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Ordering Behavior in Retail Stores and Implications for Automated Replenishment

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Abstract

Retail store managers may not follow order advices generated by an automated inventory replenishment system if their incentives differ from the cost minimization objective of the system or if they perceive the system to be suboptimal. We study the ordering behavior of retail store managers in a supermarket chain to characterize such deviations in ordering behavior and investigate their potential drivers. Using orders, shipments, and POS data for 19,417 item-store combinations over 5 stores, we find that store managers systematically modify automated order advices by advancing orders from peak to non-peak days. We show that order advancement is explained significantly by hypothesized product characteristics, such as case-pack size relative to average demand per item, net shelf space, product variety, demand uncertainty, and seasonality error. Our results suggest that store managers add value. They improve upon the automated replenishment system by incorporating two ignored factors: in-store handling costs and sales improvement potential through better in-stock. We test a heuristic procedure, based on our regression results, to modify order advices to mimic the behavior of store managers. Our method performs better than the store managers by achieving a more balanced handling workload with similar average days of inventory.

Keywords: Retail operations management; Inventory management; Ordering behavior.

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1 Introduction

Inventory management in a retail store is a complex process involving many constraints and heterogeneous product attributes. The constraints include backroom space availability, handling capacity to replenish merchandize to shelves, and shelf space availability. These constraints make the inventory policies of different items interdependent. The product attributes include physical sizes of items, case pack sizes, demand forecasts, and display requirements. Differences in these attributes across products imply that inventory policies may vary across items. Since a retail store carries thousands of dissimilar items, optimal inventory policies that incorporate their product attributes and satisfy all constraints are virtually impossible to compute. Consequently, off-the-shelf automated store ordering systems employed by retailers as well as inventory policies advocated for use in retail stores in the academic literature ignore some or all of these constraints and consider each item independently.

In contrast, retail store managers are rewarded based on their total revenues and costs, which are both very sensitive to in-store merchandize handling. As a result, a store manager may try to improve her performance by managing inventory manually instead of following an automated ordering system provided by the retailer. In doing so, she may or may not be able to effectively address the shortcomings of the automated ordering system. Therefore, retailers are interested in analyzing whether store managers deviate from automated ordering systems, and if so, whether they truly add value. They are also interested in identifying the drivers of these deviations in order to improve existing ordering systems by learning from store managers' behavior. Our paper addresses this problem.

We analyze the ordering behavior of retail store managers using product attribute data and transaction level ordering and sales data from stores in a supermarket chain in Europe. The features of the problem at this supermarket chain are typical of grocery retailers, discount stores, and mass merchandisers around the world. Demand for products follows a weekly seasonality pattern, i.e., the demand rate varies across the days of the week, but the same periodic pattern repeats every week. Excess demand is lost. Stores receive 3 to 6 replenishments each week according to a fixed schedule. An automated ordering system is implemented in each store. It uses a standard myopic order up to policy to recommend order quantities to store managers. Due to demand seasonality, the volume of orders recommended by the automated system varies considerably across the days of the week. This variation contributes to the cost of in-store merchandize handling by causing store managers to employ extra labor on peak days at higher wages. Correspondingly, we find that store managers tend to bypass the automated system and use markedly different order quantities during the week. Figure 1 illustrates this phenomenon by comparing the weekly seasonality patterns of demand, orders recommended by the automated system, and actual orders aggregated across all items in one store for a one year period. Observe that the automated order advices have the highest range of seasonal variation (24.3%), followed by daily demand (16.2%). Actual orders have the lowest range (13.1%).

The evidence in Figure 1 shows that in spite of the availability of an automated ordering system, store managers may not use it. We reason that store managers advance orders from peak days to preceding non-peak days in order to compensate for incentive misalignment and inadequacy of the automated ordering system. *Incentive misalignment* occurs because store managers are assessed on revenues, but not inventory carrying cost, whereas the automated replenishment system seeks to minimize inventory related costs. Thus, store managers may be more concerned about increasing sales, and may seek to reduce stock outs by carrying more inventory on non-peak days than recommended by the automated ordering system. *System inadequacy* arises because the automated ordering system is suboptimal. For example, due to the complexity of the replenishment problem, the automated ordering system does not incorporate joint handling workload constraints. Thus, store managers may try to balance handling workload across the days of the week in order to reduce labor costs in a store.

Based on the premise of order advancement, our paper presents its analysis in three steps. First, we provide a way to measure order advancement using point-of-sale data and order transaction data (i.e., invoice data) without referring to the inventory records. Our method is robust with respect to inventory data errors. It is also intuitively appealing since we show that it is a good proxy for taking the difference between the average days of inventory for the automated ordering system and for the actual history of orders placed by the store manager.

Second, we test hypotheses to determine the effectiveness of store managers in addressing the shortcomings of the automated ordering system. If store managers are motivated to reduce the costs of handling workload and stock outs but face capacity constraints, then they should advance the orders for different items by different amounts. We identify product attributes whose values should be correlated with the order advancement decisions of store managers. In our regression model, the hypothesized attributes together explain about 45% of the variation in order advancement across items. In particular, items that have larger case pack sizes relative to mean demand, more net shelf space, more demand uncertainty, more product variety, and more errors in weekly seasonality pattern undergo more order advancement. This result shows that store managers systematically advance a part of the assortment throughout the week taking into account attributes related to handling workload and stock out considerations as expected.

Third, we propose a heuristic procedure based on our regression results to modify the orders generated by automated ordering systems to mimic the behavior of store managers. Bowman (1963) presented a theory of management coefficients models and showed how to construct a linear decision rule from the history of actual decisions made by managers using regression. Our approach is similar to Bowman's, and extends it to retail store inventory management. We show that our method leads to a substantial improvement over the store manager's ordering performance by achieving a more balanced handling workload with similar increase in average days of inventory. Thus, our results support the application of Bowman's theory to our context. Our method is efficient to implement since it works independently for each item. Alternatively, one may seek to directly incorporate handling workload constraints and the effect of inventory on sales into the automated ordering system. However, this approach would be complex. Our approach, on the other hand, learns from the actual manager's behavior and feeds it back into decision rules.

Our paper contributes to the literature in several ways. It presents evidence that retail store managers systematically deviate from an automated ordering system for reasons that are rational and predictable. The store managers add value by reducing the cost of handling workload and the incidence of stock-outs in stores. The paper builds on the existing literature on inventory data inaccuracy by presenting a metric for order advancement that is relatively insensitive to errors in inventory data and, thus, is useful in the absence of clean inventory data. It provides insights into the selection of inventory policies for items based on their characteristics so as to balance handling workload. The results of our study are useful for inventory systems with periodic demand, multiple delivery epochs per period, high stock-out costs and high labor costs. Finally, our paper adds to the sparse literature on managerial behavior when confronted with optimization models, by using detailed transaction-level data to study this behavior.

The rest of this paper is organized as follows: §2 gives a literature review. §3 describes the research environment in detail, i.e., the replenishment process at the subject retail chain; §4 defines the order advancement metric and discusses the hypotheses. §5 describes the dataset and the various checks applied to improve its integrity; §6 presents the results of our study; §7 applies the results to construct a management coefficients model of inventory replenishment; and §8 concludes the paper with a discussion of its practical insights.

2 Literature Review

The retail store inventory management problem has similarities with the capacitated lot sizing and scheduling problem (CLSP) and the periodic review inventory control problem with batch ordering. The literature on these problems being vast, we summarize results related to our paper. We also discuss previous papers that use historical operational data to evaluate managerial performance.

Due to the presence of handling capacity constraints, our problem has similarities with a CLSP for multiple items with time-varying demand at a single capacitated production facility. In particular, if the case pack size is set to 1 unit, demand is assumed to be deterministic, and shelf space constraints are ignored, then a multi-item CLSP can be mapped to our problem. Since solving a CLSP optimally is NP-hard (Florian et al. 1980, Bitran and Yanasse 1982), it explains the complexity in the retail store inventory management problem.

The literature provides many heuristics for the CLSP that have common elements with our problem. For example, Axsater (2006:§7.2) explains how cyclic schedules obtained through a periodic review policy can be used to smooth the production schedule when items have constant demand rates. Bradley and Arntzen (1999) consider a joint production capacity, scheduling and inventory management problem for time-varying demand, which is related to the CLSP, but includes the capacity investment decision as well. Their MIP heuristic for this problem suggests that to manage seasonality in a capacity constrained system a firm should build up inventory of those items that have the lowest ratio of value to processing time. In the retailing context, Gaur and Fisher (2004) find that retailers care about workload balancing at distribution centers when setting transportation schedules. We show that store managers try to balance handling workload caused by inventory replenishment in a store. Similar to Bradley and Arntzen, we show that order advancement varies significantly across items. Further, we identify various attributes of items that are related to order advancement.

One of the main drivers of ordering behavior in our setup is batch ordering, that is, an exogenous quantity such that orders are placed in integer multiples of that quantity. Several papers in the inventory theory literature consider batch ordering. If excess demand is backordered and there are no fixed costs, then an (R, nQ) policy is optimal for a single-location single-product inventory management problem with batch ordering under fairly general assumptions on demand (Veinott 1965, Chen 2000). An (R, nQ) policy periodically checks the inventory position and advises to order the minimum number of case packs required to bring the inventory position to be at or above R. Under lost sales, the optimal inventory policy under batch ordering is not known, though it is likely that the performance of an (R, nQ) policy may be close to optimal (see Hill 2006). The policy implemented in the automated ordering system at our subject retailer is an (R, nQ) policy in which the value of R is set taking into account not only inventory costs but also marketing considerations. Hence, the automated ordering system serves as a useful benchmark for us.

