Introduction
Adopting a continuum approach, local plasticity formulations are capable of simulating the behaviour of metals to a certain degree. Specific phenomena can not be modelled properly, e.g. the intense localisation of deformation in case of shear bands. From a physical point of view these phenomena are attributed to the microstructural interactions, which are not described in the local approach. Furthermore, it is also known that the microstructure plays a key role in the ongoing miniaturization of forming processes. This research mainly considers the following aspects:
- ductile failure behaviour
- scale-size effects

Method
Yield function
Classical Huber-Mises plasticity is used, enriched with a ductile damage parameter $\omega_p$:
$$ F = \sqrt{3J_2} - \sigma_y [1 - \omega_p] $$
where $\sqrt{3J_2}$ denotes the second invariant of the stress tensor, $\sigma_y$ a standard hardening rule.

Ductile damage evolution
The amount of ductile damage is related to the nonlocal effective plastic strain $\bar{\varepsilon}_p$, according to an exponential evolution law:
$$ \omega_p(\bar{\varepsilon}_p) = 1 - e^{-\beta \bar{\varepsilon}_p} $$
Accordingly, the amount of ductile damage will gradually decrease the initial yield strength from its initial value ($\omega_p = 0$) towards zero ($\omega_p = 1$) upon complete failure.

Higher-order continuum
In addition to the equilibrium condition the nonlocal field is determined by solving the following partial differential equation of the Helmholtz type:
$$ \bar{\varepsilon}_p - \ell^2 \nabla^2 \bar{\varepsilon}_p = \varepsilon_p $$
The Laplacian $\nabla^2 \bar{\varepsilon}_p$ implicitly incorporates an infinite series of higher-order gradients of $\varepsilon_p$, which is consistent with a higher-order continuum approach to maintain the well-posedness of the problem [1]. Moreover, this type of equation has a nonlocal character [2], implying long-range interactions of a material point with its surrounding material.

Results
Length parameter
The length parameter $\ell$ is a material parameter, which governs the spatial interactions within the continuum (see figure 1).

Regularization
The local quantity $\varepsilon_p$ in figure 2 marks the plastic zone. The long-range interactions of $\bar{\varepsilon}_p$ into the elastic region allow the shear band to maintain a certain (finite) volume.

Evolution of failure
During the process of ductile failure two stages are noticeable:
- formation of the first and second shear band (1st row in figure 3)
- further localisation within the two shear bands (2nd row)

Conclusion
The presented gradient plasticity model is well capable of describing the process of ductile failure in a regularized fashion.

Future work
- Assessing the quality of the gradient model from a physical point of view
- Extension to geometrically nonlinear problems

References: