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**Citation for published version (APA):**

Wattamwar, S. K., Weiland, S., & Backx, A. C. P. M. (2010). State information based identification methods towards low order modeling. In *Proceedings of the 2010 IEEE International Conference on Control Applications (CCA), 8-10 September 2010, Yokohama, Japan* (pp. 665-670). Institute of Electrical and Electronics Engineers. <https://doi.org/10.1109/CCA.2010.5611246>

**DOI:**

[10.1109/CCA.2010.5611246](https://doi.org/10.1109/CCA.2010.5611246)

**Document status and date:**

Published: 01/01/2010

**Document Version:**

Publisher's PDF, also known as Version of Record (includes final page, issue and volume numbers)

**Please check the document version of this publication:**

- A submitted manuscript is the version of the article upon submission and before peer-review. There can be important differences between the submitted version and the official published version of record. People interested in the research are advised to contact the author for the final version of the publication, or visit the DOI to the publisher's website.
- The final author version and the galley proof are versions of the publication after peer review.
- The final published version features the final layout of the paper including the volume, issue and page numbers.

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# State information based identification methods towards low order modeling

Satyajit Wattamwar and Siep Weiland and Ton Backx

**Abstract**—In this paper we propose a model reduction framework for obtaining low order linear and non-linear models for large scale non-linear, reactive fluid flow systems. Our approach is based on the combination of the method of Proper Orthogonal Decomposition (POD), and System Identification techniques. The proposed methods involve two steps. In the first step POD is used to separate the spatial and temporal patterns and in the second step different model structures of linear and of non-linear types are proposed to approximate the temporal patterns and corresponding model parameters are identified. In particular, model structures of LTI, LPV and of tensorial or multi-variable polynomial type in lower dimensional subspace are identified. It is shown here that the POD modal coefficients can be viewed as the states of the reduced model that is to be identified. This has allowed us to propose different reduced model structures. The resulting lower dimensional models need significantly low computation time. The methods are of generic nature and are promising to different large scale applications characterized by existence of coherent patterns. Moreover, to accommodate the existing knowledge in the form of plant output measurements in the reduced order modeling framework, a new approach is proposed. The efficiency of proposed methods are illustrated on a large scale benchmark problem depicting an Industrial Glass Manufacturing Process. The results show good performance of the proposed methods.

## I. INTRODUCTION

Industrial processes which are characterized by more than one independent variable, viz. space and time are often referred to as Distributed Systems (DS). Numerical solution techniques of such a system involves decomposition of spatial and temporal components. Spatial discretization of DS is done by means of Finite Volume or Finite Element methods which transfer the original DS described by the Partial Differential Equations (PDEs) into a set of Ordinary Differential Equations (ODEs) which are eventually integrated to obtain the temporal dynamics. The mathematical models of DS which consider flow, energy and mass balance using conservation laws are often referred to as the First Principle Models (FPM) or Rigorous Models. The solution techniques in the form of spatial and temporal discretizations approximate the steady state and dynamic process behavior reasonably well, but the large number of ODEs formed as result of spatial discretization of DS leads to increased model order which further leads to increased computational complexity. It needs considerable computational efforts (time, CPU requirement) to simulate such FPM and therefore such process models can not be used for online plant optimization and control purposes. The alternative way to to develop the control

relevant models relies extensively on plant data obtained by performing identification tests on actual plant. But for some chemical processes, such tests are highly expensive due to loss of products during the tests. Moreover the validity of the identified models from the plant data is limited to the domain of tests performed on the plant and the states of the identified black-box models from the plant data do not have any physical interpretation. Therefore, on one hand there are reliable, but computationally slow FPM and on the other hand there are simpler models with many limitations. Model Order Reduction (MOR) to infer reduced order or simplified models from the FPM is therefore a necessary step. Once the low dimensional, simplified black-box models are obtained, one can proceed to controller and optimizer design, see e.g. [Shvartsman and Kevrekidis(1998)].

Among different model reduction techniques, the method of Proper Orthogonal Decomposition (POD) is widely used for deriving lower dimensional models from the First Principle Model. The POD method is applicable to the processes which exhibit coherent spatio-temporal patterns. The POD method searches the dominant coherent patterns in the given process and defines an optimal, data-dependent basis, that is subsequently used as a projection space to infer the reduced order models through Galerkin type of projections, see [Astrid(2004)] and the references therein. There are some drawbacks associated with the POD methods; e.g. POD methods are empirical (data dependent) in nature and therefore these methods are susceptible to changes in the process inputs and process parameters. Moreover, the reduced model obtained by POD techniques using Galerkin projections are usually very dense and one loses the original sparse model structure. Along with the necessity of evaluating the nonlinear functions in full dimensional space, the computational gain of POD models obtained by Galerkin projection are limited and not very attractive from the point of online usage. These motivate us to look for other possible approaches that can give computationally efficient, reliable POD models that can be used for the online control and optimization purpose.

