HAWT Near-Wake Aerodynamics, Part I: Axial Flow Conditions

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An improved physical understanding of the rotor aerodynamics of a horizontal axis wind turbine (HAWT) is required to reduce the uncertainties associated with today’s design codes. Wind tunnel experiments contribute to increased knowledge and enable validation and construction of models. The present study focuses on the near-wake of a model HAWT in both axial and yawed flow conditions. At three downstream planes parallel to the rotor plane, single-sensor hot-film traverses are made. The phase-locked unsteady three-dimensional flow velocity vector is determined by a novel data reduction method. A series of two papers discusses the near-wake aerodynamics of a model HAWT. The main goals are to obtain a detailed understanding of the near-wake development and to arrive at a base for model construction and validation. The first paper presents the experimental setup, data reduction and the results for the baseline case (axial flow conditions). In the second paper, the results for the yawed flow cases are presented and the effect of yaw misalignment on the near-wake development is discussed. Copyright © 2008 John Wiley & Sons, Ltd.

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Introduction

The rotor aerodynamics of a horizontal axis wind turbine (HAWT) consists of strongly coupled blade and wake aerodynamics. Hence, for accurate predictions of HAWT performance and loads, adequate modelling of the wake and blade aerodynamics is essential. To reduce the uncertainties of today’s design codes, the need for an improved physical understanding of rotor aerodynamics is generally acknowledged.1–3 Advanced rotor aerodynamics codes, such as vortex wake codes and Reynolds-averaged Navier–Stokes codes, and experiments in controlled conditions, hence in wind tunnels, contribute to increased knowledge. Furthermore, wind tunnel measurements are required to validate rotor aerodynamics codes and could be used to construct models as well.

Combined blade load and rotor wake recordings are desired because of the coupling between the blade and wake aerodynamics. In general, however, measurements focus either on the rotor wake or on the blade loads. In the present study, rotor wake measurements are discussed. The experiments, conducted at the Delft University of Technology (TUDelft), focus on the near-wake behind a model HAWT subject to both axial and yawed flow conditions.

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Axial flow conditions form the baseline for yawed flow conditions. Whereas axial flow conditions yield nearly axisymmetric flow with the rotor operating in the wind turbine state, yawed flow conditions are characterized by cyclic flow unsteadiness and a non-axisymmetric wake. Yaw misalignment is relevant as it occurs frequently on full-scale HAWTs, poses a challenge for today’s rotor aerodynamics codes and can be tested in a wind tunnel.

A limited number of wind tunnel experiments focused on the wake of model HAWTs have been performed. For a recent overview of rotor aerodynamics studies in axial flow conditions, see Vermeer et al. An extensive experimental study of the near-wake three-dimensional velocity field downstream of a model rotor in axial flow conditions was performed by Ebert and Wood using phase-locked average (PLA, also referred to as ensemble average) hot-wire constant-temperature anemometry (CTA). However, large, uncorrected wind tunnel wall effects complicate comparison with models; the turbine diameter equals the ‘diameter’ of the octagonal open jet outlet. The measurement domain is given by \( z/R_t = (0.08; 0.80) \), with \( z \) the axial distance from the rotor plane and \( R_t \) the radius of the rotor blade tip. These measurements were used to compare velocity trends found in the current experiments.

Regarding yawed flow conditions, the tip vortices in the wake of model rotors are both visualized and measured with particle image velocimetry (PIV) to study the skewed tip vortex geometry. PIV is also used to determine the velocity and circulation of the tip vortices and to measure the downstream development of time-averaged axial velocity profiles in the wake.

Evidence of low-frequency wake meandering, similar to bluff body vortex shedding, was found by Medici and Alfredsson for both axial and yawed flow conditions using hot-wire CTA in the far wake, with \( z/R_t \geq 2 \). The downstream extent of current measurements is \( z/R_t \leq 0.15 \). Because of the proximity to the rotor plane, the flow is considered cyclic with the rotor blade frequency and the effect of possible low-frequency wake meandering is neglected.

Vermeer performed wake studies on the same model rotor and open jet wind tunnel used for the current experimental campaign, see Reference 14 for an overview of Vermeer’s work. PLA hot-wire CTA in the near-wake is used to derive an engineering inflow model for yawed conditions. Hot-film measurements in the present study are more extensive, utilize a novel data reduction method and are analysed in more detail than Vermeer’s experiments. However, Vermeer’s experimental database provides an excellent opportunity to check repeatability.

The present hot-film measurements are part of a larger experimental campaign, using a multitude of measurement techniques; for a range of operating conditions, with both zero and non-zero yaw angles, the thrust coefficient is determined from strain gauge recordings, and a general overview of the wake geometry is obtained by quantitative smoke visualization of the tip vortices. A limited subset from the range of operating conditions is selected for study in more detail. Tuft visualizations on the blade are performed, in conjunction with hot-film CTA in the near-wake, to observe dynamically stalled flows. Furthermore, the PLA three-dimensional flow velocity vectors in the near-wake are determined by means of hot-film CTA. At three downstream planes parallel to the rotor plane, single-sensor hot-film traverses are made. A novel data reduction method enables the derivation of the PLA three-dimensional flow velocity vector, as discussed in Haans et al., where a brief overview of results is given as well.

This series of two papers concerns the hot-film CTA measurements. The two main research goals are obtaining a detailed understanding of the flow development in the near-wake and arriving at a base for model construction and validation. Consequently, the flow conditions are well documented and the data reduction includes corrections for wind tunnel wall effects and estimates of the measurement uncertainty. After presenting the experimental setup and the data reduction, this first paper discusses the near-wake aerodynamics of the baseline axial flow case. The second paper will treat the effect of yaw misalignment.

