Gradient-enhanced damage model for quasi-brittle fracture

Citation for published version (APA):
1. Introduction

Numerical simulations based on classical continuum damage concepts show a severe dependence on both the size and orientation of the spatial discretisation. This lack of objectivity is caused by the inability of these models to correctly describe the localisation of deformation which accompanies the fracture process. It may be removed by the introduction of higher-order deformation gradients in the constitutive description. This approach has been studied in the context of quasi-brittle fracture.

2. Gradient damage model

Classical approach. In continuum damage mechanics the effect of microstructural material degeneration is represented by a continuous field variable. In quasi-brittle damage models this damage variable, $D$, reduces the elastic stiffness. $D$ is zero for the initial, undamaged material and grows under the influence of mechanical loading until $D = 1$, which indicates the complete loss of material integrity. The evolution of the damage variable is governed by a scalar measure of the local strain state, the equivalent strain $\varepsilon_{eq}(\varepsilon)$.

Gradient enhancement. Instead of to the local equivalent strain, $\varepsilon_{eq}$, damage evolution is linked to the non-local equivalent strain, $\tilde{\varepsilon}_{eq}$, which is the solution of an additional partial differential equation. This equation, which can be derived from non-local damage theory, regularises the localisation of deformation and damage.

Model summary.

<table>
<thead>
<tr>
<th>Constitutive relation</th>
<th>$\sigma = (1 - D)H : \varepsilon$</th>
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</thead>
<tbody>
<tr>
<td>Equivalent strain</td>
<td>$\varepsilon_{eq} = \varepsilon_{eq}(\varepsilon)$</td>
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<tr>
<td>Non-local eq. strain</td>
<td>$\tilde{\varepsilon}<em>{eq} - c \nabla^2 \tilde{\varepsilon}</em>{eq} = \varepsilon_{eq}$</td>
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<tr>
<td>History variable</td>
<td>$\kappa(t) = \max(\kappa_0; \tilde{\varepsilon}_{eq}(\tau), \tau \leq t)$</td>
</tr>
<tr>
<td>Damage evolution</td>
<td>$D = D(\kappa)$</td>
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Numerical treatment. The equilibrium equation and the non-local equivalent strain definition are both cast in a weak form and discretised. The resulting non-linear equations are solved simultaneously by a full Newton-Raphson procedure.

3. Numerical example

4. Application

5. Conclusions

- Mesh-objectivity with respect to element size and orientation
- Efficient and robust numerical procedure
- Model gives a gradual transition from diffuse damage to a discrete crack

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