Spare parts management at the Royal Netherlands Navy: Vari-metric and beyond
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Abstract

This paper explores the applicability of sophisticated models and techniques for spare parts inventory management within a highly technology driven environment, viz. the Royal Netherlands Navy. In particular, we discuss the structure of the VARI-METRIC models as a tool that aims at a high availability rate of complete systems, and compare it with more classical inventory management approaches, aiming at a high availability of individual items. Unfortunately, the VARI-METRIC models suffer from a series of limiting assumptions that are not fulfilled within the Navy. We identify these shortcomings and suggest a number of extensions, to provide a firm basis for both the initial supply and the spare parts management during the exploitation period of systems.

Keywords: Inventory management, multi-echelon systems, indenture levels
1. INTRODUCTION

Similar to many industrial and governmental organizations, the Royal Netherlands Navy (RNN) is confronted with seriously declining budgets. At the same time, the RNN is expected to fulfill its tasks at the same level as earlier. As a result, many projects have been started, aiming at an increase of the overall efficiency and quality, a better responsiveness and at the same time a leaner organization. One project in particular is devoted to streamlining the materials management organization, including an investigation of the use of more sophisticated inventory management techniques. The goal is to guarantee a sufficiently high availability of ships and technical systems while reducing inventory investment. Roughly speaking, the operational availability of e.g. a warship is defined as the percentage of time that a ship is able to carry out its tasks. Although the fleet of the RNN consists of many ship types (frigates, submarines, counter-mine vessels), in the sequel we restrict ourselves to the operational availability of frigates, or even the main technical systems on board of these frigates.

A frigate is not available when a key system or subsystem is failing or when the ship is undergoing a maintenance operation. Basically, we distinguish between several sorts of preventive maintenance (e.g. regular preventive maintenance, a major overhaul) and corrective maintenance. Scheduled preventive maintenance periods are often tuned with the operations or mission schedule. A sudden failure of a system during a operation, however, requires corrective maintenance which may cause serious problems during a mission. For that reason, the Navy follows a 'repair by replacement' policy wherever possible, meaning that a failed part or subsystem is immediately replaced by an identical, ready-for-use part or subsystem. Obviously, this requires the presence of spare parts of all kinds. That is what we focus on in this paper. In particular, we aim at a high logistic availability, defined as the percentage of time that a ship or group of ships is not hampered in its operation due to a lack of spare parts. Whenever we use the word availability in this paper, it will be the logistic availability we refer to.

The availability strongly depends on the budgets for spare parts, both the available budget for initial supply and the available resupply budgets during the exploitation period. The initial supply takes place when a new class of ships is introduced. The suppliers of technical systems and installations together constitute a so-called 'Recommended Spare Parts List'. The problem is that the amount of money needed to procure all recommended parts usually exceeds the available budget considerably. It is common practice that the eventual selection of which parts to purchase, and in what quantities, takes place on a rather intuitive basis, with little idea on the ultimate impact of these decisions on the availability. Here, the availability denotes the long-term average availability of the frigates that is obtained in the exploitation period if during the exploitation period all spare parts inventories are kept at the levels which are (implicitly) set by the initial supply. So, what is needed is a tool to support the procurement decisions in the initial supply phase.

The exploitation period starts when a class of ships becomes operational. Theoretically, when spare parts could always be repaired after a failure (such parts are said to have zero condemnation rates) no further budget would be needed for resupply during the exploitation period. Unfortunately, life is not so easy. In practice, many parts are consumables that are fully used and, if anything left, disposed after use, while furthermore the condemnations rates of many repairable items are often substantial, leading to disposal again (and hence replacement by new, externally procured, items). Every year, the RNN receives a limited resupply budget to keep the corresponding stocks of spare parts at a sufficiently high level. Again, we wish to maximize the availability of technical systems and ships, given this limited budget.
The problems the RNN is facing with respect to spare parts management (of both consumable and repairable items) are and have been observed in many organizations. For instance, both the NASA and the US Airforce have been investigating the use of more sophisticated methods, that can be characterized as system approaches as opposed to the conventional item approach. The latter approach uses the conventional inventory control rules (see e.g. Silver et al. [1998]), which generally seek to attain a target customer service level while minimizing inventory holding costs. Such techniques, in a somewhat adapted manner, are currently applied at the RNN. The disadvantage of the item approach is that, albeit high individual service levels of the items are obtained, the overall system availability of a complex system, consisting of many items, may still be relatively low, while the total inventory investments may be considerable. More important, there is no clear relationship between the overall system availability and the total amount of money spent. The aim of a system approach, on the other hand, is exactly to establish a close link between availability and inventory investment, so that questions can be answered like: what is the maximum availability that can be obtained given a certain budget constraint, or, alternatively, what is the minimal budget needed to meet a given target system availability?

An important contribution, which can be seen as one of the first system approaches, is the Multi-Echelon Technique for Recoverable Item Control (METRIC), which originally has been developed by Sherbrooke [1968] for the US Airforce. The purpose of this method is to make optimal procurement decisions in the period of initial supply. During the last three decades several important improvements of METRIC have been developed. Muckstadt [1973] was the first to recognize the importance of the product structure with respect to recoverable item control. He extended the existing METRIC model, which may be characterized as a two-echelon, single-indenture model, to a two-echelon, two-indenture model, which is also referred to as MOD-METRIC. Another variant of METRIC is VARI-METRIC, a two-echelon, single-indenture model developed by Slay [1984]. In the core part of the analysis of the initial METRIC model, it is assumed for each product that the number of items in repair follows a Poisson distribution (for which the variance equals the mean). In his VARI-METRIC method, Slay derives an approximate expression for the variances. Next, for each product, he fits a negative binomial distribution on the first two moments in order to obtain a more accurate approximation for the number of items in repair. Graves [1985] independently developed a slightly simpler approximation for the variance of the number of items in repair. Next, he also continued with fitting a negative binomial distribution on the first two moments. Sherbrooke [1986] generalized the original VARI-METRIC method and developed a two-indenture, two-echelon version of VARI-METRIC. By simulation, it has been shown that the results produced by this method are very accurate.

