each turn has a pin-connection on one end and a hole-connection on the other to make the coil interruptible through a 56 pins connector. This allows the stator tube to be brought in and out of the translator without the necessity of cutting the primary.
The full assembly mounted to a base plate is depicted in Fig. 10.3a. The stator rod is mechanically connected to a stainless steel base by means of two stainless steel braces. When the connection of the stainless steel shaft to the stainless steel base through the stainless steel braces is made, the stainless steel structure forms an additional, undesired, tertiary, short-circuited transformer winding. Therefore, the connection of the shaft to the braces is electrically isolated from the braces by means of plastic rings. Apart from the bearings the structure is completely constructed out of stainless steel to reduce eddy current losses that originate from magnetic stray fields from the primary.
10.2 System description of the integrated tubular topology

The full system overview is schematically shown in Fig. 10.4. The integrated coaxial transformer of the CET is fed from a dc source, $V_{dc}$, through a MOSFET-operated, full-bridge inverter. The inverter operates in resonance by compensating the primary leakage inductance, $L_p$, of the transformer with a series resonant capacitor, $C_{res}$, of which the value is tuned to the operating frequency. Operating the CET in resonance causes the CET currents to be nearly sinusoidal. The secondary voltage is rectified by a passive, single-phase, diode-bridge. The rectified secondary voltage powers the power amplifier of the motor via a dc-bus capacitor.

The rectifier, dc-bus capacitor, and power amplifier are attached to the translating mover. Additional, moving, electric apparatus can be connected to the dc-bus for power.

The power amplifier (Copley Accelnet R20) contains on-board electronics for controlling and driving the motor. The encoder head, for the retrieval of position information, is attached underneath the platform on top of which the mover is mounted. The encoder head is powered by the amplifier through a D-sub connector. Communication with the stationary PC is established through the CAN communication protocol. To realize operation without any wires attached, the communication to the stationary PC is obtained by a Kvaser BlackBird SemiPro HS that allows CAN communication with a PC over an ad-hoc wireless network. The Kvaser BlackBird SemiPro HS is powered from the dc-bus by a dc-dc converter. The PC is used to transmit set-points for positioning and receive data on the
status of operation of the amplifier. The actual control of the motor is handled locally within the amplifier. The complete structure is portrayed in Fig. 10.3b.

The input voltage of the power amplifier may not exceed 180 V. When the motor decelerates, kinetic energy, contained within the moving mass, is converted into electric energy and stored in the dc-bus capacitor. As the stored energy in the dc-bus increases, so does the voltage over the dc-bus. Conversely, during acceleration the voltage of the dc-bus drops accordingly. To prevent an overvoltage on the dc-bus from damaging the amplifier, the secondary dc-bus voltage at constant speed is chosen to be \( V_{\text{bus}} = 150 \) V. The dc-bus capacitance, \( C_{\text{bus}} \), is determined by the mass, \( m \), and maximum speed, \( v_{\text{max}} \), of the entire moving platform, and the maximum increase or decrease in voltage, \( \Delta V \), as a result of deceleration or acceleration, respectively. The value of the capacitance can now be obtained by calculating the energy change in the capacitor, i.e.

\[
\begin{align*}
E'_{\text{bus}} &= E_{\text{bus}} \pm E_{\text{kin}}, \\
\frac{1}{2} C_{\text{bus}} (V_{\text{bus}} \pm \Delta V)^2 &= \frac{1}{2} C_{\text{bus}} V_{\text{bus}}^2 \pm \frac{1}{2} m v_{\text{max}}^2 \Rightarrow \tag{10.1a} \\
C_{\text{bus}} &= \frac{m v_{\text{max}}^2}{\Delta V (2V_{\text{bus}} \pm \Delta V)}. \tag{10.1c}
\end{align*}
\]

With a total moving mass of \( m = 10.5 \) kg the bus capacitance must equal \( C_{\text{bus}} = 14.5 \) mF in order to keep \( \Delta V \leq 10 \) V. It has to be observed that any losses during charging and recharging of the capacitor have been neglected. Therefore, a smaller value of the bus capacitance suffices in practice as to not exceed the maximum change in the bus voltage. The dc-bus voltage is not actively controlled.

### 10.3 Electromotive force measurements

Before the electromotive force (emf) is determined, it is convenient to make the (artificial) distinction between a transformer emf and a motion emf. The difference between the emfs is determined by the manner in which the emf is induced. Here, a transformer emf is the emf induced in a stationary coil by a time-varying electromagnetic field, whereas the motion emf is the emf induced due to a movement of a coil in a magnetostatic field.

#### 10.3.1 Motion emf

First, the motion emfs induced in the phase coils of the TSM and the primary and secondary coil of the CET are measured as a result of moving the translator by means of an external force. One period of each of the three-phase motion emf wave-shapes of the TSM is shown in Fig. 10.5a and compared to the results of transient, nonlinear, 3D, FEM simulations. The motion emf is normalized by dividing
the actually measured emf by the instantaneous velocity. Normalization allows
the emf machine-constant to be estimated directly from the wave-shape by taking
the amplitude of the periodic wave-shape. It can be seen that the FEM results
and the measurements are in good correspondence. The emf machine-constant
of the TSM as predicted by the FEM is equal to \( k_e = \hat{\mathcal{E}}_{\text{tsm}} = 39.0 \, \text{V} \, \text{s}^{-1} \), and the
measured emf machine-constant of the TSM equals \( k_e = 37.2 \, \text{V} \, \text{s}^{-1} \); which is a
relative difference of 4.8% between the measured and calculated value. Further-
more, the normalized, induced, motion emfs in the CET coils are determined as
well during the same measurement and are displayed in Fig. 10.5b. Theoretically,
the motion emfs of the CET are zero, but due to imperfections associated with
the manufacturing tolerances of parts and assembly a motion emf is present in
the CET. However, the peak value of the motion emf in the CET is maximally
\( \hat{\mathcal{E}}_{\text{cet}} = 67.5 \, \text{mV} \) in the primary coil and \( \hat{\mathcal{E}}_{\text{cet}} = 27.8 \, \text{mV} \) in the secondary coil.
Hence, the TSM and CET are not completely decoupled. Nevertheless, the flux
linkage between the permanent magnets and the CET coils is maximally \(-54.8 \, \text{dB}\)
of that between the permanent magnets and the phase coils.
10.3.2 Transformer emf

The transformer emf induced in the phase coils of the TSM as a result of the mutual inductances between the CET and TSM coils are determined in a similar way as the motion emf. To that end, the translator is locked and a sinusoidal, ac voltage is imposed on one of the CET coils; and the transformer emfs induced in the phase coils are measured. The applied ac voltage has an amplitude of $\hat{V} = 50.0\,V$ and a frequency of $f = 4.95\,kHz$. The amplitude of 50.0 V is lower than the intended operating peak voltage of 200.0 V. The amplitude is determined by the maximum output voltage of the linear amplifier that has been used during the measurements. The frequency, on the other hand, is determined by the value of the resonant capacitor that is connected in series with the primary coil, of which the value is determined by the transformer tests as described in section 10.5. The resonant capacitor compensates for the voltage drop over the primary leakage inductances to ensure that the same amount of flux is present in the core regardless of the ac source being connected to the primary or secondary.

First, the ac voltage source is connected to the primary and the transformer emf induced in the phase coils is measured for different, relative displacements of the translator with respect to the stator. The same procedure is repeated with the ac voltage source connected to the secondary instead of the primary. The wave-shapes of the transformer emf induced in the phase coils for a relative displacement of the translator with respect to the stator equal to $\Delta z = 0\,mm$ are shown in Fig. 10.6. The amplitude of the induced transformer emf as function of different
displacements with an imposed ac voltage on the primary and secondary is shown in Fig. 10.7. The emfs are only shown for a displacement of two pole pitches. Nevertheless, the curves show similar behavior over the full length of the stroke. The maximum variation with respect to the average value of the transformer emfs in the phase coils as function of position is 2.38% when originated from the primary, and 5.19% when originated from the secondary. Again, the mutual inductances between the CET and TSM coils should theoretically be zero, but they are not in practice. Furthermore, it can be seen from Fig. 10.7 that the average coupling of the CET coils with phase B is $\tilde{E}_B = 0.80$ V and $\tilde{E}_{A,C} = 0.70$ V for the other phases, which is a difference of approximately 14% in coupling. Differences in the coupling are the result of each individual phase coil having a different, non-orthogonal, relative orientation with respect to the CET coils and local anisotropy in the SMC. The flux linkage of the phase coils with the CET coils on account of mutual inductances does not exceed $-35.4$ dB of the imposed flux of the CET.

