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Sliding friction dynamics of hard single asperities on soft surfaces

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The sliding friction dynamics of hard single asperities on soft polyester surfaces has been studied. Detailed time-resolved measurements of the sliding dynamics are presented. Of particular interest are normal displacements during “stick–slip”. Furthermore, the usual transitions from “stick–slip” to steady sliding behaviour are measured for increasing driving speed and increasing driving spring stiffness. Similar transitions were encountered for decreasing asperity radius and increasing normal force, that are taken to reflect the single-asperity nature of the contact. It is argued that the normal component of displacement is crucial for the sliding dynamics. Based on this observation a simple description in a dynamical system is proposed and its relation to rate-and-state formulations discussed. It is concluded that, apart from being relevant for friction and wear of polymers in general, the system is also interesting as a model system for the study of rate-and-state dynamics.

Keywords: single asperity, stick–slip, rate-and-state, friction dynamics, sliding friction

1. Introduction

Experiment and theory analysing dynamic friction have shown that the behaviour of a sliding system depends not only on the physical properties of the contact but also on the dynamic parameters of slider and loading equipment (e.g., slider mass $m$, driving speed $v_0$ and loading spring stiffness $k_l$). It is now realised that drastically different qualitative behaviour can be intrinsic to such systems, and also that this complicated behaviour offers possibilities to better understand the sliding physics [1]. Experimental studies of dynamic behaviour, such as time dependent “stiction” and transitions from steady sliding to “stick–slip”, have uncovered remarkable parallels between widely different materials at widely different time- and length-scales. Phenomenological dynamical systems, so called “rate-and-state” models, have proven to be rather adequate in reproducing this observed behaviour. Usually these descriptions couple evolution of the position and speed of the slider to evolution of one or more “state variables”, $\dot{\vartheta}_i$. A general form is

$$m \ddot{x}(t) = k_l (v_0 t - x(t)) - F_l(t)$$

(1)

with

$$F_l = F_l(\dot{x}, \vartheta_1, \ldots, \vartheta_n), \quad \vartheta_i = \vartheta_i(\dot{x}, \vartheta_1, \ldots, \vartheta_n)$$

and

$$\dot{\vartheta}_i = \vartheta_i(\dot{x}, \vartheta_1, \ldots, \vartheta_n).$$

Here $k_l (v_0 t - x(t)) = F_l(t)$, the measured lateral force and $F_l(t)$ the friction force. The evolution equations are usually postulated and not derived from knowledge of the sliding phenomenon. To some extent the form of the coupled nonlinear differential equations and certainly the physical interpretation of the state variable(s) are system dependent. For the type of sliding in this paper, often called “dry” friction, the state variable is usually associated with the contact area, $A_s$, between slider and substrate evolving by creep. Providing explicit physical background to these models is a significant issue. It is also of practical importance, because it is linked to possible predictive use of the descriptions.

Most of the quantitative dynamic experiments performed in dry friction have been on multisurface contacts. In these experiments, normal loads on the order of 10 mN have been estimated for asperities with contact areas on the order of $10–100 \mu m^2$ [2]. Quantitative experiments with single asperities at this scale are an interesting way to proceed. The experiment can be comparatively very well characterised and offers simplicity in theoretical descriptions. Here we will briefly present such measurements, and indicate a way to describe the experiment in a dynamical system.

2. Experimental

2.1. Method

The system studied consists of hard single asperities sliding on soft polyester coatings. It is a model system, relevant to sliding friction and wear of polymers in general. The measurements were performed with a home-built device, the lateral force apparatus, LFA. A detailed description of the LFA has recently been published [3], and so only the crucial characteristics will be mentioned here. The LFA uses two perpendicular double leaf-springs combined in a single “leaf-spring unit” as force probes, one for $F_n$ and one for $F_l$. Two optical-focus, error-detection heads are used to measure the deflection of the double leaf-springs. The optical heads combine a 10 nm sensitivity with a useful range of about 100 $\mu m$. The optical heads are calibrated in situ prior to, or following an experiment. There is minimal coupling between the leaf-spring deflections in the lateral and normal direction, usually 1% or less. This coupling is calibrated in situ. Normal spring constants, $k_n$, are in the range 20–4000 N/m, lateral spring constants, $k_l$, in the range of 7–1000 N/m. $k_n$ and $k_l$ are calibrated ex situ with
a home-built nanoindentation instrument [4]. The force and displacement readings of this nanoindentation instrument have been calibrated in turn against traceable sources. The proven range of normal forces of the LFA is 400 nN–150 mN. The range of driving speeds is limited, from 1 to 40 μm/s. Absolute values of normal and friction forces are estimated to have typical errors less than 10%.