Towards the purpose of identifying the low order models from the information of states evolution alone, lately we have proposed few methods. In this paper we present these methods in a *common (simplified/reduced) modeling framework*. The proposed model reduction methods result into Reduced Order-Linear Time Invariant(RO-LTI) models, Reduced Order-Linear Parameter Varying (RO-LPV) models and Reduced Order-Tensorial (ROT) models. ROT model is a nonlinear model and can also be interpreted as a multi-

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variable polynomial model. All the forms of the reduced order models presented in this paper are computationally more than 1000 times faster than the full order rigorous models. These reduced models allow to use a commercially available software package in the form of a black-box which generates the dynamic state information, and therefore the proposed methods do not need access to the governing model equations that are necessary for other POD with Galerkin projection based methods. The methods presented here are partially presented in the PhD thesis, see [Wattamwar(2010)].

Apart from proposing a combined model reduction framework, in this paper we propose a way to combine the existing process knowledge in the form of plant output measurements to the reduced modeling framework. This novel approach allows to minimize the mismatch between the actual process and the reduced model.

This paper is organized as follows. The method of POD is explained in section II. Assuming a generic form of the full order model, the POD section explains the Galerkin projection of governing model equations to infer the reduced order model. The notions and the ideas developed in POD section are further used to propose the identification based reduced modeling framework in subsequent sections. The application/motivation is a benchmark example of Industrial Glass Manufacturing Process (IGMP) and is explained in the section IV. Some results of the implementation of the proposed methods on the benchmark example are presented in the section V which is followed by conclusions and references.

## II. PROPER ORTHOGONAL DECOMPOSITION

One of the most promising and successful techniques for an efficient reduction of large-scale nonlinear systems is the method of Proper Orthogonal Decompositions (POD) also known as the Karhunen-Loève method, see [Holmes et al.(1996)Holmes, Lumley, and Berkooz]. POD is extensively applied to the systems involving fluid flow. The method is based on the observation that for many large scale processes, the flow characteristics reveal coherent structures or *patterns*. This has led to the idea that the solutions of model equations may be approximated by considering a small number of dominant coherent structures (called *modes* or *basis*) that are inferred in an *empirical* manner from the measurements or the simulated data. Given an ensemble of  $K$  measurements  $\mathbf{T}^k(\cdot)$ ,  $k = 1, \dots, K$  with each measurement defined on some spatial domain  $\Omega$ , the POD method amounts to assuming that each observation  $\mathbf{T}^k$  belongs to a Hilbert space  $\mathcal{H}$  of functions defined on  $\Omega$ . With the inner product defined on  $\mathcal{H}$ , it then makes sense to call a collection  $\{\varphi_j\}_{j=1}^{\infty}$  an *orthonormal basis* of  $\mathcal{H}$  if any element, say  $\mathbf{T} \in \mathcal{H}$ , admits a representation

$$\mathbf{T}(k, z) = \sum_{j=1}^{\infty} a_j(k) \varphi_j(z), \quad z \in \Omega, k \in \mathbb{T} \text{ with} \\ \langle \varphi_i, \varphi_j \rangle = 1, \forall i = j, \text{ else } \langle \varphi_i, \varphi_j \rangle = 0 \quad (1)$$

Here, the  $a_j(k)$ 's are referred to as the time varying *modal coefficients* (MC) and the  $\varphi_j$ 's are the *modes* or *basis* of the

expansion. The truncated expansion

$$\mathbf{T}_n(k, z) = \sum_{j=1}^n a_j(k) \varphi_j(z), \quad (2)$$

causes an approximation error  $\|\mathbf{T} - \mathbf{T}_n\|$  in the norm of the Hilbert space. We will call  $\{\varphi_j\}_{j=1}^{\infty}$  a *POD basis* of  $\mathcal{H}$  whenever it is an orthonormal basis of  $\mathcal{H}$  for which the *total approximation error* in some norm over the complete ensemble is

$$\sum_{k=1}^K \|\mathbf{T}^k - \mathbf{T}_n^k\| \quad (3)$$

is minimal for all truncation levels  $n$ . This is an *empirical basis* in the sense that every POD basis depends on the data ensemble. Using variational calculus, the solution to this optimization problem amounts to finding the normalized eigenfunctions  $\varphi_j \in \mathcal{H}$  of a positive semi-definite operator  $R: \mathcal{H} \rightarrow \mathcal{H}$  that is defined as,

$$\langle \xi_1, R \xi_2 \rangle := \frac{1}{K} \sum_{k=1}^K \langle \xi_1, \mathbf{T}^k \rangle \cdot \langle \xi_2, \mathbf{T}^k \rangle \quad (4)$$