### 10.4 Cogging force measurements

The cogging force measurement is conducted indirectly by moving the translator back-and-forth over the full length of the stroke with a constant speed. For that purpose, the motor is controlled by an external PI²-controller to minimize the error on the speed. Hence, the power amplifier only receives the set-point values of the phase currents from the external controller. It has to be noted that during
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Fig. 10.8: Measured cogging force profiles including end-effects compared to 3D, nonlinear, FEM result.

the measurement of the cogging the amplifier is powered by an external power source and not by the CET. To minimize the error on the speed even further and to minimize the influence of dynamic effects, such as eddy current damping, the speed is chosen to be low, i.e. \( \nu = 1.0 \text{mm s}^{-1} \). The force set-point that is generated by the output of the PI\(^2\)-controller is now a measure for the force profile. However, this profile still contains an off-set component that originates from mechanical friction. The final cogging profile is obtained by taking the average of the forward and backward motion-profile. The result for one period of the cogging profile, including end-effects, is depicted in Fig. 10.8. The measured profile is compared to a 3D, nonlinear, FEM simulation. Both profiles show the same overall tendency in terms of the shape of the profiles, i.e. peaks in the force profile occur at the same positions. However, the amplitudes of the peaks for the simulated and measured profile differ strongly in value at certain positions. These discrepancies between the profiles originate, among others, from manufacturing tolerances, local differences in hard and soft-magnetic material properties, and disturbance forces from the bearings.

10.5 Equivalent transformer model

The transformer test on the coaxial transformer under no-load conditions of the TSM is conducted to calculate the values of the equivalent transformer model of Fig. 10.9. The equivalent transformer model is different from the classical model, because an additional loss component, \( R_e \), is connected in parallel with the primary leakage inductance, \( L_p \), to account for the eddy current losses in the permanent magnets as a consequence of the primary leakage flux. Furthermore, the series resonant capacitor, to compensate for the primary leakage inductance, is shown in gray. To determine the values of the components of Fig. 10.9 the
wave-shapes of the currents and voltage must be purely sinusoidal. Therefore, the transformer test is carried out with the same linear amplifier as the emf measurement of section 10.3.2. Hence, the amplitude of the applied voltages for all performed open circuit tests is $V = 50.0 \text{ V}$.

### 10.5.1 Resistance measurements

First, dc-resistance measurements of the primary and secondary coils are performed to determine $R_p$ and $R_s$. The measured values are $R_p = 1.90 \Omega$ and $R_s = 0.41 \Omega$. The theoretically obtained values for the ohmic loss calculation, which were used to determine the copper losses of (8.15), equal $R_p = 0.57 \Omega$ and $R_s = 0.35 \Omega$. Especially for the primary, there is a large discrepancy between the resistance value assumed in the model during the design procedure and the actual one. The requirement of making the primary interruptible through the 56-pins connector necessitated a design modification in the cross-section of the wire the primary coil consists of. The connector restricts the copper wire of the primary to have a maximum wire radius of 0.65 mm. Furthermore, the secondary is not tightly wound around the SMC core, but it is wound around the plastic end-plates as well resulting in a longer wire length. Also, a different flat wire had to be applied instead of the anticipated one, causing the filling factor of the secondary to be lower.
10.5.2 Open circuit tests

Next, the open circuit test from the primary side is carried out at a frequency of \( f = 5.00 \text{kHz} \). When the open circuit test from the primary side is performed, the ac current through the primary causes both the core losses and the eddy current losses in the permanent magnets. The amount of active power, consumed by the transformer under no-load conditions, is the result of the resistance of the wire of the primary, \( R_p \), the core losses, \( R_{Fe} \), and the eddy current losses in the permanent magnets, \( R_e \). The amount of consumed reactive power is determined by the combined inductance formed by the primary leakage inductance, \( L_p \), and magnetization inductance, \( L_m \). Therefore, from the open circuit test from the primary side the individual values for \( R_e \), \( R_{Fe} \), \( L_p \), and \( L_m \) cannot be determined directly, and an additional open circuit test from the secondary side is required. The active and reactive power for different positions is measured with a Zimmer LMG500 power analyzer. The average value of the active and reactive power for different positions is found to be \( P = 295 \text{mW} \) and \( Q = 4.67 \text{VAR} \) with a maximum relative variation as a function of position of 3.95\% and 0.57\%, respectively.

By measuring the secondary voltage, \( V_s \), while performing the open circuit test from the primary side, the ratio between the primary leakage inductance and magnetization inductance can be found through

\[
\frac{V_s}{V_p} = \frac{L_m}{L_m + L_p}, \tag{10.2}
\]

where \( V_p \) is the imposed primary voltage. The voltage drop over the resistance of the primary has been neglected in (10.2). The average amplitude of the measured secondary voltage equals \( \hat{V}_s = 28.0 \text{V} \). With \( \hat{V}_p = 50.0 \text{V} \), this means that 56.0\% of the flux of the primary is coupled with the secondary.

The open circuit test from the secondary side with open primary for different positions is performed next. By performing the open circuit test from the secondary side the imposed flux is predominantly confined to the SMC core. Therefore, the losses on account of the eddy currents induced in the permanent magnets originating from the secondary current are negligible. By neglecting the voltage drop over the wire resistance all measured active power consumption is attributed to core losses. Hence,

\[
R_{Fe} = \frac{\hat{V}_s^2}{2P}. \tag{10.3}
\]

Because of the coaxial configuration of the transformer all flux originating from the secondary current is linked with the primary. So, the secondary leakage inductance can be neglected, i.e. \( L_s = 0 \text{H} \). This assumption is corroborated by measuring the open primary voltage, which equals \( \hat{V}_p = 50.2 \text{V} \). The primary voltage is actually 0.2 V higher than expected on account of imperfections in the winding configurations. As is the case for the core losses, all reactive power is
attributed to the magnetization inductance, which in that case is given by

$$L_m = \frac{\hat{V}_s^2}{4\pi fQ}.$$  \hspace{1cm} (10.4)

The measured active and reactive power for different positions equal $P = 615$ mW and $Q = 8.05$ VAr with a maximum relative variation as a function of position of 3.55% and 1.73%, respectively. The value of the measured core losses is extrapolated for the nominal, sinusoidal, operational, primary voltage of $\hat{V}_p = 200.0$ V. To the end, the empirically determined Steinmetz coefficient, $c_2$, of Table 9.5 for $5.0 \cdot 10^3 \leq f \leq 1.0 \cdot 10^4$ is needed. The extrapolated core losses are calculated through

$$P_{Fe} = P \left( \frac{200.0}{50.0} \right)^{c_2}.$$ \hspace{1cm} (10.5)

The corresponding average values for the equivalent core loss resistance and magnetization inductance now become $R_{Fe} = 1.74$ kΩ and $L_m = 5.04$ mH with a maximum relative variation of 3.45% and 1.74%, respectively. Since the turn ratio is one, all impedances seen from the secondary side are numerically equal to those seen from the primary side. Hence, the average value for $L_m$ and (10.2) are applied to find the average value of the primary leakage inductance, which equals $L_p = 3.96$ mH. Now that the primary leakage inductance is known, the value of the series resonant capacitor can be calculated as explained in section 8.1.2 via

$$C_{res} = \frac{1}{4\pi^2 f^{-2} L_p^{-1}} = 256 \text{nF}.$$\hspace{1cm} The practical value of the capacitor is realized by a series and parallel connection of multiple capacitors of which the equivalent value is $C_{res} = 268 \text{nF}$. With this value for the resonant capacitance the resonant frequency becomes $f_{res} = 4.89$ kHz. The frequency being different is the reason for the transformer emf measurements of section 10.3 to be conducted at $f_{res} = 4.95$ kHz. The frequency is chosen to be slightly higher to ensure zero voltage switching of the switches in the legs of the full-bridge inverter that is applied when the CET operates normally.

