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Resilience in optical ring-resonant switches

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Abstract: Phase-modulated ring resonant switches are receiving increasing attention for monolithic Silicon photonic networks. Resilience to fabrication variations and operational tolerances are however required to create networks with sufficient connectivity and bandwidth. In this work we use the combination of vectorial optical-mode propagation and transfer matrix calculation to map fabrication-level feature size variation to the optical switch performance metrics for extinction ratio, bandwidth and power penalty. Fabrication tolerances may be relaxed considerably through the combination of moderate size directional couplers of up to 30 µm, moderate 400 GHz free spectral range resonator design and the use of fifth order resonance. High speed 10Gb/s, wavelength-multiplex-compliant, optical signal routing is predicted with on-state power penalties of 0.2 dB – 0.7 dB and off-state signal extinctions of – 62dB.

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References and links

1. Introduction

Electronically-tuned ring resonators are offering exciting opportunities for a new generation of compact, integration-compliant, electronically efficient Silicon photonic components [1, 2]. Phase-modulated ring-resonators have been demonstrated with data modulation rates and

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extinction ratios of order 12.5 Gbit/s and 9 dB for monochromatic optical carrier signals [3]. Spectrally-aligned networks of such modulators are being proposed for photonic on-chip interconnection networks [4, 5]. However high connectivity networks may advantageously exploit broadband optical switching: A modest number of switches can avoid the over-provisioning of multiple point to point photonic waveguides and the very high numbers of associated modulators. Broadband switches have been proposed and demonstrated [6,7] using higher order resonance to give 1 nm on-state bandwidth, low-penalty routing for 40 Gbit/s signals, and a route to high-capacity wavelength multiplexed routing. However so far only a modest – 20 dB extinction ratio has been achieved and without direct electronic tuning [7]. It is therefore of particular current interest to identify and quantify designs for creating fabrication-resilient, refractive-index controlled switch elements with both broadband on-state transmission and excellent off-state signal extinction.

Ring resonant photonic components require strikingly different optimizations. Filters and broadband switches require flat-top transmission bandwidths and excellent out-of-band signal extinction. Narrow-band data modulators can in principle operate with modest signal extinction and pass-band width but losses must be critically controlled. High cavity resonances have allowed modulators to operate within modest limits for refractive index tuning [2, 8], but performance for pass-band broadened switches is not so well understood. No direct quantitative comparison of switch metrics has been performed for higher order resonant switch designs and dependence on fabrication critical parameter is not quantified.

In this work, we present the first detailed theoretical study into the quantitative performance and fabrication resilience of high order resonant switch elements. The ring-resonator spectral transfer functions are first reviewed in section 2 to identify the parameters for inter-ring power coupling. The fabrication sensitivity of operational variables such as off-state signal extinction, on-state bandwidth and pass-band ripple is then presented in section 3. A vectorial mode solver analysis is presented to relate directional coupler design to small variations in feature size. The impact of path length error within the resonator is then studied statistically in section 4 to quantify the control precision requirements. Section 5 defines switch operation and section 6 finally quantifies data integrity in terms of power penalty for the routing and switching of broadband data.

2. Spectral characteristics

Schematic switch designs are shown in Fig. 1 implementing first, third and fifth order resonance. Optical waveguides are shown as dark lines, and electrodes are shown schematically as lightly shaded regions. The circuits are schematically drawn in elongated form to distinguish the directional couplers from the phase modulators in the optical resonator waveguide cavities. Close proximity waveguides represent the directional couplers which transfer optical signals back and forth between the ring resonators. For the first order example in Fig. 1(i), fixed wavelength light at the input is switched between the by-pass output and the ring-coupled output by electronically applied phase shifts within the resonator waveguides. The fourth port may be similarly used as an input with reciprocal properties. The 1 × 2 switch mode of operation is therefore studied in detail, and four port operation is revisited in section 5.
Bandwidth, loss and extinction ratio may be calculated by defining transfer matrices in optical frequency space. A base transfer matrix may be defined by subdividing the circuit into sections to describe one directional coupler and one optical waveguide phase shifter for each section. Figure 1(i) shows the partitioning for the simplest case of the single ring into two such sections. An \(N^\text{th}\) order switch consists of \(N + 1\) such sections. The transmitted optical field \(T\) is calculated after each directional coupler \(n\) with power coupling coefficient \(\kappa_n\).