Batch ordering has been shown to significantly influence the value of information sharing for a supply chain. Cachon and Fisher (2000) show this result for non-perishable items ordered in integer multiples of a batch size, and Ketzenberg and Ferguson (2006) for perishable items with order size restricted to one batch. We show that batch size is highly correlated with order advancement. Furthermore, the vast majority of items in our setting are ordered in a single batch at a time and the amount of order advancement is stronger for such items compared to those ordered in multiple batches at a time. Thus, our result implies that the value of information sharing under batch ordering may increase further if we fully consider the effect of batch sizes on the store managers' ordering behavior.

Our paper is also distinct from the literature cited above since we do not consider normative optimization of the inventory management problem in a retail store. Instead, we analyze the ordering behavior of store managers and construct a linear decision rule from this behavior. In this way, our paper builds on Bowman (1963), who introduced the management coefficients approach to support decision making. Bowman shows that a potential advantage of this approach, compared with managerial decision making based on optimization models and/or human judgment, is its consistency obtained by filtering out the noise in human decisions. He supports his theory with real-life examples based on a production smoothing model applied to a single item. We apply Bowman's theory to a more complex setting of multi-item inventory replenishment. Other papers have argued for realistic decision rules. For example, Little (2004) discusses several reasons why models are not used widely by managers and proposes the features of a decision calculus that make it more amenable to implementation. Wagner (2002) contends that automated replenishment rules are often not implemented because of data related challenges, and proposes that tests using real-life data should be used to determine the applicability of different replenishment rules.

In contrast to the theory proposed by Bowman, many researchers have examined managerial behavior using real-life data and laboratory experiments, and found that managers' decisions do not correspond to the expected profit-maximizing decisions due to the complexity of the situation (Deshpande et al. 2003, Keizers et al. 2003) or some form of decision bias (Schweitzer and Cachon 2000, Croson and Donohue 2006). Unlike the settings in these papers, store managers in our setting are provided an automated system and voluntarily choose to deviate from it. We focus on system inadequacy and incentive misalignment as the motivating reasons for these deviations. While order advancement could be motivated by other causes such as moral hazard or biases in the store managers' ordering behavior, the study of these factors is outside the scope of our paper. We do show that a decision rule based on the store managers' behavior leads to a more balanced handling workload than the decisions by the store managers or the automated ordering system.

Our paper has a potential limitation that it uses data from a small number of stores in a single firm. This controlled data set, however, enables us to obtain cleaner results than would be possible from comparing dissimilar firms in a single estimation model. In this respect, our paper falls in the realm of previous studies that have used real-life data from a single research site, see for example, Fisher and Ittner (1999), Deshpande et al. (2003), Keizers et al. (2003), or Fisher et al. (2006).

3 Research Setting

Our analysis uses data on product attributes, point-of-sale (POS) transaction history, and ordering transaction history at the item-store level for five retail stores in a supermarket chain. We refer to an item-store combination as a stock-keeping unit or an SKU. We discuss two order streams, those from the automated store ordering system and those from the actual historical orders, since we are interested in the differences between these streams. This section describes the automated store ordering system and the store managers' motivation to deviate from it.

3.1 The Automated Store Ordering system

The automated store ordering (ASO) system determines the order quantity for each item i in store s given the delivery schedule, demand forecast, and case pack size. The delivery schedule specifies the days of the week when a store can generate replenishment orders; it is identical for all items in a store that are ordered through the ASO system. A store receives at most one shipment per day from the central warehouse consisting of all the items to be delivered that day. Orders generated by the store in the morning are delivered and replenished on the shelves in the evening of the same day in order to meet demand for the following day(s). The lead time is therefore one sales day. Let L denote the lead time and D_s denote the set of days of the week on which store s is allowed to place orders with the central warehouse.

The inventory control policy in the ASO system resembles an (R, nQ) periodic review policy, where R denotes the reorder level, and Q the case pack size. The value of R is set based on input from marketing and from a standard myopic policy commonly used in the literature. The length of the review period is the time between two deliveries; it varies across the days of the week according to the delivery schedule. Let $r_s(k)$ denote the number of days until the next order date for the k-th day of the week for store s. On the k-th day of the week such that $k \in D_s$, the ASO system determines the reorder level as the sum of the minimum inventory level and the forecast of demand for the lead time plus review period, i.e., the demand forecast for days indexed from k to $k + L + r_s(k) - 1$. Thus, the reorder level is equal to $M_{si} + F_{si}(k)$ consumer units, where M_{si} denotes the minimum inventory level and $F_{si}(k)$ denotes the forecast of mean demand over the days from k to $k + L + r_s(k) - 1$ for item i in store s. After determining the reorder level, the ASO system checks the current inventory position and advises to order the minimum number of case packs required to bring the inventory position after ordering to be at or above $M_{si} + F_{si}(k)$.

In this computation, the minimum inventory level is exogenously set by the merchandizing department on the basis of a targeted service level and marketing considerations such as having a minimum number of facings of merchandize to create an appealing shopping experience for customers. The demand forecast is based on an exponential smoothing model taking into account the weekly seasonality pattern.

3.2 Deviations from the ASO System

We designate the store manager as the prime decision-maker. Actual ordering may be done by other employees who report to the store manager and, hence, take decisions that are consistent with the incentives of the store manager. We identify system inadequacy and incentive misalignment as the main reasons for store managers to deviate from the ASO system.

The store manager's performance is evaluated using a score card comprised of several financial indicators, namely, net sales per square meter of sales area, revenues and per cent gross margin for each department, leakage/shrinkage, labor costs (productivity, gross wages, illness), and others (such as maintenance, energy, packaging). Also included are non-financial indicators such as adherence to the retail format, planograms, stock-outs, and handling of complaints.

Note that the store manager is assessed on labor costs, whereas the ASO system does not take these costs into account. Since the reorder levels are based on lead-time demand, the handling workload under the ASO system varies with the seasonality of demand as shown in Figure 1. However, the availability of low-cost handling capacity on peak days is limited by considerations such as legislation on working hours, the cost of varying handling capacity across the days of the week, and physical limitations on the number of employees who can work efficiently in one aisle simultaneously. In contrast, there is slack handling capacity available on non-peak days. Consequently, a store manager might be motivated to address system inadequacy by reorganizing orders over the week in order to balance handling workload.

Further note that the objectives of the ASO system differ from the incentives of the store manager even if we ignore the complexity induced by handling capacity constraints. The store manager is not penalized for inventory holding costs, whereas the ASO system explicitly considers them in its objective function. Thus, the store manager may prefer to carry extra inventory in the store in order to reduce stock outs and improve sales. For example, a store manager might have local knowledge about demand, which might enable her to identify items with greater forecast errors. Thus, the store manager might be inclined to interfere with the order advice so as to address forecast errors. Additionally, the store managers interviewed by us believed that fully stocked shelves stimulate demand. Thus, they preferred to have fully stocked shelves even on a non-peak day. This would affect their ordering behavior since they would carry over more unsold inventory from a non-peak day to a successive peak day.

4 Hypothesis

We first explain how we compute order advancement, and then identify its potential drivers.

4.1 Order Advancement Index

We measure the extent of order advancement for an SKU by computing the average number of days that an order is advanced forward in time compared to the benchmark ASO system. Inventory data are often prone to persistent errors (Raman et al. 2001). Therefore, we devise a method to compute order advancement by using transaction history of orders and sales without reference to the inventory data. Our method is suitable for periodic demand patterns, such as the weekly cycle in a supermarket chain, with negligible stock outs.

Let $j = ASO, ACT$ denote the two ordering systems, being the ASO system and the actual order history, respectively. Let $O^j_{si}(k)$ be the total quantity ordered during the year for item i in store s on the k-th day of the week under ordering system j. Also let $p^j_{si}(k)$ be the proportion of order quantity ordered on the k-th day of the week under ordering system j; $p^j_{si}(k) = O^j_{si}(k)/\sum_k O^j_{si}(k)$. Thus, $p^j_{si}(k)$ gives the seasonality pattern of order quantities for system j across the days of the week. Let $I_{si}(k)$ denote the cumulative difference between the actual and ASO ordering patterns up to the k-th day of the week: $I_{si}(k) = \sum_{j=1..k}[p^{ACT}_{si}(j) - p^{ASO}_{si}(j)]$. Then, we define the advancement index with respect to ASO as:

$$AI_{si} = \sum_{k=1}^{6} \left[I_{si}(k) - \min\{I_{si}(1), \ldots, I_{si}(6)\}\right]. \tag{1}$$

The first part of this definition, i.e., $\sum\limits_{k=1}^{6} [I_{si}(k)]$ gives the difference between the average days of inventory in the actual orders and the ASO system orders. The second part of the definition, i.e., $\sum\limits_{k=1}^{6} [\min\{I_{si}(1), \ldots, I_{si}(6)\}]$ provides an adjustment for the fact that orders are always advanced in time to the preceding non-peak days. Since $p^j_{si}(k)$ are proportions, $I_{si}(6) = 1 - 1 = 0$. Thus, $\min\{I_{si}(1), \ldots, I_{si}(6)\}$ is always less than or equal to zero. If it is zero, then it implies that actual orders are advanced to earlier days of the *same* week compared to the ASO orders. If it is less than zero, then it implies that actual orders are advanced from one week to the previous week. An order delay would not be sensible since it would increase the probability of stock out.