Another important line of research was initiated by Gross [1982]. The main difference between the VARI-METRIC model and the models of Gross and others (see e.g. Gross et al. [1983] and Albright [1989]) is in the consideration of the repair process. In VARI-METRIC, it is assumed that all repair leadtimes are independent variables, which corresponds with an infinite repair capacity. In the models by Gross and others, a limited repair capacity is assumed, however at the cost of other rather restrictive assumptions like a dedicated repair capacity and fixed repair routings. In these models, the circulation of products through repair, distribution and use is usually modelled as a closed queueing network, and hence, in the spare parts literature, these models are also known as the 'closed queueing network models'.

Due to the complex nature of the repair processes at the RNN, the models developed by Gross and others, are less appropriate to be applied at the RNN. It seems to be far more practical to arrange that repair jobs must be completed within fixed, possibly part-type dependent, leadtimes by the repair shops. This allows the application of uncapacitated models such as VARI-METRIC. However, the VARI-METRIC models suffer from a number of limiting assumptions (such as e.g. a zero condemnation rate and the negligence of the presence of consumable parts within larger assemblies).
that do not allow a straightforward application of VARI-METRIC. In particular, spare parts management during the exploitation period should be further investigated.

The aim of this paper is two-fold. First, we discuss the problem of setting spare parts levels during the initial supply phase under limited budget constraints and discuss in more depth the VARI-METRIC approach. Next, we identify in more detail important shortcomings of the model and suggest a number of extensions to make it suitable for use at the RNN, in particular also during the exploitation period. Therefore, this paper is explorative in nature and partly dictates an agenda for further research.

The organization of this paper is as follows. In Section 2, we formulate the initial supply problem of the RNN in terms of the VARI-METRIC model. Here, a number of simplifying assumptions have to be made. Next, in Section 3, we sketch the analysis of the VARI-METRIC model as described in Sherbrooke [1986, 1992], and we present a case for which the applicability of VARI-METRIC and the difference with an item approach is shown. After that, in Section 4, we list a number of extensions that have to be made in order to cover the real situation at the RNN. Finally, the conclusions are given in Section 5.

2. DESCRIPTION OF THE VARI-METRIC MODEL

In this section we model the initial supply problem of the RNN conform the VARI-METRIC model of Sherbrooke [1986, 1992]. In Subsection 2.1, we first give a global description of this problem. Next, in Subsection 2.2, the repair and distribution process is described in detail. Finally, in the Subsections 2.3 and 2.4, we give an overview of the assumptions and notations.

2.1 Global description of the problem

Frigates of the RNN in principle carry out missions all over the world. When a system fails during a mission, the ship's crew has some capacity to repair that system. Usually a defective part is immediately replaced by a spare part, and afterwards the defective part is repaired, preferably at the frigate itself. When the defective spare part can not be repaired at the frigate, it is sent to the Naval Maintenance Company (NMC). At the same time a request for a new one is placed at the NMC. Hence, we are dealing with a two-echelon supply system, with the NMC at a central level and the various frigates at the decentralized level. The two-echelon repair and distribution structure is depicted in Figure 1.

![Figure 1: Repair and distribution structure at the RNN](image)

A second important feature concerns the material breakdown (or product structure) of the frigates. For the moment, we assume that frigates consist of technical systems that, in turn, consists of two
indenture levels: *assemblies* on the first level and *subassemblies* on the second level. This product structure is depicted in Figure 2.

![Material breakdown of a frigate](image)

**Figure 2: Material breakdown of a frigate**

The assemblies are subject to failures. If an assembly fails, then the corresponding technical system is down, leading to a reduced operational availability of the frigate. For the analysis, it appears to be appropriate to consider the frigate as a collection of assemblies, and we say that a frigate is not available if and only if at least one of its assemblies is down.

All assemblies and subassemblies can be put on stock at the NMC as well as at each frigate, and each of the stocks is controlled by an \((S-1, S)\)-policy, or equivalently, by a basestock policy (see e.g. Silver et al. [1998]). Obviously, for each assembly and subassembly, the number of items that must be procured during the initial supply is equal to the sum of the basestock (or order-up-to) levels that are used at the NMC and at the frigates for this product. The problem is to determine the basestock levels that maximize the average availability of the frigates for a given budget.

**Remark 1.** Since each frigate serves as its own repair center, the model as described in this section is slightly simpler than the model of Sherbrooke [1986, 1992]. In his model, Sherbrooke describes the repair and distribution process at the US Airforce, where each of the repair centers at the decentralized level (also called bases) may support multiple airplanes (instead of a single frigate). The simplification in our model leads to also a slight simplification in the analysis; see Section 3.