with  $\xi_1, \xi_2 \in \mathcal{H}$ .  $R$  is well defined in this manner and corresponds to a positive semi-definite matrix whenever  $\mathcal{H}$  is finite dimensional. In that case, a POD basis is obtained from the normalized eigenvectors of  $R$ , see e.g. [Astrid(2004)]. In practise, one approximate the infinite model by a finite one using some spatial discretization scheme like Finite Element or Finite Volume, with  $n$  spatial grids. The resulting model is referred to as the full order model. And then the POD basis are translated into  $\varphi_j \in \mathcal{H}_r$ , that is,  $r$ -dimensional subspace of  $\mathcal{H}$  which is spanned by  $\{\varphi_j\}_{j=1}^r$ . The POD modal coefficients ' $a_j$ ' are then obtained by the projection of the ensemble  $\mathbf{T}_n^k(\cdot)$  on the span of dominant spatial POD basis as,

$$a_j(k) = \langle \varphi_j(z)^\top, \mathbf{T}_n(k, z) \rangle \quad (5)$$

A generic form of a full order model that results from the spatial discretization of PDEs of DS may be represented by,

$$\frac{dT_n}{dt} = \mathcal{A}(T_n) + \mathcal{B}(u) + \mathcal{F}(T_n, u, d) \quad (6)$$

where observation  $T_n(\cdot, k) = \mathbf{T}_n(k) \in \mathbb{R}^n \quad \forall k$ ,  $\mathcal{A}$  is the spatial operator for convection and diffusion, and is of linear nature,  $\mathcal{B}$  defines input matrix and  $\mathcal{F}$  is nonlinear source term. In the specific case of a POD basis, the finite dimensional subspace  $\mathcal{H}_r = \text{span}\{\varphi_j\}$ ,  $j = 1, \dots, r$  where the  $\varphi_j$ 's denote the POD basis functions.

Classical POD method involves a Galerkin projection. Towards this purpose let  $P_n: \mathcal{H}_n \rightarrow \mathcal{H}_r$  and  $I_n: \mathcal{H}_r \rightarrow \mathcal{H}_n$  denote the canonical projection and canonical injection maps or operators respectively, between full and reduced finite dimensional spaces. The injection map projects back the dynamic state evolutions from the reduced space to the full dimensional space. Often,  $P_r = [\varphi_1^\top, \dots, \varphi_r^\top]$  and  $I_r = [\varphi_1, \dots, \varphi_r]$ . This allows to represent

$$a_r(k) = \langle P_r, T_n(k, z) \rangle \quad a_r(k) \in \mathbb{R}^r \quad (7)$$

$$\text{and, } \tilde{T}_n(k, z) = I_r a_r(k) \quad (8)$$

The reduced model is then given by Galerkin projection of (6) on lower dimensional space spanned by the POD basis functions using the projection operator as,

$$P_r \frac{dT_n}{dt} = P_r \mathcal{A}(I_r a_r) + P_r \mathcal{B}(u) + P_r \mathcal{F}(I_r a_r, u, d) \quad (9)$$

In this case using (7) (9) becomes an *ordinary differential equation* in the coefficients  $a_r(k)$  as,

$$\frac{d\langle P_r, T_n \rangle}{dt} = P_r \mathcal{A} I_r a_r + \mathcal{B}_r u + P_r \mathcal{F}(I_r a_r, u, d) \quad (10)$$

or equivalently,

$$\frac{da_r}{dt} = \mathcal{A}_r a_r + \mathcal{B}_r u + \mathcal{F}_r(P_r^{-1} a_r, u, d), \quad a_r(k) \in \mathbb{R}^r \quad (11)$$

Eq. (11) is reduced order model (ROM) and the POD modal coefficients  $a_r$  appear as the *states* of the ROM. Therefore the POD MC can also be viewed as *dominant temporal patterns/dynamics* along which system evolves. Often approximation order  $r$  is decided by using a criterion based upon projection energy as,

$$P_{tol} = \frac{\sum_{k=1}^r \lambda_k}{\sum_{k=1}^n \lambda_k} \quad (12)$$

where  $\lambda_k$  is the  $k^{th}$  eigenvalue of the correlation operator as defined in (4),  $r$  is the order of ROM and  $n$  is the order of finite dimensional full order FPM.

### III. LOW ORDER MODELS

In this section we present few novel ways to estimate the reduced order models by using the computed modal coefficients  $a_r(k)$  from (7) and known inputs  $u(k)$ . That is, we will present the ways to infer reduced models without invoking the Galerkin projection that is used in classical POD technique as presented in (9) to (11).