Since we assume that orders are advanced by up to 1 week, our definition bounds the value of AI between 0 and 5. Further, as shown in the appendix, the formula in (1) is equal to the difference between long run average days of inventory of the actual orders and the ASO system under certain assumptions. Thus, our metric of order advancement is related to days of inventory even though it is not directly computed from inventory data.

We use (1) rather than historical inventory data because we found that the latter have a large number of errors at our subject retailer. Raman et al. (2001) and DeHoratius and Raman (2006) have discovered the incidence of inventory data inaccuracy at several retailers. Our experience is consistent with their findings. Examples of errors in our data set included missing shipment records in the invoices, wrong items being shipped, mismatch between ordered and shipped quantities, etc. Note that inventory levels are highly sensitive to errors in inventory records because such errors propagate till the end of the time-series data. In contrast, our computation is robust with respect to errors in inventory records because it is based on orders data instead of inventory data and on observations aggregated to the annual level rather than on comparison of individual orders. This robustness enables us to utilize transactional data even though both inventory and transactional data are not 100% accurate.

We use proportions $p^j_{si}(k)$ in (1) rather than cumulative order quantities $O^j_{si}(k)$ because the data for the actual system and ASO system may differ from each other with respect to the dates of the first and last orders in the data-set. Thus, the inflows and outflows in the two systems may not match precisely for finite time intervals.

To apply (1), data for actual orders are directly obtained from the invoices and shipments databases at the retailer. However, data for the ASO system cannot be obtained in this manner because the ASO system resets each time the manager changes the order advice. Thus, we generate the unaltered recommendations of the ASO system by simulating it on the sample path of actual sales realizations without any intervention from the store manager. We conduct this simulation setting all input parameters equal to those in the real-life system. Additionally, since the starting inventory level in the real-life system is not known accurately, we affix a starting inventory level for each SKU that is high enough to ensure no stock-outs in the inventory system based on actual orders and actual sales data. The order history generated from each simulation is used in (1). Performance measures obtained from each simulation (like service level and inventory) are measured per SKU from the day the first ASO-order is generated till the day just before the last ASO-order is generated for that SKU.

Even though we use actual sales to simulate the ASO system, our simulation provides a good proxy for ASO order patterns for two reasons. First, sales history is a reasonable approximation of the demand history since store managers tend to advance orders. Second, we we found the simulation to yield a very high fill rate. We simulated the ASO system using the POS-data for one year and reorder levels as described above and found that the average service level (defined as the P_2 fill rate) was 99.15% for all SKUs in our data set.

The following example taken from our data set illustrates the order advancement phenomenon and the computation of AI_{si}.

Example 1. For a body lotion item at one of the stores, the total sales quantity on different days of the week over a 52-week period is $(16, 12, 16, 38, 40, 18)$ consumer units (CU), i.e., 16 CU sold in the year on Mondays, 12 on Tuesdays, and so on. The total order quantities recommended by ASO on different days of the week are (0, 6, 12, 36, 66, 18), and the actual order quantities on different days of the week are (60, 42, 6, 6, 6, 18). Note that the actual orders are substantially shifted to non-peak days compared to the ASO system: 75% of the quantity is ordered on Mondays and Tuesdays, whereas ASO suggests 4%. From (1), we obtain $AI_{si} = 2.23$. Thus, the retail store carries 2.23 days more of inventory than recommended by the ASO system.

4.2 Drivers of Order Advancement

We identify seven operational factors and item characteristics that are related to handling workload and revenue maximization in a store. Thus, we present hypotheses relating these variables to the amount of order advancement, AI_{si}. Hypotheses 1-3 are motivated by handling workload considerations caused by system inadequacy. Hypotheses 4-7 are motivated by incentive misalignment, i.e., by factors such as demand-stimulating effect of inventory, reduction of stock-outs, or the store manager's ability to incorporate local knowledge in forecasting.

Hypothesis 1. AI_{si} is positively correlated with case pack cover, Q_i/μ_{si}.

In inventory systems with batch ordering, case pack size leads to infrequent ordering. We define case pack cover as the ratio of case pack size Q_i to average weekly sales μ_{si}, i.e., the number of weeks of sales covered by one case pack. SKUs with a larger case pack cover are ordered less frequently than SKUs with smaller case pack cover. We expect that the store manager can systematically advance orders for infrequently ordered SKUs to non-peak days by following an ordering policy different from the ASO system. For example, if an SKU is ordered once a week, then the store manager may use a fixed-period ordering policy and order the SKU on a Monday each week instead of a Thursday or a Friday. Likewise, if an SKU is ordered twice a week, then the store manager may order it on Saturdays and Wednesdays to avoid ordering on one of the peak days. Thus, for SKUs with larger case pack cover, a larger proportion of orders can be advanced by a larger number of days than for SKUs with smaller case pack cover. Thus, we expect the amount of order advancement to be increasing in case pack cover.

Hypothesis 2. AI_{si} is positively correlated with the amount of net shelf space (expressed in average weeks of sales) available for an SKU, NSW_{si}.

If sufficient shelf space is not available for an SKU, some of the inventory cannot be positioned on the shelves and must be placed in the backroom. A number of authors have reported on the disadvantages of using backroom inventory: increased labor costs due to the double handling of items, reduced service levels (Corsten and Gruen, 2003), and inventory inaccuracy (Raman et al., 2001). Therefore, under the objective of smoothing the workload during the week, SKUs with larger net shelf space are more suitable candidates to be ordered earlier.

We measure net shelf space for an item as the difference between the shelf space allocated to that item and the maximum inventory on hand for that item. For any item i in a given store, the maximum inventory on hand is equal to the sum of the maximum reorder level across the 6 days of the week and $Q_i - 1$, where Q_i is the case pack size for item i. We then scale the net shelf space by mean demand μ_{si} in order to obtain the number of days for which the store manager can advance item i while still ensuring that it fits on the shelf. Net shelf space can be negative for items that do not have enough space to fit a case pack. Such items are replenished from the backroom. See Broekmeulen et al. (2007) for a study on net shelf space in retail stores.

Hypothesis 3. AI_{si} is positively correlated with the physical volume of the item, $Volume_i$.

Transporting voluminous items is relatively time-consuming because a fewer number of such items fit on a roll container, so that more trips to the backroom are required to replenish a given number of larger volume items than to replenish smaller volume items. Thus, items with large physical volume require greater handling effort, so that the store manager may have a greater incentive to advance orders for large volume items from peak to non-peak days. Our reasoning is related to a result in the previous literature which shows that the distance between shop floor product location and the backroom location is a key driving factor of workforce demand (Broadbridge 2002).

Hypothesis 4. AI_{si} is positively correlated with variety in a product subgroup, $Variety_{s,j(i)}$.

A fully stocked shelf can be expected to stimulate demand since it is more appealing to consumers, see for example, Balakrishnan et al. (2004) or Smith and Achabal (1998). Further, variety can also stimulate demand as modeled in classical papers by Dixit and Stiglitz (1977) and Baumol and Ide (1956). The retailer maintains a large variety for certain subgroups because demand for these subgroups is stimulated by variety. A stock out of one item in such a product subgroup creates empty space on the shelf, which is easily seen by customers, and thus, affects customer perception of variety. Thus, the store manager may wish to avoid stock-outs for such items. We measure variety for each item i in store s by the number of items in its subgroup $j(i)$ at the store. We obtained data on subgroup classifications of items from the retailer.

Hypothesis 5. AI_{si} is positively correlated with absolute profit margin.

Order advancement of an item leads to an increase in its average inventory level, and hence, its expected service level. We expect that all else remaining equal, the store manager will prefer to advance orders for higher margin items than for lower margin items because the increased service level obtained from advancing orders translates into a larger profit increase for higher margin items. This behavior would be consistent with the incentives of the store manager which reward for sales and gross profit but do not penalize for inventory costs. This behavior would also be consistent with the result from the standard newsvendor problem that the optimal inventory level is increasing in the gross margin of an item. Indeed, Gaur et al. (2005) show that retailers have lower inventory turnover for higher margin items than for lower margin items.

Hypothesis 6. AI_{si} is positively correlated with seasonality error, $CycleErr_{s,j(i)}$.

We compute seasonality error as the root mean squared error of the difference between the seasonality pattern of demand input into the ASO system and the actual seasonality pattern of demand estimated from the sales realization in the subgroup $j(i)$ of the item in the store. A store manager, by observing the incidence of stock outs in the store and through his local knowledge about demand, may be able to identify SKUs with large seasonality error of demand. Thus, we expect the store manager to advance the orders for such SKUs more than for other SKUs to avoid taking the risk of stock outs due to poor seasonality estimates.

Hypothesis 7. AI_{si} is positively correlated with forecast dispersion, $CVFcstErr_{si}$.