Optical fields propagating along the reverse path \(R\) are calculated in the same manner. Losses are accounted for with the scalar value \(\gamma\). Output fields \(T_{N+1}\) and \(R_{N+1}\) may therefore be expressed in terms of the inputs \(T_0\) and \(R_0\) through matrix multiplication [9]:

\[
\begin{pmatrix}
T_{N+1} \\
R_{N+1}
\end{pmatrix} = \prod_{n=0}^{N-1} \begin{pmatrix}
1/(1-\kappa_n)z\gamma \exp(-j\phi_n) & -\kappa_n z\gamma \exp(-j\phi_n) \\
-\kappa_n z\gamma \exp(-j\phi_n) & (1-\kappa_n)z\gamma \exp(-j\phi_n)
\end{pmatrix} \begin{pmatrix}
T_0 \\
R_0
\end{pmatrix} .
\]

Recurring switch pass-bands with a free spectral range \(\Delta f_{fsr}\) are conveniently described using the \(z\) transform description of discrete optical frequency space \(z = \exp(j2\pi f / \Delta f_{fsr})\).

For a normalized input \(T_0(f) = 1\), the two optical power transfer functions for \(N^\text{th}\) order switches may be simply expressed: \(H_{N,\text{ring-coupled}}(f) = T_{N+1} R_{N+1}\) and \(H_{N,\text{by-pass}}(f) = R_0 R_0^\ast\). The phase shifts \(\phi_n\) in Eq. (1) define the switch state. The spectral transfer function for first order and higher order resonant switch elements are now quantified using an analytical and numerical approach for first and higher order switches respectively to identify suitable power coupling coefficients.

2.1 First order resonators

The first order ring resonator is algebraically quantified by making a trigonometric substitution for discrete optical frequency \(z\). In the lossless limit with no applied phase shifts, Eq. (1) may be conveniently rearranged to give on-state bandwidth and off-state extinction for the ring-coupled path. Defining frequency detuning \(f\) as a function of transmission allows an estimate of on-state bandwidth in Eq. (2). Defining transmission for the frequency detuning of \(f = \Delta f_{fsr}/4\) allows a concise expression for signal extinction in the off-state in Eq. (3):

\[
f = \frac{1}{2\pi} \cos^{-1}\left[\frac{1+(1-\kappa^2)^2 - \kappa^2 H_{1,\text{ring}}}{2(1-\kappa)}\right] .
\]

\[
H_{1,\text{extinction}}(f = \Delta f_{fsr}/4) = (1+(1-\kappa)^2/4)\kappa^2
\]

For comparative purposes, on-state bandwidth and off-state extinction ratio are quantified for a 1dB passband and \(f = \Delta f_{fsr}/4\) frequency detuning respectively. Power coupling

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coefficients of 0.001, 0.01 and 0.1 lead to pass-band widths of 0.0002 Δf_{pr}, 0.0016 Δf_{pr}, and 0.0171 Δf_{pr} with signal extinctions of order $-63.0$ dB, $-43.0$ dB, and $-22.6$ dB. Simultaneously good on-state spectral bandwidth and off-state signal extinction therefore leads to a very high free spectral range and thus poor spectral utilization for wavelength multiplexed inputs. While this may be acceptable for modulators operating on single narrow-band optical inputs, performance is severely compromised for switches operating on wavelength multiplexes of broadband optical input data. Higher order resonance avoids this trade-off.

2.2 High order resonators

Higher order resonators are simulated by numerically calculating the matrix products in Eq. (1) across z-space. To understand the photonic physical limits, loss and phase error are omitted in the initial optimization. The influence of these parameters is addressed independently in section 4. The spectral responses for third order and fifth order resonators are generated and compared with ideally flat spectral pass-bands with 60 dB/decade and 100 dB/decade roll-offs for the third and fifth order filters. Correlations between the idealized and calculated spectral responses are quantified in terms of the root-mean-square difference. Figure 2 shows the calculations for the most relaxed calculations where the zero-loss bandwidth for the ring-coupled path corresponds to 0.126 Δf_{pr}. Symmetric designs are considered to allow the optimizations to be visualized as a two-dimensional contour map and a three-dimensional surface plot for the third and fifth order designs respectively.

(i) Third order resonator
(ii) Fifth order resonator

Fig. 2. Mapping root mean square difference between target transfer functions and idealized filter responses (i) Third order resonator compared to 0.126Δf_{pr} flat pass-band-width filter with −60 dB/decade roll-off (ii) Fifth order resonator compared to 0.126Δf_{pr} flat pass-band-width filter with −100 dB/decade roll-off.