Finally, with all the inductances known the magnetic coupling, $k$, can be calculated via

$$k = \frac{L_m}{\sqrt{(L_m + L_p)(L_m + L_s)}}.$$ \hspace{1cm} (10.6)

where $L_m + L_p$ is the inductance of the primary coil and $L_m + L_s$ is the inductance of the secondary coil. The average value of the magnetic coupling equals $k = 0.75$. No variation with position is observed, since $L_p$ is calculated by using the fixed value of $\hat{V}_p V_s^{-1} = 0.56$ in (10.2). As a result, position variations in $L_m$ and $L_p$ are identical and, therefore, cancel.
Table 10.1: Calculated and measured values of the position dependent electric components in the equivalent transformer model of Fig. 10.9

<table>
<thead>
<tr>
<th>parameter</th>
<th>calculated value</th>
<th>measured value</th>
<th>unit</th>
<th>variation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_p$</td>
<td>-</td>
<td>3.96</td>
<td>mH</td>
<td>1.74</td>
</tr>
<tr>
<td>$L_o$</td>
<td>0.00</td>
<td>0.00</td>
<td>mH</td>
<td>-</td>
</tr>
<tr>
<td>$L_m$</td>
<td>7.57</td>
<td>5.04</td>
<td>mH</td>
<td>1.74</td>
</tr>
<tr>
<td>$R_p$</td>
<td>0.37</td>
<td>1.90</td>
<td>Ω</td>
<td>0.00</td>
</tr>
<tr>
<td>$R_o$</td>
<td>0.35</td>
<td>0.41</td>
<td>Ω</td>
<td>0.00</td>
</tr>
<tr>
<td>$R_{Fe}$</td>
<td>1.36</td>
<td>1.74</td>
<td>kΩ</td>
<td>3.45</td>
</tr>
<tr>
<td>$R_c$</td>
<td>-</td>
<td>2.65</td>
<td>kΩ</td>
<td>0.54</td>
</tr>
<tr>
<td>$k$</td>
<td>-</td>
<td>0.75</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

10.5.3 Short circuit test

To determine the value of the equivalent eddy current loss resistor, $R_c$, the short circuit test is conducted. The secondary of the transformer is shorted while the primary voltage is slowly increased until the secondary rms current equals $I_s = 3.0$ A. Normally, the short-circuit test is performed under nominal, secondary rms current conditions, which would be equal to $I_s = 5.7$ A. However, the total losses turned out to be significantly higher during testing than predicted by the models of chapter 9, which force the short circuit test to be conducted with a reduced secondary current to prevent the structure from overheating. The contribution of the eddy current losses is determined by subtraction of the ohmic losses in the primary and secondary coils from the measured, total active power consumed by the transformer. The contribution of the core losses to the total losses is neglected. The average eddy losses are calculated to be $P = 52.1$ W with a maximum relative variation of 0.66%. The value of the equivalent eddy current loss resistor relates to the measured losses and the primary rms current, $I_p$, as

$$P = \frac{I_p^2 X_p^2 R_c}{R_c^2 + X_p^2},$$

where $X_p = 2\pi f L_p$ is the reactance of the primary leakage inductance. Solving (10.7) results in the average value of the equivalent eddy current loss resistor being equal to $R_c = 2.65$ kΩ with a maximum relative variation of 0.54%.

An overview of the average values of the measured equivalent transformer components is given in Table 10.1. Also, the calculated values from the combined harmonic model of chapter 9 are tabulated. The primary leakage inductance could not be calculated analytically and, therefore, no value is included in Table 10.1. As a result, the magnetic coupling and the equivalent eddy current loss resistance can also not be calculated, since their calculation requires the primary leakage inductance to be known as is evident from (10.6) and (10.7). Furthermore, the
10.5: Equivalent transformer model

Table 10.2: Comparison of the calculated and measured values of individual loss components of the CET.

<table>
<thead>
<tr>
<th>description</th>
<th>parameter</th>
<th>$I_s$ [A]</th>
<th>calculated $P$ [W]</th>
<th>measured $P$ [W]</th>
<th>error $\epsilon$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>primary ohmic</td>
<td>$P_{Cup}$</td>
<td>3.0</td>
<td>5.16</td>
<td>17.10</td>
<td>−69.82</td>
</tr>
<tr>
<td>losses</td>
<td></td>
<td>5.7</td>
<td>18.33</td>
<td>60.80</td>
<td>−69.85</td>
</tr>
<tr>
<td>secondary</td>
<td>$P_{Cuu}$</td>
<td>3.0</td>
<td>3.17</td>
<td>3.69</td>
<td>−14.09</td>
</tr>
<tr>
<td>ohmic losses</td>
<td></td>
<td>5.7</td>
<td>11.27</td>
<td>13.12</td>
<td>−14.10</td>
</tr>
<tr>
<td>core losses</td>
<td>$P_{Fe}$</td>
<td>3.0</td>
<td>14.69</td>
<td>11.46</td>
<td>+28.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.7</td>
<td>14.69</td>
<td>11.46</td>
<td>+28.19</td>
</tr>
<tr>
<td>eddy current</td>
<td>$P_e$</td>
<td>3.0</td>
<td>1.86</td>
<td>52.13</td>
<td>−96.43</td>
</tr>
<tr>
<td>losses</td>
<td></td>
<td>5.7</td>
<td>6.62</td>
<td>185.35</td>
<td>−96.43</td>
</tr>
<tr>
<td>total losses</td>
<td>$P_{cet}$</td>
<td>3.0</td>
<td>24.88</td>
<td>84.38</td>
<td>−70.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.7</td>
<td>50.92</td>
<td>270.73</td>
<td>−81.19</td>
</tr>
</tbody>
</table>

Relative variation on account of position dependency expressed as a percentage of the average of each measured value is shown as well. It is observed that there are significant discrepancies between the calculated and practically determined values. The discrepancy in the values of the primary and secondary coil resistance has already been explained. Both the magnetization inductance and core losses are overestimated. In sections 9.3 and 9.4 this was expected for the core losses, but not for the magnetization inductance. The error in both the calculation of the magnetization inductance and core losses is caused by the inaccuracy with which the field distribution in the soft-magnetic core can be estimated. This problem has already been addressed in the same sections.

10.5.4 Comparison of theoretically and practically obtained losses

The differences in calculated and measured values of the components of the equivalent transformer model are obviously the result of discrepancies in the calculated and measured losses. The individual loss components originating from the CET operation as calculated by the models of chapter 9 and the measured ones are listed in Table 10.2. By comparison, the calculated losses are determined for $I_s = 3.0$ A and for the nominal current of $I_s = 5.7$ A. The losses that actually have been measured during the transformer tests with $I_s = 3.0$ A are tabulated as well in Table 10.2 along with the error, $\epsilon$, of the calculated values with respect to the measured ones. Furthermore, the measured values of the losses under nominal operation current have not been obtained experimentally, but are found by scaling of the measured values found for $I_s = 3.0$ A to allow the comparison
with the results of the optimal design as calculated by the electromagnetic model.