We compute forecast dispersion as the standard deviation of the daily forecast error of sales divided by average sales. Forecast dispersion differs from seasonality error, $CycleErr_{s,j(i)}$, in the sense that forecast dispersion is a measure of demand uncertainty, whereas seasonality error could be corrected ex ante by using the correct seasonality pattern parameters.

Higher forecast dispersion indicates higher variability of demand. We expect that the store manager would protect himself against potential forecast errors and resulting stock outs by advancing orders for those items that have higher forecast dispersion. This reasoning is consistent with traditional inventory theory since the amount of safety stock for an item with a sufficiently large underage cost is increasing in the standard deviation of its demand.

We test Hypotheses 1-7 against the null hypothesis that differences between the simulated ASO system and the actual orders are uncorrelated with the hypothesized factors.

5 Data Description

We obtained detailed sales and replenishment data from 50 stores of the retail chain over the year 2002. For our in-depth analysis, we selected five stores that represented a diverse set of values of selling space and turnover, and were considered by the retail chain's management to be well operated and representative of the chain. We validated the data for these five stores by visiting them, measuring shelf space allocations, and verifying the local planogram against the central planogram. We also augmented invoice data to track changes in SKU number, new product introductions and termination of products, and differences in order and shipment dates and quantities.

We restricted our download to SKUs that are ordered from the central warehouse through the ASO system and have dedicated shelf space in the store. This excludes merchandizing categories for tobacco and frozen food because the SKUs in these categories share storage capacity on the shelves or in bins. This also excludes perishables such as newspapers, bread, meat, dairy, vegetables, and fruits because they are not ordered via the ASO-system. Delivery through the central warehouse ensures that we have invoice data (and, consequently, the ordering and delivery data) for all SKUs in the data set. Ordering via the ASO system ensures that we have the inventory control parameters. The selected 28 merchandizing categories (of the total of 55 at the retail chain), which we list in Table 1, resulted in an assortment of 7,700 items, and 27,136 SKUs for the five selected stores. More than 80% of the subset consists of dry groceries.

We obtained daily sales data for each SKU from the cash register systems of the stores. To obtain the deliveries data, we collected from the invoices to the stores the amount ordered and the amount delivered, measured in number of case packs. For less than 2% of the order lines, the central warehouse could not ship the ordered amount. Since this fraction is small, we ignored this error. We used the delivery date and quantity to construct the order advancement metric.

The retail chain follows an Every Day Low Pricing strategy. As a result, only the manufacturers initiate sales promotions. Sales promotions are identical across all stores and typically last a week. The sales during promotions (in CU) remained below 3% of total sales in the five selected stores. We did not correct the data for promotions because it is impossible to make a unique link between the sales in the promotion week and the deliveries in the weeks before the promotion and in the promotion week (if sales during promotions are to be deleted, the corresponding deliveries should also be deleted). We also did not filter the data for phasing-in and phasing-out effects; these effects are not dominant in our subset because we focus on the most stable part of the assortment, and restrict ourselves to SKUs with a long selling history.

We required the active trading period of each SKU to be greater than or equal to 40 weeks. We defined the start week of the trading period as the maximum of the week containing the first actual order day and that containing the first selling day in our download. We defined the end week of the trading period as the minimum of the week containing the last selling day and that containing the last actual order day. Thus, we use a conservative trading period in order to avoid SKUs with very few data points, which would lead to estimation errors in all variables. Roughly 8,000 SKUs were eliminated due to the restriction on the length of the trading period, leading to 19,417 SKUs for further analysis. While a large number of SKUs were eliminated, they represented a much smaller fraction of sales, and the remaining data set was sufficiently large to be of interest in itself.

Table 2 describes store-wise characteristics of the final data set used in our study. Stores are numbered 1 through 5 for the remainder of this paper. The average lengths of the review periods for the five stores are $r_1 = r_4 = 0.167$, $r_2 = r_3 = 0.25$, and $r_5 = 0.333$ weeks, because stores 1 and 4 receive shipments everyday, while stores 2 and 3 receive shipments on four out of six days each, and store 5 receives shipments on 3 out of six days.

Table 3 provides summary statistics of the variables used in the study. The median case pack size is 12 CU, the median sales are roughly 1.5 CU per day per SKU, and, for a median SKU, a case pack represents 1.28 weeks of sales. The median order advancement for an SKU is 0.59, which implies that the store carries 0.59 days of excess inventory for an SKU at the median. Table 3 does not include summary statistics for the profit margin variable because it is categorical. For reasons of confidentiality, the company did not provide us the exact selling prices and profit margins of the selected SKUs, but instead, provided a classification of margin data on a six-point scale. For this purpose, margin was defined as selling price minus purchasing price for the supermarket, based on the cost price of the SKU when it leaves the central warehouse.

Table 4 shows that order advancement impacts workload balancing in all five stores. For store 1, the range of the weekly pattern of orders is lower under actual orders (0.131) than under ASO (0.243). The standard deviation of the weekly pattern of orders is also lower under actual orders than under ASO. This result holds consistently across stores. On the flip side, order advancement increases the average inventory level in the stores. We computed the average inventory (in days) by simulating the ASO system to be 4.2, 5.5, 7.2, 4.1, and 9.3 days, respectively, in stores 1 through 5. Order advancement as a fraction of average inventory ranges between 5.4% and 14.3% across the five stores. The weighted average order advancement for all five stores is 0.5 days. It represents a 9.6% increase over the ASO system.

6 Estimation and Results

6.1 Model Specification

Our objectives in this section are to test the hypotheses regarding the drivers of order advancement and make predictions for order advancement that can be used for applying the management coefficients model in §7 to all stores 1-5. Since our data are categorized into store-subgroup combinations, we can use categorical variables to build a detailed model to test whether our hypotheses hold within store-subgroup combinations, as well as examine the usefulness of simpler alternative models for prediction. Thus, we estimate four regression models as specified below using the notation defined in §2. We first estimate these models for stores 1-3, and then validate the prediction results on stores 4 and 5.

$$
\begin{aligned}
\text{Model 1:}\quad AI_{si} = {} & a_{1,s} + a_{2,j(i)} + a_{3s,j(i)} + \epsilon_{si},\\
\text{Model 2:}\quad AI_{si} = {} & a_{1,s} + a_{2,j(i)} + a_{3s,j(i)} + b_{11}Q_i/\mu_{si} + b_{12}\min(Q_i/\mu_{si} - x_1, 0) + b_{13}\max(Q_i/\mu_{si} - x_2, 0)\\
& + b_2 NSW_{si} + b_3 Volume_i + b_{4,m_{si}} + b_5 CVFcstErr_{si} + \epsilon_{si},\\
\text{Model 3:}\quad AI_{si} = {} & a_{1,s} + b_{11}Q_i/\mu_{si} + b_{12}\min(Q_i/\mu_{si} - x_1, 0) + b_{13}\max(Q_i/\mu_{si} - x_2, 0) + b_2 NSW_{si}\\
& + b_3 Volume_i + b_{4,m_{si}} + b_5 CVFcstErr_{si} + b_6 CycleErr_{s,j(i)} + b_7 Variety_{s,j(i)} + \epsilon_{si},\\
\text{Model 4:}\quad AI_{si} = {} & a_4 + b_{11}Q_i/\mu_{si} + b_{12}\min(Q_i/\mu_{si} - x_1, 0) + b_{13}\max(Q_i/\mu_{si} - x_2, 0) + b_2 NSW_{si}\\
& + b_3 Volume_i + b_{4,m_{si}} + b_5 CVFcstErr_{si} + b_6 CycleErr_{s,j(i)} + b_7 Variety_{s,j(i)} + \epsilon_{si}.
\end{aligned}
$$

Here, the coefficients $a_{1,s}, \ldots, a_4$ denote the fixed effects and the intercept, and the coefficients $b_{11}, \ldots, b_7$ denote the slopes of explanatory variables. Since margin is a categorical variable, its coefficients $b_{4,m_{si}}$ are dummy variables for different margin classes.

Model 1, our base model, is an ANOVA model. We use store identifiers, subgroup identifiers and their interactions as explanatory variables denoted by fixed effects $a_{1,s}$, $a_{2,j(i)}$ and $a_{3s,j(i)}$, respectively. We use this model to investigate mean differences in order advancement across stores and product subgroups. Model 2 tests the role of the explanatory variables in a conservative manner by controlling for differences across store-subgroup combinations. It includes store identifiers, subgroup identifiers, their interactions, and all but two of the explanatory variables listed in Hypothesis

1. We do not include $CycleErr_{s,j(i)}$ and $Variety_{s,j(i)}$ in model 2 because they are identical for all items i in the subgroup $j(i)$ in the store, and hence, are linearly dependent on the store-subgroup interaction effects. Model 3 is obtained from model 2 by adding $CycleErr_{s,j(i)}$ and $Variety_{s,j(i)}$ and by removing the subgroup-level fixed effects and the store-subgroup interaction effects. Model 4 further removes store-level fixed effects. We test both models 3 and 4 in order to separately investigate whether store-level and subgroup-level fixed effects impact the coefficients' estimates of the explanatory variables. Model 4 is a pooled model with a common intercept for all stores. It is more parsimonious than models 2 and 3. It is also practically more useful because we can use it to make predictions for stores other than 1-3. We will use this model to validate our results on stores 4 and 5, and to compute predicted values of order advancement in the implementation of the management coefficients model.