\[ F_n \] is kept constant during an experiment in the scanning probe microscopy fashion: a feedback loop including a piezo moves the leaf-spring unit along \( z \), keeping the deflection \( F_n/k_n \) of the normal leaf-spring constant. Just as in an SPM this allows us to measure the normal, \( z \)-displacement of the tip during a scan along the surface.

The coatings consist of hexakis(methoxymethyl)amine (HMMM) crosslinked polyesters, deposited with thickness of about 20 μm on Al substrates [4]. Mechanical properties of the coatings were measured with a nanoindentation apparatus. Values for the yield stress (100 MPa), and the reduced modulus (4 GPa) were obtained by standard methods [6]. The calibration of the nanoindenter stiffness and of fitting constants used in the determination of the yield stress and reduced modulus from the force–displacement curves was performed along the lines described in [6], using indentations on W, Al and fused silica. Experience with a number of metallic and ceramic substrates of known modulus and yield stress indicate that estimates of yield stress and reduced modulus are usually correct to within 10–20%. A disadvantage of using thin films of visco-elastoplastic material is that there are no standard ways of measuring their nonlinear or rate-dependent mechanical properties. Considering the indentation speed and time during the nanoindentation experiments (ca. 10 nm/s and 30 min, respectively) we assume that the measured values may be reasonable estimates for the deformation during “stick” in the experiments described below.

The tips or asperities used here are electrochemically etched tungsten wires. With a high-resolution (effectively about 5 nm) SEM (XL30 FE-SEM), no protrusions were observed on the surface of these tips. The radius \( R \) of the tips is estimated from SEM micrographs. Here we have used asperities with radii in the 5 μm range, and normal forces ranging from 100 μN to 10 mN. All experiments were carried out under ambient conditions. The following experimental conditions were varied in a systematic way: \( R, F_n, v_s, k_l \).

### 2.2. Results

During the experiments, interesting dynamic behaviour was encountered. It shares a number of qualitative characteristics with multisasperity “stick–slip” systems that have been studied in terms of rate-and-state models. We find transitions from steady sliding to “stick–slip” for decreasing \( v_s \) and decreasing \( k_l \), and increasing \( F_n \) but also transitions from steady sliding to “stick–slip” for decreasing tip radius \( R \). All of the trends above can be deduced from figure 1. In figure 1 “maps” of dynamic behaviour in dynamical parameter space, \( (v_s, F_n) \) in this case, show transitions from steady sliding to “stick–slip”. Open circles indicate points in \( (v_s, F_n) \) space for which steady sliding was stable, closed circles indicate points where “stick–slip” occurred. All three maps show a transition from steady sliding to stick–slip for decreasing \( v_s \) and increasing \( F_n \). A transition to stick–slip for decreasing \( v_s \) is quite gen-

![Figure 1](image-url)

Figure 1. “Maps” of dynamical parameter space, showing transitions from steady sliding to “stick–slip”. (a) Steady sliding, (●) “stick–slip”. All three maps show this transition for decreasing driving speed \( v_s \) and increasing normal force \( F_n \). Comparison of (b) and (c) shows the shift of the transition to \( F_n \) for increasing tip radius \( R \). Comparison of (a) with (b) and (c) shows a shift of the transition to higher \( F_n \) for higher driving spring stiffness \( k_l \).

