Turbulent transport in tokamak advanced scenarios

PROEFSCHRIFT

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1 Introduction

1.1 Motivation

The quest for harnessing fusion power - the energy source of the stars - is one of paramount importance. Achieving terrestrial controlled fusion has the promise to solve one of the world’s more pressing concerns: the provision of environmentally friendly, sustainable sources of energy [1]. In comparison to wind and solar energy, fusion offers the added benefits of high baseline loads and a lack of geographical constraints for reactor construction.

The fusion reaction is a nuclear process whereby light nuclei collide and form a heavier nucleus, releasing some of their mass as energy. This excess energy is manifested in the kinetic energy of the fusion products. In a reactor this energy can be recovered to drive electricity producing steam turbines. The temperatures required for the nuclei to overcome their electric repulsion and fuse at a high enough rate is enormous, on the order of $10^8$ K ($\sim 10$ keV). At such temperatures matter is in the plasma state.

However, confining plasmas at sufficiently high temperatures and densities for commercial fusion has proven to be a difficult task. The necessary high pressure gradients in fusion plasmas tend to lead to instabilities. Turbulent transport arises, leading to leakage of heat and particles at a rate higher than expected from collisional transport. Understanding and predicting the turbulence increases our confidence in extrapolating operational scenarios to future reactors, greatly aiding their design. The research carried out in this thesis is firmly with this goal in mind.

1.2 The tokamak

A leading approach for achieving controlled fusion is ‘magnetic confinement fusion’ (MCF). The most successful present-day application of MCF is the tokamak, where the plasma is contained in a toroidal magnetic chamber by a helical magnetic field [2]. This is illustrated in Fig. 1.1. The toroidal magnetic field is generated by external field coils. Toroidal current is then necessary for the formation of the poloidal component of the helical magnetic field. In standard operation this is induced by transformer action, where the plasma loop acts as the secondary circuit. This leads to an inherent limitation in the tokamak discharge pulse time since the primary coil current is limited. This is not ideal for a reactor due to the importance in maximising availability and minimising cyclic thermal loads on the plasma facing components. For more detailed background information on tokamak physics, the reader is referred to Chap.2.
1 Introduction

Figure 1.1: Principles of tokamak operation. An externally produced toroidal magnetic field is combined with a poloidal magnetic field generated by toroidal plasma current. Additional field coils are used for plasma shaping and positioning. (Source: EFDA-JET)

1.3 Research Questions

The aforementioned pulse time limitation issue brings us directly to the underlying question motivating this thesis:

How can a tokamak operational scenario be set-up which allows for significantly longer pulses than standard operation, compatible with both reactor fusion power and availability requirements?

By ‘operational scenario’ we mean the choices and timings of the various parameters and actuators for a tokamak discharge such as: transformer current, magnetic fields, plasma shaping currents, fueling, heating schemes, and external current drive schemes.

Fully steady-state scenarios have been developed on existing tokamaks, where 100% of the toroidal current is generated by non-inductive means. This is primarily done by increasing the plasma current internally produced by pressure gradients, by suppressing plasma turbulence in narrow regions of the plasma and creating regions of high pressure gradients called ‘internal transport barriers’ (ITBs). These ITBs are formed in addition to the edge transport barrier (pedestal) formed during standard high-power, high-confinement operation. However, the formation and maintenance of a stable ITB de-
1.3 Research Questions

mands the simultaneous control of plasma current and pressure profiles. This may not be possible in reactors, which have decreased scope for diagnostics due to the increased neutron and radiation flux compared with present-day experiments [3].

Additional scenarios have been developed on present-day tokamaks which allow for significantly increased pulse times while not being purely steady state. These are named ‘hybrid scenarios’. Many of the challenges associated with scenario control are alleviated by not demanding ITB formation. The discharge pulse time is increased by both reducing the total plasma current compared to standard high-confinement operational scenarios, and by increasing the fraction of externally driven current (compared to standard operation) by neutral beam injection (NBI) and coupling with radio-frequency (RF) waves.

Tokamak confinement empirical scaling laws [4] predict reduced confinement at lower current. Hybrid scenarios are thus expected to display reduced confinement compared to standard high-confinement operational scenarios. However, hybrid scenarios are characterised by better confinement than expected from the empirical scaling laws, resulting in confinement times comparable with standard high-confinement operation. This holds great promise for the extrapolation of this scenario to future devices, and the hybrid scenario may potentially form the basis of a viable long-pulse reactor scenario. There are numerous possibilities which can explain the improved confinement. Hybrid scenarios are characterised by broad radial profiles of toroidal current. Such current profiles reduce the extent of large-scale instabilities which can induce confinement decreasing magnetic field structures in the plasma. Broad current profiles may also reduce the impact of small-scale plasma instabilities which drive the outward turbulent transport of heat and particles, limiting plasma confinement. The relative height of the plasma edge transport barrier (pedestal) associated with high input power tokamak operation is often increased in hybrid scenarios compared to fully inductive scenarios. Finally, present-day hybrid scenarios are typically set-up with significant NBI, leading to significant plasma rotational flow shear. This is predicted to reduce plasma turbulence, and this effect is not taken into account in the empirical scaling laws. In future devices the rotation resulting from NBI will be significantly lower than in present-day devices, and the question arises whether high-performance hybrid scenarios can then be sustained.

To confidently extrapolate the hybrid scenario to future machines, the physics of each of the above factors must be fully understood and incorporated into the integrated modelling codes which simulate tokamak discharges. This thesis tackles one aspect of this task, and concentrates on the question:

What is the effect of the broad current profile which characterises hybrid scenarios on small-scale plasma instabilities which drive the turbulent transport?

The research has been split up into a number of sub-questions. Since the extrapolation of the hybrid scenario to reactors is the underlying goal, we first apply the question above to hybrid scenario extrapolation to ITER - the next-step tokamak currently under construction. ITER is designed for significant fusion power production, and the hybrid scenario target is a fusion gain of $Q > 5$, where $Q = \frac{P_{\text{fus}}}{P_{\text{input}}}$. Obtaining high performance of
the hybrid scenario in ITER may verify if the scenario is indeed suitable for long-pulse reactor operation. The more conservative goal of the hybrid scenario in ITER is for long-pulse operation, maximising the discharge neutron fluence for reactor relevant component testing [5]. Simulations of the ITER hybrid scenario have been carried out previously with integrated modelling codes, but no systematic optimisation of the scenario has been performed [6]. Since we are concerned with the current profile effect on the small-scale instabilities, we pose our first sub-question as follows:

**What is the optimum application of external current drive in the ITER hybrid scenario for obtaining the current profile shape that minimises turbulent transport?**

The tool used to examine this question is the integrated modelling code CRONOS [7], which simulates tokamak discharges, coupled to the quasi-linear transport model GLF23 [8]. An overview of these tools is given in Chap. 2. The primary small-scale instability for this class of discharges is driven unstable by the ion-temperature-gradient (ITG). This instability is characterised by critical threshold behaviour, where above a critical ion temperature gradient the instability ignites. The value of the critical threshold is predicted to be parameterised in part by the current profile. This behaviour is captured by the GLF23 model. In Chap. 3, ITER hybrid scenario simulations are performed with varying current drive inputs from actuators such as neutral beam injection (NBI) and electron cyclotron current drive (ECCD). The impact of the current drive choices on the fusion power is assessed, based on the ITG instability predictions inherent in the GLF23 model. This optimisation is carried out under a number of constraints (introduced below). A secondary question is thus asked in this work, also addressed in Chap. 3.

**What are the limits for defining an ITER hybrid scenario operational window given the scenario constraints?**

By ‘operational window’ we mean the range of plasma parameters such as density and current within which a scenario can be set-up that satisfies its constraints. The constraints are as follows. Firstly, specific current profile shapes which can trigger large-scale magnetohydrodynamic (MHD) instabilities must be avoided. Secondly, a minimum fusion power is required for the scenario. The current profile constraint is easier to satisfy at low plasma current. The fusion power constraint is easier to satisfy at high plasma density. However, the plasma current and density are linked. In tokamak operation the plasma average density is typically constrained to remain below an empirically derived limit (Greenwald density limit [9]) to ensure stability. This limit is proportional to the total plasma current. Thus, even with optimum current profile shaping for the reduction of turbulent transport, it may transpire that the necessary density for sufficient fusion power is incompatible with the avoidance of large-scale MHD instabilities due to the value of the total current associated with the necessary average density. An important factor in determining this is the temperature at the top of the edge transport barrier (pedestal). The core plasma temperatures are closely linked to the pedestal temperatures. The current profile optimisation task is thus carried out for a number of different pedestal
assumptions. If the pedestal height is too low, satisfying the fusion power constraint (and thus the current profile constraint) is hampered. This work thus also predicts what the necessary pedestal height is such that an operational window for the ITER hybrid scenario can be defined.

Given the results in Chap.3, a natural question to ask is then:

**Is the predicted impact of the current profile on the turbulent transport validated by present-day experimental discharges?**

This question is addressed in Chap.4. A pair of hybrid scenario discharges from both the JET and ASDEX-Upgrade tokamaks were chosen for analysis. The discharges within each pair is characterised by similar pedestal heights but differences in both total confinement and current profile. The degree to which the differences in confinement can be attributed to the current profile is studied. CRONOS and GLF23 are used for predictive simulations of the discharges, as in the ITER case in Chap.3. These predictions are then compared to the actual experimental data. The effect of the current profile can be isolated in the simulations since all the various parameters and profiles in the simulations can be interchanged independently.

The analyses in Chap.3-4 are focused on the critical threshold variation of the ITG instability with the current profile shape. The degree that the temperature gradients can depart from the critical threshold at a given heat flux is called ‘stiffness’. GLF23 is a ‘stiff’ transport model, meaning that the predicted temperature gradients tend to be close to the critical gradient thresholds. However, in certain cases, the stiffness is reduced. This is seen in investigations with full non-linear turbulence simulations, where for flat current profiles (also known as low magnetic shear) typical in hybrid scenarios, the stiffness is reduced compared to the expectation of stiff transport models. This has been clearly observed in a comparison between a relatively new quasi-linear transport model - QuaLiKiz [10] - and full non-linear simulations using the GENE code [11]. QuaLiKiz predicted heat fluxes were significantly larger than the non-linear predictions at low magnetic shear, while for other parameter scans the agreement was satisfactory. The QuaLiKiz model is based on more realistic assumptions than GLF23, increasing the general confidence in its predictions and motivates its incorporation as a model routinely used for transport predictions. It was thus decided within the framework of this thesis to set out to improve the QuaLiKiz model in low magnetic shear parameter space, facilitating the future use of the model in hybrid scenario extrapolation. We then ask two questions:

**What is the non-linear physics at low magnetic shear that leads to reduced stiffness in ITG turbulence?**

**How can the QuaLiKiz model be improved such that increased agreement with non-linear ITG turbulence predictions is obtained at low magnetic shear?**

These questions are answered in Chap.5. The non-linear turbulence code GENE is used as the main investigation tool. The QuaLiKiz assumptions are systematically guided and validated by GENE calculations, which are based on a more complete physical model.
1 Introduction

Experimental observation of reduced ion temperature profile stiffness at low magnetic shear has recently been reported at JET [12]. It is hypothesised that this is not purely due to the low magnetic shear, but rather due to the concomitant low magnetic shear and high rotational shear present in the specific discharges. The reduced stiffness has not been reproduced by transport models. In this thesis we set out on a more systematic investigation of this effect with non-linear modelling, and ask the question:

Can the observed reduced stiffness observed in JET discharges at low magnetic shear and high rotational flow shear be understood through non-linear simulations?

This is the subject of Chap.6, using the GENE non-linear code. This question is again deeply related to hybrid scenario extrapolation. If high rotational flow shear at low magnetic shear is indeed an important factor for improved hybrid scenario confinement, then this may pose concern for the extrapolation of the scenario to future machines where the flow shear is expected to be significantly reduced compared to current machines.

We can summarise the research questions as follows. Chap.3-4 deals with the prediction of the impact of hybrid-scenario relevant current profiles on ion-temperature-gradient instability critical thresholds. Chap.5-6 deals with the impact of the low magnetic shear inherent to hybrid-scenario relevant current profiles on the transport stiffness. We will evaluate the progress made in answering the research questions in Chap.7, where an outlook towards follow-up work and future potential research directions uncovered by the research within this thesis is also presented.

1.4 Publications

Chap.3-6 have all been published or submitted to peer reviewed journals. These are listed below, together with additional contributions related to this thesis.

Refereed journal papers (as main author)


1.4 Publications

Refereed journal papers (as co-author)


Conference proceedings (as main author)


Conference proceedings (as co-author)


1 Introduction


Conference/workshop oral contributions

1.4 Publications


Seminars

- Comparison between ion temperature profile stiffness observations and nonlinear gyrokinetic simulations. Plasma Theory Group Seminars, Oxford University, Oxford, UK. July 2012

- Quasilinear transport modeling at low magnetic shear. CCFE Fusion Theory Meeting, Culham, UK. April 2012


- Analyzing the role of the $q$-profile in JET hybrid improved confinement with current-overshoot. CEA Cadarache, France. September 2010

- Optimization of ITER Hybrid Scenario performance by integrated modelling. CEA Cadarache, France. March 2010
1 Introduction

References

2 Theoretical Background

2.1 Fusion basics

The most accessible nuclear fusion reaction combines the hydrogen isotopes deuterium and tritium into helium and a neutron:

\[
\frac{2}{1}D + \frac{3}{1}T \rightarrow \frac{4}{2}He (3.5\,MeV) + \frac{1}{0}n (14.1\,MeV)
\]  

(2.1)

Deuterium reserves in seawater are practically inexhaustible. Tritium must be produced within fusion reactors themselves, through the activation of lithium by the fusion neutrons in a blanket surrounding the fusion plasma, in conjunction with neutron multipliers such as beryllium [1]. Currently extractable world reserves of lithium are sufficient for centuries of tritium production for fusion, and the development of efficient lithium extraction from seawater could extend the reserves to be effectively limitless [2]. Radioactive waste issues are highly mitigated in fusion compared to nuclear fission. The tritium fuel itself is radioactive but has a short half-life of 12.3 years. The reactor wall components can be engineered such that neutron activation of wall materials leads only to short half-life byproducts [3]. Therefore no transport of nuclear waste for long-term storage would be necessary. Furthermore, no dangerous runaway nuclear reactions are possible in fusion reactors, as opposed to fission reactors.

The necessary conditions for maintaining a self-sustaining fusion burn is quantified by the “triple product” Lawson criterion [4], which for the DT reaction at fusion relevant temperatures can be approximated as a constant:

\[
nT\tau_E \geq 5 \cdot 10^{21}\,\text{keV}\,\text{s/m}^3
\]  

(2.2)

\(n\) is the plasma density, \(T\) the temperature (in keV) and \(\tau_E\) is the confinement time. This is defined in stationary-state as \(\tau_E = \frac{W}{P_{\text{loss}}},\) the ratio between the total stored thermal energy \(W = \frac{1}{2} \int PdV\) (where \(P\) is the plasma pressure), and the total power loss rate \(P_{\text{loss}}\) from the volume boundaries. In stars, the fusion plasma is confined by gravity. For terrestrial fusion plasmas, gravity is clearly insufficient. Two mainstream approaches exist for satisfying the Lawson criterion. ‘Inertial confinement fusion’ (ICF) purports to achieve pulsed fusion burn of dense plasmas at the cost of shortened confinement time. The leading ICF approach is the interaction of a high energy laser (MJ scale) with mm scale DT fuel pellets [5]. The second approach, which forms the physics framework in this thesis, is ‘magnetic confinement fusion’ (MCF). This consists of relatively long timescale (order of s) confinement of relatively diffuse (10^{19} – 10^{20} m^{-3}) plasmas. The plasma is confined by magnetic fields which provides insulation from the surrounding walls.
2 Theoretical Background

2.2 Basic tokamak physics

Plasmas are ‘quasineutral’; at length scales above the Debye length \( \lambda_D \equiv \sqrt{\frac{\epsilon_0 e^2}{n_e m_e}} \) and at frequencies below the plasma frequency \( \omega_{pe} \equiv \sqrt{\frac{n_e e^2}{m_e \epsilon_0}} \), the plasma effectively screens charge separation and the relation \( \sum_s n_s q_s = 0 \) holds. \( s \) is the species label (i.e. electrons, ions). All units are in MKS.

The charged particles in the plasma are susceptible to the Lorentz force, \( \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \). When placed in a magnetic field, the particles are free to stream parallel to the magnetic field, while perpendicularly they rotate with a gyroradius \( \rho_s = \frac{m_s v_{\perp s}}{q_s B} \) and gyrofrequency (also called cyclotron frequency) \( \omega_{cs} = \frac{q_s B}{m_s} \). This constraint of perpendicular motion is the primary confinement mechanism of MCF. A tokamak is such a magnetic confinement device, in which the plasma is contained in an axisymmetric toroidal magnetic chamber by strong helical magnetic fields with magnitudes on the order of several Tesla, as previously illustrated in Fig. 1.1.

The toroidal plasma current responsible for the poloidal magnetic field can be formed by: a toroidal electric field induced by transformer action, in which the plasma loops acts as the secondary circuit; so-called ‘bootstrap current’, which is self-generated by a combination of radial pressure gradients and collisions between particles in the plasma [6]; radio-frequency (RF) waves coupled to the plasma; beams of neutral particles injected into the plasma which are subsequently ionized and thermalized. Neutral beam injection (NBI) and RF coupling at electron cyclotron (EC), ion cyclotron (IC), and lower hybrid (LH) frequencies are used for plasma heating as well as current drive. This is necessary for raising plasma temperatures to the required magnitudes for fusion, due to the \( T^{-3/2} \) dependence of plasma resistivity. Ohmic (resistive) heating becomes negligible at fusion relevant temperatures.

2.2.1 Particle drifts

Under external forces and/or due to field inhomogeneities, particles can drift across the magnetic field [7]. This provides a mechanism for confinement loss. The most important of these drifts are the \( \mathbf{E} \times \mathbf{B} \) drift

\[
\mathbf{v}_E = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \tag{2.3}
\]

which occurs for crossed electric and magnetic fields, and the combined \( \nabla B \) and curvature drifts, brought about by inhomogeneous magnetic fields.

\[
\mathbf{v}_{Bs} = \frac{m_s (T_{\perp s} + 2T_{\parallel s}) \mathbf{B} \times \nabla B}{2q_s B^3} \tag{2.4}
\]

\( T_{\perp s} \) and \( T_{\parallel s} \) are the perpendicular and parallel velocities. The \( \mathbf{v}_E \) drift is the same for all plasma species. \( v_B \) is of opposite sign for electrons and ions, and has a temperature dependence. This can give rise to charge separation and subsequent \( \mathbf{E} \times \mathbf{B} \) drifts. This is the physical mechanism behind the most unstable branch of the ITG instability, discussed in section 2.3.
2.2.2 Tokamak geometry

Due to toroidicity the magnetic field is inhomogeneous, with a field magnitude dependence of \( B \propto \frac{1}{R} \), where \( R \) is defined from the centre of the torus. The ‘high-field-side’ of the tokamak plasma is the inner side of the plasma ring closest to the centre (to the transformer in Fig. 1.1), and the ‘low-field side’ is the outer side of the torus. If the magnetic field were purely toroidal the combination of the aforementioned drifts would lead to outward radial bulk plasma motion. This is what necessitates a helical magnetic field for plasma confinement. The helical field lines connect the high and low-field-sides, and form nested flux surfaces, illustrated in Fig. 2.1. The radial component of the plasma drifts during motion along the helical field averages out such that confinement is attained. The equilibrium 2D flux surface geometry is set by the balance of pressure and magnetic forces: \( \nabla p = j \times B \).

![Figure 2.1: Illustration of tokamak geometry. The toroidal angle is \( \psi \), the poloidal angle is \( \theta \). A flux surface is defined by its tangential helical magnetic field lines. A tokamak is comprised of nested flux surfaces. The major radius \( R_0 \) is defined as the length between the centre of the torus and the magnetic axis (where the poloidal field is zero). The minor radius \( a \) is defined from the magnetic axis to the plasma edge.](image)

An important concept is the ‘q-profile’, which relates the relative magnitude of poloidal and toroidal magnetic fields. The q-profile is the ratio of toroidal rotations to one full poloidal rotation along the helical magnetic field on a given flux surface. For general flux surface geometries this is defined as:

\[
q(\psi) = \frac{1}{2\pi} \int_0^{2\pi} db \frac{\mathbf{b} \cdot \nabla \psi}{\mathbf{b} \cdot \nabla \theta}
\]

where \( \psi \) is a ‘radial’ coordinate that labels the flux surfaces. For circular flux surface geometries, the q-profile can be written as:

\[
q(r) = \frac{r B_\phi}{R B_\theta}
\]

(2.5)

(2.6)
where at a given flux surface, $r$ is the radius from the magnetic axis, $B_{\varphi}$ is the toroidal magnetic field, and $B_\theta$ the poloidal magnetic field. The normalized inverse gradient length of the $q$-profile is called the ‘magnetic shear’:

$$\hat{s} = \frac{r}{q \frac{dq}{dr}}$$

(2.7)

The $q$-profile and the magnetic shear play an important role in setting the onset and saturation level of the plasma instabilities which determine transport.

In modern tokamaks, the magnetic geometry towards the walls is tailored such that the majority of the plasma exhaust is confined to field lines that branch out from the last closed flux surface (the ‘separatrix’) at a prescribed location (the ‘$x$-point’), and strike plasma facing components designed to withstand the heat load. This is known as ‘divertor’ geometry, seen in Fig. 2.2, which serves to reduce the influx of impurities from the wall into the plasma. The study of plasma-wall interactions and divertor physics is critical for tokamak development. Reactor divertor concepts which can withstand the high reactor exhaust heat loads must be developed and validated. However, in the research presented in this thesis we concern ourselves with core plasma physics, and from henceforth limit our discussions to that realm.

2.2.3 Mirror forces

The parallel (to the magnetic field) trajectory of particles in the plasma leads to motion in the direction of the magnetic field gradient, which sets up a ‘mirror force’ opposite the direction of motion. This can be understood by considering the conservation of the magnetic moment $\mu = \frac{mv^2}{B}$, which is the adiabatic invariant associated with the rapid (compared with the time variation of the magnetic field in the particle frame) periodic gyromotion of the particles, combined with conservation of energy [7]. As $B$ changes in the particle frame, $v_\perp$ must compensate to preserve $\mu$. $v_\parallel$ must then also change to conserve energy. This effect separates each plasma species into ‘passing’ and ‘trapped’ populations. The trapped population, set by the fraction of particles with insufficient parallel velocity to overcome the magnetic barrier, is confined to the low-field side and move with banana shaped orbits due to the $v_B$ drift. This ‘bounce’ motion undergoes toroidal precession. This is illustrated in Fig. 2.2.

The passing particles traverse the poloidal angle with a transit frequency $\omega_T(e,i) = \frac{e\Omega}{B}$. $qR$ is known as the ‘connection length’, since this is the length scale that connects the low-field-side and high-field-side of a given flux surface. $v_{th,s} = \sqrt{T_s/m_s}$ is the thermal velocity of species $s$.

2.2.4 Diamagnetic drifts

The diamagnetic drifts arise from pressure inhomogeneities:

$$V_{\ast s} = \frac{B \times \nabla P_s}{n_s q_s B^2}$$

(2.8)
2.3 Tokamak transport

Figure 2.2: Trapped particles undergo a banana shaped motion on the low-field-side of the tokamak. This motion undergoes toroidal precession. The separatrix is also illustrated here, with open field lines diverging from the x-point onto dedicated divertor plates which absorb the bulk of the plasma exhaust. (Source: EFDA-JET)

These are not strictly single particle drifts but are rather fluid flows set up by the inhomogeneity of the gyrorotating particles [7]. Density perturbations propagating as a wave tangentially to the pressure gradient have phase velocities on the order of the diamagnetic drift velocities. These are collectively named ‘drift waves’. The drift wave frequency is $\omega_{\ast s} = k \cdot V_{\ast s}$. The various classes of unstable drift waves are the primary causes of tokamak turbulent transport [8]. For gyroradius scale fluctuations, $\omega_{\ast s}$ is approximately two or three orders of magnitude lower than the corresponding gyrofrequencies $\omega_{cs}$. This temporal separation is of great practical use for reducing computational expense in drift wave calculations, as will be further explained in section 2.4.2.

2.3 Tokamak transport

On a flux surface the parallel particle diffusivity and heat conductivities are sufficiently high such that the densities and temperatures can be considered as flux functions, i.e. a function with a unique value over a flux surface. The plasma density and temperature profiles are thus ‘radial’. The transport across the flux surfaces, in conjunction with the core heat and particle sources, sets the value of the confinement time $\tau_E$. This cross-field transport is observed to be orders of magnitude greater than the expectations from the theory of collisional transport in toroidal geometry - known as ‘neoclassical’ theory [9]. This ‘anomalous’ transport is due to turbulent transport driven by the high temperature and density gradients in the tokamak plasma [8]. The underlying instabilities which drive the transport are typically characterised by critical thresholds of temperature or density.
Theoretical Background

Gradients, above which the instability ignites. The values of the critical thresholds are calculated by analysing the dispersion relation derived by linearising (i.e. assuming small fluctuations) the system of equations governing the plasma behaviour. Above the critical threshold, the dispersion relation will support unstable modes. Such ‘linear’ instabilities arise at ion or electron gyroradii scales (depending on the underlying instability drive). These scales are much smaller than the system size, and these instabilities have thus been termed ‘microinstabilities’.

The plasma fluctuations saturate into a self-organized quasi-stationary state following mode-coupling in the non-linear regime. For electrostatic perturbations (when magnetic fluctuations are negligible), the transport can be intuitively understood as a $E \times B$ drift driven random walk through the saturated fluctuating electric potential field. An important regulating and saturation mechanism of the turbulence is through non-linear mode coupling to zonal flows [10]. These are linearly stable predominantly poloidally symmetric modes which provide radial plasma flow shear. They draw energy from the unstable drift waves and also reduce the radial extent of the turbulent eddies.