#### A. Reduced order LTI models

The first two terms of eq. (11) on RHS are linear and the third non-linear term does not appear for the systems defined by linear ODEs. For the system governed by linear ODEs the differential equations eq. (11) can be transformed in equivalent discrete time form as:

$$a_r(k+1) = A_r a_r(k) + B_r u(k) \quad (13)$$

At this point one can observe that given the ensemble  $\mathbf{T}_n^k$ , from (4) and (5) one can obtain the POD basis and corresponding MC. As MC  $a_r(k)$  are computed and inputs  $u(k)$  are known, the system parameters  $A_d$  and  $B_d$  can be easily estimated by ordinary least square (OLS) estimation techniques. Once  $A_d$  and  $B_d$  are estimated, the LTI system can be simulated for any other input trajectory and the corresponding estimated modal coefficients can be represented as  $\tilde{a}_r$ . From (8) the states  $\tilde{T}_n(k, z)$  of the full order model can be reconstructed. It is clear from this discussion that even without the availability of governing model equations one can obtain an equivalent expression for the reduced order model.

If the actual plant data in the form of outputs measurements  $y(k)$  is available, then this knowledge can be used

to compensate the mismatch between the actual plant and reduced model solutions by expressing the plant outputs as,

$$y_{plant}(k) = C_T \tilde{T}_n(k) = C_T I_n \tilde{a}_r(k) = C_r \tilde{a}_r(k) \quad (14)$$

This output structure is linear in parameter. As  $y_{plant}(k)$  and  $\tilde{a}_r(k)$  are already known,  $C_r$  can be estimated by least square estimation techniques again.

#### B. Reduced order LPV models

Linear Parameter Varying (LPV) modeling framework offers a link between LTI and nonlinear model structures. Operation domain of LPV model is larger than the LTI model and often smaller than the nonlinear models. LPV modeling framework assumes that the dynamics of a system varies in certain polytopic region such that the corners of the polytope represent the LTI reduced models obtained in subsection III-A. The approach presented here is similar to the gain scheduling principles and is already presented separately with greater details in another paper, see [Wattamwar et al.(2010)Wattamwar, Weiland, and Backx]. In the proposed framework, the Reduced Order-LPV is presented as a middle step between low order LTI and low order nonlinear models.

Throughout, we take the variable  $h(k)$  as a scheduling parameter, and assume  $h(k) \in \Theta \subset \mathbb{R}^+$ . Further consider a finite set  $\mathbb{H} = \{h_j\}$  for  $j = 1, \dots, M$  with  $M > 0$ , discrete values. Below, subscript  $h$  and  $h_j$  shows the implicit parameter dependence while explicit dependence on the parameter is expressed with brackets as,  $(h)$  or  $(h_j)$ . As this RO-LPV method is based on matching the input-output behavior and blending of RO-LTI models, for the purpose of illustration we use the transfer operators.

For a given set of local linear time-invariant (LTI) parameterized reduced models  $G_{h_j}(q)$ ,  $j = 1, \dots, M$ , the  $j^{th}$  input-output relation can be represented as:

$$y_{h_j}(k) = G_{h_j}(q)u(k) := \sum_{l=0}^{\infty} G_{l,h_j} u(k-l) \text{ for } j = 1, \dots, M. \quad (15)$$

Here  $G_{h_j}(q) \in \mathbb{R}^{n_y \times n_u}(q)$ ,  $G_{l,h_j} \in \mathbb{R}^{n_y \times n_u}$ ,  $n_u$  and  $n_y$  are number of inputs and outputs respectively,  $M$  is the number of local RO-LTI models which on weighted blending gives RO-LPV model. In other words, if we interpret  $M$  models as  $M$  vertices of a polytope in which the original nonlinear system dynamics evolve, then the RO-LPV model describes the original nonlinear system behavior in the polytopic region defined by the convex combination of the local RO-LTI models obtained in section III-A as follows:

$$y_{lpv,h}(k) = \sum_{j=1}^M \alpha^j(h(k)) y_{h_j}(k), \text{ or equivalently} \quad (16)$$

$$y_{lpv,h}(k) = \sum_{j=1}^M \alpha^j(h(k)) G_{h_j}(q)u(k), \text{ that is,} \quad (17)$$

$$G_{lpv,h}(s) = \sum_{j=1}^M \alpha^j(h(k)) G_{h_j}(s) \quad \forall h(k) \subset \Theta. \quad (18)$$

Equation (18) also defines  $G_{lpv,h(k)}(q)$ , as long as  $h(k) \in \Theta$ . Here  $\alpha^j$  are the weights such that  $\alpha^j : \Theta \rightarrow \mathbb{R}$ , which are used

for the combination of the local LTI models which need to be determined. This RO-LPV modeling amounts to approximation of global dynamics using weighted combination of local dynamics, whose weights are varying with a time varying parameter. Here we present weights  $\alpha^j(h(k))$  in the form of nonlinear splines which are presented as

$$\alpha^j(h(k)) = \sum_{i=1}^{k_n} \theta_i^j \varphi_i^j(h(k)) \quad (19)$$

where  $\theta_i^j \in \mathbb{R}$  are the spline coefficients and  $\varphi_i^j(h(k))$  are the basis functions, and  $h(k) \in \Theta \subset \mathbb{R}^+$  is the time varying parameter. The parameter can also be a working point/operating point of the process. In-stead of spline in (19), other type of scheduling function, for instance radial basis function or membership functions from fuzzy modeling as explained in [Babuska(1998)] can be used.

