Multi-time multi-scale correlation functions in hydrodynamic turbulence

Citation for published version (APA):

DOI:
10.1063/1.3623466

Document status and date:
Published: 01/01/2011

Document Version:
Publisher’s PDF, also known as Version of Record (includes final page, issue and volume numbers)

Please check the document version of this publication:

• A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher’s website.
• The final author version and the galley proof are versions of the publication after peer review.
• The final published version features the final layout of the paper including the volume, issue and page numbers.

Link to publication

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal.

If the publication is distributed under the terms of Article 25fa of the Dutch Copyright Act, indicated by the “Taverne” license above, please follow below link for the End User Agreement:
www.tue.nl/taverne

Take down policy
If you believe that this document breaches copyright please contact us at:
openaccess@tue.nl
providing details and we will investigate your claim.

Download date: 17. Mar. 2021
I. INTRODUCTION

A comprehensive theory of the Eulerian and Lagrangian statistical properties of turbulence is one of the outstanding open problems in classical physics. Most studied quantities concern either measurements performed at the same time in multiple positions (Eulerian measurements) or along one or several particles moving with the flow (Lagrangian measurements). The latter are optimal to study temporal properties of the underlying turbulent flows but cannot simultaneously also disentangle spatial fluctuations, being based on single point, as for the case of acceleration. The main idea is to set up an ensemble of probing stations riding the flow, i.e., measuring correlations in a reference frame centered on the trajectory of distinct fluid particles (the quasi-Lagrangian reference frame introduced by Belinicher and L’vov [Sov. Phys. JETP 66, 303 (1987)]. In this way, we reduce the large-scale sweeping and measure the non-trivial temporal dynamics governing the turbulent energy transfer from large to small scales. We present evidences of the existence of the dynamic multiscaling properties of turbulence - first proposed by L’vov et al. [Phys. Rev. E 55, 7030 (1997)] - in which multi-time correlation functions are characterized by an infinite set of characteristic times. © 2011 American Institute of Physics. [doi:10.1063/1.3623466]
in a cubic three-periodic domain via a pseudo-spectral algorithm and second order Adams-Bashforth time marching. The forcing \( \mathbf{f} \), defined as in Ref. 46, acts only on the first two shells in Fourier space (\( |k| \leq 2 \)) and keeps constant in time the total (volume averaged) injected power, \((\mathbf{f} \cdot \mathbf{u})_V = \text{const} \). We report data coming from a set of simulations with \( N^3 = 256^3 \) and \( 512^3 \), corresponding to \( \text{Re}_x \approx 140 \) and 180, respectively (see Table I for relevant parameters characterizing the flows). The simulation, e.g., at \( \text{Re}_x \approx 140 \) has been carried on for 40 Eulerian turnover times, \( T = (3/2)u_{rms}^2/\nu \). We also integrated numerically \( N_p = 3.2 \cdot 10^4 \) tracers evolving with the local Eulerian velocity field \( \mathbf{x}(t) = \mathbf{u}(\mathbf{x}(t), t) \). At fixed temporal intervals, we evaluate the fluid velocity also at \( \mathbf{x}(t) + r_i \), with \( i = 0, \ldots, M \) (spatial distances from each tracer). The vectors are chosen to be always along one fixed direction, \( \mathbf{r} \), and are logarithmically spaced in the range between zero and half of the box-size (we use \( M = 20 \)), see Figure 1. Similar measurements are done also at fixed positions uniformly spaced in the fluid domain. These two set of data are denoted, respectively, as quasi-Lagrangian (\( L \)) and Eulerian (\( E \)).

### C. Notations and measurements

We focus our attention on the longitudinal increments of velocity a displacement \( \mathbf{r} \)
\[
\delta u_r(\mathbf{x}, t) = (\mathbf{u}(\mathbf{x} + r, t) - \mathbf{u}(\mathbf{x}, t)) \cdot \mathbf{r}.
\]

Notice that we have adopted a unifying notation, for us \( \mathbf{x} \) can represent either a fixed point in space \( \mathbf{x} = \mathbf{x}_0 \) or a point following a fluid particle: \( \mathbf{x}(t) = \int_0^t \mathbf{u}(\mathbf{x}(t), t_0) \, dt + \mathbf{x}_0 \) (a trajectory passing from \( \mathbf{x}_0 \) at time \( t_0 \)). We distinguish between the two cases by the superscript labels: \( \mathbf{u}_R(\mathbf{x}, t) \) or \( \mathbf{u}_E(\mathbf{x}, t) \). Note that \( \langle \delta u_r^E \rangle = \langle \delta u_r^E \rangle_0 \) (therefore, for such a quantity \( E \), \( L \) labels will be dropped). We define the generic multi-scale, multi-time correlation functions
\[
C^{(q,p,q)}_{K,R} (\tau) = \frac{\langle \delta u^R_R(\tau) \rangle^q \langle \delta u^E_E(t + \tau) \rangle^p \langle \delta u^E_E(t) \rangle^q}{\langle \delta u^E_E(\tau) \rangle^q \langle \delta u^E_E(t) \rangle^p \langle \delta u^E_E(t + \tau) \rangle^q},
\]
where \( R \) and \( r \) denote separation vector fixed in space and with different magnitude. Note that the \( L \), \( E \) distinction must be kept for the average of the multi-time product in the numerator. Given the correlation functions, we can define an
Eulerian and Lagrangian (integral) correlation time as follows:\textsuperscript{21}

\[
T_{[L,E]}^{[q,p,q]}(R,r) = \int_0^{\infty} C_{R', r, [L,E]}^{[q,p,q]}(\tau) d\tau. \tag{4}
\]