The frequency width of the microinstability fluctuations has been experimentally observed to be on the order of the frequency itself [11, 12]. For example, for ion scale turbulence, $\Delta \omega \sim \omega^*$. Assuming random walk diffusivity, an estimate of the diffusion coefficient is $D_{turb} \sim \frac{\Delta r^2}{\Delta t}$. $\Delta r$ is the random walk step size and $\Delta t$ the time between steps. Taking $\Delta t \sim \frac{1}{\omega^*}$ and $\Delta r \sim \rho_i$, we obtain an order of magnitude estimate for the ion heat diffusivity, $D_{turb} \sim \frac{\Delta r^2}{\Delta t}$. This fundamental unit of diffusivity is named the ‘gyroBohm’ unit. Throughout this thesis, we normalise turbulent diffusivities by this quantity. GyroBohm scaling clearly points to the advantage of increasing the system size for reducing turbulent diffusivity, since the turbulent eddies scale with the normalised gyroradius.

Kinetic theory is necessary for an accurate description of the instabilities. This is due to the low collisionality in fusion plasmas, and due to wave-particle interactions. Resonances (Landau damping [7]) occur at the passing particle transit frequency, trapped particle precession frequency, and particle magnetic drift frequencies. The perpendicular scale length of the microinstabilities are on the order of several gyroradii: $k_\perp \rho_s \sim 0.1 - 1$. At higher $k$ the instability is stabilised by finite Larmor radius effects, whereby the average magnitude of the field fluctuations ‘felt’ by a gyrating particle is reduced as the scale of the fluctuations decreases below the gyroradius. The parallel scale length of the fluctuations is ordered as $k_\parallel \ll 1$, since field line bending is energetically unfavourable. This anisotropy in the fluctuations simplifies calculations greatly, as will be shown in section 2.4.

The microinstabilities limit plasma confinement even in regimes where the plasma is stabilised against larger scale magnetohydrodynamic (MHD) instabilities - i.e. instabilities that can be adequately described by fluid plasma equations. However, even if the microinstabilities are quenched, MHD instabilities lead to practical limits of the achievable pressure in fusion plasmas, as characterised by experimentally observed limits in achievable $\beta = \frac{p}{\frac{1}{2} \mu_0 B^2}$, the ratio between the plasma kinetic and magnetic pressure [13]. An approximate empirical limit in the achievable plasma density is also observed, the ‘Gre-
2.3 Tokamak transport

This phenomenological limit is \( n_G = I_p / \pi a^2 \), with \( n_G \) in \( 10^{20} \) m\(^{-3} \), \( I_p \) in MA, and \( a \) the minor radius in m. The Greenwald fraction is \( f_G \equiv \bar{n}_e / n_G \), with \( \bar{n}_e \) the line averaged density. By controlling microinstabilities, the aim is to then operate tokamak scenarios in the vicinity of the MHD driven limits.

2.3.1 The ITG instability

An instability of particular relevance to fusion plasmas is that driven by the ion temperature gradient, the ITG instability [15–18]. This mode is thought to be the primary source of ion heat transport, which is critical in setting the eventual fusion power in a reactor. When not considering magnetic field curvature (the ‘slab limit’) the mode is a drift wave driven unstable by parallel compression due to ion acoustic waves [19]. When considering field curvature, the drift wave is also driven unstable due to curvature effects on the low-field-side of the tokamak, where the magnetic field gradient, curvature vector, and temperature gradient are all in the same direction. This ‘bad-curvature’ driven drive then dominates over the slab drive. The physical mechanism for the curvature drive - the Rosenbluth-Longmire picture [20] - is illustrated in Fig. 2.3. For simplification, the ion temperature has been split into two distinct regions, hot and cold. An initial fluctuation (drift wave) is set up on the boundary between the regions. The \( \nabla B \) and curvature particle drifts are proportional to temperature. The ion velocity imbalance then sets up an electric field profile, which through the \( E \times B \) drift then leads to an amplification of the original perturbation. On the high-field-side the \( E \times B \) drift leads to a suppression of the original perturbation.

There exists a critical ion gradient threshold above which the instability ignites. The thresholds can be calculated from kinetic theory, invoking the quasineutrality constraint on the kinetic responses of the combined electron and ion density fluctuations, for example as in Ref. [21]. An often applied analytical parameterisation of the ITG critical gradient thresholds for circular flux surfaces in the limit of low \( r/R \) and low density

![Figure 2.3: The Rosenbluth-Longmire picture of the physical mechanism behind the bad-curvature driven ITG instability](image-url)
2 Theoretical Background

Theoretical Background

Where $L_{Ti} \equiv \frac{R}{T_{Te}}$. The $T_{i}/T_{e}$ dependence can be intuitively understood by considering that at lower relative electron temperatures, the electron density response is stronger for a given electrostatic perturbation. Thus, it is then ‘easier’ for the electrons to cancel out the ion density building up due to the magnetic drift velocity imbalance, which reduces the electric field and weakens the instability drive. The threshold is increased at low $q$ due to the decreased connection length $qR$, increasing the transit frequency of the particles from the ‘bad-curvature’ low-field-side to the stabilising ‘good-curvature’ high-field-side. $s/q$ also stabilises the mode due to decorrelation.

A similar bad-curvature-drive instability can occur for the trapped electron species at similar spatial scales to ITG turbulence, since trapped particles are predominantly situated in the region of bad curvature. This trapped electron mode (TEM) can lead to significant outward ion and electron heat and particle transport at high $R/L_n$ and/or $R/L_{Te}$. A very similar instability also exists at electron gyroradius scales, driven unstable by electron temperature gradients [22]. This is named the ETG mode, and can drive electron heat transport.

2.3.2 H-mode

A breakthrough in achieving increased tokamak confinement was the discovery of the so-called ‘H-mode’ (high-confinement-mode) in 1982 at the ASDEX tokamak [23]. In this regime, achieved when heating power above a certain threshold is introduced, the plasma self-organises into a state where turbulence is highly suppressed in a narrow region towards the plasma edge. This region is named the ‘edge transport barrier’, or the ‘pedestal’. In the region between the pedestal and the magnetic axis, the standard turbulent transport regime still holds. Tokamak temperature profiles tend to be constrained towards their critical inverse gradient thresholds, a property called ‘stiffness’. The core temperatures are thus often effectively proportional to the pedestal height. The increase in core plasma pressures, due to the H-mode pedestal and profile stiffness, approximately doubles the confinement time. Understanding the precise physics mechanisms which lead to pedestal formation is still an intense area of research. There is strong evidence that the formation of strong plasma rotation shear (flow shear) in the pedestal region plays a critical role in stabilizing the underlying turbulence [24].

The pedestal gradient is limited by MHD modes, thought to be peeling-ballooning modes [25]. These modes give rise to periodic violent outbursts of plasma from the pedestal, known as edge localised modes (ELMs). This poses a challenge to wall survivability and must be controlled in a reactor, a topic of ongoing research [26].
2.3 Tokamak operational scenarios

Worldwide progress in tokamak research has been steady since the concept was established as the leading MCF concept following the successful measurements of $\sim 1 \text{keV}$ temperatures at the T3 tokamak in the Soviet Union in 1969 [27]. Progress is quantified by both the Lawson triple product $n_e T \tau_E$, as well as the fusion gain factor $Q \equiv \frac{P_{\text{fus}}}{P_{\text{in}}}$, the ratio of the fusion power produced to the tokamak input power. This is illustrated in figure 2.4. The world record for fusion power generation is 163 MW, set in 1997 at the Joint European Tokamak (JET) [28]. The next step in tokamak research is ITER (Latin for the way), presently under construction in Cadarache, France, which aims (at the time of writing) to produce first plasma by 2020. The goal of ITER is to produce 500 MW of fusion power with 50 MW power input, thus $Q = 10$, for a discharge time of 300 s [29]. The first fusion DT experiments are planned by 2027. ITER also plans to test necessary reactor technology, such as tritium breeding blankets. If ITER is successful, the construction of electricity producing demonstration fusion reactors can follow. From a wide database of tokamak discharges, a widely used empirical scaling law has been established that predicts the confinement time based on dimensional parameters such as total plasma current and magnetic fields [30]:

$$\tau_{\text{IPB}}(y, z) = 5.62 \times 10^{-2} I^{0.93} B^{0.15} P^{-0.69} n^{-0.41} M^{0.19} R_0^{1.97} \epsilon^{0.58} \kappa^{0.78}$$

(2.10)

where $I$ is the total plasma current, $B$ the magnetic field, $P$ the input power, $n$ the line averaged plasma density, $M$ the average ion mass, $R_0$ the major radius, $\epsilon$ the separatrix aspect ratio $\epsilon \equiv a/R$, and $\kappa$ the separatrix elongation $\kappa \equiv \frac{Z_{\text{max}} - Z_{\text{min}}}{R_{\text{max}} - R_{\text{min}}}$, where $R$ and $Z$ are separatrix coordinates. The ‘H-factor’ is a measure of plasma performance compared to the scaling law:

$$H_{98} \equiv \frac{\tau_E}{\tau_{\text{IPB}}(y, z)}$$

(2.11)

where $\tau_E$ is the actual confinement time of the discharge.

Plasma discharges can be classified into a number of different scenarios, of which a few are listed below. They differ primarily in the heating power and in the $q$-profile. Schematics of the pressure profiles and $q$-profiles associated with the various scenarios are shown in figure 2.5.

**L-mode** A typically low power scenario without a edge transport barrier (pedestal). $H_{98} \sim 0.5$ in this case. Typically the initial and final phase of tokamak discharges, the current ramp-up and ramp-down phases, are in L-mode. Once sufficient power is applied to reach high plasma stored energy, the H-mode can form.

**Inductive H-mode** High current, high power discharges that typically provide the highest confinement times for a given machine. $H_{98} \sim 1$. The ITER $Q = 10$ target is planned with this scenario. The disadvantage of the scenario is the existence of periodic MHD instabilities which can reduce confinement. The high currents associated the scenario leads to a $q$-profile that dips below 1 in the inner core region. This instigates a periodic instability called the ‘sawtooth’ which reorganises the
plasma profiles, flattening the pressure profiles within the \( q < 1 \) region [31, 32]. If \( \beta \) is sufficiently high, the sawteeth can seed neoclassical tearing modes (NTMs) which lead to the formation of magnetic islands, reducing plasma confinement. In future devices such as ITER, it is crucial to control these NTMs by the dedicated injection of RF heating and current drive onto the magnetic islands [33], or by controlling the triggering sawtooth instability [34].

**Steady State Scenario** Another drawback of the standard H-mode scenario is need to draw inductive current from the central transformer, which limits the pulse time of the machine and reduces reactor relevance. Furthermore, thermal fatigue in a pulsed machine could increase the necessary machine maintenance. In the steady state scenario, the toroidal current is generated by non-inductive means. The external current drive is increased, and critically the internal bootstrap current is increased significantly. The necessary bootstrap current is created by inducing an internal transport barrier (ITB), where in a narrow region of the core plasma the microinstability transport is significantly reduced, similarly (and often in addition to) the H-mode edge barrier [35]. The high pressure gradients within the ITB significantly increases the bootstrap current fraction. The ITB is typically formed by setting up a region of negative magnetic shear, which is highly stabilising for the microinstabilities [36]. Plasma rotation has also been observed to be correlated with ITB formation [37]. The target q-profile of the steady state scenario demands...
lower plasma current, and thus according to Eq. 2.10 the confinement time is reduced. This is compensated by the ITB, which raises $H_{98}$ to high values, up to $H_{98} \sim 2$. The drawback of the steady state scenarios is the need for combined control of the pressure profile and current profile. The scenarios are also susceptible to MHD instabilities.

**Hybrid scenario** The middle ground between the standard H-mode and steady state scenarios is named the ‘hybrid scenario’ [38]. Many of the problems associated with scenario control are abated by removing the need for an ITB. The discharge pulse time is significantly increased by both reducing the total plasma current (although not to the degree done by the steady state scenario), and increasing the external current drive fraction. Hybrid scenario q-profiles have a wide region of $q \sim 1$, and high magnetic shear towards the edge. Maintaining $q \sim 1$ reduces the risk of the deleterious MHD modes such as sawteeth and NTMs. High $s/q$ in the outer radii is also predicted to improve the stability of ITG turbulence, as according to Eq. 2.9, $H_{98}$ factors reached in hybrid scenarios range from $H_{98} = 1 - 1.5$. The reasons for the improved confinement are still under debate. It is thought that low core $s$ at inner radii, high $s/q$ at outer radii, high rotation shear (which is typically associated with hybrid discharges), and the lack of deleterious MHD activity can all play a role in improved hybrid confinement. This question provides the motivation for the research carried out in this thesis, as laid out in Chap. 1. The improved $H_{98}$ in hybrid discharges holds great promise for the extrapolation of this scenario to ITER and beyond, and may pave a role towards a long-pulse reactor scenario.

![Figure 2.5](image-url)  
*Figure 2.5:* The schematic dependence of plasma pressure profiles on transport barriers is shown in (a). The range of (schematic) q-profiles corresponding to the various operational scenarios are illustrated in (b). Steady-state scenarios can correspond to both weakly and strongly reversed q-profiles with negative magnetic shear. ITG physics with hybrid-like q-profile is a recurring theme in this thesis. (Source: EFDA-JET)
2 Theoretical Background

2.4 Transport modelling

The ability to predict tokamak current, temperature, density, and rotation profiles (collectively known as kinetic profiles) is important for both the interpretation of current tokamak discharges and extrapolation to future machines. For brevity, we will exclude rotation (angular momentum) transport from the subsequent discussions, as no angular momentum transport calculations were carried out within the framework of this thesis.

The transport equations can be approximated as 1D diffusion equations, where the metric of the 1D grid is set by the self-consistently calculated 2D flux surface geometry. Codes which operate within this framework are thus known as 1.5D transport codes. 'Integrated modelling' codes also include calculations of the various sources in the equations, such as current drive and heating sources, and can simulate entire tokamak discharges. Much of the work in this thesis is carried out with the CRONOS integrated modelling code [39], introduced briefly in section 2.4.1. The calculation of the ‘anomalous’ heat and particle transport coefficients due to turbulence is a major aspect of such simulations. For speed and convenience empirical or semi-empirical transport coefficients can be used. The most accurate theoretical transport predictions are however provided by full non-linear simulations, as performed for example by the GENE [40] code introduced in section 2.4.2. Unfortunately, for integrated modelling applications such non-linear calculations are computationally prohibitively expensive since the transport time-scales are orders of magnitude greater than the fluctuation timescales of the underlying turbulence. More tractable theory based calculations, still based on first-principles, are thus commonly used. Two such ‘quasi-linear’ models used and studied in this thesis are introduced in section 2.4.3.

2.4.1 Integrated modelling - CRONOS

The heart of CRONOS is the 1D diffusion equation solver, to which multiple physics modules are attached which calculate the various terms in the equations. The 1D ‘radial’ coordinate is the toroidal flux coordinate $\rho = \sqrt{\frac{\phi}{\pi B_0}}$, where $\phi$ is the flux surface dependent toroidal magnetic flux, and $B_0$ is the vacuum magnetic field in the geometrical centre of the tokamak poloidal cross section. In CRONOS, this grid is normalized as $x = \frac{\rho}{\rho_m}$, where $\rho_m$ is the value of $\rho$ at the last closed flux surface, which may change in time if the volume and/or shape of the plasma itself changes.

The current diffusion equation is solved in terms of the poloidal flux $\psi$, where $\psi = \frac{\phi}{\pi B_0} \cdot d\mathbf{S}$, and the surface integral is carried out on the toroidal midplane up to the edge of a given flux surface. The current diffusion equation is as follows:

$$\frac{\partial \psi}{\partial t} = \frac{\langle |\nabla \rho|^2 / R^2 \rangle}{\mu_0 \sigma \parallel \rho^2_m \langle 1/R^2 \rangle} \frac{\partial^2 \psi}{\partial x^2} + \left\{ \frac{\langle |\nabla \rho|^2 / R^2 \rangle}{\mu_0 \sigma \parallel \rho^2_m \langle 1/R^2 \rangle} \frac{\partial}{\partial x} \times \left[ \ln V' \langle |\nabla \rho|^2 / R^2 \rangle F \right] + \frac{x}{\rho_m} \frac{d \rho_m}{dt} \right\} \frac{\partial \psi}{\partial x} + \frac{B_0}{\sigma \parallel F \langle 1/R^2 \rangle} j_{ni} \tag{2.12}$$
2.4 Transport modelling

The geometric quantity $\nabla \rho$, the volume enclosed by two neighbouring flux surfaces $V'$, and the flux surface averaging operator $\langle \cdot \rangle$, all depend on the solution of the flux surface geometry. This is typically carried out in CRONOS by the HELENA code [41], which solves the flux surface equilibrium for a prescribed separatrix boundary. $\sigma_\parallel$ is the parallel conductivity, $F = RB\phi$ is the 'diamagnetic function', and $j_{ni}$ is the noninductive current density at radius $x$ set by the bootstrap current and external current sources such as NBI and RF wave injection. The conductivity and bootstrap current is calculated in a separate neoclassical transport module, such as NCLASS [42]. The external current sources are also calculated in dedicated modules for each of the separate current drive sources applied.

In CRONOS, the particle transport is formulated for the electron species only. The ability to separately model multiple ion species in CRONOS is currently in development. Presently, if multiple species are present, the quantity $Z_{\text{eff}} \equiv \sum_j q_j^2 \tilde{n}_j$ is prescribed (where $j$ is an ion species index, $q_j$ the ion charges, and $\tilde{n}_j$ are the ion densities normalised to the electron density), relative ion density ratios are prescribed (when more than 2 ion species are present), and the ion densities are then calculated by invoking the quasineutrality constraint. The exception is helium transport in simulations that include fusion reactions, where a separate helium transport equation is solved. The particle transport equation is as follows:

$$\frac{\partial}{\partial t} (V' n_e) + \frac{\partial}{\partial \rho} \left( V' \langle |\nabla \rho|^2 \rangle \Gamma_e \right) = V' S_{ne} \quad (2.13)$$

$\Gamma_e$ is the electron flux, and $S_{ne}$ is the particle source due to NBI or gas puffing.

The electron and ion heat equations are as follows:

$$\frac{3}{2} \frac{\partial}{\partial t} \left( P_{e,i} V'^2 \right) + V'^2 \frac{\partial}{\partial \rho} \left[ V' \langle |\nabla \rho|^2 \rangle q_{e,i} \right] = V'^2 Q_{e,i} \quad (2.14)$$

$P_{e,i}$ are the electron and ion pressures. $q_{e,i}$ are the total electron and ion heat fluxes, and $Q_{e,i}$ are the heat sources. External heat sources are calculated in dedicated modules. Electron-ion heat exchange can also be an important heat source or sink if the collisionality is sufficiently high.

2.4.2 Gyrokinetic modelling - GENE

The most accurate calculations of the particle and heat fluxes are from direct numerical simulation of the kinetic equations. The 'Eulerian' approach solves the particle distribution functions directly. The starting point for this approach is the second lowest order truncation of the BBGKY hierarchy derived from the full N-particle Liouville equation, the collisional Vlasov equation [43]:

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla_x f_s + \frac{E + \mathbf{v} \times \mathbf{B}}{m_s} \cdot \nabla_v f_s = C (f_s) \quad (2.15)$$

which describes the evolution of the single particle 6D distribution function, under the influence of the Lorentz force and purely binary collisions (i.e. ignoring 3 particle correlations or more - well justified for hot and diffuse fusion plasmas). This system is closed.
by Maxwell’s equations, where the charges and currents are self-consistently calculated from the distribution function.

\[
\rho = \sum_j q_j \int f_j \, dv \\
j = \sum_j q_j \int v f_j \, dv
\]

(2.16)

The complexity of the system can be further reduced. The dynamics of interest (microinstabilities) are of low frequency compared to the gyrofrequencies. The particle trajectories can thus be spatially averaged over their gyromotion. The kinetic description is then reduced from a description of the gyrating particles to a description of rings of charge around the gyrocentre. This constitutes gyrokinetics [44]. The dimensionality of the system is reduced to 5D, and the minimum simulation timestep necessary to resolve the dynamics is significantly increased. This allows for tractable simulation of the system, albeit on massively parallel large-scale computing systems with typically \(10^3 - 10^5\) CPU hours per run (depending on the additional approximations) to obtain the turbulent fluxes for a given input parameter set.

The anisotropy of the turbulent structures in tokamaks, with \(k_\parallel \ll k_\perp\), allows the formulation of field line aligned coordinates with a reduced number of grid points in the parallel direction, further reducing the computational time. The distribution function \(f_s\) can also be split into a slowly evolving mean background distribution (typically Maxwellian) and a small, rapidly fluctuating part responsible for the transport: \(f_s = F_s^0 + \delta f_s\). For a given background, the (now 5D) Vlasov equation can be recast for \(\delta f_s\). The validity of these approximations is encapsulated by the gyrokinetic ordering with small parameters \(\epsilon_B\), \(\epsilon_\omega\) and \(\epsilon_\delta\) (from Ref. [44]):

\[
\frac{\rho_i}{L_B} \sim \epsilon_B \\
\frac{\omega}{\Omega_i} \sim \epsilon_\omega \\
\frac{\delta f_s}{F_s^0} \sim \frac{e\delta\phi}{T_e} \sim \frac{|\delta B|}{B_0} \sim \epsilon_\delta \\
k_\perp \rho_i \sim \epsilon_\perp \sim O(1) \\
k_\parallel \rho_i \sim \epsilon_\parallel \sim O(1)
\]

(2.17)

\(L_B\) is the magnetic field scale length, \(\omega\) the instability frequency, \(\Omega_i\) the ion gyrofrequency, \(\delta\phi\) the electrostatic potential perturbation, and \(\delta B\) the magnetic field perturbation. Both \(\epsilon_\omega\) and \(\epsilon_\delta\) are of the order \(10^{-3}\) for typical tokamak parameters. The last relation is motivated by the requirement that \(\omega \sim k_\parallel v_{th}\) such that Landau damping in the parallel dynamics can be captured.

The \textsc{gene} code (Gyrokinetic Electromagnetic Numerical Experiment) is one such Eulerian implementation for evolving the non-linear gyrokinetic system of equations [40].
In GENE, a linearized version of the equations can also be evaluated such that the growth rates and frequencies of the underlying instabilities can be studied. For a given set of input parameters (plasma gradient lengths, q-profile, flux surface geometry, etc.), $\delta f$ is evolved, and the particle and heat diffusivities are calculated from the time averages of the eventual saturated state of the turbulence. More detailed information on the underlying structure of the turbulence, e.g. the frequency and k-space spectra of the fluctuations, can be calculated directly from the simulated distribution functions. The code can simulate multiple particle species simultaneously, important for a full kinetic description of electrons and impurity species. In the electrostatic limit for the study of ion heat fluxes, a reduction in computational complexity can be gained by evolving only the ion distribution function, and assuming an adiabatic response for the electrons - i.e. neglecting electron inertia, and assuming an instantaneous Boltzmann response to the electrostatic potential: $\frac{dn_e}{dt} = \frac{e\delta \phi}{T_e}$. For the work carried out in this thesis, GENE has been used in its local version, where the turbulence around a single flux surface is calculated assuming that the background gradient scale lengths are larger than the turbulence scale lengths. For more complete descriptions of the underlying theory and numerical implementation in GENE, see Ref. [40] and references therein, as well as Ref. [45–48].

Given the computational expense for the simulations, it is impractical to routinely apply non-linear gyrokinetic simulations for $\Gamma_e$ and $q_{e,i}$ calculations within an integrated modelling framework. Such a task demands multiple calculations for different radii and for time-evolving parameters. Gyrokinetic simulations are however vital for detailed study of the non-linear physics of tokamak turbulence, which can be compared to experimental results and also used to validate simplified transport models. GENE has been used in this thesis for both these purposes. The framework for simplified ‘quasi-linear’ transport models is introduced in the next section.

### 2.4.3 Quasi-linear transport models

Quasi-linear transport models assume that the non-linear saturated turbulent state can be described partly from linear characteristics calculated by linearising the underlying equations. Added assumptions guided by non-linear simulations then incorporate the unavoidable non-linear physics. To illustrate this, we write the general definition of the various fluxes for electrostatic turbulence at a given scale length denoted by wavenumber $k$. We assume harmonic fluctuations, $\delta n, \delta P_{e,i}, \delta \phi \propto e^{-i(\omega t - k \cdot x)}$. For the particle flux we obtain:

$$\Gamma_e(k) = \text{Re}\langle \delta n_e \delta v_r \rangle = \text{Re}\langle \delta n_e \frac{ik\delta \phi}{B} \rangle$$

(2.18)

$\delta v_r$ is from the fluctuating $E \times B$ drifts, hence $\delta v_r = \frac{E_y}{B} = \frac{ik\delta \phi}{B}$, where $y$ denotes the direction binormal to the magnetic field and the radial direction. The averaging is over time and the flux surface. The time averaging is carried out on a time-scale between the fluctuation and transport time-scales, such that the diffusivities are defined for static zeroth order parameters. Similarly for the heat fluxes:

$$Q_{e,i}(k) = \text{Re}\left(\frac{3}{2}\delta P_{e,i} \frac{ik\delta \phi}{B} \right)$$

(2.19)
If the fluctuating quantities have the same phase, then the transport is equal to zero due to
the averaging over $\cos(\omega t) \sin(\omega t)$. Now comes the crux, $\delta n$, $\delta P_{\text{e,i}}$, and $\delta \phi$ can differ in
phase and relative amplitude. The quasi-linear assumption is that these relations are taken
from the solutions of the linearised equations describing the system. Summing over all
unstable modes and frequencies (since due to non-linear broadening mechanisms more
than one frequency may be present for a given $k$ [43]), the quasilinear fluxes can be
decomposed as:

$$ (\Gamma_{\text{e}}, Q_{\text{e}}, Q_{\text{i}}) = \sum_{k, \omega} (\text{Linear Response})_{k, \omega} (\Gamma_{\text{e}}, Q_{\text{e}}, Q_{\text{i}}) \otimes |\delta \phi|^2_{k, \omega} \quad (2.20) $$

The saturated electrostatic potential, $|\phi|^2_{k, \omega}$, is a quantity that is not possible to gauge
from the linear physics. The choice for $|\phi|^2_{k, \omega}$ typically invokes a random walk argument
where the random walk time-step at each spatial scale is assumed to be $\frac{1}{\gamma_k}$ ($\gamma_k$ is the
growth-rate for the mode with wavenumber $k$), and the step size is assumed to be $\frac{1}{k}$. Thus $D_k \sim |\delta \phi_k|^2 \sim \frac{1}{\gamma_k}$. The precise details of the mixing length rule [49], involving
the assumptions on the k-space and frequency spectra of $\delta \phi$, as well as other factors which
approximate additional non-linear phenomena such as zonal flows, depend on the precise
model. These choices are guided by or fitted to the results of full non-linear simulations.