We present two types of splines viz. cubic and trigonometric. They have different properties. The cubic spline is of the form,

$$\alpha^j(h) = \beta_1^j + \beta_2^j h + \sum_{i=2}^{k_n} \beta_{i+1}^j |h - b_i|^3. \quad (20)$$

$b_i \in \mathbb{R}^{k_n}$  are spline knots which are distributed in  $k_n$  different (disjoint) elements, over an interval  $[h_{min}, h_{max}]$  such that  $h_{min} \in \Theta$  and  $h_{max} \in \Theta$ , and  $h_{min} < h_{max}$ .

$\beta_i^j$  are the spline coefficients corresponding to each knot. Here we define unknown spline coefficients as a parameter vector as,

$$\theta^j = \text{col}(\beta_1^j, \dots, \beta_{k_n}^j) = \text{col}(\theta_1^j, \dots, \theta_{k_n}^j). \quad (21)$$

With  $col$  as the column operator. Note that there can be various possible spline structures other than the cubic spline as shown in equation (20). The knot distribution can be of various types as well. The simplest knot distribution is an equidistant covering the whole domain of scheduling parameters, as in (20). Another spline structure we have proposed is trigonometric of the form

$$\alpha^j(h) = \beta^j \cos(h - h_j) \prod_{i=1}^M (h - h_i) \quad (22)$$

If all the necessary information to identify a RO-LPV model viz.  $y_{plant}(k)$  (or  $y_{FPM}(k)$ ),  $y_{h_j}(k)$ ,  $h(k)$ ,  $G_h(q)$  and  $u(k)$  is available then the problem of RO-LPV identification can be transformed into a problem of estimation of spline coefficients  $\beta_i^j$ , see eq. (21). The quality of an identified RO-LPV model will then be decided by the accuracy of estimation of spline parameters,  $\theta_i^j$ . For this purpose we define the output error of the RO-LPV model as follows:

$$e_h(k) = y_{plant}(k) - y_{lpv,h}(k) = y_{plant}(k) - \sum_{j=1}^M \alpha^j(h(k)) y_{h_j}(k), \quad (23)$$

or equivalently,

$$e_h(k) = y_{plant}(k) - \sum_{j=1}^M \sum_{i=1}^{k_n} [\varphi_i^j(h(k)) y_{h_i}^j(k)] \theta_i^j. \quad (24)$$

It is desired to minimize the error in (24) by formulating an optimization problem as

$$\hat{\theta} := \arg \min_{\theta} \sum_{k=1}^K \|e_h(k)\|_2^2 \quad (25)$$

As the error model (24) is linear in the spline parameters  $\theta_i^j$ , we can attain a solution of the optimization problem (25) in least square sense as:

$$\hat{\theta} = [\Phi^T \Phi]^{-1} \Phi^T Y \quad (26)$$

where,  $Y = \text{col}(y(1), \dots, y(K))$  and

$$\Phi = \begin{bmatrix} \varphi_1^1(h)y(1) & \dots & \varphi_{i+1}^1(h)y(1) & \dots & \varphi_i^j(h)y(1) \\ \vdots & & & & \vdots \\ \varphi_1^1(h)y(K) & \dots & \varphi_{i+1}^1(h)y(K) & \dots & \varphi_i^j(h)y(K) \end{bmatrix} \quad (27)$$

$\hat{\theta}$  is the estimated value of  $\theta$ . From (27) it is clear that the splines are dependent on the process data. This suggests that it is necessary to have plant data sufficiently rich to capture the plant dynamics corresponding to the complete space in which parameters vary. This can be achieved only when the plant data contains the information of transition from one working point to another, i.e. there should be an excitation signal during the transition as well. Eq. (17) of LPV model can be written in usual state space form as

$$\begin{aligned} a_{r,lpv}(k+1) &= A_{r,lpv} a_{r,lpv}(k) + B_{r,lpv} u(k) \\ y_{lpv,h}(k) &= C_{r,lpv,h} a_{r,lpv}(k) \end{aligned} \quad (28)$$

$$A_{r,lpv} = \text{diag}(A_r^1, A_r^2, \dots, A_r^M), B_{r,lpv} = \text{col}(B_r^1, B_r^2, \dots, B_r^M)$$

$$\begin{aligned} C_{r,lpv,h} &= [\alpha^1(h)C_r^1, \alpha^2(h)C_r^2, \dots, \alpha^M(h)C_r^M] \text{ and} \\ a_{r,lpv} &= \text{col}(a_r^1, a_r^2, \dots, a_r^M) \end{aligned} \quad (29)$$