**Experiments**

**Setup and Conditions**

Hot-film measurements in the near-wake of a rotor model are performed in an open jet wind tunnel at the TUDelft. A schematic of the setup, including the global cartesian and cylindrical coordinate systems, attached
Readings from three interconnected Pitot-static tubes mounted in the jet exit plane, combined with ambient pressure and jet temperature recordings, yield the velocity magnitude in the jet exit plane $|V_{\text{jet-exit}}|$, when applying the perfect gas law and Bernoulli’s equation. The flow is assumed to have an axial component only, $|V_{\text{jet-exit}}| = W_{\text{jet}}$.

The main characteristics of the open jet wind tunnel are a diameter of the jet exit plane of 2.24 m, a maximum attainable wind speed $W_{\text{jet,max}}$ of 14.5 m s$^{-1}$ and a freestream turbulence intensity $Tu$ of 1.2 ± 0.2% at $W_{\text{jet}} = 5.5$ m s$^{-1}$, the speed at which the measurements are taken. The wind tunnel is not equipped with a separate return channel; the hall is used for recirculation instead.

The main geometric features of the two-bladed rotor model are given in Table I. The model offers the possibility of altering the blade pitch angle automatically, by modifying the tip pitch angle $\theta_{\text{ctip}}$. The axial force $T$ on the rotor, the force normal to the rotor plane, is recorded by means of strain gauges on the rotor shaft. The rotor plane is positioned 1 m downstream of the jet exit plane, with the rotor hub in line with the jet centre. The distance from the rotor plane to the downstream tunnel wall is 11 m or 18.4 rotor radii $R_t$.

The unsteady velocity field in the near-wake of the rotor model is measured using hot-film CTA. A single-film hot-film probe is mounted on a traversing rig that makes an automated traverse through a plane parallel to the rotor plane. Two similar types of single-film hot-film probes are used, apart from the hot-film orientation: the ‘normal’ type, with the hot-film perpendicular to the probe (TSI 1201-20), and the ‘parallel’ type with the hot-film parallel to the probe (TSI 1211-20)—see Figure 2. The probes are manufactured by TSI Incorporated, located in Shoreview, MN.

The main characteristics of the hot-film probes are given in Table II. The hot-film recordings are synchronized with the rotational frequency of the blades. The measurement resolution of $\Delta \theta_b = 2$ corresponds to...
approximately 2.1 kHz. Hence, the frequency resolution is 1.0 kHz. The CTA upper frequency limit depends on the flow velocity; at 10 m s\(^{-1}\), which is at the same order of magnitude as the flow velocity experienced in the near-wake experiments, a value of approximately 75 kHz is found for CTA using a cylindrical hot-film. The frequency resolution is hence significantly smaller than the upper frequency limit.

The yaw angle \(Y\), being the angle between the normal vector to the rotor plane and the undisturbed velocity vector, is set to 0° in order to obtain the axial flow condition studied in this paper. The blades rotate at \(f = 11.67\) Hz and \(W_{\text{jet}}\) is set to 5.5 m s\(^{-1}\). Hence, the tip speed ratio \(\lambda\), defined as

\[
\lambda = \frac{2\pi f R}{W_{\text{jet}}} \tag{1}
\]

is 8. \(\theta_{\text{tip}}\) is fixed at 2°. The power coefficient \(c_P\) and thrust coefficient \(c_T\) of the rotor model are given by

\[
c_P = \frac{P}{\frac{1}{2} \rho W_{\text{jet}}^3 \pi R^2} \tag{2}
\]

and

\[
c_T = \frac{T}{\frac{1}{2} \rho W_{\text{jet}}^3 \pi R^2} \tag{3}
\]

respectively, with \(P\) representing the rotor power and \(\rho\) the air density. A performance study by Vermeer with the rotor model set to a range of values for \(\lambda\) and \(\theta_{\text{tip}}\) shows that the currently selected operating condition of \(\Psi = 0°\), \(\lambda = 8\), \(\theta_{\text{tip}} = 2°\) yields \(c_{P_{\text{max}}}=0.32\). Measurements by Haans et al. yield \(c_T = 0.80\) at the currently selected operating condition.

For a point fixed on the blade, located at 0.7\(R\), the chord Reynolds number \(Re_{0.7R}\) is determined. To estimate \(Re_{0.7R}\), the flow velocity relative to the blade is assumed to consist of the undisturbed velocity and the blade rotation components only. \(Re_{0.7R}\) is written as

\[
Re_{0.7R} = c\sqrt{\frac{W_{\text{jet}}^2 + (2\pi f 0.7R)^2}{\nu}} \tag{4}
\]

with \(c\) the airfoil chord and \(\nu\) the kinematic viscosity. \(Re_{0.7R} = 1.6 \times 10^5\) is found. As argued by Vermeer et al., the low Re compared with that of a full-scale wind turbine is acceptable since the characteristics of the
NACA0012 airfoil are known for the Re-range on the model rotor. The experimental results can still be used for comparison with and validation of numerical models.

Note that the turbulence intensity for a point fixed on a rotating blade is reduced compared with the freestream turbulence intensity as a consequence of the increased apparent flow velocity due to blade rotation. The blade rotates through a non-uniform, non-axisymmetric freestream flow field (however, see Figure 3), resulting in additional ‘turbulence’ at a point fixed on the blade. The axial velocity reduction near the centre of the jet is associated with the wake trailed from the nacelle of the wind tunnel fan that is located upstream of the test section.