In summary, quasilinear theory assumes that the turbulence can be described by an
ensemble of modes which retain their linear characteristics, but where the precise spectra
and amplitudes of the modes are set by non-linear physics, which must then be incor-
porated in an ad-hoc manner into the model with the guidance of non-linear theory and
simulations. The details of two specific quasilinear models applied in this thesis are now
briefly presented.

**GLF23**
The system solved is the four-moment GLF (Gyro-Landau-Fluid) system of equations
[50, 51]. Moments are taken of the Vlasov equation, which leads to a fluid description.
The fluid closures involve terms which replicate the full kinetic response, taking into
account effects such as Landau damping. The electrostatic potentials in the model are
multiplied by factors that approximate the finite Larmor radius effects, where small-scale
perturbations are zeroed out by the gyroaveraging. Solving a fluid system of equations
provides a significant reduction in computational complexity compared with the gyrokin-
etic equation.

The GLF23 model [52] solves the eigenvalue problem of the linearised system of
GLF equations. The mode structure (eigenmode) is not solved self-consistently, and is
prescribed by trial functions with fitting parameters chosen such that the growth rates
agree with full linear gyrokinetic calculations. The GLF23 mixing length rule is:

$$ |\phi_k|^2 \propto \frac{1}{\gamma_{\text{net}}^{1/2}} \frac{1}{k^2} \quad (2.21) $$

The ‘net’ growth rate $\gamma_{\text{net}}$ takes into account the stabilising effect of flow shear:

$\gamma_{\text{net}} \equiv \gamma_k - \alpha_E \gamma_E$, where $\gamma_k$ is the calculated growth rate at wavenumber $k$, and $\gamma_E$
is the flow shear. For pure toroidal rotation (as is often the case for NBI driven rotation) \( \gamma_E = \frac{q}{\rho_s} \), where \( \Omega \) is the angular velocity. The precise value and parameterisation for \( \alpha_E \) is uncertain, and values ranging from 0.5 – 1.5 are typically used in simulations. \( \gamma_d = 0.24 \frac{q}{\rho_s} |\omega_d| \) approximates the damping effect of zonal flows, where \( |\omega_d| \) is the magnetic drift frequency. No frequency broadening for the fluctuating potential is assumed in the GLF23 model.

QuaLiKiz

The QuaLiKiz transport model [53] is based on the linear electrostatic gyrokinetic eigenvalue solver Kinezero [54]. Solving the linear gyrokinetic equation directly is computationally more expensive than the fluid system, but simultaneously involves a minimum of fitting factors compared to the GLF models. The formalism in which both the growth rates and the quasi-linear fluxes are calculated is now very briefly expounded.

The starting point is again the Vlasov equation. The physical picture can be sufficiently elucidated by taking the electrostatic, collisionless case:

\[
\frac{\partial f_s}{\partial t} + v \cdot \nabla_x f_s + \frac{q}{m_s} E \cdot \nabla_v f_s = 0 \tag{2.22}
\]

where \( E = \nabla \phi \) is an \( O(\epsilon) \) fluctuating quantity. Setting \( f_s = F_0 + \delta f_s \), and linearising with a single harmonic fluctuation \( \delta f = \delta f_k e^{-i(\omega t - k \cdot x)} \), we obtain the \( O(\epsilon) \) linear response for \( \delta f_k, \omega \):

\[
\delta f_{s,k,\omega} = -\frac{q_s m_s}{\omega} \left( k \cdot \nabla_v F_0 - v \cdot k \right) \delta \phi_k \tag{2.23}
\]

The quasineutrality constraint for a given mode is:

\[
\sum_s q_s \delta n_{k,s} = \sum_s q_s \int d\omega \delta f_{s,k,\omega} = -\sum_s q_s^2 \int d\omega \frac{k \cdot \nabla_v F_0 - v \cdot k}{\omega} \delta \phi_k = 0 \tag{2.24}
\]

Where we have substituted the linear response for \( \delta f_{k,\omega} \) from Eq.2.23 in the last step above. The equation is reduced to solely depend on \( \omega \) by integrating over space. This integrates over the radial mode structure of \( \delta \phi_k \) and also over the space-dependent drift velocities in \( v \). We are then left with the dispersion equation for \( \omega(k) \):

\[
\sum_s q_s^2 \int d\omega d\nu \frac{(k \cdot \nabla_v F_0)}{\omega - v \cdot k + i \gamma} \delta \phi_k = 0 \tag{2.25}
\]

The solution is \( \omega = \omega_r + i \gamma \), obtained for a given wavenumber \( k \). For positive \( \gamma \), this constitutes an instability. The equation can be reduced to a 5D coordinate system by averaging over the gyroangle coordinate. In Fourier space this will simply result in Bessel functions \( J_0(k_{\perp} \rho_s) \) multiplying the electrostatic potential, capturing the finite Larmor radius effects. To reduce computational complexity, the mode structure of \( \delta \phi_k,\omega(x) \) is not self-consistently calculated in QuaLiKiz. The mode structure is prescribed according
to a solution of the fluid limit of the gyrokinetic equation. The $i0^+$ term in the denominator of Eq. 2.25 determines the position of the pole when carrying out the integration, and physically corresponds to a vanishing small growth rate such that the fluctuation disappears as $t \to -\infty$, necessary for causality.

To illustrate the calculation of the quasilinear fluxes we take the particle flux as an example. For a given mode, the definition of the particle flux is:

$$\Gamma_{sk} = Re\langle \frac{\delta n_{sik}}{B} \phi_{sik} \rangle$$ \hspace{1cm} (2.26)

$\delta n_s$ is the first moment (velocity integration) of the linear response Eq. 2.23. This is substituted into Eq. 2.26, to obtain:

$$\Gamma_s = -Re \left( \frac{\theta_s}{Bm_s} \sum_k \int d\omega \int dv |k| (k \cdot \nabla_v F_0) J_0(k \cdot \rho_s) |\delta \phi_k|^2 \frac{i0^+}{\omega - \nu \cdot k + i0^+} \right)$$ \hspace{1cm} (2.27)

where all unstable modes are summed over (practically speaking, a finite number of modes are summed over in the flux calculations), as well as the frequency spectrum for each mode. $F_0$ is taken as a Maxwellian function. A broadened frequency spectrum has been introduced here for $\delta \phi_k$, assumed to be a Lorentzian with a width equal to the growth rate. This assumption approximates stochastic non-linear effects which lead to the resonance broadening. The mixing length rule for $|\delta \phi_k|^2$ is of the form $\gamma_k/k^2_\perp$, with an assumed spectral shape that simulates the non-linear k-space flux spectrum. A single constant normalises the fluxes to equal the gyrokinetic non-linear prediction of a standard benchmark case, the ‘GA-standard case’. For deeper details of the specifics of the model, see Refs. [55, 56], as well as Chap. 5 in this thesis.
References

2 Theoretical Background

3 Impact of heating and current drive mix on the ITER Hybrid Scenario

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Abstract

Hybrid scenario performance in ITER is studied with the CRONOS integrated modeling suite, using the GLF23 anomalous transport model for heat transport prediction. GLF23 predicted core confinement is optimized through tailoring the $q$-profile shape by a careful choice of current drive actuators, affecting the transport due to the predicted dependence of the turbulence level on the absolute $q$-profile values and magnetic shear. A range of various heating and current drive choices are examined, as are different assumptions on the pedestal height. The optimum $q$-profile shape is predicted to be one that maximizes the ratio of $s/q$ throughout the bulk of the plasma volume. Optimizing the confinement allows a minimization of the plasma density required in order to achieve a defined target fusion power of 350 MW. A lower density then allows a lower total current ($I_p$) at the same Greenwald fraction ($f_G$), thus aiding in maintaining $q > 1$ as desired in a hybrid scenario, and in minimizing the flux consumption. The best performance is achieved with a combination of NBI and ECCD (e.g. 33/37 MW NBI/ECCD for a scenario with a pedestal height of 4 keV). The $q$-profile shape and plasma confinement properties are shown to be highly sensitive to the positioning of the ECCD deposition. Comparisons to the lower performing cases where some or all of the ECCD power is replaced with LHCD or ICRH are shown (e.g. 33/20/17 MW NBI/ECCD/LHCD or NBI/ECCD/ICRH). The inclusion of LHCD reduces confinement due to deleterious shaping of the $q$-profile, and the inclusion of ICRH, particularly in a stiff model, does not lead to significantly increased fusion power and furthermore does not contribute to the non-inductive current fraction. For the optimum NBI/ECCD current drive mix, the predictions show that a satisfactory ITER hybrid scenario ($P_{fus} \sim 350$ MW, $Q \geq 5$, $q_{min}$ close to 1) may be achieved with $T_{ped} \geq 4$ keV. In addition, predicted performance sensitivity analysis was carried out for several assumed parameters, such as $Z_{eff}$ and density peaking.

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3.1 Introduction

The ITER “hybrid” advanced scenario aims to provide a discharge with an extended burn time (> 1000 s) while still maintaining a moderately high fusion power ($P_{\text{fus}} > 350$ MW) and significant α-particle heating ($Q > 5$), without the drawback of the stringent active control requirements which characterize steady-state scenario discharges [1]. The primary mission of this scenario is to maximize the neutron fluence per pulse, for effective testing of reactor-relevant components [2]. The extended burn time is achieved via reducing the flux consumption by operating at both reduced current ($I_p = 11 - 13$ MA) and a higher non-inductive current fraction (~50−60%) compared to the ELMy H-mode ITER reference scenario.

Scenarios that may extrapolate to the ITER hybrid scenario have been developed over this past decade at ASDEX-U [3, 4], DIII-D [5], JET [6], and JT-60U [7]. These discharges, particularly at ASDEX-U, DIII-D, and recent experiments at JET [8], have routinely displayed thermal energy confinement improvement beyond the IPB98(y,2) scaling expression [9]:

$$\tau_{\text{IPB98}(y,2)} = 5.62 \times 10^{-2} I_p^{0.93} I_0^{0.15} P_{\text{fus}}^{-0.69} n_0^{0.41} M_0^{0.19} R_1^{0.78} \epsilon_0^{0.58} \kappa_0^{3.3}$$ (3.1)

Or, expressed in dimensionless parameters:

$$\omega_{ci} \cdot \tau_{\text{IPB98}(y,2)} \propto \rho^*^{-2.70} \beta^*^{-0.90} \nu^*^{-0.01} M_0^{0.96} q^{-3.0} \epsilon_0^{0.73} \kappa_0^{-3.3}$$ (3.2)

Confinement enhancement factors of $H_{\text{98}(y,2)} \sim 1 - 1.4$ have been achieved, where $H_{\text{98}(y,2)}$ is defined as the ratio between the measured thermal energy confinement time and $\tau_{\text{IPB98}(y,2)}$. These enhanced confinement scenarios may extrapolate to ITER hybrid scenarios with $Q$ significantly in excess of the defined minimum $Q = 5$ for the ITER hybrid scenario mission [10]. A key concept in these discharges is the maintenance of a broad safety factor profile above or close to unity throughout ($q > 1$), avoiding the instigation of sawteeth thus allowing operation at higher $\beta_N$ (up to the no-wall limit), without triggering deleterious NTM’s. Achievement in ITER of a high $Q$, long pulse discharge would widen the hybrid mission by assessing the potential of the pulsed tokamak for energy production [1].

The reasons for the reported confinement enhancement are not fully understood, complicating the extrapolation of the hybrid scenario to ITER. Microrubulence theory predicts that the typical shape of the hybrid scenario $q$-profile - a broad $q \sim 1$ volume with increased shear towards the edge - should lead to more favourable core confinement [11, 12]. This points to a potential over-simplification in using purely $I_p$ in the $IPB(y,2)$ scaling expression, due to the influence of the current profile shape itself on the confinement properties. Experimental support for the role of the current profile shape in improving core confinement has been seen at ASDEX-U [13], as well as in recent $q$-profile shaping experiments at JET [8], and at HL-2A [14]. Other analysis of ASDEX-U discharges has pointed to the role of pedestal confinement improvement at high power as being responsible for the global confinement enhancement [15, 16], pointing to the potential non-applicability of the $IPB98(y,2)$ scaling expression at high $\beta_N$. However, in
3.2 Modelling techniques and methodology

3.2.1 General philosophy of this work

The central point of this work is assessing and comparing the ability of various heating and current drive systems to provide a 'satisfactory' hybrid scenario for ITER. The target, 'satisfactory' hybrid scenario that we define here is one that provides a fusion power performance of \( P_{\text{fus}} = 350 \, \text{MW} \), \( Q > 5 \) in order to still maintain a significant degree of \( \alpha \)-heating, \( P_{\text{loss}} < \sim 110 \, \text{MW} \) in order to avoid a heat flux to the divertor beyond the operational limit, and \( t_{\text{discharge}} > 1000 \, \text{s} \) in order to achieve high neutron fluence per
3 Impact of heating and current drive mix on the ITER Hybrid Scenario

pulse. The target power of $P_{\text{fus}} = 350$ MW was specifically chosen to provide a neutron flux just above the minimum required value suitable for component testing [2], and serves as a baseline when comparing the results of simulations with varying pedestal and current drive mix prescriptions. In ITER, deleterious NTM’s are predicted to be unstable even for $\beta_N$ as low as 1.8, a significantly lower limit compared to present tokamaks [26]. Hence, if we are to assume a $\beta_N > 1.8$ scenario without transport reduction from NTM’s, or alternatively to assume that no auxiliary power need be diverted to NTM control, we must avoid the initiation conditions of NTM triggering sawteeth. Towards this end, we aim for $q > 1$ throughout the simulations, or at least to maximize the time until a $q = 1$ surface forms and/or minimize the $q = 1$ radius at stationary state, i.e. following the relaxation of the current profile. Throughout this work we define the $q = 1$ radius as $x(q = 1)$. We will show in this work that devoting all auxiliary power to current drive and $q$-profile shaping is predicted to be vital for increasing both confinement and the non-inductive current fraction for pulse length optimization. We thus deem the $q > 1$ constraint as necessary for a satisfactory hybrid scenario as defined above.

For the various simulations involving different heating and current drive mixes, the plasma density is used as an actuator for controlling fusion power in order to approximately reach $P_{\text{fus}} = 350$ MW at stationary state. To maintain the same Greenwald fraction ($f_G$), the total current must then also be adjusted together with $n_e$. Therefore, we aim to determine the optimum heating and current drive mix that maximizes the core confinement and temperatures, such that a minimum $n_e$ and thus Ohmic current is needed to achieve $P_{\text{fus}} = 350$ MW. Minimizing the Ohmic current would then raise the $q$-profile towards the goal outlined above. The maximum allowed $f_G$ in the simulations is set at 0.95, slightly below the empirical Greenwald density limit [27]. If optimum simulations are achieved at $f_G < 0.95$, then the lower values are used. All radial profiles in this work are represented as a function of $x$, the normalized toroidal flux coordinate.

3.2.2 Models used

The core of CRONOS is a 1.5D transport solver, whereby 1D current diffusion, particle, energy, and momentum transport equations are solved up to the separatrix, self consistently with 2D magnetic equilibrium, which in these simulations is calculated by the HELENA [28] fixed boundary solver. The same fixed boundary was used for both the flat-top and ramp-up phases, and is described by $R = 6.2$ m (major radius), $a = 2$ m (minor radius), $\kappa = 1.89$ (elongation), $\delta_{\text{upper}} = 0.454$ (upper triangularity), $\delta_{\text{lower}} = 0.516$ (lower triangularity). Neoclassical transport, resistivity and bootstrap current is calculated by NCLASS [29]. Heating and current drive sources are determined as follows: NBI by NEMO, ECRH/ECCD by REMA [30] with an analytic approximation for the current drive efficiency [31], LHCD by either DELPHINE [32] or LUKE [33], and ICRH by PION [34]. Cyclotron radiation emission is calculated by EXACTEC [35]. Fusion reactions are determined by the Bosch-Hale formulation [36]. The contribution of supra-thermal fusion reactions is not considered. Anomalous transport is primarily calculated with the GLF23 model.
3.2.3 Assumptions made

The basic assumptions of these simulations are consistent with those made in the CRONOS simulations carried out for a recent hybrid scenario benchmarking study [37]. Maintaining these assumptions allows for a comparison to be made with the previous work. The assumptions are as follows: Equal ratios of D and T are assumed. Only electron and ion heat transport is predicted, whereas the density profile is prescribed flat (with $n_e(0)/\langle n_e \rangle = 1.03$). Rotation is set to zero - a conservative assumption. GLF23 is applied with $\alpha$-stabilization off - equivalent to setting $\alpha \equiv -q^2 \mu_0 \frac{B^2}{\rho R_0} = 0$ within the GLF23 model. The sensitivity to this particular setting, which leads to more optimistic predictions compared with $\alpha$-stabilization on, is mentioned in section 3.4.1. The pedestal temperature following the L-H transition is prescribed, and the top of the pedestal is set at $x = 0.92$. This same pedestal width is prescribed throughout all the simulations. This choice allows a comparison of the effect - at a constant pedestal height - of various heating and current drive combinations on the core confinement and $q$-profile evolution, while altering neither the temperature boundary conditions for GLF23 at the pedestal top nor the total value of the edge bootstrap current. The helium density profile is obtained in CRONOS by the solution of a 1D diffusion equation, where the helium diffusivity is chosen as $D_{He} = \max(\chi_e, \chi_i)$, with the anomalous component of $\chi_e, \chi_i$ calculated by GLF23. The recycling source is adjusted such that we impose the He particle confinement time to global energy confinement time ratio as $\tau_{He}/\tau_E = 5$. Impurities are tied to the $n_e$ profile, at assumed fractions of 2% Be and 0.12% Ar. This prescription leads to typical $Z_{eff}$ values of $\sim 1.7$. MHD effects are entirely ignored in these simulations, which excludes both the potential deleterious transport effects of NTM’s and beneficial anomalous current transport effects which could in principle facilitate the maintenance of $q > 1$.

GLF23 calculates the anomalous transport in the core for radii inside the pedestal top, i.e. $x < 0.92$. However, at lower radii, within the approximate region of $x < 0.25$, no anomalous transport is predicted by GLF23, as the $T_{e,i}$ profiles retreat below the critical gradients due to the zero-derivative boundary condition on the magnetic axis. In order to correct this feature, which is not compatible with experimental observations, a moderate degree of anomalous transport is typically assumed, and in our case is set to $\chi_{ia,ea} = 0.5 \ m^2 \ s^{-1}$ throughout the simulations, to which the neoclassical transport is then also added. Sensitivity to this assumption is discussed in section 3.4.2, but in anticipation we note here that the volume within $x = 0.25$ corresponds to only $\sim 7.5\%$ of the total plasma volume, and a critical degree of fusion performance sensitivity is not found even if the transport is set to neoclassical levels within this region. However, the assumptions do have a significant effect on the inner $q$-profile.

Another factor which may affect the inner $q$-profile predictions is the breakdown, in the near region of the magnetic axis, of the thin banana orbit assumption intrinsic in the NCLASS resistivity and bootstrap current calculations. In this work, the NCLASS predicted resistivity and bootstrap current profiles are taken as calculated, without correction, in spite of the reduced validity of the assumptions in the magnetic axis region. On the magnetic axis itself the resistivity is calculated with a cubic spline extrapolation.
of the NCLASS predicted profile. The bootstrap current on the magnetic axis is zero. In addition, we do not calculate the self-driven asymmetric current [38], which could further increase the current density in the magnetic axis region, lowering the inner $q$-profile.

The present study makes assumptions on the heating mix that are beyond what is projected for the ITER baseline, since one of the aims of the study is to indicate which realistic system upgrade could be applied in order to optimise the performance of the ITER hybrid scenario. The H/CD mixes assumed are comprised from the following components: 0-50 MW NBI (1 MeV, full off-axis injection [39]), 0-50 MW ECRH (equatorial launchers, angles varied between $20^\circ - 45^\circ$ [40]), 0-20 MW ICRH (53 MHz, 2nd T harmonic), and 0-40 MW LHCD (5 GHz, $n_\parallel = 2$). The LHCD directivity is set at 100% (i.e., the full LH power is contained in the co-current lobe of the injected spectrum at $n_\parallel = 2$). This optimistic assumption may lead to an overestimation of the LHCD driven current, as discussed in section 3.3.3.

3.2.4 Pedestal heights

In a stiff model such as GLF23, the fusion $Q$ is typically a strong function of the pedestal temperature [24]. In light of the uncertainty in extrapolating the pedestal width and thus height to ITER [41], we carry out the optimization described above for a range of pedestal heights between 3-5 keV (set at $x = 0.92$). The sensitivity of the hybrid scenario to the pedestal height is twofold: lower core temperatures necessitate an increase of $n_e$ and thus $I_p$ in order to obtain $P_{fus} = 350$ MW; furthermore, at lower pedestals, the bootstrap current - primarily concentrated in the pedestal - would decrease, lowering the non-inductive current fraction and raising the Ohmic current further, with a deleterious consequence on our $q > 1$ goal.

3.2.5 L-mode treatment

The simulations presented in this work run between either 40-1200 s or 40-3000 s. The energy transport during the 40-100 s ramp-up phase is calculated by GLF23, in conjunction with a $T_i$ and $T_e$ boundary condition of typically ~700 eV set at $x = 0.92$, such that the $H_{98}$ factor during the ramp-up ranges between 0.4 and 0.6. $I_p$ is ramped linearly from 4-11.5 MA, with a ramped $n_e$ maintaining $f_G \approx 0.5$. Different ramp-up scenarios with various H/CD mixes and waveforms were carried out, such that a scan of various initial $q$-profile conditions at the flat-top may be performed. An example of one such scenario is shown in Fig.3.1. The input powers in this scenario is comprised of a slow ramp of electron cyclotron injection up to a total of 17 MW at the L-H transition, with a quicker ramp to 37 MW at 140 s. 33 MW of neutral beams is initiated at the end of the ramp-up phase. In such a manner we obtain $P_{loss} = P_{NBI} + P_{EC} - P_{Brem} - P_{cyclo} - 1/3P_{rad} > P_{L-H}$ at 100 s. The $1/3$ factor preceding $P_{rad}$ is an approximation to take into account only the radiative loss power (without Bremsstrahlung and cyclotron radiation loss terms) emitted at lower radii than the pedestal. The L-H transition is treated by initiating the full pedestal prescription at the ordained L-H transition time (usually 100 s), while simultaneously ensuring that $P_{loss} > P_{L-H}$ at the transition. $P_{L-H}$ is taken from an empirical scaling
law [42], and is typically \( \sim 45 \) MW at the transition. Following the L-H transition, \( n_e \) is rapidly ramped up to its flat-top value within 20 seconds. The H/CD sources are shifted to their flat-top values within 40 seconds. In some simulations the \( I_p \) values are also shifted to their final flat-top values within 40 seconds, if it is found that the necessary \( I_p \) for optimum flattop parameters differs from the 11.5 MA value set at the L-H transition.

Self-consistency during the ramp-up with regard to the ITER \( l_i(3) \) operational range of 0.65-1.2 [43] is also examined. It was found that an L-H transition before the conclusion of the \( I_p \) ramp may cause an \( l_i \) drop below the operational limit, as seen in Fig.3.2, due to an inordinate amount of current building up at high radii with slow inward diffusion due to the high temperatures.

This L-mode treatment is not intended as a definitive study of the ITER hybrid scen-
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Figure 3.2: Comparison of $l_i(3)$ evolution for 2 otherwise identical ramp-up scenarios, separated by the time prescribed for the L-H transition. The red (solid) curve, corresponding to $t_{L-H} = 100$ s, has a small kink at 100 s, due to the relatively quick redistribution of the edge current profile following the L-H transition. The blue (dashed) curve, corresponding to $t_{L-H} = 80$ s, drops to levels significantly below the $l_i$ operational limit, peaking at 100 s, due to continued current ramp-up during the H-mode phase, during which the current diffusion time is higher.

Figure 3.3: Otherwise identical ramp-up scenarios separated by the initial prescribed $l_i(3)$ condition, which sets the initial prescribed current profile at 40 s from which the current diffusion commences.
3.3 Results

Numerous simulations have been carried out in this study, systematically surveying the various H/CD mixes. For brevity, we will first present the optimum H/CD mix results for the various pedestal heights, and then outline the trends that have led to the optimum, by comparing a selected number of various H/CD mixes at the same pedestal height. Unless otherwise stated, the various simulations presented all share the same L-mode H/CD strategy: a gradual EC ramp up to 17 MW, and a rapid NBI ramp up to 33 MW just before the L-H transition at 100 s.

3.3.1 Optimum cases for various pedestal heights

The H/CD mix of NBI and EC was found to provide the best combination of a high degree of non-inductive current drive, due to the NBI, as well as $q$-profile shaping capability for core confinement maximization, through the deposition radius flexibility inherent in the EC system. This will be discussed more in detail in section 3.3.2. The simulation results for the optimum $T_{ped} = 4$ keV case are presented in detail in Figs.3.4-3.5. This case will henceforth be regarded as the ‘reference case’. The $T_{ped} = 5$ keV case is outlined in Fig.3.6, and the $q$-profile evolution from the $T_{ped} = 3$ keV case in Fig.3.7. Each of the cases was designed to approximately reach the target of $P_{fus} = 350$ MW. The global parameters of these simulations are summarized in table 3.1. For comparison, results of a pedestal scan with all other parameters remaining identical to the $T_{ped} = 5$ keV case are also displayed in table 3.2.

In the $T_{ped} = 5$ keV optimum case, $q > 1$ at stationary state, and the lower amount of ECCD compared to the $T_{ped} = 4$ keV optimum case is deposited at a more inner radius to lock $q_{min}$ towards 1. In the $T_{ped} = 4$ keV optimum case, $x(q = 1) = 0.03$ even at stationary state. There is also a local minimum in the $q$-profile at $x=0.35$, where $q=1.005$ at stationary state. In the $T_{ped} = 3$ keV case, $x(q = 1) = 0.44$ at 1200 s, which is unsatisfactory for our target ITER hybrid scenario as defined in section 3.2.1.

The $T_{ped} = 4$ keV case is marginal for the $P_{loss} < 110$ MW constraint. A high T, low n discharge with a high $P_{aux}$ for current drive maximizes the non-inductive current fraction, and may be critical for the hybrid scenario, but may be at odds with the requirement of maintaining a sufficiently low $P_{loss}$. Increased margin in the operational space is obtained with more optimistic pedestal assumptions.