The spline weight must be included in the matrix  $C_{r,lpv,h}$  which reconstructs the states of the full order FPM from the states of the RO-LPV model. The RO-LPV model presented here is already based on minimizing the mismatch between the plant outputs and the outputs of RO-LPV model and therefore it does not need any further treatment. Some other aspects like bounds on parameter variation, its effect on nonlinear spline structure and stability of resulting RO-LPV models is still under investigation.

### C. Reduced order nonlinear models

Identification of nonlinear ROM aims at getting a substitute for the reduced model in (11) using an alternative approach other than the classical POD with Galerkin projection. Here we use an approach that is similar to the one mentioned in subsection III-A. Availability of the *states*, that is the POD modal coefficients of the reduced model is again exploited to determine the parameters of a nonlinear model.

There are many possible ways to approximate the nonlinearities like black-box, neural net, fuzzy logic, grey box, and many other input-output based fit of Wiener-Hammerstein type, see [Giannakis and Serpedin(2001)]. It is also well known that the Taylor series expansion can be a good approximation of a non-linear function. The use of Taylor Series is not very often considered in input-output type of identification methods due to lack of state information. But as explained earlier in section III-A, the states (i.e. POD modal coefficients) of ROM are accessible and therefore one can make use of Taylor series to approximate

the non-linear terms in (11). For the purpose of illustration, we briefly explain the Taylor series expansion for a scalar valued function, later for a vector valued function and finally its implementation for the computation purpose.

For a scalar valued function,

$$\dot{x} = f(x), \text{ where } f: \mathbb{R} \rightarrow \mathbb{R} \ \& \ f(x^*) = 0 \quad (30)$$

Taylor series expansion in  $x$  as a nominal variable and  $\tilde{x}$  as a deviation variable,  $\tilde{x} = x - x^*$

$$\dot{\tilde{x}} = f(x^*) + f'(x^*)\tilde{x} + (1/2!)f''(x^*)\tilde{x}^2 + \dots \quad (31)$$

where,  $f'(x) = \mathcal{J}(x) : \mathbb{R} \rightarrow \mathbb{R}$ , the Jacobian operator and  $f''(x) = H(x) : \mathbb{R} \rightarrow \mathbb{R}$ , the Hessian operator.

For a vector valued function  $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ , the first derivative is defined as a map:  $f': \mathbb{R}^n \rightarrow \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ , and when the first derivative is evaluated at  $x^* \in \mathbb{R}^n$  then  $f'(x^*) \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ , i.e.  $f'(x^*)$  is a linear operator, and when it acts on the  $n$  dimensional vector  $x$  then its image is  $\in \mathbb{R}^n$ , i.e.  $f'(x^*)(x) \in \mathbb{R}^n$ . This lets us to understand first derivative as a map,  $f': \mathbb{R}^n * \mathbb{R}^n \rightarrow \mathbb{R}^n$ . As  $f'(x^*)$  is constant term (fixed operator), we better write it as  $[f'(x^*)](x) \in \mathbb{R}^n$ . This operator is usually referred as the system Jacobian matrix,  $[f'(x^*)] := \mathcal{J}(x^*)$ . The same procedure is repeated for computing the second derivative of the function,  $f'': \mathbb{R}^n * \mathbb{R}^n * \mathbb{R}^n \rightarrow \mathbb{R}^n$ , i.e.  $f'': \mathbb{R}^n \rightarrow \mathcal{L}(\mathbb{R}^n, \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n))$ , i.e.  $f''(x^*) \in \mathcal{L}(\mathbb{R}^n, \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n))$ , i.e.  $f''(x^*)(x) \in \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n)$ , i.e.  $f''(x^*)(x)(x) \in \mathbb{R}^n$ , i.e.  $[f''(x^*)](x, x) \in \mathbb{R}^n$   $[f''(x^*)] := H(x^*)$ , Hessian operator. It is clear from the above discussions that the Hessian operator is a tensor with argument from two domains while its codomain remains the same that of the Jacobian operator. The linearity of Hessian operator allows us to compute it like the Jacobian operator, but now with one more argument as:

$$[f''(x^*)](x, x) = \begin{bmatrix} \sum_{k=1}^n \sum_{j=1}^n \frac{\partial^2 f_1(x^*)}{\partial x_k \partial x_j} x_k x_j \\ \vdots \\ \sum_{k=1}^n \sum_{j=1}^n \frac{\partial^2 f_n(x^*)}{\partial x_k \partial x_j} x_k x_j \end{bmatrix} \quad (32)$$