For the calculation of the quantities that are dependent on an accurate result of the magnetic field distribution in the core, large discrepancies between measured and predicted values have been expected on account of the limitations associated with the harmonic model. However, the measured eddy current losses are 28 times higher than what was to be expected based on the results of chapter 9. The large discrepancy originates, among others, from eddy currents that are associated with stray fields that penetrate the stainless steel base frame and braces of the machine. Due to the complexity of the structure these have not been anticipated during the design stage. Post-design, 2D, FEM simulations show that the eddy current losses on account of stray fields penetrating the base frame equal $P_e = 15.4 \text{ W}$ under nominal load conditions, i.e. $I_s = 5.7 \text{ A}$. Furthermore, the contribution of the return path of the primary to the eddy current distribution has been neglected in the model of chapter 9. Moreover, the primary is not coaxially mounted into the hollow shaft, but it lies on the bottom of the shaft. As a consequence the eddy current losses in the stator increase significantly. The results of post-design, 2D, FEM simulations for loss calculations in the stator-rod are shown in Fig. 10.10. The eddy current losses in the stator as a result of a displacement of the primary out of the center of the shaft by a distance $\Delta r$ are shown for the part enclosed (in.) and not enclosed (ex.) by the core. Simulations have been conducted where the return path of the primary is (with ret.) and is not (no ret.) taken into account. It is observed that the return path only contributes to the losses in the part of the stator-rod that is not enclosed by the translator. The simulated losses are ten times higher when the primary touches the shaft ($\Delta r = 8.0 \text{ mm}$).

Also, discrepancies between the values of the actual electric conductivities and the ones specified by the manufacturers might also contribute to increased losses.
Limiting the eddy current losses on account of the previously mentioned issues requires a sound coaxial location of the primary, and a base frame that solely consists of nonconductive, nonmagnetic material such as granite. By applying bonded permanent magnets with low electric conductivity instead of sintered ones, and by replacing the stainless steel shaft with a ceramic one, the losses can be decreased significantly.

10.6 CET behavior under varying electric loading

The variation of the quantities due to changes in the position at which the transformer tests have been performed do not exceed 3.4%. Due to this low variation, the quantities are considered constant. The behavior of the CET is identified under normal operational conditions in this section. First, the behavior is identified by the equivalent transformer model of section 10.5. The equivalent transformer model has been implemented in an electric network solver (PSIM 7.1). On the primary side the equivalent transformer model is connected to a square-wave voltage source, and on the secondary side it is connected to a diode-bridge rectifier followed by a dc-bus capacitor and a resistive secondary load. Next, the results from the electric network solver are verified by measurements, and the validity of the modified equivalent transformer model is demonstrated.

The primary is connected to the series resonant capacitor and powered by the full-bridge inverter. The applied dc-voltage is set to \( V_{dc} = 150.0 \text{ V} \). The bus capacitor comprises an electrolytic capacitor bank with a total value of \( C_{bus} = 14.4 \text{ mF} \). The secondary load consists of incandescent lamps in parallel. The identification of the CET behavior includes the measurements of the bus voltage, transferred power, and efficiency for different positions, electric loading of the motor, and secondary load. The electric loading of the motor is varied by connecting the power amplifier to the phase coils. The power amplifier is not fed by the CET, but by a different, separate power supply to quantify the behavior of the CET independently. At each position the mover is locked and the electric loading is varied by increasing the set-point of the phase current from \( I_{ph} = 0 \text{ A} \) to \( I_{ph} = 4 \text{ A} \) by \( 1 \text{ A} \) at a time. With \( N_{ph} = 119 \text{ turns per phase coil} \) the current density in the slot equals \( J_{ph} = 2.4 \text{ A mm}^{-2} \) for \( I_{ph} = 4 \text{ A} \). It has to be observed that position dependent commutation of the phase currents is taken into account. The secondary load is varied by increasing the number of lamps from one to four.

First, the bus voltage is calculated and measured, and the results for a different number of lamps as secondary load are depicted in the plots of Fig. 10.11. The values that are obtained by the equivalent transformer network are indicated by the dashed curves. For the measured values the envelope that is formed by the curves for the different phase current set-point are shown instead of the individual curves. Plotting the individual curves results in an indistinct view of tangled up curves in which no distinguishable tendency is observed as a function of the electric loading. Therefore, the lower or upper boundary of the range indicated...
Fig. 10.11: Measured range of the bus voltage under varying electric loading of the TSM as function of position for $R_{\text{load}} = 204 \, \Omega$ (a), $100 \, \Omega$ (b), $66 \, \Omega$ (c), and $48 \, \Omega$ (d) compared to the bus voltage calculated through the equivalent transformer model (- - -).

by the gray areas are not predominantly determined by measurements belonging to a particular value of the electric loading. This entails that the cross-coupling effect of the influence of the electric loading of the TSM on the CET is negligible.

The calculated and measured values are in good correspondence. The maximum error, $\epsilon$, in percentage of the calculated values of the bus voltage with respect to the measured ones for the different secondary loads, $R_{\text{load}}$, is listed in the second column of Table 10.3 on page 214. The maximum error is given by the largest, vertical distance between the dashed line of the calculated values and the contour of the envelope in Fig. 10.11.

Variation in the bus voltage for a given secondary load originates from permeability redistribution in the core as a consequence of the position dependency on one hand, and the electric loading of the TSM on the other hand as discussed in section 8.3. From the 3D, nonlinear FEM simulation of Fig. 9.19 it has been concluded that the variation in flux linkage of the secondary on account of dis-
placement and electric loading is maximally 3.2%. Hence, the bus voltage should vary accordingly under locked mover conditions. The maximum variation in the bus voltage on account of the electric loading of the TSM is quantified by determining the relative thickness of the envelope with respect to the lower level of the envelope for each position. The maximum relative thickness of the envelope occurs for \( R_{\text{load}} = 48 \, \Omega \) at \( \Delta z = 18 \, \text{mm} \) and equals 0.47%. This is significantly lower than the 3.2% expected by the FEM simulation. Hence, the contribution of the electric loading to the variation in flux linkage is small. The assumption that SMC is an isotropic material of which the \( BH \)-curve is a scalar function, that is only dependent on the length of the total field vector and not on its orientation, is erroneous. In reality, the material properties of the SMC may exhibit identical material properties in each direction; that does not necessarily mean that the material is isotropic. The magnetic behavior of the material is determined by a tensor rather than a scalar function [65]. Meaning that the flux density component of the CET is hardly influenced by that of the TSM when the entries
Table 10.3: Maximum error in percentage between the calculated and measured bus voltage, $V_{bus}$, secondary power consumptions, $P_{sec}$, and efficiency, $\eta_{cet}$.

<table>
<thead>
<tr>
<th>$R_{\text{load}}$ [Ω]</th>
<th>$V_{bus}$</th>
<th>$P_{sec}$</th>
<th>$\eta_{cet}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>0.65</td>
<td>1.24</td>
<td>0.42</td>
</tr>
<tr>
<td>66</td>
<td>0.24</td>
<td>0.42</td>
<td>0.94</td>
</tr>
<tr>
<td>100</td>
<td>1.04</td>
<td>2.23</td>
<td>1.98</td>
</tr>
<tr>
<td>204</td>
<td>1.14</td>
<td>1.91</td>
<td>2.42</td>
</tr>
</tbody>
</table>

on the diagonal of the permeability tensor are much larger than the nondiagonal ones. Since points on the upper and lower level of the envelope originate from any level of electric loading, the actual influence of the electric loading on the bus voltage is even lower, and the 0.47% variation might just be ascribed to noise in the measurements. Consequently, all variation on the flux linkage or, equivalently, bus voltage is ascribed to the permeability redistribution associated with position dependency. The quantification of which for each secondary load is obtained by calculating the relative increase to the maximum value of the voltage of the upper level of the envelope with respect to the minimal value on the lower level. The highest relative variation in the bus voltage that is observed when $R_{\text{load}} = 48$ Ω equals 0.86%, which is still significantly lower than 3.2%. This is in correspondence with the small variations on the values of the equivalent transformer components of the previous section. However, the influence of the transferred power, or secondary load, is more apparent. The voltage drop is mainly attributed to the high resistance of the primary and the series impedance formed by the resonant capacitor and primary leakage inductance. Their respective contributions to the voltage drop are 35% and 58%. The voltage drop over the resonant impedance can be reduced by operating the CET closer to the resonant frequency of $f_{\text{res}} = 4.89$ kHz.