Detailed time-resolved measurements of the track (●) are shown in figure 3.
erally encountered in experiments described in literature (for reviews see, e.g. [7,8]). It is associated with “velocity weakening”, i.e., a decrease of $F_f$ for increasing $v_s$. This was also observed here. The transition for increasing $F_n$ has been observed in inertially loaded ($F_n = mg$) sliding multiasperity interfaces, for example those described in [7,8].

Comparison of the two maps in figure 1 (b) and (c) with the map in figure 1(a) shows a shift of the transition to higher $F_n$ for higher $k_l$. Again, this is a trend that has been found more generally.

Comparison of figure 1 (b) and (c) shows the shift of the transition to $F_n$ for increasing tip radius, $R$. This particular transition is an indication of the single asperity nature of the contact, as will become clear from a more detailed view of the sliding dynamics in figures 2 and 3.

In figure 2, constant amplitude tapping mode AFM measurements of “post-mortem” sliding traces clearly show the occurrence of normal displacements during sliding. In the following discussion the positive $z$ axis is defined to point into the substrate, in the direction of the applied $F_n$.

In figure 2(b), an increase in $F_n$ leads to a transition from “smooth” tracks, to tracks with regularly spaced “indents”, darker in the images. The smooth tracks are characteristic for steady sliding (for which $F_l = F_f$), the indented tracks for “stick–slip” (see the discussion below). Also, it can be observed that increasing $F_n$ leads to an increase in depth $z$ of the smooth tracks. In figure 2(a) the $v_s$ increases from left to right, and a decrease in amplitude of the normal displacements is observed.

The absolute value of the normal displacements is quite large, and the depth of the indents is of the order of a micron. This means that a considerable volume of material is being deformed during sliding and this deformation is assumed to be responsible for the largest part of the occurring dissipation. Furthermore the penetration depth is usually at least an order of magnitude larger than any roughness present on the surface of the tips (that could not be resolved in the SEM). We argue therefore that the tips can be regarded as single asperities in these experiments.

The following picture arises. During steady sliding the asperity moves at some equilibrium depth $z$ ("ploughs") through the surface. This depth increases with increasing $F_n$ and decreasing $v_s$. At a certain combination of $F_n$ and $v_s$, steady sliding becomes unstable. In such cases material entering the contact can apparently no longer be deformed quickly enough for the asperity to pass at the driving speed.

From figure 2 it is clear that the subsequent mode of movement involves normal displacements of the asperity. A detailed picture of this movement can be formed with

![Figure 2](image2.png)

Figure 2. Constant amplitude tapping mode AFM images of sliding traces showing the occurrence of normal displacements. In (a) the $v_s$ increases from left to right, and a decrease in "stick–slip" period and amplitude of the normal displacements is seen. In (b) the increase of $F_n$ leads to a transition from steady sliding to "stick–slip". Note that the steady sliding traces get deeper for increasing $F_n$.

![Figure 3](image3.png)

Figure 3. Time-resolved LFA measurements of "stick–slip" motion. (a) Measured lateral displacement ($F_l/k_l$) vs. $t$. In this case $k_l = 137$ N/m, $R = 4$ μm. $F_n = 7.2$ mN, $v_s = 28$ μm/s. (b) Piezo displacement vs. time. (c) $\Delta z$ vs. lateral displacement ($F_l/k_l$) of the tip. This measurement corresponds to the large closed circle in figure 1(a).
the measurements presented in figure 3, which are LFA measurements performed during a “stick–slip” trace. The measurements correspond to the large closed circle at \((v_n, F_n) = (28 \mu m/s, 7.2 \text{ mN})\) in figure 1(a), so \(k_1 = 137 \text{ N/m}\) and \(R = 4 \mu m\). Monitoring the voltage applied to the piezo by the feedback loop in order to keep \(F_n\) constant, the movement of the tip normal to the surface can be observed during sliding. (To this end the piezo response was calibrated [3].) Furthermore, fast sampling (in this case 10 kHz) enables measurements during slip.