### 2.2 Repair and distribution process

Before we actually start to describe the repair and distribution process in more detail, we introduce some notation for the repair and distribution structure and the system structure. The repair and distribution structure consists of \(N+1\) (stock)points, where the NMC is denoted by \(n = 0\) and the frigates are denoted by \(n = 1, \ldots, N\). Each frigate may consist of \(I\) different assemblies, which are numbered from 1, \ldots, \(I\). Each assembly \(i\) consists of \(J(i)\) subassemblies (SA's), which are numbered from 1, \ldots, \(J(i)\). For each of these subassemblies, it is assumed that one or more items of it occur in the configuration of assembly \(i\). The \(j\)-th subassembly of assembly \(i\) is denoted as \(\text{SA}_i j\), or as product \(i_j\). Assembly \(i\) itself is also denoted as product \(i 0\). The number of occurrences of assembly \(i\) at frigate \(n\) is denoted by the nonnegative, integer-valued variable \(Z_{i n}\). If assembly \(i\) really occurs in the configuration of frigate \(n\), then \(Z_{i n}\) has a positive value, and else \(Z_{i n}\) is equal to zero.

Failures of assemblies \(i\) at frigate \(n\) occur according to a Poisson process with rate \(m_{i n}\). When on hand the defective assembly is replaced with a ready-for-use one and action is taken to increase the stock for ready-for-use assemblies \(i\) again. If possible the ship's crew repairs the assembly itself. This happens with probability \(r_{i n}\). The corresponding repair leadtime is assumed to be deterministic and is denoted by \(T_{i n}\). The number of items in repair is also referred to as the repair pipeline of the frigate.
When the defective assembly is not ship repairable, it is sent to the NMC for repair; the corresponding repair leadtime is assumed to be deterministic and is denoted by $T_{p_o}$. At the same time, a ready-for-use one is requested at the NMC. The requests for assembly $i$ arrive in a central queue where they are handled according to a FCFS policy.

When a request from frigate $n$ for an assembly $i$ arrives at the NMC, and finds a (still not allocated) ready-for-use assembly $i$ available, the latter one is shipped to the frigate. The order and ship time for assembly $i$ from the NMC to frigate $n$ is assumed to be deterministic and is denoted by $O_{i,n}$. Analogously to the repair pipeline, for each frigate $n$, the number of unfilled requests at the NMC plus the number of parts that is being shipped to the frigate, is referred to as the resupply pipeline of frigate $n$. We assume that a frigate can only be resupplied from the NMC (so, no lateral supply takes place).

Until now we have only paid attention to the assemblies and we have ignored the fact that failures of assemblies are due to failures of subassemblies. Therefore, we now focus on the second inden­ture level.

Let us return to the repair process at the frigates again. It is assumed that a failure of an assembly is due to the failure of precisely one subassembly. When a defective assembly $i$ at frigate $n$ appears to be ship repairable, there is a probability $q_{i,n}$ that SA $i_j$ is the cause of the defect. The defective SA is disassembled (in a negligible amount of time) and is immediately sent into repair. At the same time, a ready-for-use item of SA $i_j$ is requested from the spare parts inventory at frigate $n$. If there is a ready-for-use SA $i_j$ available, then the repair time $T_{i,j,n}$ of assembly $i$ immediately starts, otherwise it starts as soon as the required SA $i_j$ is available.

With probability $r_{i,n}$, the defective item of SA $i_j$ can be repaired at frigate $n$ itself. The corresponding repair time may be random and its mean is denoted by $T_{i,n}$. With probability $1-r_{i,n}$ the defective item of SA $i_j$ must be repaired at the NMC. At the NMC, we also have spare parts. So a ready-for-use item of SA $i_j$ is immediately requested and the defective one is sent into repair. The repair time for a defective item of SA $i_j$ may be random and its mean is denoted by $T_{i,j,n}$. The requests for ready-for-use items of SA $i_j$ are handled according to a FCFS policy. Once the request from frigate $n$ is fulfilled, which may be the case immediately or after some delay, it takes a deterministic order and ship time $O_{i,n}$ for the ready-for-use item of SA $i_j$ to arrive at frigate $n$.

Let us now return to the repair of assemblies at the NMC. There is a probability $q_{i,n}$ that a defective item of SA $i_j$ is the cause of the defect on an assembly $i$, in case this assembly is only repairable at the NMC. Also here, the defective SA is disassembled (in a negligible amount of time) and is immediately sent into repair. At the same time, a ready-for-use item of SA $i_j$ is requested at the NMC. If there is a ready-for-use SA $i_j$ available in the spare parts inventory of the NMC, then the repair time $T_{i,j,n}$ of assembly $i$ immediately starts, otherwise it starts as soon as the required SA $i_j$ is available.

Notice that, at the NMC, we have two sources of SA demand: the requests for SA's from the frig­ates and the requests stemming from assembly repairs at the NMC.

An overview of the repair process is given in Figure 3.

We receive a once-only budget $C$ for the initial supply, i.e. for the initial procurement of the spare parts (later on, no procurement is needed, since it is assumed that all parts can be repaired). The objective of this model is to determine basestock levels $s_{i,j}$ for the products $i,j$ such that the average availability across the frigates, denoted by $A_i$, is maximized given the limited budget $C$. 

2.3 Assumptions

In this subsection, we summarize the main assumptions that have been made, apart from the assumptions with respect to the material breakdown of the frigates. They are as follows:

1. At each of the frigates, the failures for the different assemblies occur according to independent Poisson processes;
2. All defective items can be repaired;
3. The repair leadtimes for the subassemblies, both at the frigates and at the NMC, are independently and identically distributed random variables;
4. The repair leadtimes for the assemblies, both at the frigates and at the NMC, and the order and ship times are deterministic;
5. A one-for-one replenishment strategy is applied for all products and at all stockpoints;
6. There is no lateral supply.