For the sake of completeness, the envelopes for the amount of power that is transferred to the secondary load, $P_{sec}$, as function of electric loading and displacement are shown in Fig. 10.12. Again, the dashed lines indicate the value as they are predicted by the equivalent transformer model. A comparable behavior is observed for the secondary power as for the bus voltage. By measuring the power that is delivered to the system from the primary side, the overall efficiency of the CET, $\eta_{cet}$, including the inverter, rectifier, and capacitances, can be determined. The predicted and measured, total efficiency of the CET system as a function of position and electric loading are portrayed in Fig. 10.13. The maximum error of the values predicted by the equivalent transformer model and measured values of both the secondary power and efficiency are also listed in Table 10.3. The efficiency of the CET is $76.8 \leq \eta_{cet} \leq 82.9\%$. 
10.6: CET behavior under varying electric loading

The analyses of the measurements of this section justify the use of position-independent components in the modified equivalent transformer model of Fig. 10.9. The position independency is allowed on account of the variations in the losses and inductances being small due to the orthogonal orientation of the magnetic fields of the CET and the TSM. Furthermore, the modification in the transformer model by adding the eddy current loss resistor in parallel to the primary leakage has been validated. The error between the values obtained by the transformer model and the measurements does not exceed 2.42%. Additionally, it has been demonstrated that cross-coupling effects due to permeability redistribution associated with displacement and changes in electric loading affect the CET functionality less severely than anticipated by the simulations of chapter 9. A maximum variation in the bus voltage on the secondary side, which is a measure for the flux linkage variation, for varying positions and electric loading of the TSM of maximally 0.86% has been observed.
10.7 Dynamical performance

So far, only stationary tests with a locked translator have been addressed. In this section the cross-coupling under dynamical performance is examined. To that end, the amplifier is fed by the CET while the TSM executes a movement. The translator repeatedly follows a third-order trajectory from the starting position \( \Delta z = 0.0 \text{mm} \) in the middle of the stator to position \( \Delta z = 200.0 \text{mm} \) and back. This trajectory is carried out for different values of the maximum acceleration, \( a_{\text{max}} \), while the speed and jerk are kept constant at \( v = 2.0 \text{m/s}^{-1} \) and \( j = 2.5 \text{km/s}^{-3} \), respectively. During the movement the dc-bus voltage, the position error, and speed error are measured. Since the electric loading of the TSM is dependent on the maximum acceleration, the influence of the electric loading on the CET performance under dynamical conditions is mapped by measuring the dc-bus voltage. Conversely, to identify the influence of the CET on the TSM the position and speed error are compared for the situations where the amplifier is fed through the CET, and where the amplifier is directly fed by a dc voltage-source. In both situations the same dc voltage-source is applied, and the voltage of the dc-bus at standstill is equal to \( V_{\text{bus}} = 150.0 \text{V} \). In the latter situations the dc-voltage source is directly connected to the input of the rectifier.

The measured, maximum position error, \( \hat{\epsilon}_z \), as a function of the maximum acceleration is depicted in Fig. 10.14. The position error is shown for the forward motion from \( \Delta z = 0.0 \text{mm} \) to \( \Delta z = 200.0 \text{mm} \), and for the backward motion from \( \Delta z = 200.0 \text{mm} \) to \( \Delta z = 0.0 \text{mm} \). The maximum position error is not only shown for the nominal operating frequency of the CET at \( f = 4.95 \text{kHz} \), but also for an operating frequency of \( f = 4.90 \text{kHz} \), and for the situation in which the amplifier is directly powered by the dc voltage-source. The maximum speed error, \( \hat{\epsilon}_v \), is shown in Fig. 10.15. For both the maximum position and speed error it is observed that for \( a_{\text{max}} \leq 20 \text{m/s}^{-2} \) the discrepancies in the errors as a result of powering the amplifier directly or through the CET are negligible. For the forward motion this discrepancy is negligible over the full range of the maximum acceleration. For the backward motion, however, the discrepancy suddenly increases for \( a_{\text{max}} = 20 \text{m/s}^{-2} \) and \( a_{\text{max}} = 25 \text{m/s}^{-2} \). This discrepancy most probably originates from the high cogging force of the TSM and additional disturbance forces from the bearings. In combination with a decreased bus voltage the amplifier is unable to generate the current needed to reach the maximum acceleration and simultaneously overcome the cogging and disturbance forces. This is apparent from Fig. 10.16 where the maximum and minimum bus voltage values are displayed. It can be seen that the bus voltage drops significantly when the amplifier is powered by the CET on account of the impedances of the CET coils. By operating the CET closer to the resonant frequency, instead of the nominal one, it is expected that the drop in the bus voltage is less severe. However, a clear difference in drop of the bus voltage is not observed. Yet, a reduction in the position error is observed when the CET is operated closer to resonance in comparison to the nominal operation conditions.
10.7: Dynamical performance

Fig. 10.14: Measured, maximum position error in case the amplifier is powered directly (no CET) or through the CET as function of the maximum acceleration for forward (a), and backward motion (b).

Fig. 10.15: Measured, maximum speed error in case the amplifier is powered directly (no CET) or through the CET as function of the maximum acceleration for forward (a), and backward motion (b).
Regardless of the amplifier being powered directly or through the CET, it has to be observed that the maximum position and velocity errors are significant. For complete wireless operation the in-built controller of the amplifier has to be applied. The controller consists of two cascaded PI-controllers, i.e. one for the position loop and the other for the speed loop. The gains of each controller have automatically been determined by the amplifier based on machine parameters. The position and speed error can be reduced significantly by applying feed-forward control. However, there is no possibility to implement feed-forward control in the existing software of amplifier.

The dynamical measurements show that the TSM is not influenced by the CET under normal operating conditions as long as the dc-bus voltage is sufficiently high to allow the power amplifier to generate the desired currents. The decrease in the bus voltage is the result of the high series impedances of the CET coils. These impedances can be lowered by operating the CET closer to the resonant frequency and by reducing the wire-resistance of the CET coils. Smaller position errors of the TSM are obtained when the dc-bus voltage is more stable.
10.8 Conclusions and remarks

The emf measurements have shown that the CET and TSM functionalities are not fully decoupled. Nevertheless, cross-coupling effects on account of mutually induced emfs are negligible. The induced motion emf in the CET coils, as a result of movement of the translator, is $-54.8$ dB of the motion emf induced in the phase coils of the TSM. Conversely, the induced transformer emf in the phase coils of the TSM has been identified to be $-35.4$ dB of the transformer emf induced in the CET coils.

The cogging force of the TSM has been identified by controlling the motor by means of a PI$^2$-controller and operating it at low speed. The output of the controller has been measured to determine the cogging force profile. The profile has been compared to results from FEM simulations. The measured and simulated profiles show the same overall tendency, and the amplitude is in the same range. However, differences in the values of the peaks of the profile have been observed.

A modified equivalent transformer model has been proposed. The components of the equivalent transformer model have been determined by performing standard transformers tests on the CET. The results have been compared to the simulated values for model verification. It has been shown that the position dependency of the components can be neglected. Loss measurements have demonstrated that the eddy current losses in the stator cannot accurately be determined by means of the electromagnetic model of chapter 9. The eddy current losses turned out to be 28 times higher than expected. It has been shown that the large discrepancy is mainly ascribed to the noncoaxial positioning of the primary coil with respect to the shaft, neglecting the return path of the primary, and losses in the stainless steel base frame. For the remaining discrepancies it cannot unambiguously be assessed whether they are ascribed to additional eddy current losses in the structure that have not been taken into account in the models or to erroneous model assumptions. A decisive quantification of the source of discrepancy can only be provided by replacing the frame with one that completely consists of nonconductive and nonmagnetic material. As expected, the calculation of the magnetization inductance and core losses is coarse. The respective values for the calculated magnetization inductance and core losses are 22% and 49% higher than the measured ones. Discrepancies in the copper losses originate from alterations in the design enforced by manufacturing issues.