In models 2-4, we represent the relationship between AI_{si} and Q_i/μ_{si} using a continuous piecewise linear function with coefficients b_{11}, b_{12} and b_{13}. Since AI_{si} is bounded between 0 and 5, a simple linear regression with a bounded dependent variable violates the assumptions of regression analysis and can produce biased estimates. A piecewise linear function reduces bias. We selected two breakpoints to separate the regions of small, intermediate and large values of Q_i/μ_{si}. Thus, the slopes of order advancement with respect to Q_i/μ_{si} in the three regions are $b_{11} + b_{12}$, b_{11} and $b_{11} + b_{13}$, respectively. The values of the breakpoints x_1 and x_2 are the 25-th and 75-th percentiles of the distribution of Q_i/μ_{si} for the entire data set. Alternative values of the breakpoints do not affect our results materially.

The coefficients of explanatory variables in models 2-4 have been pooled across stores. To test our assumption, we also estimate model 4 separately for each store and investigate sources of differences among stores by interviewing store managers.

6.2 Hypothesis Tests

Table 5 compares the overall fit of models 1-4 estimated on the data for stores 1-3 using ordinary least squares regression. Model 1 has an adjusted R^2 value of 18.64%, and models 2-4 have much larger adjusted R^2 values of 48.60%, 45.87% and 41.89%, respectively. The likelihood ratio (LR) tests comparing models 2-4 with model 1 are statistically significant at $p < 0.01$ showing the joint significance of the explanatory variables. These adjusted R^2 values and LR tests provide evidence that order advancement is explained by variables in our model in large measure.

Next we compare models 2 and 3 to examine the role of store-subgroup interactions and subgroup effects. In model 2, we find that even though the store-subgroup interactions and subgroup effects are jointly significant, the individual dummy variables for these effects are not statistically significant in most cases. The store-subgroup interactions have $p < 0.10$ in 15/297 cases and the subgroup main effects have $p < 0.10$ in 5/158 cases. This suggests that subgroup effects and store-subgroup interactions are not practically relevant for estimating the value of order advancement for a given SKU. Indeed, upon comparing models 2 and 3, we find that the hypotheses' tests are consistent across the two models and coefficients' estimates are not statistically different. Specifically, case pack cover, net shelf space, and forecast dispersion are statistically significant in both models, volume is not statistically significant in either model, and margin is collectively statistically significant in both models (F-test), but t-tests for the individual coefficients of different margin classes are not. Thus, our comparison of models 2 and 3 is unable to find any evidence that the store manager uses different rules for order advancement across different product subgroups. Model 3 being the more parsimonious is preferred to model 2. In addition, model 3 includes two new explanatory variables, variety and seasonality error, which are both statistically significant.

The coefficients' estimates for model 3 given in Table 5 yield the following inferences regarding Hypotheses 1-7. Order advancement is positively correlated with case pack cover, supporting Hypothesis 1 at $p < 0.01$. The estimates of slopes of Q_i/μ_{si} in the three parts of the piecewise-linear curve are $b_{11} + b_{12} = 0.237$, $b_{11} = 0.262$, and $b_{11} + b_{13} = 0.067$. These estimates show that the value of AI_{si} changes substantially with Q_i/μ_{si}, but has diminishing sensitivity to very large values of Q_i/μ_{si}. The coefficient estimate for net shelf space is 0.002, and is also statistically significant ($p < 0.01$), thus supporting Hypothesis 2. Thus, store managers systematically consider net shelf space when making order advancement decisions. Product variety is also statistically significant, and has a positive coefficient, supporting Hypothesis 4 at $p < 0.01$. For absolute margin class, the coefficients of dummy variables are collectively significant in each model, but are individually neither statistically significant nor monotone increasing as we go from low to high margin categories. These estimates do not support Hypothesis 5. The coefficient estimate for the error in weekly seasonality pattern is 1.628 ($p < 0.01$), and supports Hypothesis 6, showing that the store manager recognizes forecast errors in seasonality patterns and consistently corrects for them. Finally, the coefficient's estimate of forecast dispersion is 0.356 ($p < 0.01$), supporting Hypothesis 7 and showing that the store manager responds to more demand uncertainty by building more inventory in advance. Results for the remaining variable, physical volume do not support Hypothesis 3 in both models 2 and 3. In summary, the results show that store managers incorporate case pack cover, net shelf space, variety, seasonality error, and forecast dispersion in their decisions to advance orders from peak to non-peak days.

To ascertain the relative importance of the explanatory variables, we compute their standardized coefficients. For this, we re-estimate model 3 after centering the data (to remove store effects) and after omitting the categorical variables for margin. The standardized coefficients give that case pack cover has the highest relative importance among all explanatory variables in the model. It is followed by forecast dispersion, variety, seasonality error and net shelf space in that order.

We now investigate the usefulness of our results for predicting order advancement using model 4. The comparison of models 3 and 4 shows that store-level fixed effects do impact the coefficients' estimates significantly; $\min(Q_i/\mu_{si} - x_1, 0)$ is statistically significant in model 4, but not in model 3. Several other coefficients change by significant amounts in going from model 3 to model 4. For example, the coefficient of $CycleErr_{s,j(i)}$ increases by a factor of four and the coefficient of variety increases 2.5 times. In spite of these differences, the values of adjusted R^2 show that the loss of explanatory power in going from model 3 to model 4 is small. We reason that model 4 performs well because changes in coefficients from model 3 to model 4 compensate on average for the absence of store-specific intercepts in model 4. Therefore, we consider model 4 to be a reasonable approximation and to be useful for predicting the effects of our explanatory variables on order advancement.

We validate the estimation results on stores 4 and 5 by using the coefficients' estimates from model 4, which are reported in Table 5, to compute predicted values of AI_{si} for each SKU in stores

4 and 5. Table 6 presents the results obtained by comparing these predictions with the actual values. Store 4 yields a correlation coefficient of 0.55 ($R^2 = 30.3\%$) and store 5 yields a correlation coefficient of 0.47 ($R^2 = 22.1\%$), both being statistically significant at $p < 0.01$.

Finally, we estimate model 4 separately for each store. This enables us to examine differences in slope coefficients across stores and gives an alternative method for validating the model for stores 4 and 5. Table 7 reports the results obtained for all five stores. The adjusted R^2 values are 62.85%, 31.47%, 48.81%, 31.99%, and 26.77%. Their magnitudes show that a large proportion of the order advancement behavior of the store manager is explained by the drivers considered by us. Note that the adjusted R^2 values for stores 4 and 5 are only marginally better than the validation results in Table 6, thus showing that model 4 yielded good predictions of order advancement for these stores.

The coefficients' estimates and statistical significance vary across stores as shown in Table 7. Case pack cover has a positive coefficient confirming Hypothesis 1 in all stores. Net shelf space supports Hypothesis 2 for stores 3 and 5 at $p < 0.01$ and for store 4 at $p < 0.05$. Variety supports Hypothesis 4 for stores 2, 3 and 4. Seasonality error supports Hypothesis 6 for stores 1, 2 and 4. Forecast dispersion supports Hypothesis 7 for stores 3 and 4 at $p < 0.01$ and store 5 at $p < 0.05$. In all cases in which the slope coefficients are statistically significant, their signs are consistent with our hypotheses.

The differences in coefficients across stores suggest that the drivers of the store managers' decisions vary across stores. Upon presenting these results to the managers at the subject retailer, we found that differences in order advancement across stores can be explained by differences in their delivery schedules and sales volumes. Since stores 1 and 4 receive shipments 6 days in the week, their variation in handling workload corresponds to the difference between sales volumes on the busiest and the slowest days. Stores 2 and 3 have such replenishment schedules that shipments for off-peak days get combined together, providing a natural workload balancing effect. Store 5 has the most imbalanced delivery schedule because peak days 5 and 6 are combined together. Thus, store 5 shows a large amount of order advancement.

Apart from delivery schedule, sales volume also influences order advancement behavior since handling workload increases with sales volume. Stores 2 and 3 have a smaller amount of order advancement than stores 1 and 4 because they have smaller daily sales volumes. Store 5 also has a smaller daily sales volume, but its effect seems to be overshadowed by an imbalanced handling workload.

7 Application to inventory replenishment

In this section, we evaluate our estimation results on their potential to improve the managers' decision making by constructing a management coefficients model (Bowman 1963). Our method is as follows. The regression model 4 yields predicted values of the advancement index, which we denote as $\widehat{AI}_{si}$, for each store s and item i combination. Using these predicted values, we re-compute the reorder levels for each weekday via a heuristic procedure that seeks to advance orders from peak days to non-peak days independently for each SKU and to such an extent as indicated by the predicted value of the advancement index. Then, we use the new reorder levels in the ASO system to determine the replenishment orders and evaluate the effect on workload profile and days of inventory.