III. RESULTS

A. Single-scale multi-time correlation

We begin discussing the special case of a single-scale, multi-time correlation, i.e., \( R = r \). Dimensional inertial-range scaling, \((\delta u_r)^p \sim (\varepsilon r)^{p/3}\), provides the following estimate for the turnover time of inertial eddies of size \( r \), \( T_L^{[q,p,q]}(r) \sim r/\left(\delta u_r\right)^{1/p} \sim r^{2/3} \varepsilon^{-1/3} \). On the contrary, the Eulerian correlation time—due to sweeping effect—can be estimated by means of the typical velocity difference of the largest eddy, which is proportional to the mean square root velocity, \( \delta u_L \sim u_{rms} \). One has \( T_E^{[q,p,q]}(r) \sim r/\left(\delta u_L\right)^{1/p} \sim r/\varepsilon \). In the \( r \rightarrow \eta \) limit both correlation times tend to the dissipative scale \( \eta \). In Figure 2 (top inset), we show the behavior of \( C_{r,r,L}^{[1,1]}(\tau) \) for both Eulerian and quasi-Lagrangian velocity differences and for separation scales \( r \in [2.4, 245]\eta \). On the abscissa, the time increment \( \tau \) is made dimensionless through the Eulerian large eddy turnover time \( T \) (see Table I). We clearly see that after a time \( T \) all the correlations have decreased at least of a factor 50, supporting the quality and convergence of our simulations. The main panel of Figure 2 shows the integral correlation time both for the Eulerian and quasi-Lagrangian case as computed from Eq. (4), in a time integration window [0, \( T \)]. The behavior is in qualitative agreement with the expected scaling, the Lagrangian case being less steep than the Eulerian one; however, pure power-law scaling seems to be hindered by finite Reynolds number and system finite-size effects. To demonstrate this, we introduce a parametrization for the second order spatial velocity structure functions, with dissipative and large-scale cut-off (see also Ref. 45): \( T_n(r) = c_1(1 + (r/c_2)^n)^{n/2} \) for \( n = 2/3 \) and \( n = 1 \), respectively, for the Lagrangian and Eulerian case. The parameters \( c_1, c_2, c_3 \) represent the dissipative correlation time scale, the dissipative, and large cut-off scales, respectively. The good quality of the fit, shown in Fig. 2 (main panel), supports our hypothesis. Plotting the Lagrangian correlation time as a function of the Eulerian one, a procedure similar to the extended self similarity (ESS)\textsuperscript{45} does show a good scaling with slope 0.64 ± 0.02 in the range [20, 200]\( \eta \), consistent with 2/3 (Fig. 2, bottom panels). This finding again supports the idea that the limited scaling in Figure 2 is due to Reynolds number effects.

B. Intermittency and test of the bridge relations

It is well known that Eulerian statistics show intermittent corrections to dimensional scaling. For example, for structure function we have \( \langle \delta u_r^p \rangle \sim r^{\zeta(p)} \) where \( \zeta(p) \) is a nonlinear convex function of \( p \).\textsuperscript{1} In 1997, in a seminal work of L’vov, Podivilov and Procaccia\textsuperscript{21} provided a possible framework to encompass the phenomenology associated with intermittency also with temporal fluctuations. The idea consists in noticing that for time correlations the structure of the advection term of the Navier-Stokes equations suggests the relation: \( T_L^{[q,p,q]}(r) \sim r/\left(\delta u_L\right)^{1/p} \sim r^{2/3} \varepsilon^{-1/3} \) (Ref. 21). Using the scaling for the Eulerian quantities, \( \langle \delta u_r^p \rangle \sim r^{\zeta(p)} \), one gets to the so-called bridge relations (BR) connecting spatial and temporal properties:

\[
z(p) = 1 - \zeta(p) + \zeta(p - 1).
\]

Similar idea has also been successfully applied to connect the statistics of acceleration and velocity gradients. Plugging the empirical values\textsuperscript{3,6} for the Eulerian exponents in the previous expression, one predicts \( z(p) = 0.67, 0.74, 0.78, 0.80(\pm 0.01) \) for the orders \( p = 2, 4, 6, 8 \), respectively. In
between the two choices. Horizontal lines represent from bottom to top, and the BR values for confirmation to this evidence.