**Procedure**

Estimates of the velocity magnitude and direction require hot-film measurements at multiple orientations. The current research concerns the PLA flow velocity, \( \langle \vec{V} \rangle (r, \theta, z, \theta_b) \), instead of quantitative turbulence data or the instantaneous flow velocity \( V \). A single-film hot-film probe requires successive measurements at varying probe orientations to estimate \( \langle \vec{V} \rangle \). A three-film type hot-film probe can yield the instantaneous \( \langle \vec{V} \rangle \). Both measurements with the single- and three-film type hot-film require the flow conditions to be constant throughout the experimental campaign. An additional requirement for the single-film type hot-film is the repeatability of the \((r, \theta, z)\)-coordinate and of the \( \theta_b \)-phase, as multiple recordings at a given \((r, \theta, z, \theta_b)\)-point are needed to estimate \( \langle \vec{V} \rangle (r, \theta, z, \theta_b) \). Because of its relative inexpensiveness, expendability and ease of calibration, the single-film type probe is preferred to the three-film type probe.

Hot-film measurements are performed in one plane 6.0 cm upstream of the rotor plane and three downstream planes, at 3.5 cm \((0.0583R)\), 6.0 cm \((0.10R)\) and 9.0 cm \((0.15R)\) from the rotor plane. The spatial resolution of the measurement points, in cylindrical coordinates, is given by \( r = 0.24, 0.30, 0.36, \ldots 0.60 \) m \((r/R = 0.24, 0.30, 0.36, \ldots 0.60)\).
A sample size of 54 hot-film recordings is adequate; the uncertainty associated with the determination of the PLA yields a minor contribution to the total experimental uncertainty of \( \langle \dot{V} \rangle \), as is discussed later. Previous hot-film experiments in the wake of the same rotor model\(^{14} \) indicate that a measurement resolution of \( \Delta \theta_b = 2^\circ \) is sufficient to capture the PLA velocity variations observed with both blade and wake passages.

At a given \((r, \theta, z, \theta_b)\)-location, six CTA measurements each with a different orientation of the hot-film, are required for the data reduction technique proposed in Haans et al.\(^{18} \) to determine the PLA flow velocity. Because of the limited range of the traversing rig, a 30 minutes measurement run covers half of the measurement plane. Errors between intended and true \((r, \theta, z)\)-positions due to random shifts of the traversing rig accumulate and result into an offset when returning to the starting point upon completion of a traverse. After each traverse, the offset is determined and the hot-film probe is reset to the correct starting point.

The sensitivity of the hot-film output voltage \(E\) to the speed and direction of the fluid flow is determined with speed and angular calibrations, respectively. For speed calibrations, the hot-film is positioned normal to the direction of the flow. Angular calibrations determine the relation between \(E\) and flow angle for a given speed. Both speed and angular calibrations are performed regularly to compensate for hot-film ageing.

Data Reduction

Before discussing the data reduction in detail, it must be noted that three coordinate systems are utilized; the hot-film probe fixed cartesian coordinate system \((x_p, y_p, z_p)\)—see Figure 2—and the two rotor support fixed coordinate systems, a cartesian one \((x_m, y_m, z_m)\) and a cylindrical one \((r, \theta, z)\)—see Figure 1.

Velocity Determination

Speed and angular calibrations of the hot-film anemometers form the basis for velocity determination. The temperature-corrected King’s Law\(^{19} \) is used during speed calibration to relate \(E\) with \(W_p\), the flow velocity in the \(z_p\) direction,

\[
E^2 = (T_f - T_a)(A + BW_p^n)
\]

with \(T_f\) and \(T_a\) the preset hot-film and measured flow temperature, respectively. \(A, B\) and \(n\) are calibration constants. Generally, the flow velocity is unknown beforehand. The recorded \(E\) is thus related to the velocity magnitude when the flow would have been normal to the film, named effective velocity \(V_{\text{eff}}\), using equation (5) with \(V_{\text{eff}} = W_p\). The basis for angular calibration forms Jørgensen’s equation,\(^{19} \)

\[
V_{\text{eff}}^2 = h^2 U_p^2 + k^2 V_p^2 + W_p^2
\]

\(h\) and \(k\) are angular calibration constants that are determined during the angular calibration.

For a comprehensive overview of calibration procedures, see Bruun’s work.\(^{19} \) The techniques applied for the current measurement campaign are presented in Haans et al.\(^{18} \) As the employed calibration procedures are standard practice, they are not discussed in detail here.

Traditional Single-sensor Hot-film Data Reduction Method

The derivation of \(\langle \dot{V} \rangle\) with single-film hot-film type probes traditionally uses successive measurements with three orientations of the hot-film probe. Two measurements are taken with the normal probe, one with the parallel probe. For both normal probe orientations, the hot-film is normal to the axial direction \((z_p \parallel z_m)\). For one of the two normal probe orientations, the hot-film is positioned vertically \((y_p \parallel y_m, x_p \parallel x_m)\). For the other normal probe orientation, the hot-film is positioned horizontally \((x_p \parallel y_m, z_p \parallel z_m)\);
The true value of a quantity. The true value is generally unknown; hence, standard and guide.