Comparing figure 3 (a) and (b) it becomes clear that during periods of increasing \(F_i\), “stick”, the asperity is indeed deeper in the surface than during “slip”. In fact, it can be seen that during the increase of \(F_i\), \(z\) increases and the asperity decelerates and that \(z\) it subsequently increases and the asperity accelerates. It is also clear that during slip, where \(F_i\) decreases rapidly the asperity is at a relatively low depth \(z\). A linear timescale such as used in figure 3 (a) and (b) is actually quite inappropriate to present measurements at the widely different timescales apparent during this measurement, and a further clarifying graph is shown in figure 3 (c). In that figure \(F_i\) has been plotted vs. \(z\). (During slip the feedback loop is not capable of reacting fast enough, which causes an increase of \(F_n\) with about 6% in this case. As the piezo-extension does not fully determine the \(z\) movement of the asperity now, the reading \(\Delta z_{\text{piezo}}\) is corrected by adding the simultaneously measured change in deflection \(\Delta z\) of the normal force leaf-springs.)

It can be observed that the data for this regular “stick–slip” movement fall nicely onto a single limit cycle. The movement of the asperity during slip can now be clearly observed.

3. Discussion

3.1. Steady sliding

The key to a physical understanding of dynamic friction behaviour is to point out the mechanisms that tend to stabilise or destabilise steady sliding. With the data presented in the preceding section that seems possible in this case. The observation that unsteady movement is associated with normal movements evidently deserves consideration in this respect. Steady sliding can quite generally be defined as any situation for which

\[
\dot{x}(t) = v_n A \dot{z}(t) = \ddot{z}(t) = 0.
\]

Usually movement in \(y\) and \(z\) during sliding is disregarded, but \(z\) clearly has to be taken into account here. The experiments show that steady sliding is possible for a range of values \(v_n, F_n, k_1, R\) leading to specific values of \(F_i\) and \(z\). During steady sliding, unlike in a static contact, material is constantly entering and leaving the contact. Some of this material is pushed aside and some of it is pushed down. The material resists this deformation with forces that exactly balance the forces exerted on it by the driving spring via the contact surface with the asperity. Bowden and Tabor [9] proposed the following form for \(F_i\) in systems in which a hard asperity is steadily ploughing through a substrate:

\[
F_i = \tau_{\text{eff}} A_m - \sigma_x A_{xz}. \tag{2}
\]

In equation (2) \(A_m\) and \(A_{xz}\) appear, the projections of the contact surface \(A_r\) of asperity and substrate material along \(z\) and \(x\), respectively. In this view sliding or ploughing is decomposed in simultaneous uniaxial deformation, \(\sigma_r\) and shear, \(\tau_{\text{eff}}\), that are assumed to independently contribute to \(F_i\). Alternatively one can think of \(\sigma_r\) as representing the part of the deformation that is inevitably caused by repulsion during sliding, and of \(\tau_{\text{eff}}\) as “extra” deformation caused by adhesion at the interface. In any case \(\sigma_r\) and \(\tau_{\text{eff}}\) are effective values that represent the deformation of some micron-sized volume via some adhesion-controlled boundary conditions, and do not represent an interfacial quantity. It should be possible to describe them using constitutive relations relating them to strains and strain rates in the contact volume.

The value of \(z\) will to a great extent determine \(A_m\) and \(A_{xz}\), which in view of equation (2) makes clear the importance of the \(z\) position for \(F_i\).

Of course \(A_m\), and therefore \(z\), is important for the equilibrium in the normal direction as well. During steady sliding there is equilibrium in \(z\), so \(F_n = \sigma_z A_m\) with \(\sigma_z\) being the average normal stress exerted on the asperity by the material as it is deformed along \(z\). Remembering the reasonable assumption that some of the material is pushed down, and passes underneath the asperity, it follows that forward movement of the asperity and normal deformation of the substrate are coupled. Steady sliding at increasing asperity speed or depth will both necessitate deformation at higher rates. Generally speaking, materials, e.g., pseudoplastic or viscoelastic materials, resist higher deformation rates with higher pressures, which means that higher forward speeds lead to an increased upward pressure exerted by the material on the asperity. At constant \(F_n\) a new equilibrium can therefore only be reached if \(A_m\) decreases, which means that the asperity must move up. In other words \(\sigma_{zz} A_m\) contains a coupling term \(\sigma_{zz} A_m\) that depends via \(\sigma_{xz}\) on \(v_n\), or, more generally, on the forward speed \(dz/dt\) of the asperity. For the resulting restoring normal force exerted by the substrate on the tip one can write \(A_m (\sigma_{zz} + \sigma_{zz})\).