We first describe the re-computation of the reorder levels. Note that the ideal balanced workload is to have equal long run proportions of orders placed in each review cycle. Let the proportion $p^{Ideal}(t) = 1/|D_s|$ for $t \in D_s$ and $p^{Ideal}(t) = 0$ otherwise, with $|D_s|$ the number of delivery days per week for store s. Given the ASO workload $p^{ASO}(t)$ for $t = 1, \ldots, 6$ days, we interpolate between $p^{Ideal}(t)$ and $p^{ASO}(t)$ in order to determine a target workload pattern,

$$p^{Target}(t) = p^{ASO}(t) + \theta[p^{Ideal}(t) - p^{ASO}(t)], \tag{2}$$

where θ is an interpolation parameter, and for $t \notin D_s$, $p^{ASO}(t) = p^{Ideal}(t) = p^{Target}(t) = 0$. The subscripts s and i on $p^{Target}(t)$ and $p^{ASO}(t)$ are suppressed for simplicity.

Our objective is to determine θ such that the order advancement from $p^{ASO}(t)$ to $p^{Target}(t)$ is as close as possible to $\widehat{AI}_{si}$. This implies that the order advancement from $p^{ASO}(t)$ to $p^{Target}(t)$ needs to be quantified. Recall our order advancement index formula (1) for advancement from $p^{ASO}(t)$ to $p^{ACT}(t)$. Order advancement from $p^{ASO}(t)$ to $p^{Target}(t)$ can also be described by (1) if we transform the definition of $I_{si}(k)$ to:

$$I_{si}(k) = \sum_{j=1}^{k} [p^{Target}(j) - p^{ASO}(j)]. \tag{3}$$

Using (2), we rewrite (3) as

$$I_{si}(k) = \theta I'_{si}(k), \tag{4}$$

where $I'_{si}(k) = \sum_{j=1}^{k} [p^{Ideal}(j) - p^{ASO}(j)]$. Substituting (4) into (1) gives:

$$AI_{si} = \theta P_s, \tag{5}$$

where $P_s = \sum_{k=1}^{6} [I'_{si}(k) - \min\{I'_{si}(1), \ldots, I'_{si}(6)\}]$. The variable P_s has a nice intuition as it is equal to the order advancement from $p^{ASO}(t)$ to $p^{Ideal}(t)$. Hence, the order advancement from $p^{ASO}(t)$ to $p^{Target}(t)$ is simply equal to the product of θ with the order advancement from $p^{ASO}(t)$ to $p^{Ideal}(t)$. Thus, we get:

$$\theta = \frac{\widehat{AI}_{si}}{P_s}. \tag{6}$$

We must also ensure that the value of θ is such that $p^{Target}(t)$ is non-negative for all days t. Using (2), this implies that we must have $p^{ASO}(t) + \theta[p^{Ideal}(t) - p^{ASO}(t)] \geq 0$ for all days. According to (6), θ is always non-negative. Thus, we get:

$$\theta \leq \min\left(\frac{p^{ASO}(t)}{p^{ASO}(t) - p^{Ideal}(t)} : t = 1, \ldots, 6 \text{ such that } p^{ASO}(t) > p^{Ideal}(t) \right). \tag{7}$$

Combining (6) and (7), we set θ as:

$$\theta = \min\left(\widehat{AI}_{si}/P_s, \min\left\{ \frac{p^{ASO}(t)}{p^{ASO}(t) - 1/|D_s|} : t = 1, \ldots, 6 \text{ such that } p^{ASO}(t) > 1/|D_s| \right\} \right). \tag{8}$$

We substitute this value of θ into the expression for $p^{Target}(t)$ to get the target workload pattern. Next, we need to re-compute the reorder levels in the ASO system to achieve this pattern. Recall that the ordered quantity on day $t \in D_s$ is based on the difference between the reorder level and the starting inventory position on day t. This inventory position includes extra inventory ordered due to the order advancement on days before t. Let $Y(t-1)$ denote this extra built-up inventory. To order on average $\Delta(t)$ extra on day t, the reorder level on day t should be increased with $Y(t-1) + \Delta(t)$. To change the percentage ordered on day t from $p^{ASO}(t)$ to $p^{Target}(t)$ we set $\Delta(t) = [p^{Target}(t) - p^{ASO}(t)]\mu_{si}$ if $t \in D_s$ and $\Delta(t) = 0$ otherwise. So, if at the beginning of a delivery week, the extra inventory due to order advancement is zero, i.e. $Y(0) = 0$, we should increase the reorder level on day t by $Y(t) = \sum_{k=1}^{t} \Delta(k)$. Using (2), this can be written as $Y(t) = \sum_{k=1}^{t} \theta[p^{Ideal}(k) - p^{ASO}(k)]\mu_{si}$. However, to guarantee that the service level is not negatively influenced by the order advancement, we must ensure $Y(t) \geq 0$ for all $t = 1, \ldots, 6$. This is achieved by raising the reorder level on each delivery day t with $Y(t) - \min(Y(k))$. Note that this method is still a simplification because the ordering pattern will further depend on case pack size. Hence, the actual workload pattern that results for each item after applying our procedure may deviate from the target workload pattern. We evaluate this difference in our numerical study.

We implement the management coefficients model of inventory replenishment in our simulations and evaluate its performance in terms of the seasonality of the workload pattern and the average days of extra inventory as measured by the order advancement metric.[1] Table 8 shows the results obtained for each store. To evaluate the sensitivity of the results to the value of θ, we computed the results for different degrees of linear interpolation by applying multipliers $0, 0.1, 0.2, \ldots, 1.9, 2.0$ to θ. A smaller multiplier means less linear interpolation, and hence, less advancement of the order quantities from peak to non-peak days, whereas a larger multiplier means more order advancement. The scenario with θ-multiplier $= 0$ represents the ASO-system. Table 8 shows the results for a subset of the tested values of the multiplier. Figure 2 graphs the results for all tested values of the multiplier.

Consider the column for θ-multiplier equal to 1. We observe that the seasonality range of order quantities is appreciably lower than that realized by the store manager in four of the five stores and is comparable in the fifth. The change in range in the five stores is 2.82 (= 13.13 - 10.31), 8.57, 11.71, -0.51 and 15.08, respectively. The seasonality range of the management coefficients model is also far lower than that of the ASO-system in all stores, the differences being 13.97 (= 24.28 -

[1] Simulation results support our notion that the order advancement index is a good proxy for the number of days of extra inventory. Since the management coefficients model is simulated, we were able to compute average inventory for the ASO system with and without the management coefficients model. The advancement index has a correlation coefficient of 0.9901 with the increase in average inventory thus computed.

10.31), 12.91, 11.95, 14.32 and 29.94, respectively. The average order advancement achieved is close to the value obtained by the store manager, showing that the aggregate value of order advancement achieved by our heuristic is close to the targeted value. Thus, the management coefficients model results in a larger reduction in range with a similar amount of extra inventory as the actual orders in all stores. The performance improvement is particularly remarkable for store 5, which had yielded the poorest regression fit for our model. Thus, we find that the management coefficients model performs better than the human process in balancing handling workload.

Now consider the results for different values of the θ multiplier. As expected, we find that order advancement increases consistently with the θ multiplier for each store. Figure 2 additionally shows that the change in range has a U-shape curve. For small values of the multiplier, an increase in order advancement results in a more balanced workload. For large values of the multiplier, this means that orders are advanced too much from peak to non-peak days, so that the erstwhile non-peak days become the new peak days and the handling workload is again imbalanced. The minimum range is not necessarily at the θ-multiplier = 1 because model 4 does not incorporate store-wise fixed effects.

In Figure 2, we also depict the actual range as obtained by the store manager (denoted by S1,. . . ,S5) against the value of θ multiplier that most closely resembles the order advancement achieved by the store. All points are above the U-shape curves of the respective stores. Thus, we find that it is possible to improve the current operations in two ways by applying the management coefficients model: first, given the current effort of order advancement, one is able to reach a lower range; second, the same range can also be obtained with less effort in order advancement.

8 Conclusions

We have shown that retail store managers systematically deviate from the orders recommended by an automated replenishment system by advancing orders from peak days to non-peak days due to inadequacy of the automated system and incentive misalignment. We found that the case pack cover has the largest impact on ordering performance: orders for items with a large case pack cover are advanced more than items with a small case pack cover. This result implies that workload imbalance as well as deviations from the ASO system may be reduced by reducing case pack sizes. Other variables having an impact on the ordering performance are net shelf space, product variety, demand uncertainty and seasonality error. We also observed significant differences in order advancement behavior across stores, which could be explained by store differences in delivery schedule and sales volume.

In all five stores in our data set, order advancement led to a more balanced handling workload, with a 42% average reduction in the range of daily handling workload. But it also led to an average increase of 0.5 days of inventory, a 9.6% increase over the performance of the automated replenishment system. This increase in inventory can be beneficial in reducing stock-outs and stimulating demand. Typically, handling workload costs in a supermarket store are larger than inventory holding costs. Thus, store managers add value by trading off higher inventory with a more balanced workload and fully stocked shelves.

The presence and significance of order advancement indicates the need to integrate the capacity element and the effects of inventory on sales in ordering decisions. Using the order advancement metric and the regression model, we proposed a management coefficients model of inventory replenishment as a simple heuristic to accommodate the store managers' considerations in the automated replenishment system. This procedure led to a substantial improvement over the store manager's ordering performance by achieving a more balanced handling workload with similar average days of inventory.