For each \((r, \theta, z, \theta_h)\)-point and each hot-film orientation, the PLA of equation (6) is taken, where contributions from the disturbance terms, \(\langle v^2 \rangle\), are neglected as \(\langle v^2 \rangle\) is, generally, several orders of magnitude smaller than \(\langle V \rangle^2\)

\[
\langle V_{\text{eff}} \rangle^2 = h^2 \langle U_m \rangle^2 + k^2 \langle V_p \rangle^2 + \langle W_p \rangle^2
\]  

(7)

A system of equations can be constructed for the three orientations per \((r, \theta, z, \theta_h)\)-point,

\[
\begin{bmatrix}
h_1^2 & k_1^2 & 1 \\
k_2^2 & h_2^2 & 1 \\
h_3^2 & 1 & k_3^2
\end{bmatrix}
\begin{bmatrix}
\langle U_m \rangle^2 \\
\langle V_m \rangle^2 \\
\langle W_m \rangle^2
\end{bmatrix}
= \begin{bmatrix}
\langle V_{\text{eff},1} \rangle^2 \\
\langle V_{\text{eff},2} \rangle^2 \\
\langle V_{\text{eff},3} \rangle^2
\end{bmatrix}
\]

(8)

and solved for \(\langle U_m \rangle^2\), \(\langle V_m \rangle^2\) and \(\langle W_m \rangle^2\). Since squared velocity components are determined, the direction of the velocity vector is not part of the solution and has to be estimated. For axial flow conditions, the axial velocity component is large throughout the wake and it can readily be assumed that \(\langle W_m \rangle > 0\). However, the velocity components in the rotor plane, \(\langle U_m \rangle\) and \(\langle V_m \rangle\), are generally small compared with the axial velocity component, and their directions are unknown beforehand. Furthermore, the combined effect of small \(\langle U_m \rangle\) and \(\langle V_m \rangle\) values and measurement uncertainties yield \(\langle U_m \rangle^2 < 0\) and/or \(\langle V_m \rangle^2 < 0\) at several \((r, \theta, z, \theta_h)\)-points. This is clearly a non-physical result.

**New Single-sensor Hot-film Data Reduction Method**

The two problems of the traditional velocity determination method, the inability to predict the direction of the \(\langle U_m \rangle\) and \(\langle V_m \rangle\) components and non-physical \(\langle U_m \rangle^2\) and \(\langle V_m \rangle^2\) values, are overcome by a new method to determine the PLA flow velocity from single-film hot-films. Only the basics are presented here; for a more detailed discussion on the new method, see Haans et al. First, it must be noted that the determination of both the magnitude and the direction of \(\langle W_m \rangle\) is identical to the approach of the traditional method; the magnitude is estimated using equation (8), the direction of \(\langle W_m \rangle\) is assumed to be \(\langle W_m \rangle > 0\).

Asymmetry in the response of the parallel hot-film probe to the flow angle in the \((y_p, z_m)\)-plane is observed during angular calibrations (see Figure 4). The determination of both the direction and the magnitude of the \(\langle U_m \rangle\) and \(\langle V_m \rangle\) components in the new method is based upon the observed asymmetry.

For a given \((r, \theta, z, \theta_h)\)-point, a total of four probe orientations are used for the parallel probe measurements. The two probe orientations in the \((x_m, z_m)\)-plane are given in Figure 5, and two similar orientations are used in the \((x_m, z_m)\)-plane. The sign of the \(\langle U_m \rangle\)-velocity component is estimated by comparison of the \(\langle V_{\text{eff}} \rangle\)-signals from the two orientations shown in Figure 5; the smallest value corresponds to the probe orientation where the sensor is shielded from the \(\langle V_m \rangle\)-component of the flow velocity by its prongs. Similar reasoning holds for \(\langle U_m \rangle\), with the two probe orientations in the \((x_m, z_m)\)-plane.

The magnitudes of \(\langle U_m \rangle\) and \(\langle V_m \rangle\) are determined sequentially. The equation solved for each velocity component is based upon two elements: the unshielded (default) situation modelled with equation (7) and the ratio of \(\langle V_{\text{eff}} \rangle\) measurements with the probe shielded and unshielded.

**Uncertainty**

The quality of the experimental results is assessed with an analysis of the measurement uncertainty, using the methodology suggested by the American Institute of Aeronautics and Astronautics (AIAA). After a brief introduction into the terminology used, the results of the analyses of the measurement uncertainty of \(c_T\) and \(\langle \hat{V} \rangle\) are presented here. For an in-depth discussion of the methodology, see the AIAA measurement uncertainty standard and guide.

The experimental error \(\varepsilon\) is defined as the difference between the experimentally determined value and the true value of a quantity. The true value is generally unknown; hence, \(\varepsilon\) must be estimated by an uncertainty \(U_\varepsilon\).
at a confidence level specified by the experimenter; the default 95% confidence interval is used for the current analyses. \( U \) is the orthogonal summation of the systematic uncertainty \( B \) and random uncertainty \( P \),

\[
U = (B^2 + P^2)^{1/2}.
\]

An uncertainty is classified as random when it contributes to the data scatter with repeated measurements; else, it is a systematic uncertainty.

The systematic and random uncertainty of \( c_r, B_{c_r} \) and \( P_{c_r} \) respectively, are estimated at \( B_{c_r} = 0.078 \) and \( P_{c_r} = 0.021 \), which results in a measurement uncertainty of \( c_r \) of \( U_{c_r} = 0.081 \). With \( c_r = 0.80 \), \( U_{c_r} \) hence is estimated at 10%.

The three-dimensional PLA flow velocity \( \langle \vec{V} \rangle \), derived through the new data reduction method, is expressed in the cartesian non-rotating model coordinate system \( (x_m, y_m, z_m) \)—see Figure 1 for the experimental setup. \( \langle \vec{V} \rangle \) is subsequently made dimensionless by division through \( W_{jet} \). The experimental uncertainty \( U_{\langle \vec{V} \rangle} \) is estimated at the 95% confidence level for each of the three components of the dimensionless PLA flow velocity, at each measured \( (r, \theta, z, \theta_p) \)-point.