Third order resonators with coupling coefficients sequenced \{κ_1, κ_2, κ_2, κ_1\} show a minimum root mean square difference for values κ_1 = 0.45, κ_2 = 0.09 with a stronger sensitivity for the inner coupling coefficient κ_2. The three-variable parameterization for the fifth order design is represented with a single valued surface plot for the root mean square difference at 0.05. This surface shows a locus of optimum values, with a relaxed sensitivity
for the outermost couplers where the power coupling is tolerated from 0.45 up to 0.65. The lower limit of $\kappa_1 = 0.45$ is considered further in this work to study the most compact directional coupler designs. The corresponding optimum inner directional coupler coefficients are $\kappa_2 = 0.09$ and $\kappa_3 = 0.05$. Insight into the critical role of coupling coefficients on the spectral transfer function is given by plotting transfer function detail at the rollover frequency for small independent variations for each coupler. The target flat pass-band, finite-roll-off transfer function for the fifth order design is given as black dashed lines in Fig. 3. The first, second and third coupler values are varied over the range ± 20% in graphs (i), (ii) and (iii) respectively.

The outermost couplers in the fifth order design $\kappa_1$ show a marked improvement in pass-band ripple as the coupling coefficient value increases but this is ultimately at the expense of spectral bandwidth and physical size. For the second $\kappa_2$ and third $\kappa_3$ couplers, a small increase in bandwidth is observed as the poles are moved across the pass-band. The optimum values will depend on the target on-state bandwidth. The residual ripple also has important implications for off-state signal extinction as quantified in section 5.

### 3. Directional couplers

Optical signals are readily coupled between resonators using short lengths of close proximity waveguides operating as directional couplers. Figure 4 shows a schematic implementation for Silicon-on-Insulator technology, highlighting the mask level design metrics which impact the power coupling coefficient. Waveguide dimensions are set to be 220 nm x 500 nm to ensure low-loss single-mode transmission. The directional couplers include s-bends of length 7 µm at the inputs and outputs to suppress spurious coupling between the resonator cavities. The target gap between the coupled waveguides is defined by the minimum feature size for the process, but the absolute value will be subject to etching and lithographic variations [10]. Therefore a target waveguide gap is specified for this analysis and the fabrication error is specified as the difference between target gap and the range of simulated gaps. Optimum directional coupler lengths are identified for four target power coupling coefficients and three gaps between the waveguides. A vectorial mode solver [11] is used to calculate the power coupling between selected input and output for the scheme in Fig. 4. Table 1 summarizes the coupler lengths which correspond to the target power coupling coefficients $\kappa$, and thence the optimum resonant switch designs.
Table 1. Calculated Coupler Lengths in Selected Waveguide Directional Coupler Designs

<table>
<thead>
<tr>
<th>Target power coupling $\kappa$</th>
<th>Target gap between coupler waveguides</th>
</tr>
</thead>
<tbody>
<tr>
<td>1$^{st}$ order</td>
<td>3$^{rd}$ order</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>0.09</td>
</tr>
<tr>
<td>0.18</td>
<td>-</td>
</tr>
<tr>
<td>-</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Coupler designs as small as 2.8 $\mu$m are achievable for the smallest waveguide gaps and coupling coefficient. The length increases to a maximum of 72 $\mu$m for the largest coupling coefficients and largest gaps. The tabulated designs are used as target inputs for the fabrication tolerance study in Fig. 5. To allow data to be plotted concisely with the same axes, coupling coefficients are divided by the target value. Normalized values therefore equal unity for zero fabrication error in Fig. 5.

The narrowest 100 nm gap directional couplers show a rapid increase in coupling coefficient as fabricated feature size increases. A feature size increase and therefore waveguide width increase of 10 nm (and corresponding gap width decrease of 10 nm) leads to an increase of approximately 10% for the targeted coupling coefficients. This arises from enhanced optical overlap between the waveguides. In contrast, the widest 280 nm gap directional couplers show a stronger increase in coupling for reductions in gap width. This is
expected to result from reductions in effective modal refractive indices for the waveguides. The intermediate-gap directional couplers show the least sensitivity to the absolute waveguide separation with a marked reduction in sensitivity to feature size error: ±30 nm error leads to a 10% variation in power coupling. The normalized horizontal grid-lines correspond to the maximum proportional variations studied in Fig. 3.