2.4 Overview of notation

In this subsection, we give an overview of the notation that is used in our model. We distinguish the following input, decision and output variables.

**Input variables**

\[ N = \text{Number of frigates}; \]
\[ I = \text{Number of (different) assemblies}; \]
\[ Z_{i,n} = \text{Number of occurrences of assembly } i \ (i=1,...,I) \text{ in the configuration of frigate } n \ (n=1,...,N); \]
\[ J(i) = \text{Number of (different) subassemblies that occur in assembly } i \ (i=1,...,I); \]
\( m_{in} \) = Total failure rate (in failures per year) for the items of assembly \( i \) (\( i=1,...,I \)) at frigate \( n \) (\( n=1,...,N \));

\( T_{dn} \) = Deterministic repair leadtime (in years) for assembly \( i \) (\( i=1,...,I \)) at (stock)point \( n \) (\( n=0,...,N \));

\( T_{pn} \) = Mean repair leadtime (in years) for SA \( i_{j} \) (\( i=1,...,I \) and \( j=1,...,J(i) \)) at (stock)point \( n \) (\( n=0,...,N \)) (this repair time may be a random variable);

\( r_{pn} \) = Probability that a defective item of product \( i_{j} \) (\( i=1,...,I \) and \( j=0,...,J(i) \)) can be repaired at frigate \( n \) (\( n=1,...,N \));

\( q_{pn} \) = Probability that SA \( i_{j} \) (\( i=1,...,I \) and \( j=1,...,J(i) \)) is the cause of a defective assembly \( i \) (\( i=1,...,I \)) when it is repaired at (stock)point \( n \) (\( n=0,...,N \));

\( O_{pn} \) = Deterministic order and ship time (in years) for product \( i_{j} \) (\( i=1,...,I \) and \( j=0,...,J(i) \)) from the NMC to a frigate \( n \) (\( n=1,...,N \));

\( c_{ij} \) = Price (in NLG) of product \( i_{j} \) (\( i=1,...,I \) and \( j=0,...,J(i) \));

\( C \) = Available budget (in NLG) for initial supply.

**Decision variables**

\( S_{pn} \) = Basestock level for product \( i_{j} \) (\( i=1,...,I \) and \( j=0,...,J(i) \)) at stockpoint \( n \) (\( n=0,...,N \)).

**Output variable**

\( A \) = average availability of the frigates.

### 3. ANALYSIS OF THE VARI-METRIC MODEL

In this section we briefly discuss the analysis of the VARI-METRIC model as performed by Sherbrooke [1986, 1992]. The analysis consists of two parts: (i) the determination of the pipeline distributions for each of the assemblies at each of the frigates; (ii) the determination of the average availability. The first part is outlined in Subsection 3.1. In the second part, we deviate from Sherbrooke's analysis. In particular, we are able to derive an exact expression for the availability as a function of the pipeline distributions and the corresponding backorder probabilities. This part is described in detail in Subsection 3.2. Next, in Subsection 3.3, we describe the optimization algorithm with which the base stock levels are determined that maximize the average availability for a given budget. Finally, in Subsection 3.4, we present a case study in which VARI-METRIC is applied on a simplified system at the RNN. Moreover, the difference between VARI-METRIC and an item approach is shown.

#### 3.1 Determination of the pipeline distributions

We restrict ourselves to an outline of the model analysis, since the primary aim of this paper is to explore the applicability of VARI-METRIC at the RNN, and not to repeat earlier work in detail. For a complete description of the analysis, see Sherbrooke [1986, 1992].

A basic observation is that the availability of each of the frigates can be expressed completely as a function of the basestock levels and the frigates’ pipeline inventories of the assemblies. These pipeline inventories do not depend on the base stock levels. Hence, first the distributions for the pipeline inventories are determined. Actually, for each frigate, first the first two moments of the numbers of assemblies in the frigate’s pipeline are determined, and next appropriate distributions are
fitted on these moments. Once these distributions have been determined, one can easily compute the numbers of backorders for assemblies at the frigates and hence also the overall availability.

For each assembly, a frigate’s pipeline inventory for an assembly consists of the following two components:

- the number of items in the frigate’s repair pipeline, i.e. the number of items in repair at the frigate plus the number of items which are in repair at the frigate but for which the repair is being delayed by a lack of a SA;
- the number of items in the frigate’s resupply pipeline, i.e. the number of items in transport from the NMC to the frigate plus the number of unfulfilled requests for ready-for-use items as placed by the frigate at the NMC.

The first two moments of the complete pipeline inventory are easily determined, once the first two moments of both components are known. These moments are determined as follows. First, the first two moments of the numbers of subassemblies that are in repair at the NMC are computed. Next, distributions on these moments are fitted, and one can determine the distributions and hence also the first two moments of the numbers of unfulfilled requests for subassemblies at the NMC. After that, one can obtain the first two moments of the numbers of subassemblies in the frigate’s pipeline as well as the first two moments of the number of assemblies in the pipeline of the NMC. Finally, again appropriate distributions are fitted on the moments for these quantities, and one can determine the first two moments for both the first and second component, as listed above. The sequence in which the various pipelines are determined, is depicted in Figure 4.