The typical range of the measurement uncertainty is \( 0.09 \leq U_{\langle \vec{V} \rangle} \leq 0.17 \). The constituting systematic and random uncertainties are estimated at \( 0.04 \leq B_{\langle \vec{V} \rangle} \leq 0.15 \) and \( P_{\langle \vec{V} \rangle} = 0.08 \), respectively. \( P_{\langle \vec{V} \rangle} \) is thus assumed to be constant, independent of the velocity component and the \( (r, \theta, z, \theta_p) \)-point under consideration.
contribution to $U_{(V)}$ from the random uncertainty associated with determining the PLA over 54 recordings per $(r, \theta, z, \theta_b)$-point, estimated at 0.036, is minor compared with the typical $U_{(V)}$ range. Hence, the sample size of 54 recordings is adequate.

### Wall Effects

The wind tunnel setup does not perfectly replicate the conditions of a model rotor subject to unbounded flow, uniform at a distance far upstream. Two possible corrections are identified:

- **Blockage correction:** the ratio of the model rotor area to the wind tunnel jet area is finite.
- **Non-uniformity correction:** the Pitot-static tubes that record the dynamic pressure for $W_{\text{jet}}$ are close to the rotor plane.

Both corrections are studied by numerically solving the discretized Euler equations with the commercial code Fluent. The domain is a simplified representation of the wind tunnel setup (see Figure 6). A uniformly loaded actuator disc model is used instead of a two-bladed rotor. The incompressible definition of the finite volume, cell-centred flow solver is used; the mass conservation and momentum equations are solved sequentially. The SIMPLEC algorithm, combined with the Rhie–Chow procedure to prevent checkerboarding of the pressure, is used for pressure–velocity coupling.

Grid and domain dependency studies are performed; both show monotonic convergence with reduced cell size and increased domain dimensions, respectively. Based hereupon, the dimensions of the domain are selected as shown in Figure 6, and the grid yields a discretization of the semi-actuator disc with 21 cell nodes. The jet area between the tunnel exit and the actuator disc is discretized with square cells of equal sizes. In the remainder of the domain, maximum cell stretching in both the $z$- and $r$-direction is set to 1.05.

To correct for wind tunnel blockage, the approach suggested by Glauert\(^ {22} \) is pursued; an equivalent unbounded air speed $W_0^*$ is determined which yields the same velocity through the actuator disc, and hence the same thrust, as the inflow velocity $V_{z0} = W_0$ set in the wind tunnel. With the superscript $*$ indicating the equivalent unbounded condition, the equivalent velocity concept can be written as

$$c_t W_0^* = c_t (W_0^*)^2$$

$$W = W^*$$
In the unbound condition, the thrust can be written as

\[ T^* = \frac{1}{2} \rho S c_T (W^*)^2 = 2W^* \rho S(W^*_0 - W) \]  

using one-dimensional momentum theory. Combining equations (10)–(12), Sørensen et al. derived \(^{23} W^*/W_0 = 1 - \frac{1}{4W/W_0} \rho c_T \) \(^{(13)} \)

Now, \( W^*_0/W_0 \) can be determined (see Figure 7) by substituting the computational results in the right-hand side of equation (13). For \( W_0 \), the area weighted average of the computed axial velocity over the actuator disc is used. The current setting with \( \lambda = 8 \), \( \theta^*_w = 2^\circ \) yields \( c_T = 0.80 \) and hence \( W^*_0/W_0 = 1.014 \).

The computed inviscid velocity in the wind tunnel exit plane, where the Pitot-static tubes are located, is non-uniform; the flow speed is reduced towards the jet centre because of the proximity of the wind turbine model to the wind tunnel exit plane. For the current setting however, the local axial velocity \( W_{jet} \) at the radial position of the Pitot-static tubes is approximately equal to \( W_0 \); \( W_{jet}/W_0 = 1.001 \) is found. The main correction thus originates from wind tunnel blockage,

\[ \frac{W^*_0}{W_{jet}} = \frac{W^*_0}{W_0} \cdot \frac{1}{W_{jet}/W_0} = 1.013 \]  

(14)

The corrected flow conditions now become \( W^*_0 = 5.57 \) m s\(^{-1} \) and \( \lambda^* = 7.90 \), with \( c_T^* = 0.77 \).

**Results**

**Azimuthal-averaged Axial Induction**

A global analysis of the measured data is performed by a study of the azimuthal-averaged axial induction factor in the rotor plane.

No measurements are performed in the rotor plane itself. The velocity data from the three downstream planes, at \( z/R_e = 0.0583, 0.100 \) and \( 0.150 \), are used to construct the induction factor \( a \) in the rotor plane, with

\[ a^* = 1 - \frac{\tilde{W}}{W_0^*} \]  

(15)
In the three measurement planes, the hot-films are positioned at identical \((r, \theta)\)-points. With the blade at a given angle \(\theta_b\), the azimuthal-averaged axial velocity \(\bar{W}\) at each \(r\)-coordinate is determined for all three measurement planes. The assumed linear function \(\bar{W} = \bar{W}(z)\) is fitted to the measured data in the Least Squares sense for equal radii and is used to extrapolate \(\bar{W}\) to the rotor plane. Using equation (15), \(a^*\) is determined in the rotor plane for each recorded blade azimuth angle \(\theta_b\). The average and standard deviation of \(a^*\) over the range \(\theta_b = 0^\circ, 2^\circ, \ldots, 358^\circ\) is shown in Figure 8.

A radial dependency in \(a^*\) can be observed. The increased standard deviation of \(a^*\) near the blade tip can be explained by small asymmetries in the strong tip vortex.