4. Propagation impairments

The propagation of optical signals through the photonic waveguides leads to changes in phase and amplitude. Phase error leads to a misalignment of resonances between resonators within the switch elements themselves and also with respect to the input optical data spectrum. Loss reduces the resonator strength. At the circuit level, the same free spectral range $\Delta f_{\text{fsr}}$ is required for each ring resonator and this should be aligned to the same wavelength grid as the broadband input optical data. However the free spectral range is sensitive to local and global refractive index variations and feature size variations at micro-bends. To quantify the role of such fluctuations across the circuit, systematic and random phase errors are introduced into transfer function calculations. The effect is visualized in terms of on-state bandwidth as a function of path length phase error in Fig. 6. To ensure reasonable on-state bandwidths of 13 GHz, 50 GHz and 50 GHz for the three orders of resonator, a free spectral range of 400 GHz is considered. This corresponds to a physical path length of order 180 μm for 1.55 μm data signal wavelengths assuming a group refractive index value of 4.2. The implications for other important switch design parameters are discussed in section 7. The comparison is made with the narrower bandwidth first order resonator with $\kappa = 0.18$ to ensure a tolerable on-state extinction ratio of ~17.1 dB. For a fixed free-spectral range, a broader on-state pass-band would further degrade the off-state extinction, but an increased free-spectral range reduces the spectral utilization.

![Graph](image)

Fig. 6. Bandwidth narrowing as a result of optical path length phase error and loss in high-order switches. Systematic error (black lines) plotted as a function of phase error Random error (red lines with error bars) plotted as a function of standard deviation for (i) first ring resonant switch, (ii) third ring resonant switch and (iii) fifth order ring resonant switch Data for varied levels of waveguide loss are also shown with solid and dashed lines.

Systematic errors resulting from a global circuit-level change in refractive index or a detuning of the signal spectrum, lead to a simple linear reduction in the overlap between the designed and achieved bandwidth. This is shown for the black lines in Fig. 6. The available bandwidth is reduced to 20% for a 0.05π phase shift error in the single ring resonator design, but remains at approximately 80% of the original value for both higher order designs. This
fourfold improvement in tolerance results directly from the broader, near-rectangular pass-bands for the higher order switches.

Random error is plotted in Fig. 5(ii) and Fig. 5(iii) for the higher order switches. Now the phase errors are varied independently for each resonator and may represent imperfectly resolved proximity effects in the mask transfer during fabrication. The x axis values now correspond to the standard deviation for independent normal distributions of phase errors for each of the higher order rings. The mean and standard deviations for the bandwidth narrowing are represented using dashed lines and the vertical error bars. One thousand calculations are performed for each data point to ensure reproducible estimates for the mean and standard deviation. The role of random phase errors for each of the ring resonators within the same higher order switch element is initially a weak effect. However, the impact of random phase errors with standard deviations exceeding 0.02π and 0.01π for third and fifth order rings would impact the usable bandwidth, and diminish the value of higher order resonant design if they are not adequately controlled.

Loss impacts the on-state bandwidth as well as the transmitted power. Figure 6 shows the impact on transmission bandwidth using the dashed and dotted lines. The bandwidth at 1dB roll-over is evaluated for each synthesized transfer function. Loss is increased from 0/cm to 3 /cm, corresponding to a range of up to 13 dB/cm and a worst case net loss of 1 dB for a fifth order design. Losses of these magnitudes have limited impact both in terms of systematic and random error. Nonetheless, state of the art processing is required to achieve these loss levels [7], and such worst case on-state losses may limit the number of on-state ring-coupled switches in an optical path.

5. Switch operation

Performance as a switch element is verified through the comparison with available experimental data, prior to performing calculations for more generalized designs. Experimentally measured switch characteristics have been reported for photo-induced phase modulation at the intersection between two of the five rings in a fifth order resonator [7]. For calibration purposes, this mode of switching is studied numerically by applying the phase shifts \( \phi_n = \{0, \phi, \phi, 0, 0\} \) in Eq. (1).