the above expression can be written as:

$$[f''(x^*)](x, x) = A_1(x \otimes x) \quad (33)$$

where,  $(x \otimes x)$  is the Kronecker product. The complete simplification procedure mentioned above is aimed to express,  $f'': \mathbb{R}^n \rightarrow \mathcal{L}(\mathbb{R}^n, \mathcal{L}(\mathbb{R}^n, \mathbb{R}^n))$  as,  $f'': \mathbb{R}^n \rightarrow \mathcal{L}(\mathbb{R}^{n^2}, \mathbb{R}^n)$ . This is possible due to the notion of the linearity of the tensor operator. From the discussion above, a nonlinear equation of the form  $\dot{x} = f(x, u)$  can be expanded in Taylor series as in (31) which can be approximated by a polynomial of the form,

$$\begin{aligned} \dot{x} = & Ax(t) + Bu(t) + A_1(x(t) \otimes x(t)) \\ & + B_1(u(t) \otimes u(t)) + Q(x(t) \otimes u(t)) \end{aligned} \quad (34)$$

Where,  $A_1, B_1, Q$  are Hessian operators,  $A, B$  are Jacobian operators.  $x \in \mathbb{R}^n, u \in \mathbb{R}^l, A \in \mathbb{R}^{n*n}, B \in \mathbb{R}^{n*l}, A_1 \in \mathbb{R}^{(n*n)*n}, B_1 \in \mathbb{R}^{(l*l)*n}, Q \in \mathbb{R}^{(l*n)*n} \otimes$  the Kronecker products. Equivalent discrete form of Eq. (34) can be written as,

$$\begin{aligned} x(k+1) = & A_d x(k) + B_d u(k) + A_{1d}(x(k) \otimes x(k)) \\ & + B_{1d}(u(k) \otimes u(t)) + Q_d(x(k) \otimes u(k)) \end{aligned} \quad (35)$$

For convenience, below we drop the superscript  $d$  from eq. (35). Note that the polynomial equation (35) is non-linear in states and inputs but it is linear in all the model parameters. This is a big advantage. As the states, i.e. the POD modal coefficients and inputs are known from (4) and (5), by fixing the polynomial model structure of (35) we can estimate the model parameters by Least Square parameter Estimation (LSE) techniques again. Therefore, replacing  $x(k)$  by  $\tilde{a}_r(k)$  gives us reduced order nonlinear model. The reduced model structure in (35) has form similar to bilinear systems, see [Wingerden and Verhagen(2009)]. Towards this, define

$$\begin{aligned} \xi_k := & \text{col}(x(k), u(k), (x(k) \otimes x(k)), (x(k) \otimes u(k)), \\ & (u(k) \otimes u(k))) \end{aligned} \quad (36)$$

then from (35),  $x_{k+1} \simeq \Theta \xi_k$  where,  $\Theta = [ABA_1B_1Q]$ . Define the parameter estimation error at each time instance as

$$e_{k+1} = x_{k+1} - \Theta \xi_k \quad (37)$$

or the error over the complete simulation horizon  $K$  is

$$E := [x_1 \dots x_K] - \Theta [\xi_0 \dots \xi_{K-1}] \quad (38)$$

equivalently,  $E := X - \Theta \Xi$ , where,  $K$  is the number of samples and  $X \in \mathbb{R}^{n*(K-1)}, \Xi \in \mathbb{R}^{(n+l+n*n+l*l+n*l)*(K-1)}$  and  $\Theta \in \mathbb{R}^{n*(n+l+n*n+l*l+n*l)}$  The least square solution is given by

$$\Theta = X \Xi^T (\Xi \Xi^T)^{-1} \quad (39)$$

Once the reduced order tensorial (polynomial) model is obtained, it can be integrated for any other input trajectory and corresponding modal coefficients  $\tilde{a}_r$  can be computed. The complete spatio-temporal information  $\tilde{T}_n$  of FPM can be reconstructed by injecting back the solution of reduced model (35) i.e.  $\tilde{a}_r$  in place of  $a_r$  in (8). The error involved here will be the sum of projection error and the statistical fit in the identification step to the few selected POD modal coefficients corresponding to the maximum energy content as per eq. (12). Similar to section III-A, to compensate the mismatch between the actual plant outputs and the outputs of the reduced model, we propose a polynomial structure again as,

$$y_{plant}(k) = C_1 \tilde{a}_r(k) + C_2 (\tilde{a}_r(k) \otimes \tilde{a}_r(k)) \quad (40)$$

$y_{plant}$  and  $\tilde{a}$  are known,  $C_1$  and  $C_2$  are linear in parameter. Therefore they can be estimated using least square estimation techniques.