The analysis in our paper has some limitations. It also leads to several questions for future research, both empirical and methodological. First, the order advancement metric developed by us fits only periodic demand patterns and also does not detect order postponement. The ordering behavior of inventory managers under aperiodic demand patterns, such as stationary demand or demand with tends, may be investigated at other research sites. We note that this research will require accurate inventory data, whereas our study could be conducted in the absence of such data. Second, our finding that store managers give different weights to the explanatory variables motivates us to study how these factors can be used to improve the effectiveness of the retailer's ordering policies. For example, the automated replenishment system could be enhanced taking into account case pack sizes, handling capacity and incentive elements in store operations. Our management coefficients model could be further developed by extending the definition of the target workload to incorporate case pack effects. Third, our study focused on common logistics variables rather than the store-wise fixed effects in the regression model. Another research question would be to study differences in performance among managers and retail stores. Fourth, a detailed model of handling workload may be constructed and validated to fully determine the effect of ordering behavior on handling costs. Finally, our analysis may be replicated at other retailers and wholesalers to further validate and generalize our results.

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Appendix

We show the relationship between the order advancement index and the increase in average days of inventory in the actual orders compared to the ASO system under the following assumptions: (i) Demand is deterministic and all demand is met. (ii) Actual orders are always advanced compared to the ASO orders. (iii) Demand and orders both follow weekly periodic patterns. Let $d_{si}(j)$ denote the demand for item i in store s occurring on the j-th day of the week. Also let and $q_{si}^{ACT}(j)$ and $q_{si}^{ASO}(j)$ denote the order quantities under the actual orders and the ASO system, respectively. (iv) The inventory at the end of the week stays constant. Thus, $\sum_{j=1}^{6} d_{si}(j) = \sum_{j=1}^{6} q_{si}^{ACT}(j) = \sum_{j=1}^{6} q_{si}^{ASO}(j)$. Let x_0^{ACT} denote the week ending inventory under actual orders and x_0^{ASO} denote the week ending inventory under ASO system orders.

Assumptions (i) and (ii) imply that inventory at the end of each day is non-negative under both actual orders and the ASO system. Further,

$$x_0^{ACT} + \sum_{j=1}^{k} \left\{ q_{si}^{ACT}(j) - d_{si}(j) \right\} \;\geq\; x_0^{ASO} + \sum_{j=1}^{k} \left\{ q_{si}^{ASO}(j) - d_{si}(j) \right\} \;\geq\; 0 \qquad \text{for all } k = 1, \ldots, 6. \tag{9}$$

Thus, the difference between average days of inventory in the two systems is given by

$$\frac{1}{\mu_{si}} \left[6x_0^{ACT} + \sum_{k=1}^{6} \sum_{j=1}^{k} \left\{ q_{si}^{ACT}(j) - d_{si}(j) \right\} - 6x_0^{ASO} - \sum_{k=1}^{6} \sum_{j=1}^{k} \left\{ q_{si}^{ASO}(j) - d_{si}(j) \right\} \right].$$

Here, μ_{si} denotes the total weekly demand for item i in store s. We now use the notation from §4 that $p_{si}^{ACT}(j)$ denotes the fraction of order quantity on the j-th day of the week under the actual orders, i.e., $p_{si}^{ACT}(j) = q_{si}^{ACT}(j)/\mu_{si}$. Likewise, $p_{si}^{ASO}(j)$ denotes the fraction of order quantity on the j-th day of the week under the ASO system. Rearranging terms, we see that the difference between average days of inventory in the two systems is given by

$$\frac{6(x_0^{ACT} - x_0^{ASO})}{\mu_{si}} + \sum_{k=1}^{6} \sum_{j=1}^{k} \left\{ p_{si}^{ACT}(j) - p_{si}^{ASO}(j) \right\} \;=\; \frac{6(x_0^{ACT} - x_0^{ASO})}{\mu_{si}} + \sum_{k=1}^{6} I_{si}(k), \tag{10}$$

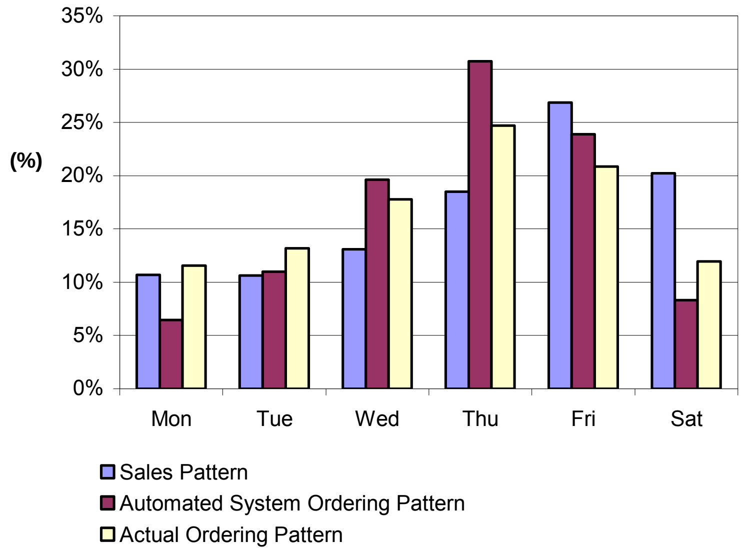
where $I_{si}(k)$ is as defined in §4.

Equation (9) implies that

$$\frac{x_0^{ACT} - x_0^{ASO}}{\mu_{si}} \geq \min\{I_{si}(1), \ldots, I_{si}(6)\}. \tag{11}$$

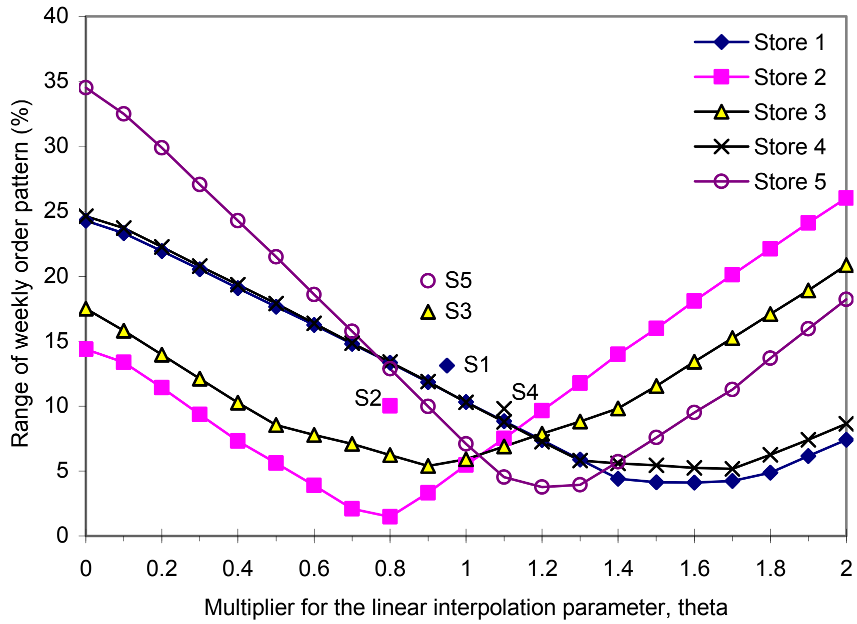
This inequality is binding if the store manager does not increase the level of safety stock in the store. Substituting into (10) and applying the definition of order advancement from (1), we see that the difference between average days of inventory in the two systems is greater than or equal to the order advancement metric. It is equal to the order advancement metric if (11) is satisfied as an equality. Alternatively, it is equal to the order advancement metric if the store manager advances orders to prior days within the same week.

Figure 1: Weekly seasonality patterns of sales, automated replenishment orders, and actual orders in a store



Note: These data are from Store 1 in our data set. The automated replenishment orders are computed by simulating the automated system over the actual sales pattern as described in §4.1.

Figure 2: Effect of management coefficients model on the range of the weekly order pattern



The vertical axis represents the range (maximum – minimum daily fractional workload) of the weekly order pattern. Range is computed as described in Table 8. The line for each store shows the range achieved for different values of the multiplier varying between 0 and 2. The 0 value of the multiplier represents no interpolation, i.e., the ASO system is used as is. Increasing values of the multiplier represent increase in the amount of workload shifted from peak to non-peak days. For comparison, the solitary point for each store shows the range of workload pattern for the actual orders placed by the store manager.

Table 1: List of merchandising categories included in our dataset

Sugar	Canned vegetables	Pet food
Coffee / tea	Canned fruits	Detergents
Flour / baking	Health food	Paper ware
Sauces / acids / oils	Baby food	Sanitary towels
Soups	Cookies	Household
Margarine / Butter	Sweets	Cosmetics
Crackers / Cereals	Chips / Nuts	Dairy keepable
Bread spreads	Coffee creamers	Eggs
Meal components	Soft drinks / Beer	
Canned meat / fish	Wines	

Table 2: Summary statistics for the five stores included in our dataset

	Store 1	Store 2	Store 3	Store 4	Store 5	Overall
# of SKUs	4862	3922	3499	4218	2916	19417
# of subgroups	159	152	147	155	139	161
# of SKUs/Subgroup (average)	30.6	25.8	23.8	27.2	21	25.8*
# of SKUs/Subgroup (std. Dev.)	30.1	22.9	21.1	25.4	18.9	24.3*
# of SKUs/Subgroup (median)	22	20	18	20	15	19*
Total weekly sales (CU/ week)	116,684.7	67,951.3	42,201.4	106,226.2	27,889.0	360,952.5
Average weekly sales per SKU	24.0	17.3	12.1	25.2	9.6	18.6
Total # of orders (per year)	362,426	203,582	130,714	331,911	82,843	1,111,476
Average # of orders per SKU per year	74.5	51.9	37.4	78.7	28.4	57.2
Review Cycle**	{1,2,3,4,5,6}	{2,4,5,6}	{1,2,4,5}	{1,2,3,4,5,6}	{2,4,6}	

*These statistics are computed at the store-subgroup level and then averaged across stores. **Days are numbered with Monday being day 1, Tuesday being day 2, and so on up to Saturday.