The switch state allowing optimum ring-coupled operation is represented by the low loss region for zero detuning \( (f = 0) \) and zero phase shift \( (\phi = 0) \). Contours at 10dB in Fig. 7(i) show how the spectral transfer function of the ring-coupled path evolves rapidly with increasing phase shift within the two switched resonators. Increasing the phase in only two of the five rings leads to a break-up of the resonator transfer function as the two ring resonators are detuned relative to the remaining three resonators. To quantify achievable signal extinction for the ring-coupled path, frequency resolved transfer functions are also shown for four values of phase shift \( \phi = 0, \pi/4, \pi/2 \) and \( \pi \) in Fig. 7(ii). Ring-coupled transfer functions are shown to be near-rectangular for the zero-detuning condition. Increasing the phase shift reduces the transmission at zero detuning to –15 dB for a phase shift of \( \pi/4 \), –26dB for a phase shift of \( \pi/2 \) and –32 dB for \( \pi \). This exceeds the –20 dB experimentally demonstrated signal extinction where the maximum implemented phase tuning was unspecified. The switch transfer functions can be improved even further if all rings are simultaneously phase modulated.
Comparative numerical simulations are performed for 1st, 3rd and 5th order designs with varying numbers of phase modulated resonators. Signal extinction is calculated for zero detuning between data and resonance ($f = 0$) with no phase modulation ($\phi = 0$) for the first switch state, and with a phase detuning ($\phi = \pi/2$) for the second switch state. The value for extinction is specified as the mean loss within bandwidths of $0.032 \Delta f_{fsr}$, $0.126 \Delta f_{fsr}$ and $0.126 \Delta f_{fsr}$ for the three orders of switch. The second switch state is achievable by phase modulating combinations of ring resonators, and is therefore represented by the number of rings which are phase modulated. Data are summarized in Fig. 8 for the case of the single ring design, a third order design with one, two and three phase-modulated rings, and finally a fifth order design, with varied numbers of phase modulated rings.

The four enumerated path states in Fig. 8 for one input to the ring-coupled output and the by-pass output show important differences:
1. The ring-coupled path for the $\phi = 0$ switch state shows losses of $-0.35$ dB, $-0.12$ dB and $-0.13$ dB for the first, third and fifth order designs respectively. This modest but finite loss may place a limit on the number of times a signal may be serially routed via ring-coupled paths.

2. The ring-coupled path for the $\phi = \pi/2$ switch state shows a clear dependence between off-state extinction and the number of phase modulated rings. The extinction level is also determined to a small extent by the selected combination of rings. For example, in the case of the fifth order design, two centrally placed phase modulators allow $-26$ dB extinction (Fig. 7) while modulators placed at the input of the switch allow only $-22$ dB extinction (Fig. 8.). The highest number of phase modulated rings in the fifth order design offers the most impressive $-62$ dB signal extinction.

3. The by-pass path for the $\phi = 0$ switch state shows a consistently poor off-state signal extinction of $-12$ dB to $-16$ dB, with only a weak dependence on the order of resonance and the number of phase modulated resonators. This is attributable to finite levels of pass-band ripple and the reciprocal signal leakage.

4. The by-pass path for the $\phi = \pi/2$ switch state is consistently very good. Transmission losses are less than $-0.1$ dB, and of order 1 dB when three or more resonators are phase tuned in the fifth order design.

The fifth order design is therefore able to implement one path state (state 1) with low loss, one path state with negligible loss (state 4) and one path state (state 2) with excellent signal extinction. This combination enables a $1 \times 2$ element which is already well suited to a range of bus coupled architectures to create $1 \times N$ switches. The fourth port could conceivably be exploited in $N \times N$ cross-point switch architectures if an appropriately high performance waveguide crossing design is implemented. The relatively poor signal extinction for path state 3 could however limit performance as a generic $2 \times 2$ switch element.

6. Data integrity

The impact of fabrication imperfection on the photonic data integrity is now considered. Broadband optical data signals are represented in frequency space by taking the Fourier transform of non-chirped 10 Gbit/second data sequences with $2^7-1$ pseudo random bit sequence. Sampling is performed at the same frequency interval for both the optical data spectrum and the resonant switch transfer function. The product in the frequency domain defines the output signal spectrum. A time domain optical receiver model is implemented with a fifth order Bessel filter and thermal noise defined to give a sensitivity of $-20$ dBm. To predict power penalty, the optical power levels which are required to achieve eye-diagram Q factors of 6 are calculated [12] with and without the ring resonators. The worst case power penalties are recorded for the full range of directional coupler designs represented in Fig. 5.
Power penalties are plotted in Fig. 9 for the conditions where mean transmission loss does not exceed 2dB through the ring-coupled path. To study the role of detuning, the spectra for the input data and the transfer function are also detuned with respect to one another. For clarity, the role of path length phase error is not considered here, but if inadequately controlled, this would lead to a further bandwidth narrowing. Worst case power penalties are simulated to be in the range 0.2 dB to 0.7 dB across the on-state pass-bands for all three orders of resonant switch. A clear correlation is observed between the pass-band ripple and the penalty performance as a function of signal detuning. The single peak in the first order response results from the rounded pass-band and non-optimum optical domain signal filtering. The multiple peaks in the fifth order response are also evident in the ripple shown in Fig. 3. The small variation in penalty is thus attributable to signal distortion, and may impose a limit on the number of times a signal may traverse a ring-coupled path in a given circuit.