The electromagnetic behavior of the CET for varying electric loading of the TSM, positions, and amount of power being transferred has been mapped to quantify the cross-coupling. These tests have been conducted with a locked translator. Fluctuations in the secondary voltage of the CET are mainly ascribed to the voltage drop over the resistances of the primary and secondary coil, and the resonant impedance formed by the resonant capacitor and primary leakage inductance. Compared to these voltage drops, fluctuations in the secondary on account of electromagnetic cross-coupling effects are marginal. The dynamical
measurements have shown that the performance of the TSM in terms of the position error is not influenced by the CET as long as the dc-bus voltage is stable. Therefore, also under dynamical operation of the TSM the integrated topology shows excellent decoupled behavior.

The feasibility and the practical verification of the proof-of-principle of CET into the TSM has been demonstrated in this chapter. It has been shown that the integration of CET into a TSM is viable, and that good electromagnetic decoupling is obtained by integration in the manner presented in this thesis. In fact, the electromagnetic decoupling is even better than anticipated by the 3D, nonlinear, FEM simulations. The good decoupled electromagnetic behavior is attributed to the 3D material properties of the SMC. However, the efficiency with which energy can be transferred contactlessly is only $\eta_{\text{cet}} = 80\%$ where $\eta_{\text{cet}} = 93\%$ was expected. As a consequence, the intended continuous, apparent power rating of the CET of $S_{\text{cet}} = 800\,\text{VA}$ had to be readjusted to $S_{\text{cet}} = 300\,\text{VA}$, which still suffices to operate the TSM as intended. The eddy current losses in the structure of the integrated topology have to be reduced to make the application fit for industrial use. To that end, the primary has to be positioned coaxially with respect to the shaft. Furthermore, bonded magnets with low electric conductivity can be applied instead of sintered ones. The shaft and base frame could be made of ceramic material and granite, respectively, to reduce the eddy current losses even further.

The innovativeness, uniqueness, and electromagnetic complexity of the integrated tubular topology called for an equivalently unique, complex and yet fast electromagnetic modeling, which has been presented in this thesis. This chapter has laid bare that both the loss and inductance calculations by means of the combined harmonic model are still too coarse in order to obtain a reliable design based on the combined harmonic model alone. Nevertheless, it has been shown that discrepancies with respect to the practical findings are to a large extent the result of seemingly minor adjustment to the topology to aid manufacturability. The combined harmonic model still provides an initial description of the electromagnetic behavior of the integrated, tubular topology. Yet, the method is promising and further research on the refinement of the method is required to ameliorate its accuracy to make it more suitable for design.
Part III

Closing
This thesis is focused on the analysis, design, realization and practical verification of the integration of dual energy conversions in one device and, more in particular, the integration of contactless energy transfer (CET) into a linear, permanent magnet actuator. Furthermore, a comparative analysis of appropriate modeling methods that are considered suitable as design tools has been presented. The conclusions and answers to the risen research questions as stated in section 1.3 and the scientific contributions of this study are addressed in this chapter. Recommendations for further research are given in the final section.

11.1 Conclusions

11.1.1 Modeling techniques for linear energy conversion devices

Four modeling methods, being the harmonic method (HM), the Schwarz-Christoffel method (SCM), the boundary element method (BEM), and the tooth contour method (TCM) have been presented in part I of this thesis. A comparative analysis of the modeling techniques has been conducted by applying each method to the same electromagnetic benchmark problem. The discrepancies with FEM simulations, behavior of the convergence in terms of accuracy, and the simulation time for the benchmark problem have been determined.

The following conclusions are drawn from the analysis of part I:

- The HM, SCM, and BEM all provide a detailed local magnetic field dis-
tribution that reverberates in a low discrepancy in terms of the calculated magnetic flux density field components in comparison to FEM simulations. Flux density calculations for these three methods are within 2.2% accurate. The TCM only provides a global flux distribution. Hence, only flux linkages of the coils can be determined. The discrepancy with the FEM for the flux linkage calculations is within 2.4%. Discrepancies between the various methods compared to the FEM simulations on grounds of physical assumptions are predominantly attributed to:

– the inability of the HM to take different material properties within one subdomain into account;
– the restriction of the SCM that the entire domain of interest must have a unity relative permeability;
– truncation of, both, the infinite series of the HM and the point-source representation for the SCM;
– the assumption of the perpendicularly orientated field component between flux tubes at boundaries of a material transitions in the TCM.

• All discussed modeling methods are applicable to the benchmark topology due to the particular choice for its geometry. However, for it to be applicable:

– the HM requires the geometry to be divisible in orthogonal subdomains and exhibit periodic or Neumann boundary condition in one direction;
– the SCM requires the geometry of the domain of interest to be representable as a closed or open polygon;
– the BEM imposes no limitations on the geometry of the problem;
– the TCM, in principle, imposes no limitations on the geometry, however, an accurate solution can only be obtained for problems that are bounded by soft-magnetic boundaries on account of the perpendicularly orientated field component between flux tubes.

• From the obtained magnetic field distribution the forces on the benchmark topology have been calculated. Three different force calculation methods - Lorentz force, Maxwell stress tensor (MST), and virtual work (VW) - have been discussed. For the HM and SCM the MST method is computationally the most efficient method for force calculations, since the field distribution is known locally. For the TCM only the VW method is applicable, because it only provides a global field distribution.

• The accuracy of the HM, SCM, and TCM has been determined by the discrepancy between the force ripple calculation of each method compared to FEM simulations, since the force ripple calculation requires the most computational effort in order for it to be satisfied. The HM performs the best, both, in terms of accuracy and overall computation time. A force ripple with a discrepancy of 5.0% within a corresponding calculation time per step of 7.7 ms has been obtained via the HM. Performance-wise as well
as calculation time-wise the SCM performs second best. For the SCM a force ripple with a discrepancy of 10.0% within a corresponding calculation time per step of 1.2 s has been obtained. The slowest and least accurate modeling technique is the TCM, where the obtained discrepancy in the force ripple is 16.1% with an accompanying calculation time per step of 92 ms. It has to be observed that the SCM and TCM require additional preparatory calculations that significantly contribute to the overall computation time, i.e. a preparatory calculation time of 1.3 s and 3.3 s, respectively.

Conclusively, despite the relatively small discrepancies in the magnetic flux (density) distribution for all methods the force ripple calculations yield significantly larger discrepancies. For electromechanical energy conversion devices with a comparable structure and physics as the benchmark topology the HM has proven to be the best option for modeling in terms of accuracy as well as computation time. The absence of preparatory calculations predominantly attributes to the reduction in computational time for the HM.

11.1.2 Requirements with respect to cross-coupling effects associated with integration of CET into a linear actuator

A qualitative magnetic field analysis with respect to the integration of an inductively coupled CET system into an electromechanical actuator has led to the formulation of five criteria that ideally should be met and apply generally, to minimize cross-coupling effects between functionalities:

1. No undesired emf is induced in the transformer coils due to the time-varying magnetic field of the motor and vice versa;
2. The magnetic coupling of the transformer is independent with position;
3. The magnetic coupling of the transformer is independent of the machine phase current;
4. The force or torque is independent of the transferred power of the CET;
5. Integration must not lead to overheating of the system.

In regard to these five criteria it has been demonstrated that:

- criterion 1 is met through a spatial positioning of the coils and permanent magnets that exploits physical symmetries in the device;
- criteria 2 and 3 can never fully be met on account of the soft-magnetic material being nonlinear. Variations in the working point of the BH-curve
due to the superimposed field components of the CET and actuator cannot be averted. However, an orthogonal, spatial, orientation of the magnetic field component of the CET with respect to the magnetic field component of the actuator minimizes the cross-coupling effects that cause criteria 2 and 3 to be violated;

• criterion 4 is the result of the leakage flux of the transformer traversing through the soft-magnetic material, which locally influences the working point on the \( BH \)-curve. Again, the extent to which criterion 4 is violated can be minimized through a mutual, orthogonal, spatial orientation of the CET and machine flux component;

• criterion 5 is met through a compromise between a concession on both the force density and power transfer rating of the integrated solution.

A simplified electromagnetic model based on initial sizing equations for synchronous motors and transformers has been presented. A quantitative analysis using this simplified model has shown that:

• the orthogonal orientation of the magnetic fields, instead of an in-plane one, simultaneously improves the performance and decoupling between functionalities of an integrated topology;

• the model provides a fast feasibility check by means of which the performance in terms of flux density levels, apparent power rating of the CET, losses, and the force density of the actuator can be estimated.