The flattop loop voltage of the $T_{ped} = 5/4/3$ keV cases is $\sim 21.5/31/49.8$ mV respectively, calculated as an average up to 1200 s. The increasing loop voltage is due to progressively lower $T_e$ and higher $n_e$ (and thus $I_p$), as well as reduced bootstrap current at the edge. For the 4 keV and 5 keV cases particularly, this loop-voltage may be sufficiently low to allow a flattop of up to 3000 s, provided that resistive flux consumption is saved during the ramp-up phase by application of early heating [44, 46].

Note that the differences in $H_{98}$ in the cases outlined above are primarily due to the different prescribed pedestal heights. In the subsequent comparisons, we will observe the more interesting cases of comparing the $H_{98}$ obtained with different H/CD mixes under the same pedestal assumption, where the differences in confinement result from first-
3 Impact of heating and current drive mix on the ITER Hybrid Scenario

Figure 3.4: Total powers and currents of the optimized $T_{\text{ped}} = 4$ keV simulation (the reference case). In the left panel we can see the evolution of the total current, bootstrap current, neutral beam driven current, and electron cyclotron driven current. The right panel displays the evolution of the $\alpha$-power, electron cyclotron injected power, neutral beam injected power, and the total emitted radiation power sink.

Figure 3.5: Various plasma profiles for the optimized $T_{\text{ped}} = 4$ keV simulation (the reference case) at $t = 3000$ s: electron temperature, ion temperature and the electron density profiles (left panel); total current, bootstrap current, current driven by 33 MW neutral beam injection, and current driven by 37 MW electron cyclotron wave injection (central panel); evolution of the $q$-profile (right panel).

principle based physics predictions of the anomalous transport $q$-profile dependence.

The absolute results, under our various assumptions and transport model applied, state that a satisfactory hybrid scenario may be achieved - where even at stationary state $x(q = 1) = 0.03$ - with a pedestal of 4 keV (assuming a EC upgrade to 37 MW). For a pedestal of 5 keV a $q > 1$ profile can be maintained at stationary state with the currently designed ITER H/CD mix. Note that for this $T_{\text{ped}} = 5$ keV case we retain some margin for achieving a scenario with both $q > 1$ and $P_{\text{fus}} > 350\,\text{MW}$. In a further run carried out, increasing $I_p$ and $f_{GW}$ in the 5 keV simulation to 11.7 MA and 0.95 respectively, leads to a result of $q_{\text{min}} = 1.04$ and $P_{\text{fus}} = 412\,\text{MW}$ ($Q = 8.2$) at 1200 s, at near
3.3 Results

Table 3.1: Summary of results for optimum performance scenarios for various $T_{\text{ped}}$ at the target $P_{\text{fus}} = \sim 350$ MW. All evolving parameters such as $Q$ and $x(q = 1)$ are quoted at $3000$ s for the $T_{\text{ped}} = 4, 5$ keV cases, and at $1200$ s for the $T_{\text{ped}} = 3$ keV case.

<table>
<thead>
<tr>
<th>$T_{\text{ped}}$</th>
<th>$I_{\text{p}}$</th>
<th>$I_{\delta}$</th>
<th>NB/EC</th>
<th>$P_{\text{fus}}/f_{\text{tot}}$</th>
<th>$P_{\text{fus}}$</th>
<th>$\beta_N$</th>
<th>$B_{\text{in}}$</th>
<th>$P_{\text{loss}}$</th>
<th>$t(q = 1)$</th>
<th>$x(q = 1)$</th>
<th>$\langle s/q \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>11.5</td>
<td>0.9</td>
<td>30/50</td>
<td>0.360/0.62</td>
<td>365</td>
<td>7.2</td>
<td>2.15</td>
<td>1.23</td>
<td>99</td>
<td>-</td>
<td>0.74</td>
</tr>
<tr>
<td>4</td>
<td>11.8</td>
<td>0.95</td>
<td>55/77</td>
<td>0.310/0.59</td>
<td>351</td>
<td>5</td>
<td>2.02</td>
<td>1.08</td>
<td>114</td>
<td>0</td>
<td>0.62</td>
</tr>
<tr>
<td>3</td>
<td>12.2</td>
<td>0.95</td>
<td>16/50</td>
<td>0.260/0.47</td>
<td>348</td>
<td>3.2</td>
<td>1.82</td>
<td>0.98</td>
<td>109</td>
<td>0.44</td>
<td>1.06</td>
</tr>
</tbody>
</table>

Figure 3.6: Various plasma profiles for the optimized $T_{\text{ped}} = 5$ keV simulation at $t = 3000$ s: electron temperature, ion temperature and the electron density profiles (left panel); total current, bootstrap current, current driven by $33$ MW neutral beams injection, and current driven by $17$ MW electron cyclotron wave injection (central panel); evolution of the $q$-profile (right panel).

stationary state conditions. For a $3$ keV pedestal, no satisfactory hybrid scenario may be obtained: a $q=1$ surface appears rather early in the discharge from $t = 360$ s, and develops into a broad surface with $x(q = 1) = 0.44$ by $t = 1200$ s, as can be seen in the simulation displayed in Fig.3.7.

The observed evolution of $P_{\alpha}$ in the current diffusion timescale, as can be observed in Fig.3.4 and is present in all simulations carried out with GLF23, is a result of the $q$-profile evolving to a more favourable predicted confinement regime, as discussed in section 3.3.2. This effect was separated and can be clearly seen in Fig.3.8. There, the red (solid) curve corresponds to the $P_{\alpha}$ of the reference case. The blue (dashed) curve corresponds to an identical case, apart from the poloidal flux profile (from which $j$ and $q$ are calculated), which is prescribed to a non-evolving state between $200-600$ s and $620-1200$ s, and evolves between the two states in the intermediate time. The prescribed poloidal flux profiles are that of the reference case at $200$ s and $1200$ s respectively. We see that the predicted time evolution of the fusion performance on the current diffusion timescale in the reference case is a consequence of the dependence of the predicted anomalous transport on the evolving $q$-profile. This observation has ramifications for the ramp-up H/CD strategy, and is discussed in section 3.3.6.

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Table 3.2: $T_{\text{ped}}$ sensitivity scan, with all other prescribed parameters remaining the same as an optimized $T_{\text{ped}} = 5$ keV, $f_G = 0.9$ case (first row) tailored for $P_{\text{fus}} = 350$ MW. All evolving parameters are quoted at 1200 s.

<table>
<thead>
<tr>
<th>$T_{\text{ped}}$</th>
<th>$I_p$ [MA]</th>
<th>$f_D$</th>
<th>NBEC [MW]</th>
<th>$f_{\text{boot}}/f_{\text{ni}}$</th>
<th>$P_{\text{fus}}$ [MW]</th>
<th>$Q$</th>
<th>$\delta_N$</th>
<th>$N_{\text{m}}$ [MW]</th>
<th>$Q_{\beta_N}$ [MW]</th>
<th>$x(q = 1)$</th>
<th>$\langle s/q \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>11.5</td>
<td>0.9</td>
<td>33/17</td>
<td>0.30/0.53</td>
<td>285</td>
<td>5.6</td>
<td>1.86</td>
<td>1.10</td>
<td>82</td>
<td>520</td>
<td>0.06</td>
</tr>
<tr>
<td>4</td>
<td>11.5</td>
<td>0.9</td>
<td>33/17</td>
<td>0.24/0.44</td>
<td>214</td>
<td>4.2</td>
<td>1.58</td>
<td>0.96</td>
<td>73</td>
<td>280</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Figure 3.7: Evolution of the $q$-profile in the best achieved $T_{\text{ped}} = 3$ keV case.

Figure 3.8: Illustration of the evolving $q$-profile affecting the confinement and thus $Q$. The red (solid) curve is the $P_{\alpha}$ of the reference case. The blue (dashed) curve is an identical case, apart from a prescribed poloidal flux profile which alternates between the states of the reference case at 200 s and 1200 s respectively.
3.3 Results

3.3.2 Critical importance of q-profile shaping

The degree of ITG dominated turbulence is predicted to increase with increasing q but decrease with increasing s (for $s - \alpha > \sim 0.5$), as can be seen in Fig.6 of reference [12], and in [47]. This points to the advantage in maximizing the parameter s/q throughout the bulk of the plasma volume where $s > \sim 0.5$, and maintaining flat or weak inverse shear in the inner region where $q \sim 1$. This recipe would remain the same even with $\alpha$-stabilization on, which only introduces a slight shift of the radial position from which $s - \alpha > \sim 0.5$. The s/q parameter can be considered as a figure of merit for the q-profile shape in the region corresponding to $s > \sim 0.5$, and appears explicitly in various approximate formulations of the ITG linear threshold [48, 49]. In the simulations considered here, the $s > \sim 0.5$ region typically corresponds to $x > \sim 0.4$. In the remainder of the region in which GLF23 is operational in our simulations, $x = 0.25 - 0.4$, it is more difficult to quantify the quality of the q-profile shape, but it is clear from the theoretical dependence that maintaining q as close to 1 as possible, and $s \sim 0$, is advantageous. Note that we are considering here only families of q-profiles without strongly reversed shear which may lead to an ITB, and where we constrain $q > 1$ for sawteeth avoidance.

The key to maximizing confinement is thus to maximize the extent of the inner $q \sim 1$ region, which then also leads to a maximization of s/q throughout the outer part of the plasma. This can be achieved with ECCD, whose position of localized deposition can be tailored to ‘pin’ down the q-profile towards 1 at a radius dependent on the EC power employed. The higher the EC power, the further out the q-profile can be pinned down to 1. A scan of EC deposition radius, with the same general parameters as the reference case apart from the EC launcher angles, can be seen in Fig.3.9, illustrating the sensitivity of the scenario performance to the q-profile shaping. The further out the ECCD deposition radius, the higher the q-profile is raised in the region of the deposition. The effect on fusion performance is significant, with a 20% difference between the extreme cases in the scan. The local changes of shape in the q-profile lead to a corresponding shift in the ion temperature gradient lengths. The volume averaged s/q parameter $\langle s/q \rangle$, as displayed in the figures, is compared in the region $x = 0.4 - 0.9$, which corresponds to the region up to the pedestal in which $s > \sim 0.5$. Note that even though the EC power remains constant throughout this scan, the total EC driven current drops as the ECCD deposition radius moves outwards. This is due to both the increasing trapped electron fraction, and the fact that at lower temperatures (resulting in this case primarily from the less optimized q-profiles), the current drive efficiencies decrease.

Optimizing a scenario with an NBI/EC mix of 50/20 MW instead of the reference case of 33/37 MW leads to very similar results, as can be seen in Fig.3.10. The average s/q is reduced in the 50/20 MW case leading to a higher necessary $n_e$ and thus $I_p$ to achieve the same target fusion power. Furthermore, the average $T_i/T_e$ parameter is higher in the 50/20 MW NBI/EC case by $\sim 2\%$ compared to the reference case, also aiding in reaching a similar degree of fusion power in spite of the lower average s/q. The higher NBI driven current leads to a similar value of $I_{ohmic}$, resulting in a similar stationary state q-profile, with t($q = 1$) increased by 23%. However, in terms of flexibility, it would be more advantageous to upgrade the EC system towards 40 MW. For the ITER steady...
3 Impact of heating and current drive mix on the ITER Hybrid Scenario

State scenario for example, it has been shown that use of NBI may degrade the ITB [50], and a significant degree of ECCD power would be required for current profile shaping.

3.3.3 Inclusion of LHCD

Here we examine the effect of departing from our optimum NBI/EC mix. We take the parameters of the reference case, and carry out a scan replacing EC power with LHCD power, as calculated by DELPHINE. The results are seen in Fig. 3.11. LHCD is effective in driving far off-axis non inductive current (at around $x = 0.7$), and this provides a significant broadening of the current profile. However, from the confinement point of view, this broadening leads to a significant decrease of $\langle s/q \rangle$ and thus to a degradation of the energy confinement properties of the discharge with respect to the reference case as pre-

Figure 3.9: Scan of ECCD launcher angles from the equatorial ports. The red (solid) curve corresponds to the reference case, and the blue (dashed) and green (dash-dot) curves correspond to identical cases apart from the launcher angles and thus ECCD deposition radius, which is progressively increased, as seen in the upper-left panel. The $q$-profiles are displayed in the upper-right panel. The ion temperatures and corresponding fusion performances are shown the lower-left panel. Ion temperature gradient lengths are displayed in the lower-right panel.
dicted by GLF23. This can be clearly seen by the reduction in $T_i$ and the corresponding fusion performance, as well as the increase in $L_{T_i}$. The decrease in fusion power in the LHCD case is almost purely due to the now non-optimal $q$ and $s$ profiles, and not due to the reduction in heating at the more inner radii where the ECCD power is deposited. This is a typical consequence of the highly stiff GLF23 model, where pure heating has a minor influence on the temperature profiles, and the primary influence of the H/CD mix on the temperature gradient lengths is through the current drive $q$ and $s$ profile shaping, setting the degree of GLF23 predicted turbulence. This can be clearly seen in Fig. 3.12, where we separate the effects of $q$-profile shaping and heating on fusion performance. Therein, the green (dash-dot) curve begins as the NBI/EC/LHCD case, and at 1160 s is prescribed the same poloidal flux profile (and thus $q$ and $s$ profiles) as the reference case, while maintaining the NBI/EC/LHCD heating deposition profiles. At 1180 s the poloidal flux profile is switched back to the NBI/EC/LHCD case. After each switch, the fusion performance relaxes to its new equilibrium within the thermal confinement timescale. The minor discrepancy between the green (dash-dot) and red (solid) curves between 1165-1180 s is thus the difference caused by the pure heating effect of switching 17 MW of power from $x = \sim 0.3$ (the EC power) to $x = \sim 0.7$ (the LHCD power), a small effect due to the stiffness of the GLF23 model. Thus, the respective impact of $q$-profile shaping and heating on the fusion performance, as predicted by GLF23, is clearly illustrated.

The time between the initial stage of the flat-top and $t(q = 1)$ is increased by $\sim 19\%$ in the 17 MW LHCD case compared to the reference case, due to the significantly broader current profile. This is in spite of the fact that $I_{\text{ohmic}}$ has actually increased by 0.14 MA from 4.86 MA (reference case) to 5 MA (17 MW LHCD case), due to the slightly reduced bootstrap current and NBCD/ECCD efficiencies at lower electron temperatures. Although the total $I_{\text{ohmic}}$ is larger, the $T_e$ and thus conductivity profiles are slightly broader in the LHCD case and the resulting Ohmic current density is slightly lower near the magnetic axis, delaying the occurrence of the $q = 1$ surface. This higher

**Figure 3.10**: Comparison of the reference $T_{\text{ped}} = 4$ keV case (red solid curves) with an optimized 50/20MW NBI/EC case (blue dashed curves). The left panel displays the total currents. The central panel shows the $q$-profiles and volume averaged $s/q$ values. The right panel displays the ion temperatures and corresponding fusion performances.
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Figure 3.11: Effect on fusion performance of LHCD inclusion. The red (solid) curve corresponds to the reference case, the blue (dashed) curve an identical case apart from transferring 17 MW of ECCD to 17 MW of LHCD, and the green (dash-dot) curve an identical case apart from a H/CD mix of 33MW NBI and 37MW LHCD. We can observe the differences in total currents (upper-left panel), $q$-profile (upper-right panel), ion temperature and fusion performance (lower-left panel), and ion temperature gradient lengths (lower right panel).

$t(q = 1)$ would lead to a number of total emitted neutrons until $t(q = 1)$ lower by only 2% in the LHCD case compared to the reference case. However, for the 17 MW LHCD case at $t=1200s$, $P_{\text{fus}}$ is below the defined target by 21%. When setting the $n_e$ and $I_p$ parameters such that we reach towards the target $P_{\text{fus}} = 350$ MW, $t(q = 1)$ and $x(q = 1)$ in the 17 MW LHCD case is noticeably worse than in the reference case, as seen in row 5 in table 3.3. In the 37MW LHCD case, the $q$-profile would remain above unity even at stationary state, in spite of $I_{\text{ohmic}} = 5.07$ MA, an increase of 0.2 MA compared to the reference case. However, at $t = 1200$ s $P_{\text{fus}}$ is only 47% of the target value.

The magnitude of LHCD driven current may have been overestimated due to the 100% directivity assumption. At lower assumed directivity the trends outlined above would still hold, but at a reduced sensitivity to the degree of LHCD power. At lower...
LHCD current, the reduction of $\langle s/q \rangle$ and thus $P_{\text{fus}}$ would be less severe. However, the increase of $t(q = 1)$ would also be diminished. We can compare the above results to simulations employing a transport model without $q$ and $s$ dependence, such as the 'kiauto' model [51]. The kiauto model is used here with a prescribed pedestal, and the core transport coefficients have a fixed profile shape renormalized such that the plasma energy follows the the IPB98($y,2$) scaling law. These results, together with all other simulations carried out at $T_{\text{ped}} = 4$ keV, are summarized in table 3.3. We can see that replacing 17 MW of ECCD with LHCD leads to marginally improved results due to the higher LHCD driven current - an improvement of $t(q = 1)$ from $810$ s to $860$ s, and a reduction of $x(q=1)$ from $0.04$ to $0.03$. However, previous work on the validation of GLF23 on experimental hybrids has been reasonably successful in reproducing the average stiffness ratio $T(x = 0.3)/T(x = 0.8)$ [52], lending confidence to the trends predicted by the GLF23 model particularly given that the variation of the $q$-profile as seen above in the ECCD and LHCD scans is concentrated within this region. Further specific hybrid scenario experiments targeting the effect of $q$-profile shaping on core-confinement would be beneficial in assessing the trends predicted here with GLF23.

We note that even though the replacement of ECCD by LHCD is detrimental for energy confinement with respect to the optimised reference case, LHCD is effective in broadening the current profile. This capability may be needed for the development of some ITER scenarios, and LHCD would add a key feature to the ITER H&CD mix (namely efficient far off-axis current drive). In particular, LHCD may be a vital component for the ITER steady state scenario [50]. ECCD from the ITER upper launcher can also drive far off-axis current, but at total current drive efficiencies lower by a factor of $\sim 7$ [53], compared with the LHCD efficiency predicted by DELPHINE.
Table 3.3: Comparison of various scenarios all with $T_{\text{ped}} = 4$ keV, and $f_{\text{GW}} = 0.95$. All evolving parameters are quoted at 1200 s. The first row corresponds to the reference case to which the others are to be compared to. Run 235 is the 50/20 MW NBI/ECCD case. Runs 352 and 353 are the ECCD launcher angle scan, runs 255, 256, and 346 the cases with LHCD. Run 256 has slightly different parameters to the reference case in order to boost $P_{\text{fus}}$ towards 350 MW (which leads to a worsened $t(q = 1)$ and $x(q = 1)$). Run 270 is the ICRH case. Runs 339 and 340 are the cases with the scaling transport model, where inclusion of LHCD leads to a marginal improvement of the scenario.

<table>
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<tr>
<th>Transport model</th>
<th>$I_p$ [MA]</th>
<th>NBI [MW]</th>
<th>EC [MW]</th>
<th>EC launcher angles [deg]</th>
<th>LHCD [MW]</th>
<th>IC [MW]</th>
<th>$P_{\text{fus}}$ [MW]</th>
<th>$\beta_u$</th>
<th>$H_{\text{aux}}$ [G]</th>
<th>$t(q = 1)$</th>
<th>$x(q = 1)$</th>
<th>$\langle s/q \rangle$</th>
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<td>33</td>
<td>37</td>
<td>34.36,37</td>
<td>346</td>
<td>2.00</td>
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<td>0</td>
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<td>0.81</td>
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<td>20</td>
<td>34,36,37</td>
<td>17</td>
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<td>0.66</td>
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3.3.4 Dependence of $P_{\text{fus}}$ on $\langle s/q \rangle$

In Fig.3.13 we summarize the predicted dependence of $P_{\text{fus}}$ on $\langle s/q \rangle$ in the ECCD and LHCD simulations described above, for the subset in which $I_p$, $f_{\text{GW}}$, and total $P_{\text{aux}}$ are kept constant. This set is comprised of the reference run, and runs 352, 353, 255 and 346 as seen in table 3.3. Runs including ICRH in the H/CD mix, as to be described in section 3.3.5, are not included in this dataset, since with on-axis ICRH we heat the region of the plasma ($x < 0.25$) where the anomalous transport is prescribed as a constant value, which leads to raised temperatures since the temperature profiles are therefore not stiff in that region. By concentrating the dataset on the ECCD and LHCD scans, where the deposition is primarily in the region $x > 0.25$ where GLF23 is operational, we isolate the effect of the GLF23 predicted dependence of transport on the $q$-profile shape. On the left panel of the figure, the data points in the lower range of $P_{\text{fus}}$ and $\langle s/q \rangle$, with $\langle s/q \rangle, P_{\text{fus}} = (0.64, 273 \text{MW})$, and $\langle s/q \rangle, P_{\text{fus}} = (0.37, 163 \text{MW})$, correspond to the runs with 17MW and 37MW of LHCD respectively, which underline the deleterious nature of LHCD on the $q$-profile and subsequent predicted transport. The strong correlation between $\langle s/q \rangle$ and $P_{\text{fus}}$ is a consequence of the linear relationship between $s/q$ and the normalized inverse ion temperature gradient scale length, $R/L_{T_i}$, which can be seen in the right panel of the figure. This linear relationship can be expected from ITG dominated turbulence in the flat density limit, particularly since in our case neither the $T_i/T_e$ nor the $Z_{\text{eff}}$ profiles vary by more than 5% between the various simulations in the dataset, further isolating the instability threshold dependence on the $q$-profile.
3.3 Results

Figure 3.13: \( \langle s/q \rangle \), the volume average of \( s/q \), and \( <R/L_{Ti}> \), the volume average of \( R/L_{Ti} \), are calculated at \( t = 1200 \) s between \( x = 0.4 \) − 0.9 (the region where GLF23 is operational and \( s > 0.5 \)).

### 3.3.5 Inclusion of ICRH

Here, we replace 17 MW of EC from the reference case with 17 MW of ICRH (2nd T harmonic). The results are shown in Fig.3.14, where we can see both a marked decrease in \( t(q = 1) \) to 420 s, and an increase in \( x(q = 1) \) at 1200 s to 0.13. The reason for this is twofold: (i) no current is being driven by the ICRH scheme assumed here, (ii) the central heating raises \( T_e \) such that the resistivity and thus current profile is sharper, decreasing \( q \) near the magnetic axis. Even though GLF23 is a highly stiff model, the temperatures are raised in the center with the on-axis ICRH due to the fact that GLF23 does not predict any anomalous transport within \( x < 0.25 \), and a constant \( \chi_{\text{a,ea}} \) has been set in that region, as explained in section 3.2.3. The constant transport coefficient, the added ICRH heating source indeed leads to a rise in temperatures and fusion \( Q \) (an increase of \( \sim 6 \% \)), which compensates the loss of \( q \)-profile shaping from the missing ECCD, but also has the deleterious effect of sharpening the current profile. However, even if the transport would have been stiff throughout, the addition of ICRH would then only serve to remove the needed ECCD without any advantageous effect on performance.

An advantage which may be expected for on-axis ICRH in the hybrid scenario is direct ion heating (on top of the strong electron-ion collisional energy exchange), which would provide higher \( T_i \) and thus fusion reactivity. This effect would only be significant if the plasma temperature profiles were not strictly stiff, which can be expected [54, 55]. In addition, in a minority \( He^+ \) ICRH scheme, suprathermal fuel ion tails may develop, also boosting fusion reactivity [56]. It remains to be seen whether such performance improvements would compensate the lack of current drive that is vital in the hybrid scenario for both reducing the non-inductive current fraction and shaping the \( q \)-profile.

The differences in average flattop loop-voltage between the reference case, the 33/20/17 NBI/EC/LHCD, and the 33/20/17 NBI/EC/IC cases is not significant, being 31/32.3/34.5 mV respectively.
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Figure 3.14: Effect on \(t(q=1)\) and \(x(q=1)\) of ICRH inclusion. The red (solid) curve corresponds to the reference case, and the blue (dashed) curve to the case where 17 MW of EC is replaced by 17 MW of ICRH. The \(T_e\) profiles are displayed in the left panel, the total current profiles in the central panel, and the \(q\)-profiles in the right panel.

Figure 3.15: Effect of initial \(l_i\) on the \(P_{\alpha}\) evolution. The three cases outlined are all an H/CD mix of 33NBI/17EC/18.5LHCD, with \(T_{ped} = 5\) keV. They differ only by the ramp-up strategy, which results in different initial \(q\)-profile conditions (affiliated with \(l_i\)) at the beginning of the flattop. The various \(l_i\) evolutions are seen in the left panel, while the right panel displays the \(P_{\alpha}\) evolution.

3.3.6 Initial \(l_i\) scan for \(q\)-profile matching

As mentioned in section 3.3.1, \(P_{\alpha}\) evolves on the current diffusion timescale as the Ohmic current diffuses inwards and raises \(\langle s/q \rangle\) throughout the bulk of the discharge, improving confinement and raising the temperatures. The average \(Q\) throughout the discharge is then dependent on the \(q\)-profile at the beginning of the flat-top phase, whereas the closer the initial \(q\)-profile to the optimised final relaxed state, the higher the average \(Q\). This is illustrated in Fig.3.15, where a scan of initial \(q\)-profiles is carried out for simulations with otherwise identical parameters, with a H/CD mix of 33NBI/17EC/18.5LHCD (with LUKE). For an optimized hybrid scenario where the final relaxed \(q\)-profile is satisfactory
3.4 Sensitivity studies

We have examined the sensitivity of the absolute predictions to the following assumptions: $\alpha$-stabilization, value of prescribed centre $\chi_{ia,ea}$, and $Z_{eff}$. In addition, we have examined cases with a prescribed peaked density profile as opposed to the flat density profile. Sensitivity to the assumptions regarding the He ash diffusivity and recycling, and to the plasma rotation, were not carried out here, but have been well studied for ITER in other works [23, 24, 57].

3.4.1 $\alpha$-stabilization

The reference case was rerun with $\alpha$-stabilization set on in the GLF23 model, which led to a reduction in fusion performance by $\sim 15\%$. Including $\alpha$ leads approximately to the effective replacement of $s$ with $s - \alpha$ within GLF23, increasing the degree of predicted anomalous transport. This is typical for monotonic $q$-profiles without an ITB, where $s > 0.5$ throughout the bulk of the plasma. Other H/CD mixes were also rerun with $\alpha$-stabilization on, and the relative trends reported in the previous sections remain identical.