Clearly, this coupling term may be destabilising and can potentially lead to “velocity weakening”. Higher deformation rates lead to higher stresses but also to smaller projected surfaces \(A_m\) and \(A_{xz}\). Whether \(F_i\), in which products of these terms appear, increases or decreases as a result of this is therefore not a priori clear: “velocity weakening” and “velocity strengthening” are possible as a result of an interplay between geometry and material behaviour.
3.2. “Stick–slip”

Let us assume that during steady sliding $F_n$ increases. As the balance in the normal direction is disturbed, theasperity will move down, increasing the projected surfaces and deformation rates. It may reach a new dynamic equilibrium at higher $F_t$ and $z$, but for large enough $F_n$ such an equilibrium does apparently not exist. Without trying to explain why there is no new equilibrium it is still possible to understand which phenomena drive the motion of theasperity in this regime. Whenever theasperity moves down, the upward force $A_m(\sigma_{xz} + \sigma_{zz})$ exerted on it by the material is too small to counter $F_n$. Two effects limit the normal travel, the increase in $A_n$ and the increase in $F_t$ caused by the driving spring. The latter term will tend to increase the forward speed of theasperity which, in turn, will lead to an increase in $\sigma_{xz}$. At a certain point, $F_n$ will be balanced and this runaway behaviour: the increase in normal pressure $F_n/A_m$, and the decrease in $F_t$ after theasperity has reached speeds higher than $v_t$.

So, starting from the requirements of the dynamic equilibrium during steady sliding and noticing the central role of the normal displacements one arrives at a picture that involves a simple combination ofasperity geometry and material behaviour and gives a physically reasonable mechanism for the observed stick–slip behaviour.

3.3. Relation to similar systems reported in literature

It has been suggested in literature that movement in the $z$ direction may affect sliding dynamics, both of multisasperity and single-asperity interfaces. Tolstoi in an early paper was the first to point it out based on experimental evidence on multisasperity interfaces [10]. More recently, experimental evidence of normal movements during sliding has been found in a system very similar to the one presented here [14]. Study of dynamical parameter space in that case did not include $R$, and transitions from steady sliding to stick–slip were not studied. Nor were detailed, time-resolved measurements of the system dynamics, such as shown in figure 3, obtained. However, it is reasonable to assume that behaviour similar to that described here would have been found if such measurements had been carried out.

Time-resolved measurements were obtained in studies of sliding on granular substrates by Gollub and coworkers [11]. Those measurements clearly point out the importance of normal movements in that system. The relation of this behaviour to the viscoelastic or viscoplastic material behaviour of the material studied here is not particularly clear.

Normal deformation has also been encountered in SFA experiments. The so-called dilatancy is then connected to a qualitative change, sometimes considered to be a solid–fluid phase transition, in the structure of a confined layer of molecules that usually has a thickness of the order of a few nm. A direct physical analogy of the mechanisms underlying stick–slip and the associated normal movements in SFA experiments with the mechanism proposed here seems inappropriate considering the fact that orders of magnitude difference in the smallest characteristic length scale are involved.

Nevertheless there are striking similarities with the systems mentioned above and it has already been mentioned that the dynamic behaviour has parallels in many other systems for which a description in terms of rate-and-state models has proven to be useful.

3.4. Relevance to rate-and-state formulations

As has been stated in the introduction, providing rate-and-state formulations with a physical basis is an important issue. The main question being what exactly constitutes the “state” of a sliding contact and what governs its evolution. It is one of the reasons why there is a pronounced trend toward the in situ real time measurement of physical and chemical properties of sliding contacts [12]. Because of the small volume of the contacts this is certainly an experimental challenge. Furthermore there have been attempts to calculate these properties in small contacts, such as those encountered in SFA [13].