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**Remark 2.** Sherbrooke fits negative binomial distributions on the first two moments of the various pipeline inventories. Here, he implicitly assumes that the negative binomial distribution can be fitted on any combination of the mean ($\mu$) and the variance to mean ratio ($\nu$) if $\nu \geq 1$. (Also Slay [1984] and Graves [1985], who independently studied the case in which the variance of the number in the pipeline overstates the mean, assumed that a negative binomial can always be fitted on the above mentioned combinations of $\mu$ and $\nu$.) However, as shown by Adan et. al. [1995], the negative binomial distribution can only be fitted on the first two moments if $1 \leq \nu < 1 + \mu$. If $\nu \geq 1 + \mu$, then a mixture of two geometric distributions may be fitted on the first two moments. The latter is what we do and constitutes a slight difference with Sherbrooke’s approach.

**3.2 Computation of availability**

From the pipeline distributions we can determine the average availability $A$. We will first compute the availability per frigate. Here, we use a different approach than Sherbrooke [1986, 1992]. Because of the slight simplification in our model, compared to Sherbrooke’s model (see Remark 1 in at the end of Subsection 2.1), we can derive exact expressions for the availability of a frigate and the overall availability, whereas Sherbrooke is only able to derive approximate expressions.
Before we compare Sherbrooke’s method and our method, we need some additional notation. The expected number of backorders for assembly $i$ at frigate $n$ appears to depend only on the decision variables $S_{j,n}, S_{j,n}^*, S_{j,n}^*, S_{j,n}^*$. Let $\mathbf{S}_n$ be the $(2j+2)$-dimensional vector $(S_{j,n}, S_{j,n}^*, S_{j,n}^*, S_{j,n}^*)$. The expected number of backorders for assembly $i$ at frigate $n$ is denoted by $EBO_{\mathbf{S}_n}$. Applying Sherbrooke’s approximation for the availability to our model yields the following formula for the availability of frigate $n$:

$$A_n = \prod_{i=1}^{I} \left( 1 - \frac{EBO_{\mathbf{S}_n}(S_{i,n})}{Z_{\mathbf{S}_n}} \right)^{Z_{\mathbf{S}_n}}$$

(1)

This formula is explained as follows. Frigate $n$ consists of $Z_{\mathbf{S}_n}$ items of assembly $i$. For each of the corresponding positions of this assembly in frigate $n$, the fraction of time in which there is a ‘hole’ at this position equals $EBO_{\mathbf{S}_n}(S_{i,n})/Z_{\mathbf{S}_n}$. In order to be available, a frigate may not have any hole. Hence, the fraction of time that the system is not down because of a hole on a position for an assembly $i$ is found to be equal to $(1 - EBO_{\mathbf{S}_n}(S_{i,n})/Z_{\mathbf{S}_n})^{Z_{\mathbf{S}_n}}$ at least if it is assumed that the periods of time in which there is a hole at one particular position for assembly $i$ are independent of the periods of time in which there are holes at the other positions for assembly $i$. Finally, the frigate’s availability is obtained by multiplying the individual assembly availabilities.

Formula (1) is an approximation, since the assumption that is mentioned is not satisfied in general. Contrary to Sherbrooke’s model (with possibly multiple airplanes per base; see Remark 1 at the end of Subsection 2.1), for our model, this assumption can be easily avoided. In our case, a positive number of backorders for assembly $i$ immediately implies that the frigate is not available. Therefore, for our model, we obtain the following exact formula for the availability of frigate $n$:

$$A_n = \prod_{i=1}^{I} (1 - PBO_{\mathbf{S}_n}(S_{i,n})),$$

(2)

where $PBO_{\mathbf{S}_n}(S_{i,n})$ is defined as the probability that there is a positive amount of backorders for assembly $i$ at frigate $n$.

The availability of the fleet is obtained by taking the average across the frigates:

$$A = \frac{1}{N} \sum_{n=1}^{N} A_n$$

(3)

### 3.3 Optimization algorithm

Sherbrooke [1986,1992] shows that maximizing availability is equivalent to minimizing the sum of the assembly backorders at the bases. Analogously we can show that maximizing availability is equivalent to minimizing the sum of the probabilities of positive amounts of backorders. This property enables us to formulate the following Non-Linear Integer Problem:

$$\text{Min} \sum_{i=1}^{I} \sum_{n=1}^{N} PBO_{\mathbf{S}_n}(S_{i,n})$$

subject to:

$$\sum_{i=1}^{I} \sum_{j=0}^{J} \sum_{n=0}^{N} C_{i,j} S_{i,n} \leq C$$

$$S_{i,n} \in \{0,1,2,\ldots\} \text{ for all } i = 1,\ldots,I \text{ and } j = 0,\ldots,J \text{ and } n = 0,\ldots,N$$

We apply a greedy heuristic to solve problem P1. We have to distinguish between two types of stock allocations: (i) stocking an item at the NMC; (ii) stocking an item at one of the frigates. In the first
case the stock allocation affects the backorder probabilities at all the frigates. Therefore, for each combination of product $ij$ ($i=1,\ldots,I; j=0,\ldots,J$) and location 0 (the NMC) the delta value $\Delta_{00}$ is defined as the decrease in the sum of $PBO_{0n}(\Sigma_{0})$ across all the frigates, divided by the cost of product $ij$, when we increase the base stock level $S_{jn}$ with 1.