• an optimum exists for the ratio between the magnitudes of the flux density components of the CET and actuator that leads to the highest force density for a given apparent power transfer of the CET;

• it cannot quantify any cross-coupling effects.

11.1.3 Study on topological configurations of integrated solutions and the use of magnetic materials

Three long-stroke, permanent magnet actuators with integrated CET have been proposed, namely:

• the flat, synchronous, linear machine with a double C-core transformer configuration that exhibits an orthogonal mutual field orientation;

• the double sided, linear, flux switching machine with a U-I core transformer configuration that exhibits an in-plane mutual field orientation;
the linear tubular, synchronous machine (TSM) with a coaxial transformer configuration that exhibits an orthogonal mutual field orientation.

Each topology has been qualitatively assessed to which extent the first four of the formulated criteria with respect to cross-coupling effects are expected to be satisfied. The results of the analyses are shown in Table 8.4. Based on the results of Table 8.4 a choice was made for a topology for further analysis, design, and prototyping. It has been concluded that:

- the integrated TSM performs the best in terms of minimal cross-coupling effects;
- the integrated flux switching topology performs the worst in terms of minimal cross-coupling effects;
- a more detailed quantitative analysis should be conducted to obtain a better founded comparison.

The following conclusions are drawn from the material survey in chapter 8:

- SMC material is the only viable soft-magnetic material for integrated solutions that employ an orthogonal mutual magnetic field orientation on account of the combined material properties in terms its isotropy, high saturation level, and low electric conductivity;
- for in-plane field orientations laminated steel can be applied, but its applicability is limited by its high electric conductivity;
- ferrite is altogether unsuitable to be applied in an integrated solution due to its low saturation level.

11.1.4 Extension to the classical modeling of electromechanical devices

An extensive electromagnetic model based on combined harmonic models has been proposed in chapter 9 as a less accurate, but significantly faster alternative to a transient, nonlinear, 3D FEM model. The model consists of two decoupled harmonic models, i.e. one for the calculation of the eddy current losses in the permanent magnets and shaft due to the leakage field of the transformer, and one for the force calculation of the TSM. It has been shown that:

- for a small relative values for the CET flux density component in comparison to the TSM flux density component decoupling into two separate models is allowed, since the degree to which the core saturates is predominantly determined by the TSM flux density;
• the decoupled, 2D, harmonic models are in good agreement with full, 3D, FEM simulations under the same model assumptions;

• the combined model is suitable to be applied in a fast design routine;

• inaccuracies in the calculations of the core losses, inductances, and force profile are the result of model assumptions associated with:
  – the limitation of the HM to only apply to periodic models, hence end-effects cannot be considered;
  – the inability of the HM to provide the field distribution inside the soft-magnetic core;
  – representing the problem as two, 2D problems that cannot fully account for all 3D effects;
  – the core being considered to consist of infinitely permeable, linear material.

The combined harmonic model has been applied in a parametric search to find the optimal solution in terms of highest average, thrust force for the tubular integrated topology that is intended to produce at least 250 N of thrust force at a speed of \(2 \, \text{m s}^{-1}\) and an apparent power transfer of 800 VA. The optimum topology obtained by the harmonic model has been verified by means of a nonlinear, 3D, magnetostatic FEM model. It can be concluded that:

• there is a discrepancy between the calculated average thrust force and force ripple of 9.3% and 13.6%, respectively, between the combined harmonic and FEM model for the optimal design;

• electromagnetic cross-coupling effects have been mapped through the FEM model of the optimal geometry and the simulations have shown that it is expected that:
  – no emf is induced in the CET coils as a result of movement of the translator or changes in the phase current of the TSM;
  – no emf is induced in the phase coils of the TSM as a result of the time-varying CET flux;
  – the variation in the magnetic coupling as a function of both the electric loading and relative position of the mover does not exceed 3.2%;
  – increasing the amplitude of the magnetization flux density of the transformer from 0.0 T to 1.0 T leads to a maximum force reduction of 0.9%.
11.1.5 Realization of a prototype for practical verification

A prototype has been manufactured and measurements have been carried out to demonstrate, firstly, the proof-of-principle and viability of the integration of CET into a linear machine and, secondly, quantify the accuracy of the combined harmonic model of chapter 9. From the findings the following can be concluded:

- between the functionalities mutually induced emfs occur on account of imperfections in the structure and magnetic material properties. However, the cross-coupling is sufficiently low to not obstruct normal operation of either functionality;

- electromagnetic cross-coupling effects in terms of criteria 2, 3, and 4 are lower than anticipated by 3D, nonlinear, FEM, simulations on account of the 3D material properties of the SMC Somaloy 5P130i. Quantification of the 3D electromagnetic behavior of the SMC Somaloy 5P130i is required to obtain more accurate FEM results;

- it is shown that a modified equivalent transformer model can be used to quantify the behavior of the CET with an error of maximally 2.42%. The values of the components of the equivalent transformer model can be considered position independent on account of negligible electromagnetic cross-coupling due to the mutual orthogonal orientation of the magnetic fields of the functionalities;

- the efficiency of the CET is 80%, but it can be increased by positioning the primary coil coaxially with respect to the shaft, applying bonded magnets instead of sintered ones, and replacing the stainless steel shaft with a ceramic one;

- the dynamical performance of the TSM is not influenced by the integrated CET as long as the variation in the dc-bus voltage is sufficiently low to allow the amplifier to generate the desired current levels;

- the combined harmonic model provides an initial description of the electromagnetic behavior of the integrated, tubular topology, but is still not accurate enough to be suitable as a reliable design tool. Yet, the method is promising and further research of the method is required to improve its accuracy to make it more suitable for design.
11.2 Scientific contributions

The main scientific contribution as a result of the study as presented in this thesis can be summarized as:

- Three novel, non-add-on, but physically integrated concepts for CET into a linear, permanent magnet, actuator have been proposed. One of the integrated designs, viz. the tubular topology, has been thoroughly analyzed, designed, manufactured, and verified by means of measurements on a working prototype. The realization of the working prototype and the measurements on it have proven the proof-of-principle. The concept of the integrated tubular topology has been patented [56].

- A modeling strategy for the complex electromagnetic problem associated with the integration of CET into a linear actuator has been formulated. The complexity of the electromagnetic problem of two superimposed magnetic fields, of which each operates in a different frequency domain, would require a transient, nonlinear, 3D FEM model for accurate modeling. Such a method is unsuitable for design on account of an unacceptably long calculation time. Instead, an analytical electromagnetic model based on the HM has been formulated, where accuracy has been sacrificed in favor of a significantly reduced calculation time. The fast model has allowed the dependency of the electromagnetic behavior on changes in the geometrical parameters to be mapped quickly for a large set of geometrical parameters.

- A quantitative, comparative, study has been conducted for four different modeling techniques. The modeling techniques as presented in this thesis are in themselves not new and no contributions with respect to the theory or mathematics of the modeling techniques have been made. However, a comparative analysis of the model techniques by applying all of them to an electromechanical problem - the benchmark problem - has been conducted.

- New state-of-the-art SMC Somaloy 5P130i material has been applied in a new and unconventional way to realize the integration of CET into a linear TSM.

11.3 Recommendations

11.3.1 Improved modeling for design

The electromagnetic modeling of the TSM motor with integrated CET has been executed by combining 2D harmonic models. The decomposition of the 3D problem into a combination of 2D subproblems introduces errors. A more accurate
11.3: Recommendations

model can be obtained by means of a 3D, harmonic vector potential model that incorporates force and eddy current calculations in a single model. A 3D harmonic model requires no preparatory FEM simulations for any tuning of the model. Since a 3D harmonic model is inherently linear, the errors in the force profile calculation due to the nonlinearity of the soft-magnetic material and the inability of quantifying the cross-coupling effects will remain. This thesis has demonstrated the validity of the decomposition of the problem into two frequency domains. Therefore, two, periodic, 3D FEM models can be applied to increase the accuracy; one nonlinear, magnetostatic model for the force profile calculation and core losses due to the TSM functionality, and one steady state, AC model for the eddy current losses and core losses due to the CET functionality.