### 7. Discussion

The impact of nanoscale fabrication errors on switch component specification may be understood to a first order by considering the resonance condition for one ring. The optical path length of the resonator defines the comb of resonant wavelengths \( \lambda_i \) supported in a first order ring resonator. Neglecting the wavelength dependence of refractive index \( n_r \) for the spectral range of interest allows an estimate of free spectral range in terms of path length for speed of light c:

\[
n_i L \approx i \frac{\lambda}{c} = i / f_i = c / \Delta f_{fsr}
\]

Substitutions may be made to relate the path length \( L \) to both the \( i \)th absolute frequency \( f_i \) in the comb and the free spectral range \( \Delta f_{fsr} \). Tolerated phase errors may then be estimated by differentiation and substitution:

\[
\delta f_{fsr} / \Delta f_{fsr} \approx n_i \delta L / \lambda_i = \phi / 2\pi
\]

The optical modes in a small-radius, closed resonator are primarily defined by the outer radius [13]. For a worst case change in bend radius \( \delta r \), a path length error may be estimated from the change in waveguide width \( \delta w \) such that \( \delta L = 2\pi \delta r = \pi \delta w \). The tolerated phase error for first and fifth order resonators of 0.01\( \pi \) and 0.05\( \pi \) in Fig. 6 would therefore correspond to feature size errors of 0.6 nm and 2.9 nm respectively, potentially dominating over the directional coupler tolerances calculated in section 3. Nanometer feature size variations do however now appear to be conceivable [10, 14]. The absolute wavelength error may similarly be estimated \( \delta \lambda = \delta L \lambda_i / L = \pi \delta w \lambda_i / L \). A 2.9 nm feature size error
corresponds to 8 pm wavelength error for the 400 GHz (3.2 nm) free spectral range proposed, and thus only a 2% reduction of the spectral pass-band-width.

Optoelectronic switching requirements may be similarly understood to a first order by expressing the effective phase shift in terms of small changes in refractive index for a fixed resonator path length.

\[ \delta f/\Delta f_{\text{rd}} \approx \delta n_{\text{r}}, \frac{L}{\lambda} = \phi/2\pi \] (6)

A phase modulation to achieve switching of order \( \phi = \pi/2 \), as per section 5, with phase modulators of length 100 µm and 1mm would lead to refractive index changes of order \( 4 \times 10^{-3} \) and \( 4 \times 10^{-4} \) respectively. While these are high values, they are promising for free carrier injection and the Kerr effect in Silicon [9]. Phase modulators in combination with directional couplers of length 28 µm in Table 1 would again allow for the free spectral range of 400 GHz considered in sections 4-6. Minimum bend radii of 5 µm could allow a minimum enclosed footprint of order \( 10^{-5} \) cm\(^2\) per ring, potentially enabling a 64 x 64 cross-point interconnect arrangement using fifth order resonators within 1 cm\(^2\). The extinction levels of \(-62\) dB for the fifth order design would lead to crosstalk levels of order \(-44\) dB, indicating a potential for even further connection scaling.

8. Conclusion

The first detailed quantitative study into the fabrication tolerance for high-performance ring resonant switches is presented. The intrinsic deleterious interdependence between on-state pass-band-width and off-state switch extinction ratio observed for first order ring resonant switches is removed through the use of higher order resonance and moderate length phase modulators. Several orders of magnitude improvement in extinction ratio are predicted while maintaining good spectral utilization of order 10% for broadband wavelength multiplexed routing. Feature size tolerance is relaxed by a factor of four when comparing fifth-order, 50 GHz pass-band, \(-62\) dB extinction switches with first-order, 13 GHz bandwidth, \(-17\) dB extinction. Even further improvement would be seen if compared with an equivalent off-state extinction or on-state spectral efficiency for the single order resonator design. This simultaneous relaxation of fabrication tolerance and decoupling of off-state extinction from on-state bandwidth offers a promising route to broadband high-connectivity monolithic switching networks.