In the second case, the stock allocation only affects the backorder probability at the frigate where the stock allocation takes place. So now, for each combination of product $ij$ ($i=1,\ldots,I; j=0,\ldots,J$) and location $n$ ($n=1,\ldots,N$), the delta value $\Delta_{in}$ is defined as the decrease in $PBO_{in}(\Sigma_{in})$, divided by the cost of product $ij$, when we increase the base stock level $S_{jn}$ with 1.

The combination of a product $ij$ and a location $n$ with the largest delta value is selected for stocking an additional item. We continue this procedure until a budget $C$ has been invested. Finally, for the resulting basestock levels $S_{jn}$, the corresponding availability $A$ is determined (cf. (2) and (3)). The formal procedure is described in Algorithm 1. (In this procedure, $\epsilon_{jn}$ is a $(2J+2)$-dimensional unit vector with a one on the $(j+1)$-th position if $n=0$ and on the $(j(j+1)+2)$-th position if $n \in [1,\ldots,N]$.)

Algorithm 1.

**Step 1:**
Set $S_{jn} = 0$ for all $i=1,\ldots,I; j=0,\ldots,J$ and $n=0,\ldots,N$.

$C' := 0$

**Step 2:**
Calculate:

$$\Delta_{00} = \left\{ \sum_{n=1}^{N} PBO_{0n}(\Sigma_{0n}) - \sum_{n=1}^{N} PBO_{0n}(\Sigma_{0n} + \epsilon_{00}) \right\} / c_{ij}$$

for all $i=1,\ldots,I; j=0,\ldots,J$ and $n=0$.

$$\Delta_{in} = \left\{ PBO_{in}(\Sigma_{in}) - PBO_{in}(\Sigma_{in} + \epsilon_{jn}) \right\} / c_{ij}$$

for all $i=1,\ldots,I; j=0,\ldots,J$ and $n=1,\ldots,N$.

$k, l, m = \arg\max \{ \Delta_{in} | i = 1,\ldots,I; j = 0,\ldots,J; n = 0,\ldots,N \}$

$S_{km} = S_{km} + 1$

$C' = C' + c_{kl}$

**Step 3:**
If $C' \geq C$ then stop else goto step 2

**Step 4:**
Calculate $A$

It appears that this algorithm provides near optimal solutions when the budget $C$ and the resulting availability $A$ are sufficiently high.

### 3.4 Case study

In our example we consider a part of a real fire extinguishing system of which the material breakdown is depicted in Figure 5. These systems are installed in the Multi-Purpose (M-) frigates of the RNN. Note that the system consists of two assemblies, a pump and an electromotor. So, $I=2$. The pump consists of three SA-s, and thus $J(1)=3$. The electromotor consists of two SA-s, and thus $J(2)=2$. 

-11-
We consider a fleet consisting of five M-frigates \((N=5)\), which are assumed to be identical.

We obtained the input data from the configuration management system and the logistic management system of the RNN. The input data are given in Table 1.

<table>
<thead>
<tr>
<th>Item</th>
<th>(m_{kn}) ((n\geq 1))</th>
<th>(T_{pm}^{*}) ((n\geq 1))</th>
<th>(T_{ girlfriends}^{*}) ((n\geq 1))</th>
<th>(r_{pm}^{*}) ((n\geq 1))</th>
<th>(q_{pm}^{*}) ((n\geq 1))</th>
<th>(q_{st}^{*}) ((n\geq 1))</th>
<th>(O_{pm}^{*}) ((n\geq 1))</th>
<th>(c_{j})</th>
</tr>
</thead>
<tbody>
<tr>
<td>pump (Ass.1)</td>
<td>17.5</td>
<td>0.02</td>
<td>0.4</td>
<td>0.10</td>
<td>-</td>
<td>-</td>
<td>0.02</td>
<td>19583</td>
</tr>
<tr>
<td>bearing (SA 1,1)</td>
<td>-</td>
<td>-</td>
<td>0.1</td>
<td>0</td>
<td>0.30</td>
<td>0.36</td>
<td>0.02</td>
<td>330</td>
</tr>
<tr>
<td>seal (SA 1,2)</td>
<td>-</td>
<td>-</td>
<td>0.1</td>
<td>0</td>
<td>0.25</td>
<td>0.28</td>
<td>0.02</td>
<td>452</td>
</tr>
<tr>
<td>casing (SA 1,3)</td>
<td>-</td>
<td>-</td>
<td>0.1</td>
<td>0</td>
<td>0.45</td>
<td>0.36</td>
<td>0.02</td>
<td>439</td>
</tr>
<tr>
<td>electromotor (Ass.2)</td>
<td>13.7</td>
<td>0.02</td>
<td>0.4</td>
<td>0.15</td>
<td>-</td>
<td>-</td>
<td>0.02</td>
<td>50821</td>
</tr>
<tr>
<td>rotor (SA 2,1)</td>
<td>-</td>
<td>-</td>
<td>0.1</td>
<td>0</td>
<td>0.33</td>
<td>0.28</td>
<td>0.02</td>
<td>459</td>
</tr>
<tr>
<td>stator (SA 2,1)</td>
<td>-</td>
<td>-</td>
<td>0.1</td>
<td>0</td>
<td>0.67</td>
<td>0.72</td>
<td>0.02</td>
<td>152</td>
</tr>
</tbody>
</table>

Table 1: Input data (the \(m_{kn}\) are given in failures per year, the \(T_{pm}^{*}\), \(T_{ girlfriends}^{*}\) and \(O_{pm}^{*}\) in years and the \(c_{j}\) in NLG)

Note that because of our assumption of identical frigates, the input variables for the various frigates have the same values. Further, since in this case the subassemblies never can be repaired at the frigates, i.e. all \(r_{pm} = 0\), the corresponding repair times at the frigates \((T_{pm})\) are not relevant and hence they do not have to be specified.