3.4.2 Sensitivity to inner prescribed $\chi_{ia,ea}$

In section 3.2.3 we discussed that GLF23 does not predict anomalous transport within $x < 0.25$, and that we assumed a constant value of $\chi_{ia,ea} = 0.5 \text{ m}^2\text{s}^{-1}$ within this region. The region between $x = 0.25$ and the pedestal contains the majority of the fusion power in ITER, as seen in Fig.3.16, thus the sensitivity of the assumptions in the $x < 0.25$ region is not expected to be critical with regard to the fusion performance. This was examined by carrying out a systematic scan of $\chi_{ia,ea}$ for the reference case. We display the results in Fig.3.17. We can see that indeed, the $P_{fus}$ sensitivity to the assumptions is quite low. However, the $q$-profile in that region is highly sensitive to the assumption on $\chi_{ea}$, particularly since electron neoclassical transport is more than an order of magnitude lower than ion neoclassical transport (whereas the total transport $\chi_{i,e} = \chi_{ineo,eneo} + \chi_{ia,ea}$). We can see that $t(q = 1)$ and $x(q = 1)$ are highly effected by $T_e$ within the sensitive region, which sets the degree of current profile sharpness. The 'worst-case' scenario is run 5, that corresponds to a case with no prescribed anomalous transport in the sensitive region $x < 0.25$, which drops down to neoclassical levels towards the axis. In run 5, $t(q = 1)$ drops to 400 s, and $x(q = 1)$ increases to 0.19. Comparison of runs 2 and 3 show that the sensitivity of $t(q = 1)$ and $x(q = 1)$ to the $\chi_{i}$ assumption is low. For values of $\chi_{ea} > 0.5 \text{ m}^2\text{s}^{-1}$, the sensitivity is highly reduced as the $T_e$ profile flattens.
3 Impact of heating and current drive mix on the ITER Hybrid Scenario

Figure 3.16: $P_\alpha$ per unit normalized toroidal flux coordinate (for the reference case at 3000 s), following multiplication by the Jacobian (volume element). We can see that the peak power is at $x = 0.36$. The vertical lines split the profile into four equal quarters of fusion power.

3.4.3 Sensitivity to $Z_{eff}$

A $Z_{eff}$ scan was carried out by changing the assumptions of Ar and Be impurity concentrations (while maintaining the same assumed Ar/Be ratio). A $Z_{eff}$ of 1.35 or 2.07 as opposed to the standard assumption of 1.67 leads to a $P_{fus}$ enhancement of +20% or reduction of -20% respectively, due primarily to the degree of impurity dilution of the DT fuel. We have assumed that the Ar and Be impurity profile shapes are identical to the (flat) electron density profile shape. Therefore, no significant alteration to the bulk ion density gradients occurred during this particular $Z_{eff}$ scan, which could in itself have affected the GLF23 predictions.

The ion and electron heat flux profiles differed throughout the $Z_{eff}$ scan, due to variations in quantities such as $P_{brem}$, $P_{rad}$, and $P_{fus}$. However, this did not have an appreciable effect on the electron and ion temperature profiles, due to the high degree of stiffness inherent in GLF23. Application of a less stiff transport model would thus increase the sensitivity of the fusion power performance to $Z_{eff}$, due to the variation of the ion temperature profile, in addition to the DT fuel dilution.

3.4.4 Prescribed peaked density profile

Cases with a $\times 1.5$ prescribed peaked density profile were examined for the optimum NBI/ECCD mix. The $\times 1.5$ peaking factor is predicted by $\nu^*$ scaling experiments [58]. Both linear and quadratic $n_e$ profile shapes were assumed, and the pedestal pressure was kept the same as the reference $T_{ped} = 4$ keV case, which resulted in $T_{ped} = 5$ keV for the peaked $n_e$ cases. $f_{GW}$ was kept at 0.95, and the EC powers and injection angles were varied such that the cases were optimized at the $P_{fus} = 350$ MW target with $Q > 5$. 
The results are shown in Fig.3.18. We can see that even though transport is slightly increased by the finite density gradient, the higher pedestals more than compensate and improves the scenario performance. Only 17 MW of EC power is sufficient for a satisfactory scenario. However, for the quadratic profile shape, this improvement is marginal due to the larger density gradients, particularly towards the edge, leading to higher transport. For even higher gradients, for example with assumed $x^2$ density peaking, the scenario performance may even be eroded. This high degree of sensitivity of predicted heat transport to the inverse density gradient length above a certain threshold is typical of GLF23 predictions with prescribed density profiles. For regions with sufficiently high inverse density gradient lengths, completely flat temperature profiles may even be predicted. For the simulations carried out in this work, the density gradients are assumed to be flat or moderate already from directly within the pedestal region. Therefore, such overly pessimistic temperature profile predictions just inside the pedestal do not occur.

![Figure 3.17: Sensitivity to prescribed inner $\chi_{i,a,ea}$ (in units of $m^2 s^{-1}$). The upper-left and upper-right panels illustrate the various pairs of $\chi_{i,a}$ and $\chi_{ea}$ assumptions, with the consequences on $T_e$ and $T_i$ shown on the bottom-left and bottom-right panels respectively.](image-url)
Impact of heating and current drive mix on the ITER Hybrid Scenario

Figure 3.18: Effect of density peaking. The red (solid) curve is the reference case with a flat electron density profile, the blue (dashed) curve a case with a linear \( \times 1.5 \) peaked density profile, and the green (dash-dot) curve a quadratic \( \times 1.5 \) peaked density profile. The various density profiles are seen on the left panel. Total pressure profiles are displayed in the central panel. The electron heat diffusivity profiles are seen on the right panel.

3.5 Discussion and conclusions

An extensive study of the H/CD mix for the ITER hybrid scenario has been performed, with the GLF23 anomalous transport model predicting transport in the energy channel, and for a range of prescribed pedestals. According to GLF23, the key point for scenario optimization is tailoring the \( q \)-profile with a judicious choice of current drive, in order to minimize the predicted \( q \) and \( s \) dependent transport. This then allows a minimization of \( n_e \) and thus \( I_p \) needed for the scenario target \( P_{fus} = 350 \) MW, raising the \( q \)-profile towards 1 as desired in a hybrid scenario. Note that for a hybrid scenario with no ITB, we have excluded the examination of cases with highly reversed shear. The recipe for such optimization is to maximize the volume of the inner low shear \( q \sim 1 \) region, which both improves confinement in itself, and also maximizes the value of the \( s/q \) profile in the remaining bulk of the plasma where \( s - \alpha > 0.5 \), which further reduces transport.

The optimum H/CD mix towards this end was found to be an NBI/EC mix. With respect to this optimal mix, the replacement of about half of the ECCD power by LHCD delays \( t(q=1) \) by 15%, but at a price of significantly worse confinement due to the far off-axis current deposition, which decreases the outer \( s/q \) values and raises the inner \( q \) values. The dependence of the total \( P_{fus} \) on the volume averaged \( <s/q> \) in the region \( x = 0.4 \)–0.9 (where GLF23 is operational and \( s >\sim 0.5 \)) was found to be highly correlated, due to the linear relationship between \( <s/q> \) and \( <R/L_{Ti}> \). Pure heating alone does not affect \( T_{pe} \) to a large degree, due to the extreme stiffness of the GLF23 model. Therefore, in the context of this transport model, 2nd T harmonic ICRH found not to be beneficial to the hybrid scenario, primarily due to the lack of current drive. For \( T_{ped} = 5 \) keV, \( q > 1 \) was predicted to be maintained at stationary state with a 33/17 MW NBI/ECCD mix, with \( Q = 7 \). For a more conservative \( T_{ped} = 4 \) keV, \( x(q = 1) = 0.03 \) was predicted at stationary state with a 33/37 MW NBI/ECCD mix, with \( Q = 5 \). This strongly informs
3.5 Discussion and conclusions

the choice of ITER H/CD upgrades towards increasing the power of the ECRH/ECCD system. Similar performance was achieved for a 50/20 MW NBI/ECCD mix. For $T_{\text{ped}} = 3 \text{ keV}$, no satisfactory hybrid scenario was predicted, with $x(q = 1) = 0.44$ at 1200 s. The $q$-profile dependence of the transport also points to the necessity of matching the $q$-profile at the beginning of the flat-top to the final relaxed $q$-profile, as long as the final $q$-profile is optimised for the hybrid scenario, in order to maximize the average $Q$ throughout the discharge. Comparison with experimental data which specifically aims to test the effect of $q$-profile shaping on core confinement is needed, in order to gauge the veracity of the importance of this effect predicted by GLF23.

The GLF23 predictions reported here do not succeed in extrapolating the high $\beta_N$ hybrid scenarios achieved in present-day tokamaks to ITER. In order to further increase $\beta_N$ towards these values it would be necessary in these simulations to either increase $n_e$ (and thus $I_p$) to values that would result in $q < 1$, or to increase $T_{\text{ped}}$ by an unreasonably high degree. However, under the assumptions outlined in section 3.2.3, the hybrid scenarios predicted here are able to achieve the ITER hybrid scenario primary mission as defined in section 6.1, for $T_{\text{ped}} \geq 4 \text{ keV}$. Sensitivity scans of these various modelling assumptions were carried out, and numerous uncertainties exist which could each potentially alter the fusion performance by $\sim \pm 20\%$. In this light, it is clear that our absolute predictions are still heavily influenced by the assumptions. However, the strength of the approach taken in this work is that the significant performance differences between the various H/CD settings and mixes are dependent primarily on the physical ITG/TEM/ETG microturbulence picture and the predicted $q$ and $s$ dependence of the transport. The trends observed towards hybrid scenario core confinement optimization should hold regardless of the specific transport model, as long as these models are based upon these same first principle physics. The predicted relative core confinement improvement achieved by careful $q$-profile shaping, as illustrated for ITER in this work, should hold in spite of the high number of assumptions made which strongly affect the absolute predictions.

Acknowledgments

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4 Predictive analysis of q-profile influence on transport in JET and ASDEX Upgrade hybrid scenarios

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\textsuperscript{*}See the Appendix of F. Romanelli et al., Proceedings of the 23rd IAEA Fusion Energy Conference 2010, Daejeon, Korea

Abstract

Hybrid scenarios in present machines are often characterized by improved confinement compared to the IPB98(y,2) empirical scaling law expectations. This work concentrates on isolating the impact of increased $s/q$ at outer radii (where $s$ is the magnetic shear) on core confinement in low-triangularity JET and ASDEX Upgrade (AUG) experiments. This is carried out by predictive heat and particle transport modelling using the integrated modelling code CRONOS coupled to the GLF23 turbulent transport model. For both machines, discharge pairs were analyzed displaying similar pedestal confinement yet significant differences in core confinement. From these comparisons, it is found that $s/q$ shaping at outer radii may be responsible for up to $\sim 50\%$ of the relative core confinement improvement observed in these specific discharges. This relative improvement is independent of the degree of rotational shear turbulence suppression assumed in the GLF23 model. However, employing the full GLF23 rotational shear model leads to an overprediction of the ion temperatures in all discharges analyzed. Additional mechanisms for core confinement improvement are discussed and estimated. Further linear threshold analysis with QuaLiKiz is carried out on both pairs of discharges. This work aims to validate recent predictions of the ITER hybrid scenario also employing CRONOS/GLF23, where a high level of confinement and resultant fusion power sensitivity to the $s/q$ profile was found.

4 Predictive analysis of q-profile influence on transport in JET and ASDEX Upgrade hybrid scenarios

4.1 Introduction

Over the past decade, an attractive operating scenario for the ITER tokamak has emerged, combining long discharge times similar to the steady-state scenario, while maintaining the reliability of the reference H-mode scenario. This so-called 'hybrid' scenario aims to maximize neutron fluence per shot by achieving an extended burn time ($t > 1000$ s in ITER) together with significant $\alpha$-particle heating ($Q > 5$) [1]. The extended burn time is achieved by operation at reduced current and at a higher non-inductive current fraction compared to the reference H-mode scenario. The basis for this scenario has been established at DIII-D [2], AUG ('improved H-modes') [3], JET [4], JT-60 [5], and NSTX [6].

In present tokamaks, the hybrid scenario is characterized by a q-profile tailored to form a broad region of low magnetic shear, with $q_0$ typically close to unity. Operation at a $\beta_N$ higher than in standard H-mode discharges is allowed by either a total avoidance of deleterious NTM triggering sawteeth ($q > 1$ everywhere), or by having sawteeth small or frequent enough such that the $\beta_N$ limit for NTM triggering is significantly raised compared to standard H-mode discharges. These discharges frequently display improved confinement compared to the IPB98(y,2) empirical scaling law expectations, with $H_{98}$ in the range of $1 - 1.5$. $H_{98} \equiv \tau_{th}/\tau_{IPB98(y,2)}$, where $\tau_{th}$ is the thermal confinement time and $\tau_{IPB98(y,2)}$ the empirical scaling law confinement time introduced in [7]. Assuming the same $H_{98}$ factor as in present experiments, this performance extrapolates favourably in ITER, beyond the minimum $Q = 5$ ITER hybrid scenario requirement [8–10]. It is clearly of great importance to understand the source of improved confinement in present machines, to reliably extrapolate the scenario to ITER.

A number of possibilities explaining the confinement improvement have been proposed. One possibility is that the confinement dependence on $\beta$ is not as strong as suggested by the IPB98(y,2) $\beta^{-0.9}$ scaling. The IPB98(y,2) scaling law was developed with discharges primarily in the range $\beta_N = 1 - 2$, lower than most hybrid discharges. Thus, if the $\beta$ dependence would be in reality weaker, then the high $\beta_N$ hybrid scenarios would display significantly higher confinement than expected from the IPB98(y,2) scaling. Indeed, a weaker $\beta$ scaling than IPB98(y,2) has been found in dedicated experiments in DIII-D and JET [11, 12]. Furthermore, pedestal confinement improvement due to an increase of pedestal width with $\beta_N$ is reported in AUG improved H-modes [13, 14]. However, in more recent experiments at JET, the $\beta_N$ confinement scaling has been observed to depend on triangularity [15]. Low triangularity discharges display a weak $\beta_N$ dependence, while high triangularity discharges display a strong edge confinement degradation with $\beta_N$ in accordance with the IPB98(y,2) scaling.

Improved core confinement is seen in DIII-D hybrids and 'late-heating' AUG improved H-mode discharges, with electromagnetic gyrokinetic calculations showing that $\beta$-stabilization may be responsible for the core confinement improvement [14]. Rotation is seen to play an important role in improved confinement. Variable torque experiments at DIII-D by simultaneous application of co and counter beams led to a decrease of $H_{98}$ from 1.5 to 1.1 in the extreme low rotation case (central $M \approx 0.15$) [16]. However, this $H_{98}$ factor is still as high as in high rotation conventional H-mode plasmas. This shows that further factors are also responsible for core confinement improvement in hybrid scen-
arios, since the hybrid scenario at low rotation should exhibit a higher $H_{98}$ factor than standard H-mode scenarios at low rotation. The precise mechanism of rotational shear core confinement improvement is also currently under investigation. Observations at JET point to the combination of low magnetic shear and high rotational shear leading to significantly reduced ion temperature profile stiffness [17, 18]. This is in opposition to the instability threshold offset observed in non-linear simulations and reproduced in reduced models such as GLF23 [19], which describe well the effect of rotation particularly for DIII-D [16]. Finally, the characteristic hybrid scenario $q$-profile shape - a central broad region of low magnetic shear, and increased magnetic shear towards the edge - may further contribute to improved confinement through a relative increase in $s/q$ at outer radii, leading to an increase in the ITG threshold. This can be seen from the ITG threshold linear gyrokinetic analytical expression, in the flat density limit [20]:

$$\left(\frac{R}{L_{T_i}}\right)_{\text{crit}} = \frac{4}{3}(1 + T_i/T_e)(1 + \frac{2s}{q}), \quad \text{with } R/L_n < 2(1 + T_i/T_e)$$

(4.1)

A similar dependency in the collisionless, flat density limit with both passing and trapped electrons was reported in modelling carried out with the Kinezero linear gyrokinetic code [21]. For $s > 0$ and for $T_i/T_e = 1$, the best fit of $R/L_{T_i}$ following parameter scans in $s$ and $q$ was found as [22]:

$$\left(\frac{R}{L_{T_i}}\right)_{\text{crit}} = 2(1.1 + 1.4s + 1.9\frac{s}{q})$$

(4.2)

Similar $s/q$ dependence has also been observed in electron heat transport experiments on Tore Supra [23, 24].

This work concentrates on isolating the impact of varying $s/q$ at outer radii on core confinement in low-triangularity JET and AUG hybrid scenario experiments. For each machine, a pair of shots has been chosen which display a similar level of pedestal confinement yet a significant difference in core confinement. For each pair, the improved confinement case is correlated with a higher average $s/q$ parameter. The aim of this work is to assess whether the predicted threshold improvement due to $q$-profile shaping is consistent with the observations. This is carried out by predictive heat and particle transport modelling using the integrated modelling code CRONOS [25] coupled to the GLF23 turbulent transport model [19, 26]. Successful validation of the GLF23 $q$-profile scaling has been carried out on DIII-D discharges at fixed magnetic shear [27]. The GLF23 modelling is supplemented by linear instability threshold calculations with the linear gyrokinetic code QuaLiKiz [28], based on the Kinezero code. One of the driving motivations of this work is to validate recent predictions of the ITER hybrid scenario also employing CRONOS/GLF23, where a high level of fusion power sensitivity to the $s/q$ profile was found, even for relatively modest improvements in $H_{98}$ [29].

This paper is organized as follows: the experimental discharges are described in section 4.2, the modelling tools and techniques in section 4.3, the GLF23 modelling results in section 4.4, additional mechanisms for core confinement improvement are discussed in section 4.5, the QuaLiKiz results in section 4.6, and the discussion and conclusions in section 6.6.
4 Predictive analysis of $q$-profile influence on transport in JET and ASDEX Upgrade hybrid scenarios

4.2 Experimental discharges

For both machines, discharge pairs were analyzed displaying similar pedestal confinement yet significant differences in core confinement. A variation in $q$-profile was experimentally achieved in each pair. For the JET pair (79626/79630), this variation was achieved via the ‘current-overshoot’ method [10, 30]. With this method, the current is ramped down to its flat-top value just prior to the main heating phase, resulting in a broader $q$-profile compared with a regular ramp-up scenario. For the AUG pair (20993/20995), the $q$-profile variation was achieved by varying the auxiliary heating timing, with the later heating case resulting in a broader $q$-profile [31]. Apart from the above-mentioned variations, all other control parameters (e.g. shaping, fueling, total heating power, PINI configuration) are kept the same between the discharges of each pair. The total flattop $P_{aux}$ for the JET case is $\approx 17\,\text{MW}$, all from NBI. For the ASDEX case, flattop $P_{aux} \approx 8\,\text{MW}$, with 5 MW from NBI and 3 MW from ICRH to reduce central tungsten impurity concentration. Temporal evolution of the total plasma current, heating power, and confinement factor $H_{98}$ for each pair can be seen in Fig.5.1.

Figure 4.1: Temporal evolution of total plasma current (upper row), total auxiliary heating power $P_{aux} = P_{NBI} + P_{ICRH}$ (center row), and $H_{98}$ confinement factor (lower row) for the JET pair 79626/79630 (left panels) and AUG pair 20993/20995 (right panels).
4.2 Experimental discharges

The vertical lines in the plots enclose the periods during which the kinetic profiles are averaged for analysis: 6.14-6.64 s for JET 79626, 7.14-7.64 for JET 79630, and 5.25-5.75 s for both ASDEX discharges. The periods differ for JET due to the different heating scheme timings. Furthermore, it is during these periods that the pedestal confinement is similar for the two discharges. Following this period, the pedestal confinement in 79626 decays in time, contributing significantly to the observed reduction in $H_{98}$ on a timescale significantly shorter than the current diffusion timescale. This behaviour is not a regular feature of JET hybrids with current-overshoot at either low or high triangularity. For example, Fig. 1 in [10] illustrates improved confinement maintained throughout the flattop phase of a JET hybrid scenario discharge. We reiterate that the main purpose of this work is to isolate and validate the expected $s/q$ impact on core confinement by comparing discharges in periods where they display similar pedestal confinement yet observed differences in both $q$-profile and core confinement. This is what is achieved by comparing the averaged kinetic profiles of the discharges within the above-mentioned time windows.

The degree of similarity of the pedestal confinement can be assessed by comparing the thermal $\beta_N$ at the location of the GLF23 boundary conditions: $x = 0.83$ for the JET pair and 0.76 for the AUG pair. The values are 0.99/0.94 for JET 79626/79630, and 1.11/0.98 for AUG 20995/20993. The impact that these differences have on the core confinement is discussed in section 4.5.4, where core profile stiffness is assumed. Note that while the boundary condition values are related to the height of the edge transport barrier (pedestal), the correlation is not direct since the boundary condition locations for the GLF23 modelling do not coincide with the pedestal top. Particularly for the AUG simulations, the boundary condition is limited by the outermost data point of the $T_e$ diagnostic, and no data on the actual $T_e$ pedestal height is available. In principle, there may be differences in core confinement between the actual pedestal top and the boundary condition taken for the GLF23 modelling. Thus we cannot relate with full certainty the differences in the GLF23 boundary condition values to differences in pedestal confinement.

All GLF23 analyses in this paper take background kinetic profiles and boundary conditions from the averaged periods defined above. Details of 0D quantities averaged over these periods can be found in table 1. $\langle s/q \rangle$ is defined as the volume averaged $s/q$ between $x = 0.4 - 0.8$, where $x$ is the normalized toroidal flux coordinate. The average is carried out from $x = 0.4$, since for $x < 0.4$ the magnetic shear is low ($s < 0.6$) and the short wavelength approximation within which Eq.5.2 is derived may no longer hold. The averaging procedure is up to $x=0.8$, the approximate location of the GLF23 boundary conditions. All discharges have fishbone activity, and are devoid of NTMs in the temporal periods studied, apart from AUG 20993, which has a 3/2 NTM in the vicinity of $x = 0.5$. The potential polluting effect of this mode on the isolation of the $s/q$ impact on confinement for the AUG pair is examined in section 4.5.

The $s/q$ and rotation profiles used throughout this analysis can be seen in Fig.5.2, and the $q$-profiles themselves in Fig.5.3. For each pair of discharges, the interpretative $q$-profiles are compared with the measured $q$-profiles obtained by MSE constrained equilibrium reconstruction. By interpretative $q$-profiles, we mean $q$-profiles predicted by CRONOS through solving the current diffusion equation with prescribed temperatures and densities (from measurement), and calculated current drive sources. For the JET case,
Table 4.1: 0D quantities for all discharges analysed in this paper. Quantities are averaged over 7.14-7.64s and 6.14-6.64s for JET 79630 and 79626 respectively, and 5.25-5.75s for AUG 20993 and 20995.

<table>
<thead>
<tr>
<th>Shot</th>
<th>$B_T$ [T]</th>
<th>$I_p$ [MA]</th>
<th>$\beta_N$ ($W_{th}$)</th>
<th>$\beta_N$ ($W_{mhd}$)</th>
<th>$(s/q)$</th>
<th>$Z_{eff}$</th>
<th>$H_{98}$</th>
<th>MHD activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>JET 79630</td>
<td>2</td>
<td>1.7</td>
<td>1.9</td>
<td>2.6</td>
<td>0.79</td>
<td>1.97</td>
<td>1.13</td>
<td>Fishbones</td>
</tr>
<tr>
<td>JET 79626</td>
<td>2</td>
<td>1.7</td>
<td>2.1</td>
<td>2.8</td>
<td>0.94</td>
<td>1.76</td>
<td>1.26</td>
<td>Fishbones</td>
</tr>
<tr>
<td>AUG 20993</td>
<td>2.4</td>
<td>1</td>
<td>1.4</td>
<td>1.9</td>
<td>0.69</td>
<td>2.42</td>
<td>0.98</td>
<td>Weak 3/2 NTM</td>
</tr>
<tr>
<td>AUG 20995</td>
<td>2.4</td>
<td>1</td>
<td>1.7</td>
<td>2.3</td>
<td>0.84</td>
<td>2.35</td>
<td>1.2</td>
<td>Fishbones</td>
</tr>
</tbody>
</table>

For $x \sim 0.35$ the interpretative $q$-profiles have significantly lower values than the measured $q$-profiles. This discrepancy is limited to a region comprising only $\sim 10\%$ of the plasma volume, and thus plays a minor role in the determination of total core confinement, which is the central point of this work. It was thus decided to use the interpretative JET $q$-profiles for the GLF23 transport modelling in this work, since it was deemed that the current diffusion model could capture with higher fidelity than the measurements the subtle effect of current overshoot on the $q$-profile. The discrepancy for $x < \sim 0.35$ may be due to MHD activity redistributing the current, clamping the $q$-profile to 1, as reported in [32–35]. The discrepancy within $x = 0.35$ is consistent with the lack of observed sawteeth in the JET pair. However in principle $q < 1$ is a necessary but not sufficient condition for sawteeth. The sawtooth trigger can also depend on the magnetic shear at the inversion radius, the location of the inversion radius and the fast ion distribution function. Thus it is possible that no sawteeth are present in spite of $q < 1$ for these specific discharges. All JET interpretative simulations started at 2 s - early enough in the ramp-up phase such that the sensitivity of the $q$-profiles at the time of interest for the predictive simulations to the initial prescribed $q$-profiles is negligible. The initial prescribed $q$-profiles corresponded to a quadratic current profile constrained to satisfy $I_c$ from EFIT reconstruction.

In the AUG pair, the interpretative $q$-profiles significantly depart from the measured $q$-profiles within a much greater bulk of the plasma volume compared to the JET pair. Such discrepancies between interpretative and measured $q$-profiles was also seen in previous AUG improved H-mode modelling [36]. This increased discrepancy may be due to a higher degree of current redistributed by MHD activity. It was thus decided to use the measured $q$-profiles for the GLF23 modelling for the AUG cases, since it is possible that neoclassical diffusion cannot account for the precise $q$-profile shape in the AUG case even at higher radii.