Another alternative is to set up the experimental situation in such a way that some measurable quantity is a useful first-order estimate of a “state” parameter. As was indicated in the introduction for “dry” friction the “state” is usually associated with the projected normal contact surface. This notion has been introduced here via equation (2), but also appears there. Assuming that $A_m$ and $A_{xz}$ are functions of $R$ and $z$ only, and noticing that $R$ is constant during an experiment, we can consider $z$ to be the “state” of this particular contact.

Entering this notion in equation (1) and realising that there is an extra degree of freedom the following dynamical system can be defined in which all of the discussed elements can be assembled:

$$m\ddot{x} = -k(x - v_x t) - \tau_{eff}(\varepsilon_x, \dot{\varepsilon}_x)A_m(z, R) - \sigma_x(\varepsilon_x, \dot{\varepsilon}_x)A_{xz}(z, R),$$

$$m\ddot{z} = F_n - \sigma_{xz}(\varepsilon_z, \dot{\varepsilon}_z)A_m(z, R) - \sigma_{xz}(\varepsilon_z)A_{nm}(z, R).$$

The relation with the general form of equation (1) becomes clearer if the projected areas $A$ are considered to be a function of $z$ and $R$ only (neglecting pile-up for example, the strains functions of $z$ only and the strain rates functions of asperity speed only). If, furthermore the influence of the inertia in $z$ is neglected one finds:

$$m\ddot{x} = -k(x - v_x t) - \tau_{eff}(\dot{x})A_m(z, R) - \sigma_x(\dot{x})A_{xz}(z, R),$$

$$0 = F_n - \sigma_{xz}(\dot{z})A_m(z, R) - \sigma_{xz}(\dot{z})A_{nm}(z, R).$$
It is evident that the form of equation (4) is equivalent to that of equation (1). It is also evident that many simplifications have been made to arrive at equation (4), regarding the stress states in the contact, the material behaviour, and the contact area. Still, equation (3) is in a sense a more general form for a rate-and-state equation with one state parameter, in this case \( z \), and from literature it is clear that such systems may already describe important characteristics of the dynamic behaviour. In fact in the discussion it was shown that a system with characteristics present in the dynamical system of equation (4) could predict stick–slip motion (for which a weakening regime must be present) and transitions from stick–slip to steady sliding at high velocities (for which a strengthening regime is sufficient).

Furthermore, we note that, although the relation to the rate-and-state formulations is clear, the usual phenomenological state parameters or evolution equations are absent. Instead, quantities that are accessible to measurement occur, such as \( z \) and relations for material constitutive behaviour hidden in \( \sigma \) and \( \tau \) are present. The fact that this particular experimental set-up allows one to measure the evolution of this state parameter makes it particularly attractive as a model system.

We therefore suggest that a combined experimental and numerical study of sliding systems such as these will prove to be useful to understand more general sliding systems that show similar dynamic behaviour.

Also it seems worthwhile to investigate experimentally, whether multiasperity systems that are known to show similar dynamic behaviour, show normal displacements. The measurements of Tolstoi [10] would suggest that this might be the case.

We are currently studying particularly simple versions of such dynamical systems in which stresses \( \sigma \) and \( \tau \) are functions of estimated strain rates only [15,16]. These systems seem to reproduce all trends qualitatively.

4. Conclusion

Dynamic friction behaviour of hard single asperities ploughing through soft substrates has been studied. Behaviour similar to that found in multiasperity situations has been encountered with the notable difference that importance of geometry (asperity radius \( R \)) for the dynamic behaviour was clearly measured, we believe for the first time. Normal displacements were observed that were shown to be essential for the dynamics of the system. The interpretation of the role of normal displacements reveals a direct analogy with the role of the projected contact surface in multiasperity interfaces. Based on these observations a dynamical system is proposed that is able to describe both “velocity weakening” and “velocity strengthening”, and therefore in principle the observed behaviour, including stick–slip and transitions from steady sliding to stick–slip. This dynamical system is equivalent to a rate-and-state equation with one state variable. The system is concluded to be an interesting model system for the study of friction dynamics and rate-and-state formulations.

References