We have run Algorithm 1 for various budget constraints. The results are shown in Figure 6. The pattern of the curve is intuitively quite clear. First there is a large increase in availability per additional invested NLG. Later on the curve flattens because it takes more budget to accomplish the same increase in availability.
We have also compared the VARI-METRIC approach, which is a system approach, with an item approach. In the latter approach we set fixed service levels for each product at the NMC as well as at the frigates. On the basis of these service levels we determine the base stock levels $S_{ij}$ and the corresponding availability $A$. We also generated an Investment - Availability curve for this item approach and compared it with the curve obtained by the system approach. The results have been depicted in Figure 7.

From Figure 7, we observe that the system approach outperforms the item approach until an availability of approximately 92%. Especially for investments that correspond with moderate availabilities, the differences are significant. The question is: What is the specific reason that the system ap-
proach is so much better than the item approach for this region of the investments? To answer this question, we study the basestock levels obtained by both approaches at an availability of approximately 75%. These results are given in Table 2.

<table>
<thead>
<tr>
<th>Item</th>
<th>System approach (Availability = 77%)</th>
<th>Item approach (Availability = 74%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Investment = 590,000 NLG)</td>
<td>(Investment = 710,000 NLG)</td>
</tr>
<tr>
<td></td>
<td>Base stock levels at NMC</td>
<td>Fill rates at NMC</td>
</tr>
<tr>
<td>pump (Ass. 1)</td>
<td>3</td>
<td>0.41</td>
</tr>
<tr>
<td>bearing (SA 1,1)</td>
<td>5</td>
<td>0.79</td>
</tr>
<tr>
<td>seal (SA 1,2)</td>
<td>4</td>
<td>0.77</td>
</tr>
<tr>
<td>casing (SA 1,3)</td>
<td>6</td>
<td>0.76</td>
</tr>
<tr>
<td>electromotor (Ass. 2)</td>
<td>3</td>
<td>0.34</td>
</tr>
<tr>
<td>rotor (SA 2,1)</td>
<td>4</td>
<td>0.78</td>
</tr>
<tr>
<td>stator (SA 2,2)</td>
<td>7</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Table 2: Base stock levels and fill rates obtained by the system and the item approach

From Table 2, we observe that in the system approach less electromotors (the most expensive products) but more pumps and second indentured items (the cheaper products) are procured, compared to the item approach. More generally, it holds that in the system approach low service levels of expensive products are compensated by high service levels of cheaper products. This enables us to achieve the same availability with less budget. In the item approach the service levels are fixed, so we cannot make this kind of compensation. We conjecture that the benefits of the system approach are primarily a consequence of the multi-indenture structure combined with the fact that the greedy heuristic explicitly aims at a maximum return (decrease of PBO-s) per NLG invested. We see the expected benefits of the multi-echelon structure in both approaches: relative high fill rates at the frigates and lower fill rated at the NMC.

Remark 3. The resulting fill rates in Table 2 for the item approach were achieved by setting a target fill rate of 20%. It seems surprising that, despite this low target, the resulting availability equals 75%. The explanation for this phenomenon is the following. The algorithm increases the stock of a certain product until the target fill rate is reached. In order to achieve a fill rate of 20%, at least one item has to be stocked. However, when demand is low, the fill rate can be rather high when one item is stocked, as we see in Table 2. Due to the high fill rates at the bases, the availability can reach the value of 75%.

4. NECESSARY EXTENSIONS

In the previous sections, we found that the (adapted) VARI-METRIC model in principle can be used to model the repair and distribution process at the RNN. However, in order to cover a number of important characteristics, several substantial extensions are needed. These extensions are listed below. As we shall see, the first six extensions are related to additional problems in the phase of initial supply. The other two extensions concern the presence of condemnation and consumable
parts within the assemblies, and models for the resupply during the exploitation period. In particular, the latter two extensions will require a substantial research effort.

**More than two indenture levels**

There exist many systems on the frigates that contain four or even more indenture levels. Especially Sensor-, Weapon- and Command Systems tend to have a more complicated product structure. Clark [1981] was the first to incorporate more than two indenture levels. To that end, he generalized the MOD-METRIC model to incorporate a random number of indenture levels. Our aim is to generalize the VARI-METRIC model to an arbitrary number of indenture levels.

**Criticality**

As shown in Figure 2, a frigate consists of many technical systems. Naturally, one system may be more critical to the operational availability than another one. For instance, the disposal press system is less important than the fire extinguishing system.

In the theoretical case that the inventory budget would be sufficient to achieve 99.9% percent availability across all the systems, criticality differences are not necessary. So the need for different criticality levels becomes apparent as the budgets are relatively low. As mentioned in the introduction, this is the case at the RNN.

A framework for determining the criticality levels for systems used in the RNN, is described by Van der Eijnden [1994].

**Redundancy**

To prevent losing a ship’s function when a system falls down, redundancy is incorporated in the design of the system. Naturally, only the systems that are very critical to the operational availability, are designed redundantly.

Redundancy is often described as follows. Suppose there are $L$ systems, designed to carry out the same function. When $K$ of these $L$ systems have to be operational in order to carry out the function properly, the redundancy is said to be $L$ over $K$.