11.3.2 Structural limitations

The stroke of the integrated, tubular topology is limited as a result of the stiffness in the radial direction of the suspended stator. On the other hand, the suspended stator allows a gapless transformer that ensures better magnetic coupling as compared to a gapped core. For extended strokes a gapped, integrated, tubular solution could be investigated. Alternatively, the integrated flat or integrated flux switching topology can be considered for extended strokes. Gapped solutions will generally lead to a more constant magnetic coupling due to the fact that the gap forms the dominant part of the magnetization permeance. Depending on the geometrical configuration, gapped solutions allow multiple segmented primary coils to reduce the ohmic losses. However, a segmented primary requires a switching algorithm for adequate powering of the primaries.
11.3.3 Separation of fluxes

To decrease cross-coupling effects of the integrated, tubular topology the flux paths could be separated as shown in Fig. 11.1. Due to the segmented TSM the CET field is concentrated in the outer ferrite ring. The soft-magnetic core material of the TSM could be SMC or laminated steel. However, additional eddy current losses will be present in the core of the TSM due to the leakage flux of the CET.

11.3.4 Identification of the 3D material properties of SMC

In this thesis it has been assumed that the SMC is isotropic and that its behavior is only determined by the magnitude of the field and not its orientation. To predict the behavior of the SMC Somaloy 5P130i more accurately by means of the FEM, it is recommended that the 3D electromagnetic behavior of the SMC Somaloy 5P130i, in which two orthogonal fields with different frequency and amplitude are present, is quantified. To that end, the identification of material properties ($BH$-curve tensor and loss-tensor) has to be conducted by constructing a test set-up which can impose two orthogonal fields of different frequency and amplitude.

11.3.5 Reduction of the eddy current losses

The eddy current losses in the stator of the integrated, tubular topology can be reduced by positioning the primary coil coaxially with respect to the shaft, applying bonded magnets, and use nonmagnetic, nonconductive materials for the shaft and frame.


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Integration of Dual Electromagnetic Energy Conversions
Linear Actuation with Integrated Contactless Energy Transfer

Voor lineaire, lange-slag machines met bewegende spoelen vindt de energievoorziening conventioneel plaats via een bewegende kabelboom. De kabelboom heeft een nadelig effect op de prestaties en dynamica van het systeem. Door de bekabeling te vervangen door een draadloos energieoverdrachtsysteem kunnen deze nadelige effecten ondervangen worden. In de high-precision-industrie worden voor nauwkeurige positionering vaak synchrone, permanente magneet machines toegepast. De voorkeur voor synchrone, permanente magneet machines vloeit voort uit hun uitstekende servokarakteristieken. Het onderzoek naar de electromagnetische integratie van een draadloos energieoverdrachtsysteem in een lineaire, synchrone, permanente magneet machine wordt in dit proefschrift uiteengezet. Daarbij ligt de nadruk niet alleen op de identificatie en analyse van de fysische gevolgen die gepaard gaan met de integratie, maar ook op de magnetostatische modellering van permanent magneet machines in algemener zin.

In het eerste deel van dit proefschrift worden vier magnetostatische modelleringsmethoden op kwantitatieve wijze met elkaar vergeleken, namelijk de Fourier-, Schwarz-Christoffel-, randelementen- en tandcontourmethode. Op basis van de bevindingen van deze vergelijking kan er een gefundeerde keuze gemaakt worden voor de geschiktste modelleringsmethode om het complexe, electromagnetische probleem dat zich als gevolg van de integratie van het draadloze energieoverdrachtsysteem in een lineaire machine aandient mee te modelleren. De uiteenzetting van iedere modelleringsmethode wordt ingeleid door een wiskundige afleiding. Om een kwantitatieve vergelijking te maken tussen de technieken, is iedere techniek getoetst op hetzelfde testprobleem. Het testprobleem bestaat uit een lineaire, synchrone, permanente magneet machine. Met behulp van iedere techniek is de ruimtelijke magnetische fluxverdeling in de luchtspleet van het testprobleem bepaald. Vervolgens is het krachtprofiel bepaald via verschillende methoden voor krachtberekeningen. De resultaten van zowel de fluxverdeling als het krachtprofiel zijn niet alleen onderling vergeleken, maar ook met resultaten die verkregen zijn via eindige elementenmethodesimulaties.
Het tweede deel van dit proefschrift is gewijd aan de elektromagnetische integratie van het draadloze energieoverdrachtsysteem in de lineaire machine. De integratie wordt gerealiseerd door twee magnetische veldcomponenten met verschillende amplituden en frequenties te superponeren in één magnetische kern. Wisselwerkingen tussen de energieoverdracht- en de machinefunctionaliteit zijn geïdentificeerd. De identificatie van de te verwachte wisselwerkingen heeft geleid tot de formulering van vijf criteria waaraan een geïntegreerde oplossing idealiter zou moeten voldoen. De criteria hebben betrekking op mutueel geïnduceerde electromotorische krachten in het overdrachtsysteem ten gevolge van de motorfunctionaliteit en vice versa, verstoringkrachten op de machinefunctionaliteit als gevolg van de overdrachtsfunctionaliteit en de beïnvloeding van de overdrachtscapaciteit door de machinefunctionaliteit. Aan de meeste criteria kan worden voldaan door de magnetische veldcomponenten in de kern onderling orthogonaal in de ruimte te oriënteren. De wijze waarop de integratie gerealiseerd wordt, stelt specifieke eisen aan de materiaaleigenschappen van de kern. Daarom is er een vergelijkende analyse van beschikbare zacht magnetische materialen uitgevoerd. De realiseerbaarheid van een geïntegreerde oplossing is toe te schrijven aan de specifieke materiaaleigenschappen van zacht magnetisch composietmateriaal. Door de isotropie en lage elektrische weerstand van het composietmateriaal kunnen de twee velden, die twee ordegroottes in frequentie uit elkaar liggen, gesuperpondeerd worden. Drie geïntegreerde structuren zijn voorgesteld. De geïntegreerde structuren zijn ieder onderworpen aan een vergelijkende, kwalitatieve analyse om de mate waarin zij voldoen aan de vijf criteria te toetsen. Uit deze analyse is geconcludeerd dat de tubulaire structuur het beste alternatief vormt. Tevens is bepaald met welke modelleringstechniek uit deel één iedere geïntegreerde structuur het beste gemodelleerd kan worden.

De tubulaire structuur heeft een orthogonale veldoriëntatie en is ontworpen om 800 VA aan vermogen over te dragen bij een maximale kracht van 250 N met een topsnelheid van 2 m s\(^{-1}\). Als modelleringstechniek is een gecombineerd Fouriermodel opgesteld als alternatief voor de tijdrovende, 3D, eindige elementenmethode. Door de ontkoppeling tussen beide functionaliteiten als gevolg van de orthogonaliteit van de velden en de kleine amplitude van de fluxdichtheid van de overdracht ten opzichte van die van de machine, kunnen de functionaliteiten los van elkaar gemodelleerd worden. Iedere functionaliteit is gemodelleerd met een apart 2D Fouriermodel. Deze modelaanname is geverifieerd met 3D eindige elementensimulaties. De optimale geometrie is verkregen door een 'parametric search'. Wisselwerkingen tussen de functionaliteiten zijn geïdentificeerd met behulp van niet-lineaire, magnetostatische, 3D eindige elementensimulaties. Deze simulaties kwantificeren tevens de fout die optreedt ten gevolge van de eis om het zacht magnetisch materiaal in de Fouriermodellen oneindig permeabel te veronderstellen.

Een prototype is vervaardigd waarop metingen ter verificatie zijn uitgevoerd. De metingen hebben het principe van de elektromagnetische integratie van draadloze energieoverdracht in een lineaire, synchrone, permanente magneet machine, zoals beschreven in dit proefschrift, aangetoond.
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