Regarding $q$-profile measurement errors, the MSE measurement error for the AUG $q$-profiles are $\sim 10\%$. For the JET $q$-profiles, the modelled values are sensitive to the measured ($Z_{eff}$), $Z_{eff}$ profile shape, and bootstrap current calculations. A thorough assessment of the $q$-profile sensitivity to the modelling assumptions has not been carried out for this work, apart from a $Z_{eff}$ sensitivity check shown in section 4.4. However, both for the AUG and JET cases, the proximity of the shots (each pair was carried out...
4.2 Experimental discharges

Figure 4.2: s/q profiles (left panel) and toroidal rotation profiles (right panel) for all discharges analysed in this paper.

Figure 4.3: Comparison of interpretative and experimental q-profiles for the JET discharges (left panel) and for the AUG discharges (right panel).

in the same experimental session) means that it is likely that any systematic errors are the same for both discharges in each pair. Since we are concerned with differences in the q-profiles, this increases the reliability of the results. Nevertheless, it is indeed possible that the error in the difference between the q-profiles in each pair is of the order of the difference itself. This important caveat underlines this entire work.

The rotation profiles for the JET case are similar. For the AUG case, the 20993 (lower confinement) case has a significantly flatter rotation profile in the low magnetic shear region $x < 0.4$. It is possible that this flatter rotation profile is due to the magnetic braking induced by the 3/2 mode present in shot 20993.
A representative overview of the kinetic profile data and fit quality for all discharges analyzed in this paper can be seen in Fig. 5.4 for single timeslices. Spline fits were carried out for each timeslice. The fit order (typically cubic or quartic) and break-points (typically near the pedestal if at all) varied between the different profiles. Note that the standard $\left.\frac{d}{dx}\right|_{x=0}$ constraint on the profiles was not applied by the fitting routine used for this work. This constraint does not fundamentally change the profile shape in the region of interest for comparison with the GLF23 modelling ($x \sim 0.3 - 0.8$). JET $T_e$ was measured by a combination of core and edge charge exchange (CXFM and CXSE) diagnostics. The CXSE is vital for determining the approximate equivalence of the $T_i$ pedestal top values between the pair. However, the finite radial extension of the diagnostic does not allow the full resolution of the pedestal $T_i$ profile. JET $T_e$ was measured by both ECE and high resolution Thompson scattering (HRTS). The spline fit used both datasets on an equal footing. This is not ideal since the datasets are not fully mutually consistent. However, since the pedestal region (which will serve as the modelling boundary condition) is only resolved by HRTS, the choice of carrying out a combined data fit has no impact on the
modelling results. Nevertheless, the lack of full agreement between the HRTS and ECE measurements should be maintained as a caveat when comparing the $T_\text{i}$ modelling results to the fits from the experimental data. JET $n_e$ was measured by HRTS. For AUG, $T_\text{i}$ was measured by CX. $T_e$ was measured by ECE. $n_e$ was measured by a combination of interferometry and lithium beam spectroscopy, and the data combined with the Integrated Data Analysis (IDA) approach [37].

For comparison of the kinetic profiles with the GLF23 predictions, the profiles were averaged for 0.5 s during the time windows displayed in Fig.5.1. This is more than an energy confinement time for both machines. This averaging also significantly reduces the RMS error of the data points, to below 2% for example for both the JET and AUG $T_\text{i}$ and $V_{tor}$ measurements. The boundary conditions for the GLF23 simulations were also taken from this averaging. From this averaging we can also analyze $R/L_{Ti}$, to determine the location and robustness of the $R/L_{Ti}$ differences between the shots of each pair. This can be seen in Fig.5.5. These curves were made by time averaging the $T_\text{i}$ spline fits made at each timeslice. $R/L_{Ti}$ is calculated with respect to the average flux surface minor radius on the midplane, $\tau = (R_{\text{out}} - R_{\text{in}})/2$. The error from the scatter in these fits is 10% for the JET data, and 15% for the AUG data. We can see that the JET $R/L_{Ti}$ differences are localized to the region $x = 0.6 - 0.8$, while the AUG $R/L_{Ti}$ differences are localized from $x = 0.25 - 0.7$ (where $x=0.25$ is the approximate location of the innermost reliable $T_i$ datapoint). While the error bars lead to overlap over much of the $R/L_{Ti}$ profiles, the errors are not independent. The scattering of one specific data point creates an opposing change in $R/L_{Ti}$ on either side of the position of that point. This fact increases the robustness of the observed $R/L_{Ti}$ differences and locations.

Finally, we compare the ratios of these experimental $R/L_{Ti}$ profiles to the predicted analytical $R/L_{Ti}$ ratios from Eq.5.2, displayed in Fig.5.6. We note that the $R/L_n$ ratios of all discharges are low enough to satisfy the flat density profile approximation -
4 Predictive analysis of $q$-profile influence on transport in JET and ASDEX Upgrade hybrid scenarios

![Graph showing $R/L_{Ti}$ ratios](image)

**Figure 4.6:** $R/L_{Ti}$ experimental ratios compared with the analytical linear thresholds ratios (from Eq.5.2) for JET 79626/30 (left panel) and AUG 20993/95 (right panel)

$R/L_n < 2(1 + T_i/T_e)$. This can be seen in Fig.5.7, where the $R/L_n$ values of the JET discharges (calculated with respect to $r = r_{out} - r_{in}$) are compared with $2(1 + T_i/T_e)$. The constraint is satisfied to an even higher degree in the ASDEX pair, since the collisionality is higher and thus the density peaking reduced. In Fig.5.6 we also include analytical $R/L_{Ti}$ ratios also assuming constant $T_i/T_e$, to separate the effect of $s/q$ and $T_i/T_e$. Qualitatively the analytical and experimental ratios show the following similar features: in the JET case, the maximum observed difference for $x > 0.5$ is predicted by Eq.5.2, with no predicted difference (as in the observations) in the region of $x=0.4$; for AUG, the increasing difference from 0.75 in towards the core is also predicted by Eq.5.2. However, the observed difference in the AUG case for $x < 0.5$, the low magnetic shear region ($s < 0.6$), is not predicted. The $T_i/T_e$ component has a minor influence...
4.3 Modelling tools and techniques

The core of CRONOS is a 1.5D transport solver, whereby 1D current diffusion, particle and energy equations are solved up to the separatrix, self consistently with 2D magnetic equilibrium [25]. In this work, the NBI heat and current sources are calculated by NEMO/SPOT [38, 39]. The ICRH heat sources for the AUG discharges are calculated with PION [40]. The magnetic equilibrium is calculated with HELENA [41]. The neoclassical transport, bootstrap current, and neoclassical resistivity is calculated with NCLASS [42]. Turbulent transport is calculated with the GLF23 model [19, 26]. The fast particle profiles calculated by NEMO/SPOT and PION for NBI and ICRH are subtracted from the ion density profiles entered into GLF23. The calculated fast particle fractions for both the JET/AUG simulations successfully account for the observed differences in $\beta_N(W_{\text{dia}}/W_{\text{mhd}})$ and $\beta_N(W_{\text{th}})$.

For the JET simulations, GLF23 is employed within the region $x=0-0.83$. For the AUG simulations, GLF23 is employed within the region $x=0-0.76$. The simulations are carried out on the energy confinement timescale until the kinetic profiles reach stationary conditions. The background profiles and scalar quantities for the simulations are averaged over the time periods displayed in Fig.5.1. Both heat transport only simulations (with prescribed density profiles for measurement) and combined heat and particle transport simulations are carried out. As mentioned in section 4.2, the JET input q-profiles are taken from CRONOS interpretative runs (current diffusion only simulations), and the AUG q-profile inputs from MSE measurements. For each discharge, comparison simulations were carried out substituting the q-profile input with the q-profile from the other member of each pair. The magnetic shear is then recalculated by CRONOS in a self-consistent manner. In such a manner GLF23 predicts the confinement difference solely due to the changed values of the q-profile and magnetic shear. Further linear threshold analysis examining the effect of $s/q$ is also carried out with the quasilinear gyrokinetic transport model QuaLiKiz [28].

4.4 Results: GLF23 q-profile substitution simulations

In this section we isolate the degree of improved confinement which stems from the differences in $q$-profile between the two discharges in each pair, as according to the GLF23
Predictive analysis of \( q \)-profile influence on transport in JET and ASDEX Upgrade hybrid scenarios

model. A comparison of full simulations with GLF23 of each separate discharge shows a similar degree of improvement in core confinement as observed. This is particularly the case for the JET discharges, as seen in Fig. 5.8, where the predicted total pressure is compared with the observations. Both with and without the GLF23 \( E \times B \) turbulence suppression model (as specified by the \( \alpha_{E \times B} \) parameter) the degree of predicted stored energy in 79626 is higher than that predicted for 79630 by a ratio comparable with the experimental observation. However, the main point of this work is to determine whether increased \( s/q \) in the outer radii of the improved confinement discharges is a significant factor responsible for the improved confinement through the increase of ITG linear thresholds. To this end, simulations for either JET 79630 or AUG 20995 were carried out substituting the \( q \)-profile of each simulation with the \( q \)-profile corresponding to the partner discharge in each pair. The comparison of simulations with differing \( q \)-profile inputs is the central point of this work. The details of these simulations are outlined in the following sections.

**Figure 4.8:** Results of combined heat and particle transport GLF23 simulations for JET 79630 and 79626. Predicted and observed \( P_{\text{tot}} \) profiles are compared with (right panel) and without (left panel) the inclusion of the full GLF23 \( E \times B \) turbulence suppression model.

### 4.4.1 JET heat transport only

Fig. 5.9 shows the \( T_i \) and \( T_e \) predictions for JET 79630. These simulations include heat transport only. Runs were carried out both with and without the GLF23 \( E \times B \) turbulence suppression model. 79630 simulations with the substituted \( q \)-profile from 79626 (the improved confinement case) are also shown.

The inclusion of \( E \times B \) suppression leads to overprediction of \( T_i \). This overprediction is also seen in JETTO [43] simulations of the same discharge, verifying that this observation is not an artifact of the CRONOS/GLF23 methodology used in this work. However,
independently of the degree of prescribed $E \times B$ suppression, the $q$-profile substitution leads to a degree of $T_i$ increase comparable to the experimentally observed difference between the two discharges. A more quantitative analysis of these differences, and of all subsequent simulations discussed below, can be found in the tables in section 4.4.5.

Figure 4.9: Results of heat transport only GLF23 simulations for JET 79630, comparing $q$-profile inputs from both 79630 and 79626. $T_i$ profiles are on the left column. $T_e$ profiles are on the right column. Results with both no $E \times B$ suppression (top row) and full $E \times B$ suppression (bottom row) are shown.

### 4.4.2 AUG heat transport only

The AUG simulations display a similar pattern to the JET simulations: simulations with the 20995 (improved confinement case) $q$-profile display improved ion confinement compared to a simulation which is identical apart from substituting in the 20993 $q$-profile. Inclusion of the $E \times B$ suppression model leads to $T_i$ overprediction for both cases. This can be seen in Fig.5.10. In the AUG 20993 case the general degree of $T_i$ overprediction is more severe. However, regardless of the $E \times B$ suppression assumption, the $q$-profile
substitution leads to a $T_i$ difference qualitatively consistent with experimental observations.

![Figure 4.10](image)

**Figure 4.10:** Results of heat transport only GLF23 simulations for AUG 20995, comparing $q$-profile inputs from both 20995 and 20993. $T_i$ profiles are in the left column. $T_e$ profiles are in the right column. Results with both no $E \times B$ suppression (top row) and full $E \times B$ suppression (bottom row) are shown.

### 4.4.3 JET combined heat and particle transport

In Fig.5.11, the results of combined heat and particle transport simulations for JET 79630 are shown. The pattern remains similar to the heat transport only cases, although the primary effect of global confinement improvement following the $q$-profile substitution is now in the particle channel. $R/L_n$ is increased at $x > 0.4$ up until the GLF23 boundary condition at $x = 0.83$, while $R/L_n$ is decreased for $x < 0.4$. Compared to the heat transport only case, the $T_i$ profile increase is reduced when switching to the 79626 $q$-profile. This is due to the increased transport predicted by GLF23 due to the increased density gradients at $x > 0.4$, interpreted as the destabilization of TEM modes due to the...
Figure 4.11: Results of combined heat and particle transport GLF23 simulations for JET 79630, comparing $q$-profile inputs from both 79630 and 79626. $T_i$ profiles are in the left column. $T_e$ profiles are in the center column. $n_e$ profiles are in the right column. Results with both no $E \times B$ suppression (top row) and full $E \times B$ suppression (bottom row) are shown.

### 4.4.4 AUG combined heat and particle transport

The AUG heat and particle transport simulations show similar results to the JET heat and particle transport case. The AUG 20995 simulations are displayed in Fig.5.12. For the AUG case, the degree of improved particle transport is consistent with observation, although when $E \times B$ suppression is included the $n_e$ profiles are significantly higher than the measured values.
4.4.5 Summary of GLF23 runs

In tables 2 and 3, the simulation results are summarized in terms of the predicted core thermal energy content:

\[ W_{\text{core}} = \frac{1}{J} \int_{x_{bc}}^{x_{\text{in}}} (P_{\text{th}}(x) - P_{\text{th}}(x_{bc})) J \text{d}x \]

where \( P_{\text{th}} \) is the thermal pressure, \( x_{bc} \) is taken at the GLF23 operational zone boundary at \( x = 0.83/0.76 \) for JET/AUG. \( J \) is the Jacobian corresponding to the volume element. The tables summarize - according to GLF23 - the extent of the \( s/q \) effect alone in explaining the core confinement differences between each pair, both including and excluding \( E \times B \) shear stabilization.

In spite of the absolute core energy content overprediction when including \( E \times B \) shear stabilization, the degree of change when substituting the \( q \)-profile is maintained both with and without \( E \times B \) shear stabilization. The degree of change also remains the
4.4 Results: GLF23 q-profile substitution simulations

Table 4.2: Core thermal energy following GLF23 predictions for JET hybrids. Units are [MJ].

<table>
<thead>
<tr>
<th>EXP</th>
<th>Heat transport</th>
<th>Heat and particle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no $E \times B$</td>
<td>with $E \times B$</td>
</tr>
<tr>
<td>79630 (q30)</td>
<td>1.67</td>
<td>1.71</td>
</tr>
<tr>
<td>79630 (q26)</td>
<td>1.97</td>
<td>1.9</td>
</tr>
<tr>
<td>Ratio</td>
<td>1.17</td>
<td>1.11</td>
</tr>
</tbody>
</table>

Table 4.3: Core thermal energy following GLF23 predictions for AUG hybrids. Units are [MJ].

<table>
<thead>
<tr>
<th>EXP</th>
<th>Heat transport</th>
<th>Heat and particle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no $E \times B$</td>
<td>with $E \times B$</td>
</tr>
<tr>
<td>20995 (q93)</td>
<td>0.2</td>
<td>0.249</td>
</tr>
<tr>
<td>20995 (q95)</td>
<td>0.294</td>
<td>0.293</td>
</tr>
<tr>
<td>Ratio</td>
<td>1.47</td>
<td>1.18</td>
</tr>
</tbody>
</table>

same when including particle transport, due to the aforementioned increase in heat transport when increasing the density gradients. Taking the average of the GLF23 simulated core energy content for all runs, we obtain a ratio for 1.105 for the JET runs, and 1.17 for the AUG runs. This corresponds to the $s/q$ effect alone contributing to 60% of the core confinement difference for the JET pair, and 35% of the core confinement difference in the AUG pair, according to GLF23. In section 4.5 we discuss other possible contributions to the core energy content differences.

4.4.6 Sensitivity of JET results to choice of $Z_{eff}$ and $\alpha_E E \times B$

The $Z_{eff}$ taken for the JET simulations had an assumed form of $Z_{eff} \propto n_0^{-0.4}$, and was normalized such that the line average $\langle Z_{eff} \rangle$ agrees with Bremsstrahlung measurements. However, as seen in Fig.5.13, this assumption does not agree with $Z_{eff}$ profiles calculated from carbon concentration measurements carried out by recombination charge exchange spectroscopy (CX). For $x > 0.8$, where no CX measurements were available, an assumed near constant CX $Z_{eff}$ profile was prescribed.

The sensitivity of the calculated q-profile to $Z_{eff}$ was examined by repeating the interpretative run for 79630 with alternative $Z_{eff}$ profiles based on the discrepancy between the Bremsstrahlung and CX measurements. Since the CX measurements were only available following application of the neutral beam injection, neither precise information on the $Z_{eff}$ profile shape in the early phase of the discharge, nor an alternative value of $\langle Z_{eff} \rangle$ was available. Therefore, we tested the sensitivity by simply assuming a $Z_{eff}$ profile identical to the CX profile from Fig.5.13, and reducing $\langle Z_{eff} \rangle$ in the interpretative run by a constant factor $\sim 1.3$, typical of the discrepancy between the Bremsstrahlung...
4 Predictive analysis of $q$-profile influence on transport in JET and ASDEX Upgrade hybrid scenarios

The impact of using the CX $Z_{\text{eff}}$ profile in the GLF23 simulations was also examined. Both the 79630 heat transport only and combined heat and particle transport simulations were repeated using CX $Z_{\text{eff}}$ profiles averaged over the 7.14-7.64 s time window. Finally, the same simulations were repeated when reducing the GLF23 $\alpha_{E\times B}$ parameter to 1. The heat transport only results are seen in Fig. 5.14, and the combined heat and particle transport results in Fig. 5.15.

The $q$-profile sensitivity to the $Z_{\text{eff}}$ profile, within the range studied, is small. However, the variation of the $T_i$ predictions when switching to the CX measured $Z_{\text{eff}}$ profile is not negligible, and is primarily due to reduced dilution. When combined with switching to $\alpha_{E\times B} = 1$, the ion temperature overprediction, while still apparent, is significantly diminished. This more minor level of $T_i$ overprediction is consistent with GLF23 results reported in [44], where both CX $Z_{\text{eff}}$ profiles and $\alpha_{E\times B} = 1$ were employed. Also in the combined heat and particle transport case, the degree of averaged $n_e$ overprediction is reduced when switching to the CX $Z_{\text{eff}}$ profile, although the profile peaking still remains well above the experimentally observed value. We reiterate that the relative differences due to the $s/q$ effect - the central point of this work - are invariant to the choice of $Z_{\text{eff}}$ and $\alpha_{E\times B}$.

4.4.7 Discussion of GLF23 $E\times B$ suppression model

The absolute $W_{\text{core}}$ values are significantly overpredicted when including $E\times B$ shear suppression in the GLF23 model. It is difficult to determine whether this overprediction is due to a deficiency in the $E\times B$ suppression model, or due to an intrinsic overpredic-

**Figure 4.13:** Comparison of $Z_{\text{eff}}$ profiles for JET 79630 averaged between 7.14-7.64 s, both from line averaged Bremsstrahlung and an assumed profile, and from carbon concentration CX measurements.
4.4 Results: GLF23 q-profile substitution simulations

Figure 4.14: Results of heat transport GLF23 simulations for JET 79630, testing the sensitivity of the results to $Z_{eff}$ and the $\alpha_{E \times B}$ parameter. Progressive results for $T_i$ (left panel) and $T_e$ (right panel) are shown where the experimental profile is compared to: the original 79630 simulation with Bremsstrahlung $Z_{eff}$ and $\alpha_{E \times B} = 1.35$, a simulation where the q-profile is substituted with the q-profile obtained with lowered $Z_{eff}$ profiles, additionally reducing $Z_{eff}$ itself in the predictive simulation to the CX measured $Z_{eff}$ profile, and finally additionally switching the $\alpha_{E \times B}$ parameter to 1.

Figure 4.15: Results of combined heat and particle transport GLF23 simulations for JET 79630, testing the sensitivity of the results to $Z_{eff}$ and the $\alpha_{E \times B}$ parameter. Progressive results for $T_i$ (left panel), $T_e$ (center panel), and $n_e$ (right panel) are shown where the experimental profile is compared to: the original 79630 simulation with Bremsstrahlung $Z_{eff}$ and $\alpha_{E \times B} = 1.35$, a simulation where the q-profile is substituted with the q-profile obtained with lowered $Z_{eff}$ profiles, additionally reducing $Z_{eff}$ itself in the predictive simulation to the CX measured $Z_{eff}$ profile, and finally additionally switching the $\alpha_{E \times B}$ parameter to 1.
tion of the underlying instability thresholds. We note that GLF23 simulations for high triangularity JET hybrids also display a similar $T_i$ overprediction as seen in this present work [45]. However, in previous work, the JET hybrid discharge 58323 was shown to be satisfactorily predicted by GLF23 [46], even when including $E \times B$ suppression. Similar satisfactory predictions of JET hybrid discharges have also previously been reported [47].

A simulation of discharge 58323 was repeated with the same CRONOS/GLF23 methodology performed in this work, and the previous result was indeed recovered. However, additional simulations taking into account the uncertainties in $Z_{eff}$, combined with the resultant differences in the modelled $q$-profile, also led to $T_i$ overpredictions as seen here. For other JET discharges analyzed in [46], numerous examples of significant $T_i$ deviations can be seen in figure 4. Also in this work, the degree of overprediction - while still significant - is reduced when using the $Z_{eff}$ profile from charge-exchange measurements as shown in section 4.4.6.

These results highlight the modelling sensitivity to the input parameters, which helps explain the scatter in the literature with regard to the degree of predicted GLF23 $T_i$ overprediction. We also note in recent TGLF [48] modelling of DIII-D hybrid discharges [49], $T_i$ was reported to be significantly underpredicted when not employing $E \times B$ suppression. However, the ion energy transport was reported to be reduced to neoclassical levels, while the neoclassical transport was taken from a modified Chang-Hinton model. This model yields diffusivities larger than that calculated by first-principle methods such as by the NCLASS [42] model employed in this work. This may lead to more pessimistic $T_i$ predictions. Further validation of the various transport models on discharges from different machines, while employing using the same modelling methodology to reduce scatter induced by different modelling assumptions, would be highly beneficial.

With regard to potential explanations for a deficiency in the GLF23 $E \times B$ suppression model, we note that discharges considered here are highly NBI-driven discharges, with a relatively high degree of rotational shear. It has been shown that due to parallel velocity shear the efficiency of $E \times B$ shear in decreasing transport decreases at high rotation [50–52]. This may be relevant also for the increased $q_{95}$ hybrid discharges. At higher $q$, the parallel (to the field line) component of the toroidal velocity is increased. While parallel velocity shear destabilization is included in GLF23, the destabilization effect is minor for the rotation velocities considered here. This is seen by comparing JET 79630 GLF23 $T_i$ predictions (with heat transport only), for cases where the $E \times B$ suppression model is not included ($\alpha_{E \times B} = 0$), and where the toroidal velocity itself is set to zero ($V_{tor} = 0$). The difference between the predictions corresponds to the parallel velocity shear destabilization, which in this case only reduces the $T_i$ profile by $\sim 2\%$. However, it would be worthwhile to reexamine the effect of parallel velocity gradient destabilization with gyrokinetic codes at experimentally relevant levels of turbulent fluxes for improved transport model verification.

Recent results at JET show that the effect of rotational shear in increasing $R/L_{Ti}$ (due to decreased stiffness) is located only in the low magnetic shear region, in our case $x < 0.4$ [17, 18]. However, this is inconsistent with the main effect of including GLF23 $E \times B$ suppression in our simulations, which increases $R/L_{Ti}$ primarily in the high mag-
4.5 Additional contributions to core confinement differences

In this section we explore possibilities - other than the $s/q$ effect - for explaining the observed differences in core confinement for both the JET and AUG pairs.

4.5.1 Density profiles

For the heat transport only simulations, the prescribed density profile may have an effect on the predicted core confinement. This may be due to both the inherent different stored energy content, and also the $R/L_n$ parameterization of the predicted transport. In Fig.5.16 a comparison of AUG 20995 heat transport simulations prescribing both the...
4 Predictive analysis of $q$-profile influence on transport in JET and ASDEX Upgrade hybrid scenarios

Figure 4.16: Comparison of AUG 20995 heat transport simulation with GLF23, with different prescribed $n_e$ profiles. The dashed lines correspond to $T_e$, and the solid lines to $T_i$. The prescribed $n_e$ profiles are seen in the left panel, the temperature profiles without $E \times B$ suppression in the center panel, and the temperature profiles with full $E \times B$ suppression in the right panel.

20995 and 20993 density profiles is displayed. This sensitivity test was carried out for AUG since the $\Pi$ difference for the AUG pair is greater than the JET pair: 6% as opposed to 3%.

The resulting difference in $W_{\text{core}}$ is a negligible $< 1\%$ increase when prescribing the 20993 $n_e$ profile as opposed to the 20995 profile, for the no $E \times B$ suppression case. The increase in $T_{i,e}$ compensates for the decreased $n_e,i$ profiles. For the full $E \times B$ suppression case, prescribing the 20993 $n_e$ profile led to a 6% decrease in $W_{\text{core}}$. The discrepancy between the two cases can be traced to the flat temperature profiles in the 20995 $n_e$ case located at $x = 0.4$. This is an artifact of GLF23 modelling, where typically above a certain $R/L_n$ threshold the sensitivity of the transport to $R/L_n$ increases greatly. Evidently the 20995 $R/L_n$ is at that threshold around $x = 0.4$. However, these TEM modes, which leads to a flat temperature profile in the region of $x = 0.4$ in the $\alpha_{E \times B} = 0$ case, are stabilized by the $E \times B$ suppression model when $\alpha_{E \times B} = 1.35$. Thus the flat temperature profile in the $x = 0.4$ region in the $\alpha_{E \times B} = 0$ case is not evident in the $\alpha_{E \times B} = 1.35$ case.

4.5.2 $Z_{\text{eff}}$

Sensitivity of the transport to $Z_{\text{eff}}$ is examined for the JET pair, which displays a greater difference in measured $\langle Z_{\text{eff}} \rangle$ compared to the AUG case: 12% as opposed to 3%. The improved confinement discharge has lower $\langle Z_{\text{eff}} \rangle$ in both pairs. In Fig.4.17 two separate JET 79630 simulations are compared, with the differing $Z_{\text{eff}}$ profiles. The simulations include both particle and heat transport, and no $E \times B$ suppression is included in these simulations. The results show that the sensitivity is small but not entirely negligible: the 79626 $Z_{\text{eff}}$ case has an increased $W_{\text{core}}$ by 4% compared to the 79630 $Z_{\text{eff}}$ case, which is also in the correct direction of the experimentally observed improvement.
4.5 Additional contributions to core confinement differences

Figure 4.17: Comparison of JET 79630 combined particle and heat transport simulation with GLF23, without $E \times B$ suppression. The dashed lines correspond to simulations with $Z_{eff}$ from 79630, and the solid lines to $Z_{eff}$ from 79626. The prescribed $Z_{eff}$ profiles are seen in the left panel, the density profiles in the center panel, and the temperature profiles in the right panel.