One-dimensional momentum theory is used to determine the local thrust coefficient at a given radius, \(C_t^\#(r)\), in the rotor plane

\[
C_t^\# = 4a^*(1 - a^*). \tag{16}
\]

For each recorded blade azimuth angle \(\theta_b\), the rotor thrust coefficient \(C_t^\#\) is computed by integrating \(C_t^\#(r)\) across the rotor disc under the assumptions that \(C_t^\#(R_t) = 0\) and \(C_t^\#(0) = C_t^\#(r_1)\), with \(r_1\) the radius of the most inboard data point. The average over the \(\theta_b\) range is found to be \(C_t^\# = 0.72\). Force measurements on the rotor, using strain gauges, yield \(C_t^\# = 0.77 \pm 0.078\). The difference in \(C_t^\#\) from momentum theory and force measurements is within the uncertainty bounds for both approaches. An exact match is not expected, however, as the infinite blades assumption of one-dimensional momentum theory is violated by the two-bladed rotor.

**Trends in Azimuth Angle-dependent Velocity**

Not only the phased-lock average ‘true’ velocity \(\bar{V}\), but also the phase-locked standard deviation of the effective velocity \(s_{\text{veff}}\), determined over 54 blade rotations, yields information on the wake flow. \(s_{\text{veff}}\) appears to be an effective measure of the wake passage. Fluctuations due to wake turbulence result in a local \(s_{\text{veff}}\) increase when a wake passes a given data point \((r, \theta, z)\). The wake evolution in the near-wake region can be seen in Figure 9.

The deformation and diffusion of the wake, while convected downstream, can be observed. Non-uniform induction yields deformation of the wake from its original linear shape when trailed from the blade trailing edge. The concentrated tip vortices induce a strong flow field locally near the tip vortex, with corresponding large induced velocity gradients. Hence, wake deformation increases towards the tip vortices.
The analysis of PLA results commences with a comparison of $\langle V \rangle$ derived with the new and traditional single-sensor hot-film data reduction technique. Furthermore, current results are compared with results published by Schepers\textsuperscript{15}, measured on the same rotor, in the same wind tunnel, under the same conditions, $\theta_b = 28^\circ$, $q_{ctip} = 2^\circ$, $W_0^* = 5.57$ m s$^{-1}$ and $\lambda = 7.90$, that are obtained from 10 instead of 54 recordings per $(r, \theta, z, \theta_b)$-point.

The presently measured data set is processed using both the new and the traditional single-sensor hot-film data reduction technique. Comparisons are made for all three velocity components, $\langle V_r \rangle$, $\langle V_q \rangle$ and $\langle V_z \rangle$. As both data reduction techniques derive $\langle V_z \rangle$ identically, the present $\langle V_z \rangle$ data are plotted in Figure 10 using the new method only. The agreement between present and previously published $\langle V_z \rangle$ data is within the uncertainty bounds of the present measurements. For the hot-film position plotted in Figure 10, maximum and average absolute $\langle V_z \rangle$ differences are 0.3 and 0.1 m s$^{-1}$. The repeatability in $\langle V_z \rangle$ between the two measurement campaigns gives confidence in the present data set.

Figure 9. Standard deviation of hot-film effective velocity $s_{eff}$ at three downstream planes for a blade orientation of $\theta_b = 28^\circ$. The blades rotate clockwise; the freestream wind is directed away from the observer.

Figure 10. $\langle V_z \rangle/W_0^*$ versus $\theta_b$ at $(r/R, \theta, z/R) = (0.7, 180^\circ, 0.100)$ for Schepers\textsuperscript{15} and present data. Uncertainty bounds are included for present data. Blade passage: $\theta_b = 0^\circ, 180^\circ$; wake passage: $\theta_b = 90^\circ, 270^\circ$. 
Comparing the results for $\langle V_r \rangle$ and $\langle V_q \rangle$ (see Figures 11 and 12 for a typical example) yields that the order of magnitude of a velocity component is equal for all three velocity sets. The characteristics of the velocity distribution as a function of $q_b$ differ significantly, however, between the two data reduction techniques. The wiggles in both $\langle V_r \rangle$ and $\langle V_q \rangle$, present with the traditional approach, are reduced for the new technique, thereby better identifying blade and wake passages. The proposed technique yields the direction of the velocity component as part of its solution (see Figure 12). Whereas the traditional approach predicts $\langle V_q \rangle$ to be positive everywhere, the new data reduction predicts sign changes. Qualitative comparison with near-wake velocity studies by Ebert and Wood\textsuperscript{6} and Vermeer\textsuperscript{14} confirms the $\langle V_q \rangle$ shape.

The new data reduction technique yields more detail in the $\langle V_r \rangle$ and $\langle V_q \rangle$-distribution than the traditional technique. The remainder of the discussion is hence based upon the velocity derived using the new approach.

To help explain the observed trends in near-wake velocities, a typical lifting line wake vortex model is presented in Figure 13. A line of bound circulation, the lifting line, spans the blade. Circulation is trailed into the wake to form a vortex sheet.

The axial flow condition ideally results in $\theta$-independent PLA velocity $\langle \vec{V} \rangle$, when $\langle \vec{V} \rangle$ is corrected for $\theta_b$-phase shifts that occur with variations in the $\theta$-coordinate. Averaging over the $\theta$-positions yields $\langle V \rangle$. For the combination of three radial with three downstream locations, $\langle V_r \rangle$, $\langle V_q \rangle$, and $\langle V_z \rangle$ are plotted in Figures 14–16.