Table 3: Summary statistics for the dependent and explanatory variables for the pooled data for all 5 stores

	Mean	Std. Dev.	Median	Minimum	Maximum
Case pack size, Q_i	11.86	6.06	12	1	90
Average weekly sales, μ_{si}	18.59	47.92	8.55	0.18	2209.79
Case pack cover, Q_i/μ_{si}	1.99	2.17	1.28	0.005	23.00
Net shelf space, NSW_{si}	2.53	7.49	0.46	-12.21	237.97
Physical volume (cubic dm), $Volume_i$	1.04	1.36	0.69	0.02	19.60
Variety in the product subgroup, $Variety_{s,j(i)}$	63.91	46.94	54	1	280
Seasonality error, $CycleErr_{s,j(i)}$	0.03	0.01	0.03	0.01	0.24
Demand uncertainty, $CVFcstErr_{si}$	0.21	0.11	0.18	0.04	1.97
Order advancement index, AI_{si}	0.71	0.48	0.59	0	4.35

Table 4: Weekly seasonality patterns in sales and orders for each store

	Store 1	Store 2	Store 3	Store 4	Store 5
Range of weekly pattern of:					
Sales	0.1625	0.1783	0.1883	0.1128	0.1728
ASO Orders	0.2428	0.1437	0.1750	0.2462	0.3450
Actual Orders	0.1313	0.1003	0.1726	0.0979	0.1964
Standard deviation of weekly pattern of:					
Sales	0.0641	0.0698	0.0748	0.0492	0.0668
ASO Orders	0.0965	0.0705	0.0795	0.0987	0.1738
Actual Orders	0.0536	0.0436	0.0722	0.0426	0.0983
Simple Average of AI_{si}	0.7814	0.6296	0.7562	0.6981	0.6740
Weighted Average of AI_{si}	0.4813	0.4204	0.4386	0.5797	0.5031

The weekly seasonality pattern of sales for each store is computed by aggregating sales across items in the store and then calculating the fraction of the week's sales occurring on each day. Weekly seasonality patterns for ASO orders and actual orders are computed similarly. Then the range and standard deviation values are taken from the weekly seasonality patterns. For order advancement, we report simple averages as well as weighted averages obtained with weights μ_{si} applied to each item in each store. The weighted average is more meaningful, whereas the simple average is reported for comparison.

Table 5: Estimation results for all models

		Model 1		Model 2		Model 3		Model 4	
Model description		ANOVA		Within-subgroup		Pooled across subgroups		Pooled across stores and subgroups	
Adjusted R^2		18.64%		48.60%		45.87%		41.89%	
		Estimate	Std Error	Estimate	Std Error	Estimate	Std Error	Estimate	Std Error
Intercept		0.493	0.474	0.203	0.382	0.047	0.064	0.119	0.066
Store 1		-0.160	0.499	-0.092	0.397	0.235***	0.010		
Store 2		0.104	0.490	0.144	0.390	-0.012	0.009		
Q_i/μ_{si}				0.270***	0.007	0.262***	0.007	0.245***	0.007
Min$\{Q_i/\mu_{si} - x_1, 0\}$				-0.042**	0.021	-0.025	0.020	-0.101***	0.021
Max$\{Q_i/\mu_{si} - x_2, 0\}$				-0.203***	0.008	-0.195***	0.008	-0.181***	0.008
NSW_{si}				0.002***	0.001	0.002***	0.001	0.004***	0.001
$Volume_i$				0.001	0.004	-0.002	0.003	-0.007**	0.003
$Variety_{s,j(i)}$						0.0004***	0.000	0.001***	0.000
Absolute margin class (1=lowest, 6=highest)	1			0.078	0.061	0.006	0.059	-0.110*	0.061
	2			0.078	0.060	0.011	0.059	-0.087	0.061
	3			0.052	0.060	0.008	0.059	-0.081	0.061
	4			0.088	0.059	0.063	0.059	-0.019	0.061
	5			0.056	0.061	0.011	0.061	-0.038	0.063
$CycleErr_{s,j(i)}$						1.628***	0.461	6.535***	0.436
$CVFcstErr_{si}$				0.339***	0.065	0.356***	0.061	0.284***	0.063

*, **, *** denote statistical significance at p=.10, .05, .01, respectively. There are 6 profit margin classes; class 6 is used as the reference.

Table 6: Prediction accuracy of order advancement for stores 4 and 5

Store	Correlation Coefficient	R^2	Regression Results: Intercept	Regression Results: Slope
4	0.550	0.303	0.160***	0.658***
			0.013	0.015
5	0.470	0.221	0.086***	0.542***
			0.021	0.019

We predict order advancement for SKUs in stores 4 and 5 (test sample) using model 4 estimated on stores 1-3 (fit sample). We then regress actual order advancement in stores 4 and 5 against the predictions. This table shows the results obtained. *** denotes statistical significance at p=.01.

Table 7: Store-wise regression results for model 4

		Store 1		Store 2		Store 3		Store 4		Store 5	
Adjusted R^2		62.85%		31.47%		48.81%		31.99%		26.77%	
		Estimate	Std Error	Estimate	Std Error	Estimate	Std Error	Estimate	Std Error	Estimate	Std Error
Intercept		-0.227***	0.071	0.621***	0.146	0.102	0.220	0.090	0.107	0.467**	0.217
Q_i/μ_{si}		0.567***	0.011	0.120***	0.011	0.154***	0.013	0.129***	0.011	0.175***	0.013
Min{$Q_i/\mu_{si} - x_1$, 0}		-0.401***	0.027	0.166***	0.031	-0.006	0.048	-0.124***	0.024	-0.096*	0.055
Max{$Q_i/\mu_{si} - x_2$, 0}		-0.538***	0.014	-0.048***	0.013	-0.071***	0.014	-0.070***	0.013	-0.104***	0.014
NSW_{si}		0.0002	0.001	0.001	0.001	0.003***	0.001	0.002*	0.001	-0.004***	0.001
$Volume_i$		-0.003	0.004	-0.010**	0.005	0.002	0.007	-0.004	0.004	-0.011*	0.007
$Variety_{s,j(i)}$		0.0001	0.000	0.001***	0.000	0.001***	0.000	0.000***	0.000	0.000	0.000
Absolute margin class (1=lowest, 6=highest)	1	0.116	0.061	-0.282	0.142	-0.055	0.214	-0.015	0.101	-0.178	0.211
	2	0.112	0.061	-0.271	0.142	-0.043	0.214	0.001	0.101	-0.168	0.211
	3	0.093	0.061	-0.273	0.141	-0.019	0.213	0.021	0.101	-0.202	0.210
	4	0.120	0.061	-0.206	0.141	0.071	0.213	0.000	0.101	-0.145	0.208
	5	0.026	0.063	-0.246	0.143	0.098	0.215	-0.045	0.102	-0.078	0.215
$CycleErr_{s,j(i)}$		4.142***	0.660	2.969***	0.702	0.521	0.944	7.429***	0.473	0.322	0.389
$CVFcstErr_{si}$		-0.139*	0.081	-0.059	0.104	1.019***	0.108	0.376***	0.098	-0.236**	0.114

*, **, *** denote statistical significance at p=.10, .05, .01, respectively. Model 4 is estimated separately for each store using OLS regression. There are 6 profit margin classes; class 6 is used as the reference.

Table 8: Results of application of management coefficients model to each store

Store		Actual	Simulation results for different multipliers of θ					
			0	0.8	0.9	1	1.1	1.2
1	Wtd. average of AI_{si}	0.48	0	0.38	0.43	0.48	0.52	0.57
	Range (%)	13.13	24.28	13.35	11.85	10.31	8.85	7.38
2	Wtd. average of AI_{si}	0.42	0	0.33	0.37	0.41	0.45	0.48
	Range (%)	10.03	14.37	3.33	1.84	1.46	2.79	4.43
3	Wtd. average of AI_{si}	0.44	0	0.35	0.39	0.43	0.48	0.52
	Range (%)	17.26	17.50	7.01	6.23	5.55	5.59	6.48
4	Wtd. average of AI_{si}	0.58	0	0.43	0.48	0.53	0.59	0.64
	Range (%)	9.79	24.62	13.39	11.9	10.3	8.79	7.26
5	Wtd. average of AI_{si}	0.5	0	0.5	0.56	0.62	0.68	0.74
	Range (%)	19.64	34.5	10.63	7.46	4.56	3.72	4.26

The weighted average of AI_{si} and the range are calculated in the same manner as described in Table 4. The column for θ-multiplier = 0 corresponds to the ASO system.