There are several ways of looking at redundancy. Kaplan [1989] and Sherbrooke [1992] develop expressions for the fraction of time that $K$ out of $L$ systems are available as a function of the stock levels. The underlying analysis is rather complicated and the practical value is questionable. Our aim is to develop a method that is more straightforward in terms of inventory control.

**Commonality**

It regularly occurs that identical products are used in different systems. This phenomenon, which is referred to as commonality, mostly occurs for lower indentured parts. However, sometimes even complete pumps are common in different installations (to mention an example). The RNN strongly encourages the use of commonality because of the reduction in spare part investment and the reduction in the need of codification.

An analytical problem underlying models with common parts is that the arborescent system structure is affected. This type of structure usually is essential for the analysis. Clark [1981] was the first to come up with the commonality problem, but he does not provide an adequate solution. Sherbrooke [1992] does give a solution to this problem. But, whether his approach towards commonality is acceptable from a computational point of view remains to be studied.

**Generalization of Poisson failure processes**

For the analysis, the assumption that failures occur according to Poisson processes appears to be very convenient. One property of a Poisson failure process is that the number of failures in a period with a given length is Poisson distributed and hence has a variance to mean ratio $V$ equal to one. Analyses within the RNN show that this variance to mean ratio $V$ for the number of failures per
period often exceeds the value of 1 substantially. Values of $V$ between 5-10 are quite common. Values of $V$ lower than one almost never occur. Sherbrooke [1992] states that these high values of $V$ are a result of failure rates that vary with time.

In Sherbrooke [1992], some attention is paid to a generalization of the Poisson distribution for the number of failures per period (in fact, this is what is needed for the analysis). He shows how the analysis can be adjusted to binomial distributions ($V < 1$) or negative binomial distributions ($V > 1$). There are two missing links in his analysis.

1. The analysis is adjusted for other distributions but the property of memorylessness for the corresponding failure processes is still assumed.

2. Sherbrooke conjectures that the negative binomial distribution is the most accurate distribution for cases in which $V > 1$. For justification of this conjecture he uses simulation results and practical experience within the US Airforce. As stated in Remark 2 at the end of Subsection 3.1, the negative binomial distribution can only be fitted on certain combinations of the first two moments.

**Non-continuous resupply**

The VARI-METRIC theory assumes continuous resupply. This means that the frigates (=bases) can be resupplied at any point in time. For bases of an Airforce, such as the US Airforce, this assumption is quite legitimate since the geographical position of the bases is fixed.

A typical characteristic of a Navy frigate is that it carries out missions all over the world. It is not always possible to instantaneously resupply during such a mission. So the replenishments do not take place on a continuous basis but rather periodically.

Naturally, when the mission periods become longer, the theory gets more violated. Since the periods in which the frigates can not be resupplied are often substantial (3 months), we want to adjust the theory to periodic resupply. Unfortunately, we have not found any literature addressing this issue within a maintenance framework.

**Condemnation**

Sherbrooke [1992] reports that the VARI-METRIC theory will still be accurate if the condemnation rate is 5% or less. A recent study among platform articles within the RNN showed that the average condemnation rate appeared to be nearly 30% (see Pronk [1996]). This means that the probability that an arbitrary repairable item can be repaired after a failure, is only 70%. In addition, consumable parts have to be replaced by new ones anyhow. Apart from an early study of single-indenture models by Simon [1971], we are not aware of any literature considering this important aspect.

**Restricted budgets in the exploitation period**

The theory developed up to now is applicable on the initial supply phase only. The main characteristic is that we receive a once-only budget to procure spare parts. The theory provides us of a procurement and allocation strategy with which we can maximize the availability given the budget constraints.

If there would be no condemnations of repairables and if we would not use any consumable items at all, we would not need any budget for the procurement of new spare parts during the exploitation period. However the RNN does deal with condemnation (nearly 30%) and uses consumable items extensively.

Every year the NMC receives a budget to procure new spare parts (repairables and consumables). As in the initial supply phase, the inventory strategy must be focused on maximizing the availability.
of the frigates, given the available budget. Actually the inventory manager is interested in the following questions:

- What is the available budget for the procurement of new spare parts?
- What is the workload for the procurement department?
- Which frigate availabilities can be achieved during the budget year and what are the corresponding base stock levels?
- How do we maintain these base stock levels?

To our knowledge, also this problem has not been studied before in the literature.

5. CONCLUSIONS

We have established that the VARI-METRIC model, as described by Sherbrooke [1986,1992], provides a useful basis for modeling the materials management problems arising in the repair and distribution process of the RNN. Moreover, we showed that the VARI-METRIC model has explicit advantages compared to a traditional item approach. It appears that by using VARI-METRIC low service levels of expensive products are compensated by high service levels of cheaper products. This enables us to achieve a close-to-optimal availability, under prescribed budget constraints. In the item approach the service levels are fixed per item, thereby not allowing the possibility to treat distinguished items differently.

Despite the fact that the principles underlying VARI-METRIC are useful, the approach suffers from important shortcomings that prevent its direct application to the repair and distribution process at the RNN. In Section 4, we have listed a number of additional needs related to the initial supply phase, as well as specific problems related to condemnation, the presence of consumable parts within assemblies (and, hence, hybrid products) and budget constraints during the exploitation period. This list constitutes an agenda for further research. It is noted that the latter problems of this list could be also extremely useful in providing an overall life cycle assessment of complex technical systems.

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REFERENCES