The blade passages are located at $\theta_b = 90^\circ$ and $270^\circ$. At the blade passages, a local $\langle V_r \rangle$-maximum followed by a local $\langle V_q \rangle$-minimum and a local $\langle V_q \rangle$-minimum are found. The minima and maxima in $\langle V_r \rangle$ and $\langle V_q \rangle$ at the blade passages are induced by the bound vortex; the trends at the blade passages agree with the orientation of the bound circulation vector, pointing outboard as can be seen in Figure 13 using the right-hand rule. The amplitude of the peaks is reduced when going downstream for a given radial position, since the distance to the bound circulation is increased.

Three causes are identified to explain $\langle V_r \rangle$ during a blade passage. First, from the simple vortex wake model (see Figure 13), the bound circulation is expected to induce a local $\langle V_r \rangle$-minimum followed by a local $\langle V_r \rangle$-maximum, resulting in an overall $\langle V_r \rangle$-increase with a blade passage. Second, when considering a lifting surface

![Figure 11. $\langle V_r \rangle/W_0^*$ versus $\theta_b$ at $(r/R, \theta, z/R) = (0.7, 180^\circ, 0.100)$ for Schepers and Vermeer\textsuperscript{15} and present data. Blade passage: $\theta_b = 0^\circ, 180^\circ$; wake passage: $\theta_b = 90^\circ, 270^\circ$](image-url)
instead of the lifting line vortex wake model, varying bound circulation across the blade span introduces bound circulation oriented in the chordwise direction that trails into the wake. During a blade passage, chordwise-bound circulation induces a $\langle V_r \rangle$-contribution; negative for a bound circulation increase when going outboard and vice versa. Third, the pressure difference between the blade pressure and the suction side induces radially inboard $\langle V_r \rangle$ near the tip and radially outboard $\langle V_r \rangle$ near the root. Observations from Figures 14 to 16 agree with the three mentioned $\langle V_r \rangle$-trends. Note that in general $\langle V_r \rangle$ is positive, corresponding to an expanding flow.

With regard to the wake passage, the inviscid vortex wake model shown in Figure 13 only explains the $\langle V_r \rangle$-behaviour. Passage of the vortex sheet results in a step change in $\langle V_r \rangle$. The orientation of the trailed circulation determines the sign of the step change. The signs of the step changes, negative for $r/R_t = 0.4$ and
Figure 14. Non-dimensional velocity versus $\theta_b$ at $(\theta, z/R_t) = (90^\circ, 0.0583)$ for radial locations $r/R_t = 0.4, 0.6$ and 0.9. The passages of the first and second blade and their corresponding wakes are indicated by (b1, w1) and (b2, w2) respectively.

Figure 15. Non-dimensional velocity versus $\theta_b$ at $(\theta, z/R_t) = (90^\circ, 0.100)$ for radial locations $r/R_t = 0.4, 0.6, 0.9$. The passages of the first and second blade and their corresponding wakes are indicated by (b1, w1) and (b2, w2), respectively.
0.6 and positive for \( r/R_t = 0.9 \), agree with a typical variation of bound circulation on a rotating blade and the corresponding orientation of the trailed circulation in the wake.

Viscous effects are responsible for the trends in \( \langle V_z \rangle \) and \( \langle V_q \rangle \) with wake passages. The local \( \langle V_z \rangle \)-minimum and \( \langle V_q \rangle \)-maximum represent the momentum loss due to viscous drag associated with the boundary layer that trails from the blade surface into the wake. For the current high-\( \lambda \) condition, the viscous drag mainly acts in the tangential direction; its effect in the axial direction is small.

The local \( \langle V_z \rangle \)-minimum at the wake passage represents an axial momentum loss, hence the contribution to the thrust due to viscous drag. The local \( \langle V_q \rangle \)-maximum at the wake passage is larger than zero and hence in the same direction as the rotor rotation, indicating that the wake is dragged along in the rotor rotation direction. The local \( \langle V_q \rangle \)-maximum corresponds to a reduction in the angular momentum of the flow and hence a reduction in the blade torque.

**Local Angle of Attack**

Blade Element Momentum and vortex wake methods utilize airfoil models to determine the blade forces for a given velocity distribution in the rotor plane. The airfoil models require a two-dimensional angle of attack \( \alpha \).

For a two-dimensional airfoil, \( \alpha \) is defined by the angle between the undisturbed flow and the airfoil chord. On a rotating blade, however, the flow passing a blade section is bent because of the blade rotation; bound circulation induces local velocities; and three-dimensional effects are introduced by tip and root vortices. Hence, \( \alpha \) is difficult to define.

Methods to estimate \( \alpha \) include an azimuthal-averaged axial velocity method by Hansen et al.,\textsuperscript{24} a combined load-velocity method by Shen et al.,\textsuperscript{25} and a blade pressure method by, e.g. Van Rooij et al.\textsuperscript{26} Furthermore, \( \alpha \)-definition issues are avoided by using a measured local inflow angle instead, e.g. in a discussion by Schreck and Robinson on results of the National Renewable Energy Laboratory (NREL) Unsteady Aerodynamics Experiment.\textsuperscript{27}
The current experiment does not yield local blade loads; hence, only the azimuthal-averaged axial velocity method can be applied. The procedure to arrive at the azimuthal-averaged $V_z$ in the rotor plane is similar to the determination of the azimuthal-averaged induction factor in the rotor plane.

Furthermore, $\alpha$ is determined from the present measurements with a modified version of the blade passage method by Sant [28]. The method is based on the observation that, with the blade represented by a lifting line, the contribution from the bound vortex to the $\langle V_z \rangle$-signal at given $(r, \theta, z)$-location is antisymmetric around the point of blade passage, $\theta_{b,blade} = \theta$. The average of the local $\langle V_z \rangle$-minimum and maximum at a blade passage is not influenced by the local bound circulation. When determined at the rotor plane, the blade passage average axial velocity can be used to determine $\alpha$.