4.5.3 Heat and particle sources

Simulations which compare the effect of substituting only the heat and particle source profiles were carried out, to test the sensitivity of the predicted core energy content on the precise source profiles. This was done by re-running the combined heat and particle transport simulation of 79630 with the 79626 $q$-profile, and replacing the heat and particle sources in the simulation with those calculated in the 79626 interpretative simulation. However, due to the highly stiff nature of the GLF23 model, the predicted changes in $W_{core}$ following such substitutions were completely negligible ($< 1\%$).

4.5.4 GLF23 boundary conditions

The largest part of the observed difference in confinement between each pair of discharges is from the core. However, the differences in the values of the kinetic profiles at the GLF23 boundary conditions - $x = 0.83$ for the JET pair and $x = 0.76$ for the AUG pair - can still play a significant role. To test the impact of the boundary condition on the core confinement, we first note that from the definition of the gradient length $1/L_T = \frac{dT/dr}{T}$, we can obtain $T(r) = T(r_{bc}) \exp(\int_{r}^{r_{bc}} \frac{1}{L_T(r)} dr)$, where we have taken a temperature gradient length for this example, and $r_{bc}$ is the boundary condition radius. If we then assume that the inverse gradient lengths of the kinetic profiles would be maintained following slight changes in the boundary condition values, then the modification of the kinetic profiles following a change in the boundary conditions can be calculated by a simple multiplication of each profile corresponding to the change in each respective boundary condition value. In such a case, the effect of these boundary conditions on the core confinement can be easily examined. We can recalculate the experimental $W_{core}$ by simply multiplying the experimental kinetic profiles by factors corresponding to the ratios of the boundary conditions between the discharges in each
pair. This assumption was tested with GLF23 simulations, and was found to be adequate.

The 79630 experimental kinetic profiles were multiplied to match the 79626 boundary conditions, and $W_{core}$ recalculated. The recalculated $W_{core}$ was 5% higher than the original $W_{core}$. Thus, the boundary conditions alone may be responsible for ~ 30% of the perceived core confinement difference. For the AUG case, setting the 20993 experimental kinetic profiles to the 20995 boundary conditions showed that ~ 25% of the perceived core confinement difference may be due to the boundary conditions.

4.5.5 NTM effect

While the pair of JET discharges do not have any NTMs within the temporal period of analysis, AUG 20993 (the lowered confinement case) has a low amplitude 3/2 NTM located in the vicinity of $x = 0.5$, in accordance with the location of $q=1.5$ [31]. It is possible that the effect of this NTM on the kinetic profiles can explain part of the 40% difference in $W_{core}$ for the AUG pair which remains after taking into account the $s/q$ effect and the boundary condition effect. While it is difficult to precisely measure the impact of this NTM on confinement, we estimate the maximum possible impact of NTM as follows. The experimental $T_e$ of the two AUG discharges are compared (Fig.4.18), where the 20995 profile is normalized to have the same boundary condition as 20993 at $x = 0.76$. We can see that the $T_e$ differences following the normalization are indeed localized to the region $x < 0.5$, in line with a possible relation to the NTM activity. The ratio between the profiles is also seen in Fig.4.18. To estimate the effect of the NTM on the total $W_{core}$, we make a strong assumption that these $T_e$ differences are fully due to the NTM, and not due to the differing $q$-profile and rotation profiles. We can then assess the potential impact of the NTM on $W_{core}$ by multiplying all kinetic profiles ($T_{i,e}$, $n_{i,e}$) of discharge 20993 by the ratio seen in the right panel of Fig.4.18 for $x < 0.5$, and then recalculate $W_{core}$. This led to a $W_{core}$ increase of 20993 of 17% of the total difference between 20993 and 20995. We can thus conclude that while the NTM may have a non-negligible impact on core confinement, the impact is not large enough to fully explain the remaining 40% difference.

4.5.6 Predictions in the low magnetic shear region

In Fig.4.19 we compare the predicted $R/L_{Ti}$ profiles from the GLF23 simulations with the observed values, for both pairs of discharges. The results shown are from simulations including heat transport only, and the results both with and without $E \times B$ suppression are shown. The absolute predictions - particularly when including $E \times B$ suppression - deviate from the observations. However, the relative predictions - when substituting the $q$-profile - approximately agree with the observations in the region $x > 0.4$.

In the JET case, the location of the $R/L_{Ti}$ differences when substituting the $q$-profile agrees with the location of the $s/q$ differences, as seen in Fig.5.2. The location of these differences also fully agrees with the observations. For the AUG pair, this is also the case for $x > 0.4$, but the observed differences in $R/L_{Ti}$ within the region $x < 0.4$ are not predicted by GLF23. This region corresponds to the low magnetic shear region.
4.6 QuaLiKiz linear threshold analysis

Additional analysis was carried out for both the JET and AUG cases with QuaLiKiz, where we assess the sensitivity of the instability linear thresholds to the $q$-profile, at the locations $x = 0.35, 0.45, 0.55, 0.65, 0.75$. This was carried out by a QuaLiKiz $R/L_{Te}$ scan, to identify the linear threshold. The QuaLiKiz calculations took as input the parameters for 79630 for the JET pair, and 20995 for the AUG pair. The simulation at each radial point was then repeated after replacing $q$ and $s$ with the values from 79626 and 20993 respectively. The results are seen in Fig.4.20. The QuaLiKiz results are compared to the experimental $R/L_{Te}$ calculated with respect to a radial coordinate defined as $(R_{in} + R_{out})/2$, where $R_{in, out}$ are the radii to the high field and low field side flux surfaces on the midplane. $R/L_{Te}$ was kept at the observed ratio to $R/L_{Ti}$ throughout the QuaLiKiz $R/L_{Te}$ scans.

While the predicted relative differences in $R/L_{Te}$ are recovered following the $q$-profile substitution - particularly for JET - the absolute values for $x > 0.5$ are over-predicted. However, both QuaLiKiz and GLF23 utilize $s - \alpha$ geometry, only correct in a...
circular small inverse aspect ratio limit. Shaped geometry can have a significant impact on the linear threshold calculation. A parameterization of this effect has been obtained from an extensive study of ETG linear thresholds from linear gyrokinetic calculations, which included shaping [58]. The study assumed adiabatic ions. The obtained (from fitting) ETG linear threshold formula is thus isomorphic to the ITG case assuming adiabatic electrons, which we rewrite as follows:

\[
\left( \frac{R}{L_{Ti}} \right)_c = \max \left\{ \left( 1 + \frac{T_i}{T_e} \right) \left( 1.33 + 1.91 \frac{s}{q} \right) \left( 1 - 1.5\epsilon \right) \left( 1 + 0.3\epsilon \frac{d\kappa}{d\epsilon} \right) , 0.8 \frac{R}{L_{Te}} \right\}
\]

(4.3)

where \( \epsilon = r/R \) and \( \kappa \) is the elongation. Equation 4.3 is very similar to equation 5.2 apart from the increased weight of \( s/q \), and the geometrical terms. These terms are of more importance at higher radii.

We estimate the effect of shaped geometry by multiplying the QuaLiKiz calculated \( R/L_{Ti} \) by the geometrical term \( \left( 1 - 1.5\epsilon \right) \left( 1 + 0.3\epsilon \left( \frac{d\kappa}{d\epsilon} \right) \right) \) from equation 4.3. The results are seen in Fig.4.21. The predicted and observed \( R/L_{Ti} \) are now reconciled for \( x > 0.5 \), particularly for the JET cases. The measured \( R/L_{Ti} \) at \( x < 0.5 \) remain far from threshold. However, this is consistent with observations in JET plasmas with high
rotational shear, where $R/L_{T_i}$ at greater radii have been measured to be tied close to the linear threshold, while at lower radii (low magnetic shear) the profile stiffness can be significantly reduced [17].

We note that agreement between the QuaLiKiz predictions and the measured $R/L_{T_i}$ for $x > 0.5$ can be improved to the same degree as seen in Fig.4.21 by simply setting as input the experimental gradients with respect to $R_{out}$ as opposed to $(R_{in} + R_{out})/2$. This may reflect the following physics: at high magnetic shear - characteristic of hybrid scenarios for $x > 0.5$ - the ITG modes are more strongly ballooned to the low field side due to the increased ion Landau damping. We speculate here that in such situations, it may thus be more appropriate to input experimental gradients defined with $R = R_{out}$. 

Figure 4.20: Comparison of $R/L_{T_i}$ QuaLiKiz predictions following $q$-profile substitution. Results are shown for the JET pair (left panel) and the AUG pair (right panel).

Figure 4.21: Comparison of $R/L_{T_i}$ QuaLiKiz predictions following $q$-profile substitution. Results are shown for the JET pair (left panel) and the AUG pair (right panel). The QuaLiKiz predictions here have been multiplied by the geometrical factor from equation 4.3 to estimate the effect of shaping on the linear thresholds.
into the model, i.e. the gradient in the region where the mode is most unstable. This topic, as well as profile stiffness at low magnetic shear, is currently under investigation as part of the ongoing development of the QuaLiKiz transport model.

In summary, QuaLiKiz linear threshold calculations following \( q \)-profile substitution show that the \( s/q \) differences between each pair of discharges is responsible for a significant degree of the observed \( R/L_{TI} \) differences. This is in corroboration with the GLF23 results. However, the significant overprediction of \( R/L_{TI} \) for \( x > 0.5 \) may be a symptom of the well known limitation of applying \( s-\alpha \) geometry in shaped plasmas. An estimation of the shaping effect, utilizing previous linear gyrokinetic calculations which included shaping, led to a reconciliation between the predicted and observed \( R/L_{TI} \) values for \( x > 0.5 \), particularly in the JET case. This is consistent with the observation that even with no \( E\times B \) shear included (\( \alpha_{E\times B} = 0 \)), also the GLF23 model overpredicts \( R/L_{TI} \) for \( x > 0.5 \) as seen in Fig.4.17.

4.7 Conclusions

4.7.1 JET 79630/79626 \( q \)-profile substitution

A significant proportion of the core confinement difference between the JET hybrid scenarios analysed here is due to an increase in the ITG threshold due to improved \( q \)-profile shaping in the high magnetic shear region, at \( x > 0.4 \), according to GLF23. A proportion of \( \sim 60\% \) of the observed improvement in core is predicted through \( q \)-profile substitution alone. In the heat transport simulations, differences in \( R/L_{TI} \) are in the JET case only observed in the high magnetic shear region (\( x > 0.4 \)) and are correlated with differences in \( s/q \). This relative difference in \( W_{\text{core}} \) is independent of the inclusion of the \( E\times B \) suppression model in GLF23. In the combined heat and particle simulations, the primary channel of confinement improvement is \( n_e \). The majority of the remaining 40\% of core confinement difference can be linked to the differences in GLF23 boundary conditions at \( x = 0.83 \). Slight differences in \( n_e \) and \( Z_{\text{eff}} \) between the shots can account for the remaining proportion.

4.7.2 AUG 20995/20993 \( q \)-profile substitution

In the AUG hybrid scenarios analysed, approximately \( \sim 60\% \) of the observed \( W_{\text{core}} \) difference can be accounted for by a combination of the \( s/q \) effect (35\%) and the difference in boundary conditions (\( \sim 25\% \)). Again, this is independent of the \( E\times B \) suppression model. It has been estimated that the NTM in 20933 may be responsible for up to a further \( \sim 20\% \) of the difference. As in the JET case, the improvement due to the \( s/q \) effect is predicted to be primarily in the particle channel when including particle transport in GLF23. The observed \( R/L_{TI} \) differences occur both in the low and high magnetic shear regions within \( x = 0.2 - 0.6 \). The \( R/L_{TI} \) difference in the high shear region is correlated with a difference in \( s/q \). However, the difference in the low shear region cannot be explained by \( s/q \) effects, according to GLF23. Due to the difference in rotational shear for
4.7 Conclusions

$x < 0.4$ between the two discharges in the AUG case (see Fig. 5.2), it may be possible that reduced stiffness in the low shear region (not predicted by the stiff GLF23) may account for a further proportion of core confinement difference, as also observed in JET [17, 18]. It is possible that the low amplitude NTM present in 20993 at around $x = 0.5$ is responsible for a slight local difference in observed $R/L_{T_i}$ in the $x=0.5$ region. However, the NTM cannot explain the observed differences in $R/L_{T_i}$ for $x < 0.4$.

4.7.3 GLF23 $E \times B$ suppression model $W_{\text{core}}$ overprediction

The overprediction of $W_{\text{core}}$ in GLF23 when including the full $E \times B$ suppression model opens up questions regarding: the appropriate value of $\alpha_{E \times B}$ for use in GLF23, accuracy of $Z_{\text{eff}}$ for modelling input, accuracy of the GLF23 parallel velocity shear destabilization model, the mechanism of $E \times B$ shear suppression (threshold shift vs stiffness reduction), and the possible role of anomalous poloidal rotation. However, in addition the intrinsic linear thresholds are overpredicted by the GLF23 $s - \alpha$ model. Finite aspect ratio and elongation effects reduce the predicted linear thresholds. This then would reduce the degree of apparent overprediction from the $E \times B$ model. However, the critical point is that the degree of core confinement improvement due to $s/q$ is independent of both shaping effects and the value of $\alpha_{E \times B}$. Our conclusions that derive from $q$-profile shaping effects are thus independent of the abovementioned modelling uncertainties, while these results simultaneously underline the challenges raised by these same uncertainties.

4.7.4 Final conclusions

The majority of the $W_{\text{core}}$ differences can be satisfactorially predicted regardless of the $E \times B$ model, and the $s/q$ profile plays a significant role in these predictions. These GLF23 predictions are also corroborated by $R/L_{T_i}$ threshold analysis using QuaLiKiz. A general caveat is that the relatively low magnitude of the experimentally observed and calculated differences in both $R/L_{T_i}$ and the $q$-profile demands that these conclusions be treated with caution. The small magnitude of the effect - a $H_{98}$ increase of up to $\sim 0.1$ due to $s/q$ only - hinders the experimental isolation of the $s/q$ impact which can be in general be easily masked by competing effects and experimental error. Nevertheless, the results obtained here are consistent with the theoretically expected impact of $q$-profile shaping on core-confinement through threshold increase at outer radii and provides an encouraging validation in this regard. In ITER operational scenario studies, such $H_{98}$ improvements of up to $\sim 0.1$ due to $q$-profile shaping have in fact been shown to have a significant impact on the projected fusion power in the ITER hybrid scenario [29]. Validation of these ITER predictions made with GLF23 was the initial driving motivation for this work. Further $q$-profile shaping experiments aiming for larger differences would be extremely helpful for continued study of the $q$-profile impact on transport.
Acknowledgements

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4 Predictive analysis of $q$-profile influence on transport in JET and ASDEX Upgrade hybrid scenarios

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5 Quasilinear transport modelling at low magnetic shear

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Abstract
Accurate and computationally inexpensive transport models are vital for routine and robust predictions of tokamak turbulent transport. To this end, the QuaLiKiz [C. Bourdelle et al., Phys. Plasmas 14, 112501 (2007)] quasilinear gyrokinetic transport model has been recently developed. QuaLiKiz flux predictions have been validated by non-linear simulations over a wide range in parameter space. However, a discrepancy is found at low magnetic shear, where the quasilinear fluxes are significantly larger than the non-linear predictions. This discrepancy is found to stem from two distinct sources: the turbulence correlation length in the mixing length rule and an increase in the ratio between the quasilinear and non-linear transport weights, correlated with increased non-linear frequency broadening. Significantly closer agreement between the quasilinear and non-linear predictions is achieved through the development of an improved mixing length rule, whose assumptions are validated by non-linear simulations.

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5 Quasilinear transport modelling at low magnetic shear

5.1 Introduction

The availability and accuracy of computationally inexpensive first-principle physics based transport codes is vital for both interpretation of tokamak experiments and extrapolation to future devices. With this goal in mind, the QuaLiKiz quasi-linear transport model has been developed [1]. Particle and heat flux calculation time in QuaLiKiz is \( \sim 50000 \) less than in local non-linear codes. Fluxes computed by QuaLiKiz have been validated by non-linear GYRO [2, 3] simulations over a wide range of parameter space [4]. However, significant discrepancies between QuaLiKiz and non-linear simulations have been observed at low magnetic shear.

The successful prediction of transport in the low magnetic shear regime is important for the interpretation of advanced tokamak scenarios, as has been recently seen in JET hybrid scenarios at combined low magnetic shear and high rotational shear [5]. Low magnetic shear regions are also expected during ramp-up and ramp-down phases. For extrapolation to ITER, transport at low magnetic shear is important for all scenarios. This is due to the fact that in ITER, the sawtooth period is expected to be an order of magnitude greater than the confinement time [6]. Therefore even in the ITER reference H-mode scenario, low magnetic shear transport within the inversion radius between sawtooth crashes will be critical for determining the average fusion power.

The primary motivation of this work is thus to understand the physics behind the low magnetic shear discrepancy between the QuaLiKiz model as formulated in Casati et al [4], and non-linear simulations. To this end, the assumptions made in the QuaLiKiz model are systematically examined at low magnetic shear. Both linear and non-linear simulations with the gyrokinetic code GENE [7] have been used extensively for comparisons and assumption validation.

The structure of the paper is as follows. In section 5.2 the QuaLiKiz model is reviewed and the discrepancy compared to non-linear simulations at low magnetic shear is introduced. In section 5.3, a review of basic understandings of the magnetic shear impact on linear modes is presented. QuaLiKiz and GENE linear growth rates are compared at low magnetic shear, and the differences between them are not sufficient to explain the flux discrepancy. In section 5.4, QuaLiKiz assumptions on non-linear effects are explored. The validity of the quasi-linear approximation itself with regard to the ordering of the non-linear characteristic times (Kubo numbers) is examined, and is found to still hold at low magnetic shear. The delicate choice of electrostatic potential saturation level, through the mixing length rule and frequency broadening, is reevaluated at low magnetic shear. The assumptions behind the saturated potential formulation have been reexamined in great detail with the aid of 16 non-linear GENE simulations at various values of magnetic shear, \( q \)-profile and driving gradient lengths. Partial isotropization of turbulent eddies, radial correlation length dependence on the magnetic shear, and increased resonance broadening at low magnetic shear are all observed. These observations are taken into account in the quasilinear model, and agreement with the non-linear magnetic shear scans is much improved. Conclusions are discussed in section 5.5.

A number of more technical aspects have been summarized in appendixes. In appendix 5.A the analytical fluid limit applied in the QuaLiKiz Gaussian eigenmode ansatz
5.2 Review of the QualiKiz model

QuaLiKiz is based on the electrostatic eigenvalue gyrokinetic code Kinezero [8], which includes two ion species and both passing and trapped electrons. For the minimization of computational expense - critical for integrated modelling applications - the electrostatic potential shape is not calculated self-consistently. Rather, an analytical solution obtained in the fluid limit is taken and the strong ballooning representation limit applied. A Krook-like collision operator accounting for the impact of collisions on the trapped electrons has also been included [9]. Shifted circle \((s - \alpha)\) geometry is assumed for equilibrium.

The linear response calculated by Kinezero has been incorporated into the gyrokinetic quasilinear expressions for the particle and heat fluxes, resulting in the QuaLiKiz transport model [1]. From henceforth we will also refer to the eigenvalue solver as part of 'the QuaLiKiz model'. The fluxes can be schematically decomposed into contributions of the linear response and the saturated electrostatic potential:

\[
\Gamma, Q_i, Q_e = \sum_{k_\theta, \omega, \omega_{kj}} \text{Im}(R_{lin}^{\Gamma, Q_i, Q_e}(k_\theta, \omega, \omega_{kj})) |\delta \phi(k_\theta, \omega, \omega_{kj})|^2
\]  

(5.1)

Where \(R_{lin}^{\Gamma, Q_i, Q_e}\) is the linear response for the density, ion temperature, and electron temperature fluctuations respectively, \(\delta \phi\) the saturated electrostatic potential, \(\omega\) the fluctuation frequency, and \(\omega_{kj}\) the eigenfrequency of the unstable mode with poloidal wavenumber \(k_\theta = nq/r\). Multiple instabilities may coexist with the same \(k_\theta\), and are discriminated by the index \(j\). The contribution of each unstable mode (also named 'branches') to the fluxes are summed over. The linear response has been extensively validated by comparison with non-linear simulations [4, 10].

The model for the saturated electrostatic potential - a critical ingredient in evaluating the turbulent fluxes - consists of a mixing length rule weighted by a \(k\)-space spectral function validated against both non-linear simulations and experimental observations [4, 11].
The saturated potential frequency spectral shape is taken as a Lorentzian, with a width equal to the linear growth rate. This assumption has been shown to agree with non-linear simulations and experimental observations at transport relevant wavelengths [10]. Finally, the QuaLiKiz predicted fluxes are normalized by a single, constant, fitting coefficient, set such that the ion heat flux predicted by QuaLiKiz for the GA-standard case agrees with GYRO predictions.

The mixing length rule as previously applied in QuaLiKiz is now presented. We redefine for convenience the saturated electrostatic potential for each mode as $|\delta \phi_{k,j}|^2$, where $k$ is the wavenumber index and $j$ the branch index. We assume $|\delta \phi_{k,j}|^2 = S_{kj} L_j$, where $S_{kj}$ is a branch dependent k-space spectral function shape, and $L_j$ is the maximum mixing length amplitude for branch $j$: $L_j \equiv |\gamma_{k,j} \langle k^2 \rangle_{\perp}|_{\text{max}}$. The maximum is taken over the k-spectrum. The $k_0$ corresponding to the maximum is denoted $k_0|_{\text{max}}$, and represents the peak of the flux spectrum defined by $S_{kj}$, designed to reproduce the downshifted non-linear flux spectrum [12]. $S_{kj}$ is taken such that $S_{kj} \propto (k_0)^{-3}$ for $k_0 < k_0|_{\text{max}}$, and $S_{kj} \propto (k_0)^{-3}$ for $k_0 > k_0|_{\text{max}}$. $\langle k^2 \rangle_{\perp} = k_0^2 \left(1 + (s - \alpha)^2 \langle \theta^2 \rangle \right)$, where $\langle \rangle$ denotes a poloidal average. This average over the ballooning structure, when integrating over the form imposed by the fluid Gaussian eigenmode assumption, is given by $\langle \theta^2 \rangle = \frac{2 \pi m^2}{\Gamma(3/4) \Gamma(1/4)} d^2$, where $d$ is the distance between rational surfaces and $w$ is the mode Gaussian width. This choice of k-space spectrum is in agreement with nonlinear simulations, and observations [4, 11].

Fluxes computed by QuaLiKiz have been compared with GYRO non-linear simulations, and agreement for both heat and particle fluxes have been obtained in $R/L_T$, $Z_{eff}$, $T_i/T_e$ and collisionality scans around GA-standard case parameters [4]. Extensive GYRO non-linear simulations consisting of $q$-profile and magnetic shear scans have previously been carried out [13]. In Fig.5.1, the $R/L_T$ scan comparison is reproduced, and $q$-profile and magnetic shear scans around the GA standard case have been added. The $q$-profile scan also shows agreement between the predictions. However, significant discrepancies between QuaLiKiz and GYRO are evident in the magnetic shear scan, in the region $|s| < 0.5$. Note that $\alpha = 0$ in all these calculations. We now set out to uncover the sources of this discrepancy.

### 5.3 Magnetic shear impact on linear physics

In this section we review linear growth rate dependence on magnetic shear, and compare QuaLiKiz and linear-GENE growth rate calculations. Both the role of the magnetic shear on the $R/L_T$, $T_i/T_e$ and collisionality scans around GA-standard case parameters [4]. Extensive GYRO non-linear simulations consisting of $q$-profile and magnetic shear scans have previously been carried out [13]. In Fig.5.1, the $R/L_T$ scan comparison is reproduced, and $q$-profile and magnetic shear scans around the GA standard case have been added. The $q$-profile scan also shows agreement between the predictions. However, significant discrepancies between QuaLiKiz and GYRO are evident in the magnetic shear scan, in the region $|s| < 0.5$. Note that $\alpha = 0$ in all these calculations. We now set out to uncover the sources of this discrepancy.
5.3 Magnetic shear impact on linear physics

LiKiz is satisfactory at low magnetic shear. This leads to the conclusion that the reason for the discrepancy between the quasilinear and non-linear fluxes at low magnetic shear lies in the non-linear physics.

5.3.1 Linear thresholds from kinetic theory

In this section we briefly review previous results of kinetic theory regarding magnetic shear dependences on threshold, for ITG turbulence. For the $\eta_i$ ITG mode ($\eta_i \equiv d\ln T_i / d\ln n$), the $R/L T_i$ threshold has been calculated in the short-wavelength, flat density limit ($k_\theta \rho_s \approx \left( L_T / R \right)^{1/4}$, $R/L_n < 2(1 + T_i/T_e)$), as follows [14]:

$$ (R/L T_i)_c \approx \frac{4}{3} \left( 1 + T_i/T_e \right) \left( 1 + \frac{2s}{q} \right) $$  \hspace{1cm} (5.2)

Increasing the density gradient is predicted to lead to an increased $R/L T_i$ threshold. However, when including trapped electrons the increased density gradient can destabilize the TEM branch, nullifying the turbulence stabilization [15].

A study of the $(s,q)$ impact on the linear $R/L T$ threshold for ITG/TEM modes in Kinexero (and thus in QuaLiKiz) have previously been carried out with fixed $T_i/T_e = Z_{eff} = 1, \alpha = 0$, and in the flat density limit [16]. The parameterization of the critical temperature gradient lengths, $(R/L T)_c$, was as follows for ITG-like modes: $1.1 + 1.4s + 1.9s/q$ for $s > 0$, and $0.9 + 1.6|s| + 9.9|s|/q$ for $s < 0$. This mixed $s$ and $s/q$ dependency share the same qualitative dependencies as in the analytical limits above. The asymmetry with the sign of $s$ is due to the stabilization of the toroidal branch for negative shear. An extensive study of $T_i/T_e$ dependence in QuaLiKiz was carried out by Casati et al [17].
Finally, comprehensive linear gyrokinetic simulations with GS2 have led to a parameterization of ETG linear thresholds, isomorphic with ITG linear thresholds (assuming adiabatic electrons), for which it can be written as follows [18]:

\[
\left( \frac{R}{L_{T_i}} \right)_{\text{c}} = \max \left\{ \left( 1 + \frac{T_i}{T_e} \right) (1.33 + 1.91 s) \left( 1 - 1.5 \epsilon \right) \left( 1 + 0.3 \frac{d\epsilon}{d\epsilon} \right), 0.8 \frac{R}{L_{T_i}} \right\}
\] (5.3)

where \( \epsilon = r/R \) and \( \kappa \) is the elongation. In the circular cross-section limit, this formula is very similar to the analytical formula, equation 5.2.