#### IV. MOTIVATION: GLASS MANUFACTURING

This section describes the motivating example of Industrial Glass Manufacturing Process, *IGMP*. IGMP is usually carried out in large furnaces which are very well designed in order to have a desired laminar flow pattern of the glass. The flow pattern of glass determines the residence time of the glass in the melting furnace which in turn determines the quality of the glass produced. The process is an example of very large scale integrated systems. Most of the process variables like temperature, velocity, pressure, viscosity are interacting with each other. Due to this interacting nature the control of the furnace has to be done carefully. Usually pull rate (production rate), heat input and pressure valve positions are some of the control variables. Variables of

interest are temperature distribution and velocity profiles in the furnace. The product quality is largely determined by these two variables. The temperature maintained inside the furnace varies between 1400–1650 °C. Based on the process operation there are roughly three regimes - glass melting, fining to remove high concentration of dissolved gases from the molten glass and refining to remove all remaining undissolved gases from the glass. The IGMP shows large variation in the time constants, from minutes to days. Some details about mathematical modeling of glass can be found in [Huisman(2005)], [Patankar(1980)], [Post(1988)]. Due to very high process temperature and due to the viscous nature of glass, the glass furnace is a hostile environment for sensor systems. Therefore sensors are largely limited to temperature measurements in the bottom refractory of the melting furnace. 3 dimensional glass furnace model easily consist of  $10^4 - 10^6$  equations, which are not convenient to test novel methods easily, therefore we use an approximate 2D glass furnace which mimics a vertical cross section along the length of 3D glass furnace.

## V. RESULTS AND DISCUSSION

Here we present the application of the proposed methods to glass manufacturing process. We have considered only temperature as the variable of interest. The order of the first principle model is 3000. From the method explained in the section III we have obtained a fourth order linear and non-linear polynomial model. The input considered for the identification purpose is pull-rate(feed) in terms of tons/day, which varies 5% around the nominal value in the form of +/- steps superimposed by PRBS signal. The simulation horizon is 120 hours and sampling time is 16 mins. Figure 1 shows the identification result for both linear and polynomial models as proposed in this paper. Plot shows the result for four outputs which are temperature at the bottom of the four main zones of the glass. Plots show that the both models approximate the dynamic behavior very well, but in comparison to the reduced order polynomial model, the linear model fails to capture the PRBS signal dynamics with precision. For a similar experiment we validate the performance of the RO-LPV model in Figure 2. Plots shows that the RO-LPV models also preforms reasonably well. All the reduced models are > 1000 times faster than the original full scale model.

## VI. CONCLUSIONS

In this paper we have proposed a few data-driven model reduction methods using a common model reduction framework with their application to a large scale industrial problem. The proposed methods are promising and suited especially for very large scale processes where complexity reduction merely by physical insight is not possible. The methods presented here allow to infer low order LTI, LPV and nonlinear tensorial models in the absence of governing model equations. In future, we want to investigate observer and controller synthesis for the RO-polynomial model.

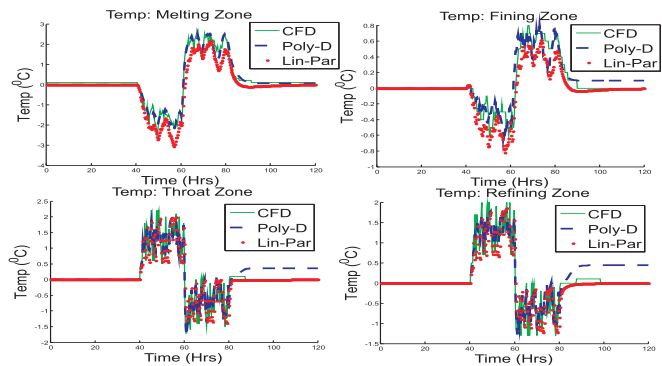


Fig. 1: Performance of RO-LTI and RO-Poly models

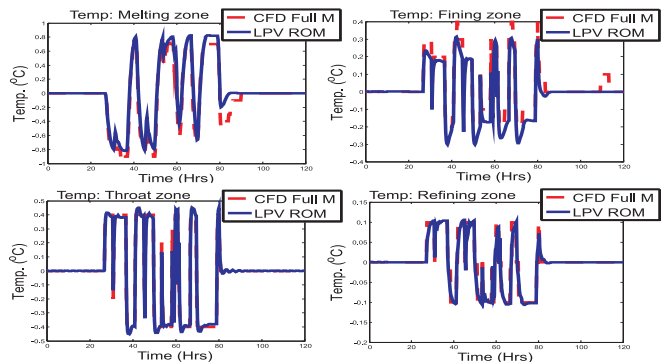


Fig. 2: Performance of RO-LPV model

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