A linear relation between the blade passage $\langle V_z \rangle$-average and $z$ is fitted in the Least Squares sense to the recordings at the three downstream measurement planes for given $r$ and $\theta$. Extrapolation of the linear function yields the value for the blade passage $\langle V_z \rangle$-average in the rotor plane that is used subsequently to determine $\alpha$. The average and standard deviation of $\alpha$ are subsequently determined for the sample of 24 $\theta$-points per given $r$.

The difference in mean $\alpha(r)$ between the two methods is smaller than the corresponding standard deviations (see Figure 17). For the model rotor, with its relatively high solidity, the difference between the azimuthal-averaged axial induction and the local axial induction, both in the rotor plane, is therefore insignificant. For a rotor having lower solidity, differences in the prediction of $\alpha$ between the two methods are expected.

$\alpha_{max}$, found at the most inboard measurement position, is 8.5°. Measurements on the NACA0012 airfoil at $Re = 0.15 \times 10^6$ yield a static stall angle $\alpha_{stall} = 10.6°$, suggesting that stall is not present on the blades. Detailed blade loads that could confirm this suggestion are not measured however.

### Wake Convection

The viscous wake is associated with a distinct increase in the standard deviation of the effective velocity $s_{V_{eff}}$. For a given hot-film location $(r, \theta, z)$, the wake passage angle $\theta_{b, wake}$ is detected using the $s_{V_{eff}}$-increase. The wake age $\Delta \theta$ is determined from

$$\Delta \theta_{b, wake} = \theta_{b, wake} - \theta_{b, blade}$$

\( (17) \)

![Figure 17. Angle of attack $\alpha$ versus $r/R_t$. $\alpha$ is determined with both the azimuthal average and the blade passage method](image)
with $\theta_{b,\text{blade}}$ the blade passage angle. For a given hot-film location $(r, \theta, z)$, $\theta_{b,\text{blade}}$ equals $\theta$ or $(\theta + 180^\circ)$, when considering the passage of the first and the second blade, respectively. The viscous wake velocity is found by averaging the measured $\langle V_z \rangle$, $\langle V_\theta \rangle$ and $\langle V_r \rangle$ over the viscous wake passage.

Wake age in the mid-span region increases when going outboard (see Figure 18). Correspondingly, the wake axial velocity decreases when going outboard (see Figure 19). Both observations agree with the higher axial induction factor seen when going outboard in Figure 8. Both the viscous wake radial and tangential velocity are positive in the mid-span region. The viscous wake thus moves outboard and rotates in the same direction as the blades, indicating it is dragged along in the rotor rotation direction.

The reduced viscous wake age and increased wake axial velocity for $r/R_\text{t} = 1.0$, compared with $r/R_\text{t} = 0.9$, shown in Figures 18 and 19, indicate a roll-up of the viscous wake into the tip vortex, since the tip vortex is convected downstream faster than the wake in the mid-span region. Without roll-up of the viscous wake into the tip vortex, the wake axial velocity would be reduced instead of increased when approaching $r/R_\text{t} = 1.0$, because of the dominant contribution of the tip vortex to the axial induction velocity near $r/R_\text{t} = 1.0$. The largest standard deviations in both the wake age and wake velocity are found for $r/R_\text{t} = 1.0$, because of the dominant influence the tip vortex has on the local flowfield, combined with the tip vortex geometry that is not perfectly axisymmetric.

**Conclusions**

A wind tunnel measurement campaign focused on a wind turbine model rotor in both axial and yawed flow conditions is conducted. Measurements of the tip vortex geometry and the velocity in the near-wake of a two-bladed model rotor are performed in an open jet wind tunnel. Additionally, axial force measurements on the rotor are taken. The near-wake velocity measurements of the baseline case, the rotor in axial flow, are discussed here. Three downstream planes are traversed by a single-sensor hot-film probe.

A new data reduction method based on the asymmetrical response of the hot-film probe to the flow vector enables the derivation of the PLA three-dimensional flow velocity vector. The traditional data reduction method
for single-sensor hot-films only yields the magnitudes of the components of the PLA flow velocity vector, not its directions.

The traditional and the new data reduction technique are applied to the same data set. When comparing the PLA radial and tangential velocity distributions, the results from the new approach are less noisy and the direction of the velocity components is part of the result. Hence, the new data reduction method is preferred to the traditional method.

The data quality is further assessed by estimating the random measurement uncertainty, by studying wind tunnel wall effects and by checking repeatability with published measurements. These assessments are needed to properly compare experimental data with models.

The experimental determination of the unsteady velocity field assists in a better understanding the near-wake aerodynamics behind the model HAWT. Both the phase-locked average and standard deviation of the velocity yield information; the standard deviation is an effective measure of the wake passage.

The observed trends in the PLA velocity in the near-wake can be explained by considering a typical inviscid vortex wake model combined with viscous wake effects. The wake deformation and roll-up into the tip vortex can be observed from the derived age and convection velocity of the viscous wake. The flow velocity measurements are also used to determine the azimuthal-averaged axial induction in the rotor plane and angle of attack on the blades. The azimuthal average and the local blade passage approach arrive at an equivalent angle of attack distribution across the blade span.

In conclusion, the analysis of the detailed unsteady flow velocity measurements enables an understanding of the near-wake aerodynamics of the model wind turbine rotor subject to the baseline axial flow condition. In a next paper, the influence of yaw misalignment on the near-wake aerodynamics is discussed.
References