The scaling behavior of the peak time $T_{\text{peak}}(R)$ for fixed $R^* = 2.4\eta$ at changing $R \in [\eta : 120\eta]$. The solid line is a guide to the eyes connecting the peak for each given $R$. Inset bottom panel: cut of $C_{R,r;L}^{(1)}(\tau)$ plot at increasing $R$ (from bottom to top) as a function of $r$. Symbol (●) marks the position of the maximal correlation value (denoted as $T_{\text{peak}}(R)$), which increases for increasing values of $R$. Bottom panel: comparison between $T_{\text{peak}}(R)$ and $T_{\text{lag}}^{(1)}(R) - T_{\text{lag}}^{(1)}(\eta^4)$. The inertial scaling $R^{2/3}$ is also drawn for comparison.

C. Multi-scale multi-time correlation

We now focus on the most general case of multi-scale and multi-time correlation functions in the Lagrangian frame. In particular, in the correlation function (3) we vary the large scale $R$ while the small scale $r$ is kept fixed $r \approx \eta$. Note that the velocity difference $\delta u_{r;\eta}$ precedes in time, the difference $\delta u_{r;\eta}$. We are, therefore, interested in the time it takes for a velocity fluctuation to cascade down from a large eddy (of size $R$) to the smallest one (of size $\eta$). In Figure 4 (top panel), we show the correlations $C_{R,r;L}^{(1)}(\tau)$ with $R^* = 2.4\eta$ as defined from Eq. (3) (except for the fact that to enhance the contrast instead of $\delta u_{r;\eta}$ we used $|\delta u_{l;\eta}|$). The presence of a peak in $C_{R,r;L}^{(1)}(\tau)$ for each given $R$, defines a time, $T_{\text{peak}}(R)$, which increases for increasing values of $R$. The presence of the peak can be directly associated with the time lag, it takes the energy to go down through scales from $R$ to $R^*$, i.e., a direct evidence of temporal properties of the Richardson turbulent cascade. In the inset of the bottom panel, we show the curves corresponding to $C_{R,r;L}^{(1)}(\tau)$, for each different $R$ at varying $\tau$.

The scaling behavior of the peak time $T_{\text{peak}}(R)$ $\sim R^{2/3}$, shown in Figure 4 (bottom panel), is in agreement with what has been measured for Fourier-space based quantities by Wan et al.\textsuperscript{38}
Also in Fig. 4 (bottom panel) a comparison of \( \tau_{\text{peak}}^{(1,1)}(R) \) with \( T_{\text{L}}^{(1,1)}(R) - T_{\text{L}}^{(1,1)}(r^*) \) (as computed for Fig. 2) is shown. It is remarkable to note that the amplitude and scaling of \( T_{\text{peak}}^{(1,1)}(R) \)-coming from the multi-scale correlation function—is close and compatible with \( T_{\text{L}}^{(1,1)}(R) - T_{\text{L}}^{(1,1)}(r^*) \)-coming from the single-scale correlation functions. This finding provides a clean confirmation that energy is transferred down-scale in the quasi-Lagrangian reference frame with a temporal dynamics consistent with what estimated from K41 theory.

IV. CONCLUSIONS

We presented an investigation of multi-scale and multi-time velocity correlations in hydrodynamic turbulence in Eulerian and quasi-Lagrangian reference frame. Our main results are the following: (1) We have demonstrated that quasi-Lagrangian measurement are able to remove the sweeping effect. The integral correlation times in the Eulerian and Lagrangian frame are shown to scale differently. (2) Lagrangian properties possess a dynamical multi-scaling, i.e., different correlation functions correlated with different characteristic time scales. (3) Bridge relations connecting single-time multi-scale exponents with multi-time single-scale exponents are valid, within numerical accuracy. (4) The locality in space and time of the energy cascade is supported by studying the delayed peak in multi-time and multi-scale correlations. Temporal fluctuations becomes larger and larger by going to smaller and smaller scales, a phenomenon that may even affect numerical stability criteria for time marching, similarly to an effect concerning spatial resolution induced by spatial intermittency. Some issues similar to the ones here discussed have also been addressed in a recent numerical study, where evidences of the Lagrangian nature of the turbulent energy cascade have been demonstrated by studying the correlation between energy dissipation and local energy fluxes in the quasi-Lagrangian frame. While the method followed in Ref. 38 requires a knowledge of the three-dimensional velocity field, the approach proposed in the present manuscript needs only the knowledge of the velocity at just a few points along a Lagrangian trajectory: a measurement which may be already accessible in current particle tracking experimental set-ups.

ACKNOWLEDGMENTS

The authors acknowledge useful discussions with R. Pandit and P. Perlekar. E.C. wishes to thank J.-F. Pinton for discussions and support to the initial stages of this work. E.C. has been supported also by the HPC-EUROPA2 project (project number: 228398) with the support of the European Commission - Capacities Area - Research Infrastructures. We acknowledge COST Action MP0806 and computational support from SARA (Amsterdam, The Netherlands), CINECA (Bologna, Italy), and CASPUR (Rome, Italy).


1. A. Pandit and P. Perlekar. E.C. wishes to thank J.-F. Pinton for discussions and support to the initial stages of this work.