Growth rate contour plots calculated by QuaLiKiz are shown in Fig.5.2. The growth rates correspond to the most unstable branch in GA-standard \( R/L_T \) and magnetic shear scans (where \( R/L_{T_i} = R/L_{T_e} \)). In the left panel, the maximum growth rate for the spectrum \( k_y < 0.8 \) is displayed. In the right panel, the growth rates for \( k_y = 0.2 \) are displayed. In both plots, the increased instability threshold for high magnetic shear and negative magnetic shear is clearly visible. The residual modes at low \( R/L_T \) at low magnetic shear are modes in the electron diamagnetic direction due to the finite \( R/L_n \) in the GA-standard case.

An interesting detail to note in the low \( k_y \) case (right panel) of Fig.5.2 is reduced \( \gamma \)-stiffness at low magnetic shear. This is evidenced by the significantly decreased gradient with respect to \( R/L_T \) at \( s < 0.2 \). A possible explanation for this decreased \( \gamma \)-stiffness is that the long-wavelength limit is reached, defined by \( k_\| s \ll (L_{T_i}/R)^{1/4} \). In this limit, the toroidal branch is stabilized [19]. This is due to the growth of the eigenfunction envelope width along the magnetic field, given by \( \theta \approx s^{1/2} e^{-1/4}(k_\| \rho_i)^{-1} \). A broad envelop-

![Figure 5.2](image_url)

Figure 5.2: Linear growth rates for the most unstable branch calculated by QuaLiKiz in a GA-STD \( R/L_T \) scan, where \( R/L_{T_i} = R/L_{T_e} \). In the left panel the maximum growth rate for the spectrum \( k_y < 0.8 \) is shown. In the right panel, the growth rate for \( k_y = 0.2 \) is shown.
5.3 Magnetic shear impact on linear physics

Magnetic shear impact on linear physics leads to decreased interaction with the bad curvature region, stabilizing the toroidal branch. In the QuaLiKiz Gaussian eigenmode approximation the envelope width is proportional to $d/w$, which increases at low $k_y$, and also increases at low magnetic shear as seen in figure 5 in Romanelli et al [9]. Decreased linear growth rates at low $k_y$ may lead to preferential quenching of long wavelength turbulence by zonal flows at low magnetic shear. Since these long wavelengths dominate the transport, this can play a major role in the determination of the fluxes.

5.3.2 Comparison of QuaLiKiz and linear-GENE growth rates

Growth rates of the dominant branch calculated by QuaLiKiz and GENE are compared in this section. In Fig.5.3 results of a $R/L_T$ scan are displayed (where $R/L_T = R/L_Ts$ throughout the scan), around the GA-standard case, at $k_y = 0.15, 0.3$ and $s = 0.1, 1$. We can see that even though the $R/L_T$ thresholds are lower for the lower shear cases, the $\gamma$-stiffness is also lower for the lower shear case, leading to a crossover of the high-shear and low-shear $\gamma$ curves at a $R/L_T$ position dependent on $k_y$. At low $k_y$, this crossing point is much closer to the threshold, at experimentally relevant $R/L_T$. Note that this effect is predicted by both GENE and QuaLiKiz, in spite of the Gaussian eigenmode ansatz made in QuaLiKiz. Indeed, it is shown in Appendix 5.C that this ansatz is valid down to $s = 0.1, k_y = 0.15$.

We also show in Fig.5.4 a comparison between linear-GENE and QuaLiKiz of the

![Figure 5.3: Linear growth rates calculated for the GA-STD kinetic electron case in a $R/L_T$ scan at various $k_y$ and magnetic shear values by both linear-GENE (left panel) and QuaLiKiz (right panel).](image)
growth rate calculation for a GA-standard magnetic shear scan at $k_y = 0.3$. The QuaLiKiz predictions agree with linear-GENE to within $\sim 30\%$ even at low magnetic shear. However, at negative magnetic shear QuaLiKiz severely underpredicts the growth rates, due to an underestimation of the slab ITG branch growth rate. This may be due to the strong ballooning representation and prescribed eigenmode assumptions. The resolution of this issue is left for future work.

In summary, while the ITG threshold decreases at low magnetic shear, the $\gamma$-stiffness is also reduced at lower magnetic shear. At low $k_y$ this effect is increased. This is reproduced both by linear-GENE and QuaLiKiz. Furthermore, dominant branch linear-GENE and QuaLiKiz calculations agree to within $\sim 30\%$ in the parameter ranges studied, in spite of the QuaLiKiz Gaussian eigenmode ansatz. Therefore a miscalculation of the dominant branch growth rate cannot be responsible for the low magnetic shear flux discrepancy between QuaLiKiz and non-linear models, where the QuaLiKiz flux is significantly higher as seen in Fig.5.1. The reduced growth rates at combined low magnetic shear and low $k_y$ is an important point, since this could have ramifications for setting the turbulence correlation length. This is explored further in the modification of the mixing length rule in section 5.4.3.

### 5.4 Non-linear effects at low magnetic shear

In this section we explore the non-linear physics which may lead to the lack of validity of the previous QuaLiKiz quasilinear computed fluxes at low magnetic shear. As described in section 6.1, the quasilinear flux can be decomposed into two parts: the linear response and the saturated electrostatic potential. This description is justified if the particles un-
dern, the QuaLiKiz mixing length model for the non-linear saturated potential was also reexamined at low magnetic shear. This includes the model for $k_\perp$, the evaluation of the $k_y$ maximum of the non-linear flux spectrum, and the frequency broadening assumption. These assumptions were extensively validated previously for a wide range of parameters [4], but not for magnetic shear scans. We extend this validation towards low magnetic shear.

A set of dedicated GENE non-linear runs were carried out to test and validate the QuaLiKiz assumptions. These non-linear runs are introduced in section 5.4.1. In section 5.4.2 the evaluation of the Kubo numbers in the non-linear simulations are discussed. In section 5.4.3 the mixing length model for the saturated potential is reexamined, and modified following comparisons with the non-linear simulations. Finally, predicted fluxes from both the modified QuaLiKiz and the set of non-linear GENE simulations are compared in section 5.4.4.

5.4.1 Description of GENE runs

A total of 16 non-linear GENE runs were carried out. The runs are comprised of 4 sets of 4-point magnetic shear scans at $s = -0.4, 0.1, 0.6, 1$. The sets are comprised of the GA-standard case, and 3 further sets with lowered inverse gradient lengths and varying $q$-profile values, consistent with typical JET hybrid scenario parameters at mid-radius. This was done in order to widen the GENE-QuaLiKiz magnetic shear scan validation in $q$-profile parameter space, down towards $q = 1$ which often coincides with low magnetic shear in experiments. Furthermore, the added data sets allow GENE-QuaLiKiz validation at lower levels of flux than the GA-standard case. Since the eventual goal is to apply QuaLiKiz as a transport model within the integrated modelling framework, validation at more experimentally relevant fluxes is critical. The parameters for these sets are summarized in table 5.1. All GENE simulations carried out were electrostatic, local, collisionless, with periodic radial boundary conditions, kinetic electrons (and real mass ratio), and with $s - \alpha$ geometry. Throughout the sets, $\alpha = 0$, $r/a = 0.5$, and $R_0/a = 3$.

In all simulations, we used 16-point discretization in the parallel direction, 32 or 48-points in the parallel velocity directions (based on linear convergence studies), and 8 magnetic moments. For these parameters, non-linear convergence tests were carried out, and the heat flux sensitivities were negligible. For each simulation, extensive non-linear convergence studies were carried out in the number of toroidal modes, the perpendicular box sizes, and in the radial grid discretization. The box sizes in the perpendicular directions range from $[L_x/\rho_s, L_y/\rho_s] = [100, 126]$ for the set A, $s = 1$ case to up to $[L_x/\rho_s, L_y/\rho_s] = [333, 210]$ for the more challenging $s = 0.1$ cases. The number of toroidal modes ($n_y$) employed ranged from 16 for the set A, $s = 1$ case, to 32 for the $s = 0.1$ cases. The $n_y = 32$ cases correspond to $k_y(max) = 1.55$ or $k_y(max) = 0.93$, depending on the perpendicular box size. In general, the $s = 0.1$ cases were more computationally intensive due to the higher radial box size needed to satisfy the parallel
boundary condition, and also due to the higher number of toroidal modes necessary for convergence.

Low heat flux sensitivity was also found to the inclusion of collisions. However, it was found that the particle transport was highly sensitive to the velocity and parallel space resolutions, particularly when a particle inward flux dominates (as in the set A, s = 1 case). This particle transport sensitivity was greatly decreased when including collisions. In general, verification of the QuaLiKiz particle transport predictions is also of great importance. However, it is preferable to carry out this verification vs non-linear runs when including collisionality. This is due both to the increased experimental relevance, and also the seemingly improved non-linear convergence properties of particle diffusivity in collisional runs. Due to the increased computational expense of collisional runs, a more rigorous validation of the particle fluxes when including collisionality is deferred to future work. Here we concentrate on the heat transport validation of the collisionless QuaLiKiz model at low magnetic shear, and maintain a caveat on the convergence of the non-linearly calculated particle fluxes in this work, particularly the inward fluxes. The discretization and box size choices, as well as the transport predictions of all 16 runs, are shown in table 5.2.

5.4.2 Kubo numbers

The validity of the quasilinear assumption depends on whether test particles in the turbulent fields undergo stochastic linearized trajectories, allowing a diffusive random-walk process to be considered as opposed to field trapping. This can be quantified by the Kubo number $K$ [20, 21], the ratio between the Eulerian autocorrelation time, $\tau_{ac}$, of the fluctuating potential and the particle flight time, $\tau_f$. $\tau_f$ is defined as $\lambda_x/\langle|v_x|\rangle$, where $\lambda_x$ is the radial correlation decay length, and $\langle|v_x|\rangle$ the root mean square radial velocity of a test particle in the turbulent field. For $K < 1$, the autocorrelation time of the turbulent field is shorter than the transit time of a test particle around a turbulent eddy, justifying the use of a random-walk diffusive model. This condition is however necessary but not sufficient. In addition, there must be enough randomness in the Hamiltonian described in the dynamics, e.g. when it includes many waves with random phases [22]. This condition though is well satisfied in the strong turbulence cases studied here. For a $R/L_T$ scan based on the GA-standard case, computed using GYRO, it was seen that $K < 1$ [4]. This was done calculating the autocorrelation time as $\tau_{ac} = 2D/\langle|v_x|^2\rangle$, where D is the

### Table 5.1: Input parameters for the GENE magnetic shear scans.

<table>
<thead>
<tr>
<th>Set name (GASTD)</th>
<th>$R/L_{Ti}$</th>
<th>$R/L_{Te}$</th>
<th>$R/L_n$</th>
<th>$T_i/T_e$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9</td>
<td>9</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>7.6</td>
<td>6.7</td>
<td>4</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>C</td>
<td>6.3</td>
<td>5.5</td>
<td>3.3</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>D</td>
<td>7.6</td>
<td>6.7</td>
<td>4</td>
<td>1.4</td>
<td>1</td>
</tr>
</tbody>
</table>
5.4 Non-linear effects at low magnetic shear

Table 5.2: Grid settings and results of converged GENE non-linear runs.

<table>
<thead>
<tr>
<th>Run</th>
<th>$n_x$</th>
<th>$n_y$</th>
<th>$k_y$ min</th>
<th>$n_z$</th>
<th>$n_{av}$</th>
<th>$L_z$</th>
<th>Sim time [$R/c_s$]</th>
<th>$\chi_i/\chi_e/D$ [GB units]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: $s=1$</td>
<td>140</td>
<td>16</td>
<td>0.05</td>
<td>16</td>
<td>32</td>
<td>8</td>
<td>100</td>
<td>950</td>
</tr>
<tr>
<td>A: $s=0.6$</td>
<td>140</td>
<td>16</td>
<td>0.05</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>140</td>
<td>960</td>
</tr>
<tr>
<td>A: $s=0.1$</td>
<td>256</td>
<td>24</td>
<td>0.05</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>200</td>
<td>560</td>
</tr>
<tr>
<td>A: $s=-0.4$</td>
<td>256</td>
<td>24</td>
<td>0.03</td>
<td>16</td>
<td>32</td>
<td>8</td>
<td>166</td>
<td>630</td>
</tr>
<tr>
<td>B: $s=1$</td>
<td>140</td>
<td>16</td>
<td>0.05</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>140</td>
<td>800</td>
</tr>
<tr>
<td>B: $s=0.6$</td>
<td>160</td>
<td>32</td>
<td>0.03</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>160</td>
<td>750</td>
</tr>
<tr>
<td>B: $s=0.1$</td>
<td>300</td>
<td>32</td>
<td>0.03</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>333</td>
<td>720</td>
</tr>
<tr>
<td>B: $s=-0.4$</td>
<td>240</td>
<td>24</td>
<td>0.03</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>160</td>
<td>640</td>
</tr>
<tr>
<td>C: $s=1$</td>
<td>140</td>
<td>16</td>
<td>0.05</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>140</td>
<td>1570</td>
</tr>
<tr>
<td>C: $s=0.6$</td>
<td>210</td>
<td>24</td>
<td>0.05</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>140</td>
<td>600</td>
</tr>
<tr>
<td>C: $s=0.1$</td>
<td>300</td>
<td>32</td>
<td>0.03</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>200</td>
<td>600</td>
</tr>
<tr>
<td>C: $s=-0.4$</td>
<td>240</td>
<td>24</td>
<td>0.05</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>160</td>
<td>610</td>
</tr>
<tr>
<td>D: $s=1$</td>
<td>200</td>
<td>32</td>
<td>0.05</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>200</td>
<td>610</td>
</tr>
<tr>
<td>D: $s=0.6$</td>
<td>240</td>
<td>32</td>
<td>0.05</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>200</td>
<td>560</td>
</tr>
<tr>
<td>D: $s=0.1$</td>
<td>300</td>
<td>32</td>
<td>0.05</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>200</td>
<td>560</td>
</tr>
<tr>
<td>D: $s=-0.4$</td>
<td>240</td>
<td>32</td>
<td>0.05</td>
<td>16</td>
<td>48</td>
<td>8</td>
<td>160</td>
<td>380</td>
</tr>
</tbody>
</table>

particle diffusivity, as carried out in [23]. This assumes a priori that the particle transport is due to a random-walk process in the oscillating field. In this work we calculate the Kubo number by a more direct method, by calculating the $\tau_{ac}$ directly from the fluctuating potential field from GENE non-linear simulations. The time correlation function is $C(\Delta \tau) = \langle \phi(x, y, t) \phi(x, y, t + \Delta \tau) \rangle / \langle |\phi|^2 \rangle$, averaging over the x and y directions. The parallel coordinate is kept fixed at the low-field side. The correlation time is taken as the $1/e$ time of this function. All $n = 0$ modes are not included in the calculation, which thus considers only the flux inducing $n > 0$ background drift-wave population. $\lambda_x$ is calculated as the $1/e$ length of the function $C(\Delta x) = \langle \phi(x, 0, t) \phi(x + \Delta x, 0, t) \rangle / \langle |\phi|^2 \rangle$. $\langle v_x \rangle = \sqrt{\langle E_x^2 \rangle / B_{ref}}$, with the same averaging procedure as $\tau_{ac}$ for consistency. $B_{ref}$ is the reference magnetic field used to normalize the GENE equations and variables.

We have calculated the Kubo numbers for the full set of GENE non-linear runs. The calculated correlation lengths, times and Kubo numbers can be found in table 5.3. For all cases, $K < 1$. We thus conclude that the quasilinear approximation indeed holds. Note furthermore that $K < 1$ but still of order unity supports the application of the mixing length rule for setting the level of the saturated potential, where the implicit assumption is that the step length of the random walk process is the eddy size. While the quasilinear approximation itself may hold, the value of the saturated electrostatic potential cannot be determined by linear theory. This value is determined by a mixing length rule, whose validation at low magnetic shear is evaluated in the next section.
5 Quasilinear transport modelling at low magnetic shear

Table 5.3: Correlation times, lengths and Kubo numbers for the \textit{GENE} non-linear runs. Units are Larmor radii for the lengths, and $R/c_s$ for the times.

<table>
<thead>
<tr>
<th>Run</th>
<th>$\sqrt{\langle E_y^2 \rangle}$</th>
<th>$\lambda_x$</th>
<th>$t_{ac}$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: $s=1$</td>
<td>8.6</td>
<td>12.60</td>
<td>1.85</td>
<td>0.42</td>
</tr>
<tr>
<td>A: $s=0.6$</td>
<td>11.78</td>
<td>11.2</td>
<td>2.05</td>
<td>0.72</td>
</tr>
<tr>
<td>A: $s=0.1$</td>
<td>12.23</td>
<td>7.12</td>
<td>0.58</td>
<td>0.33</td>
</tr>
<tr>
<td>A: $s=-0.4$</td>
<td>9.66</td>
<td>6.94</td>
<td>1.85</td>
<td>0.86</td>
</tr>
<tr>
<td>B: $s=1$</td>
<td>8.7</td>
<td>12.70</td>
<td>2.14</td>
<td>0.49</td>
</tr>
<tr>
<td>B: $s=0.6$</td>
<td>10.8</td>
<td>10.50</td>
<td>1.74</td>
<td>0.60</td>
</tr>
<tr>
<td>B: $s=0.1$</td>
<td>8.59</td>
<td>6.90</td>
<td>1.09</td>
<td>0.45</td>
</tr>
<tr>
<td>B: $s=-0.4$</td>
<td>7.93</td>
<td>6.87</td>
<td>2.14</td>
<td>0.82</td>
</tr>
<tr>
<td>C: $s=1$</td>
<td>5.37</td>
<td>13.67</td>
<td>1.90</td>
<td>0.27</td>
</tr>
<tr>
<td>C: $s=0.6$</td>
<td>8.48</td>
<td>10.92</td>
<td>3.44</td>
<td>0.89</td>
</tr>
<tr>
<td>C: $s=0.1$</td>
<td>6.69</td>
<td>7.30</td>
<td>1.44</td>
<td>0.44</td>
</tr>
<tr>
<td>C: $s=-0.4$</td>
<td>5.5</td>
<td>6.08</td>
<td>2.01</td>
<td>0.61</td>
</tr>
<tr>
<td>D: $s=1$</td>
<td>7.38</td>
<td>9.76</td>
<td>0.96</td>
<td>0.24</td>
</tr>
<tr>
<td>D: $s=0.6$</td>
<td>9.23</td>
<td>8.41</td>
<td>1.38</td>
<td>0.50</td>
</tr>
<tr>
<td>D: $s=0.1$</td>
<td>7.67</td>
<td>5.88</td>
<td>1.20</td>
<td>0.52</td>
</tr>
<tr>
<td>D: $s=-0.4$</td>
<td>6.9</td>
<td>5.93</td>
<td>1.96</td>
<td>0.76</td>
</tr>
</tbody>
</table>

5.4.3 Reevaluation of the QuaLiKiz saturated potential model

The model for the saturated potential is comprised of three parts: the model for $k_\perp$, setting the maximum and spectral shape of the non-linear flux spectrum, and frequency broadening. We briefly recall here the previous model. The $k_y$ maximum of the flux spectrum is designated $k_y|_{\text{max}}$, and corresponds to the maximum of the mixing-length flux spectrum $\langle k^2 \rangle$ [12, 24], where:

$$\langle k^2 \rangle = k_y^2 \left( 1 + (s - \alpha)^2 \langle \theta^2 \rangle \right) \quad (5.4)$$

This formulation of $k_\perp$ calculates only the effective $k_x$, due to the shearing of the mode along the field line, and assumes that the dominant instability has intrinsic $k_x = 0$.

The $k$-spectrum is $k_y^{-3}$ and $k_x$ above and below the flux spectrum maximum respectively. The frequency spectrum of the saturated potential is broadened by the linear growth rate as follows: $\phi_k(\omega) \propto \frac{1}{(\omega - \omega_k)^2 + \gamma_k^2}$. This model has been shown to lead to discrepancies both in the $k_y$ of the saturated potential peak and in the frequency spectrum width when compared to \textit{GENE} non-linear results at low magnetic shear. The $k$-spectrum model is reviewed and an improved formulation for $k_\perp$ is proposed. The frequency spectrum broadening comparison and consequences of the discrepancy are then addressed.
5.4 Non-linear effects at low magnetic shear

Choice of \( k_{\perp} \) and the model for calculating the flux spectrum peak

The correlation length of the turbulence is set both by the \( k_x \) inherent in the \( k_{\perp} \) formulation and also by \( k_y|_{\text{max}} \). The magnetic shear dependence in equation 5.4 suggests (with \( \alpha = 0 \)) that the effective \( k_x \) is lower at \( s = 0.1 \) than at \( s = 1 \) (for the same \( k_y|_{\text{max}} \)). This would correspond to increased correlation lengths as the magnetic shear is reduced. The \( k_x \) and \( k_y \) components of \( k_{\perp} \) from equation 5.4 are plotted in Fig.5.5, for the GA-standard \( s = 1 \) and \( s = 0.1 \) case, where \( \langle \theta^2 \rangle \) is calculated from the QuaLiKiz eigenmode ansatz. At \( s = 1 \) the assumed structures are approximately isotropic, particularly at lower, transport relevant \( k_y \). At \( s = 0.1 \), \( k_x < k_y \). Even though \( \langle \theta^2 \rangle \) increases as the magnetic shear decreases, this does not compensate the decreasing \( s^2 \), at least down to \( s = 0.1 \). This was corroborated by linear GENE analysis, where the eigenmodes were self-consistently calculated.

However, the magnetic shear dependence in equation 5.4 is not consistent with the non-linear results, where the correlation length decreases with decreasing shear, as seen in table 5.3. The eddy structure for both the \( s = 0.1 \) and \( s = 1 \) non-linear GA-standard cases are shown in Fig.5.6. The structures are roughly isotropic. The decreased correlation length at low magnetic shear is a result of the increased \( k_y|_{\text{max}} \) at \( s = 0.1 \).

Therefore the \( k_y|_{\text{max}} \) calculation is also critical for setting correlation length and flux values. However, it was found that due to the large weight of \( 1/k_y^2 \) at low \( k_y \), \( k_y|_{\text{max}} \) typically corresponds to the lowest (in \( k_y \)) unstable mode, in spite of the relatively low growth rates. For QuaLiKiz this is typically in the range of \( k_y = 0.05 - 0.1 \). This is in disagreement with the non-linear flux spectra maxima from the magnetic shear scans, only agreeing approximately at \( s = 1 \). Thus both the \( k_{\perp} \) and \( k_y|_{\text{max}} \) formulations must be rectified to correctly account for the magnetic shear dependencies on the flux in the

Figure 5.5: \( k_x \) as a function of \( k_y \), as calculated in QuaLiKiz for \( s = 1 \) and \( s = 0.1 \), from the \( k_{\perp} \) formulation given in equation 5.4. The dashed curve represents \( k_x = k_y \), as a measure for the degree of isotropy.
Figure 5.6: Contours of the electrostatic potential (top row) and the x and y length correlation functions (bottom row) for a typical timeslice in GENES = 1 (left column) and s = 0.1 (right column) GA-standard case simulations.

mixing length model.

The non-linear flux spectra maxima, $k_y|_{\text{max}}$, were reproduced by replacing the effective $k_x$ in $k_z$ with the actual $k_z$ values from the non-linear simulations. Thus $k_z^2 (nl) = k_y^2 + \langle k_x \rangle^2 (nl)$, where $\langle k_x \rangle (nl)$ is the averaged value of $k_x$ for each $k_y$ mode in the non-linear simulation. $\langle k_x \rangle (nl)$ is defined as $\sum_{k_x} |\phi(k_x, k_y)|^2 / \sum |\phi(k_x, k_y)|^2$, where the summation is over $k_x \geq 0$ for a given $k_y$. The values of $|\phi(k_x, k_y)|^2$ are pairwise averaged over negative and positive $k_x$, as well as over the parallel coordinate and for the times corresponding to the saturated state of the simulation. In the non-rotating, up-down symmetric plasmas considered in our simulations, $k_x$ is symmetric around 0. In Fig.5.7 these $\langle k_x \rangle$ values are plotted versus $k_y$ for the GENES non-linear GA-standard case magnetic shear scan. At low $k_y$, the eddies are not isotropic. $\langle k_x \rangle$ does not continue to drop
5.4 Non-linear effects at low magnetic shear

at low \( k_y \), and saturates at a value which increases as the magnetic shear decreases. This corresponds to reduced correlation lengths at low magnetic shear.

The \( k_y|_{\text{max}} \) corresponding to the maximum \( \gamma_k/k^2 (nl) \) corresponds well with the non-linear flux spectra \( k_y \) maximum, as seen in Fig.5.8 for the entire range of non-linear simulations carried out. \( \gamma_k \) was taken from GENE linear simulations and \( \sqrt{k}^2 (nl) \) from GENE non-linear simulations. The considerations raised in sections 5.3.1 - the stabilization of the toroidal branch at low \( s k_y \) - also plays a role in setting \( k_y|_{\text{max}} \). Not only is \( \langle k_x \rangle \) increased at low magnetic shear, but the growth rates themselves are reduced at low \( s k_y \), further increasing the obtained \( k_y|_{\text{max}} \), leading to reduced correlation lengths at low magnetic shear.

In light of these observations from the non-linear simulations, we have improved the \( k^2\perp \) formulation in QuaLikiz by taking into account the non-linear contribution to \( \langle k_x \rangle \). This simultaneously recovers the non-linear flux spectra \( k_y \) maximum, and provides the magnetic shear and \( q \)-profile dependence on the precise relation between \( k_y \) and \( \langle k_x \rangle \) at \( k_y|_{\text{max}} \). We have found that the following relation successfully reproduces the non-linear \( \langle k_x \rangle(k_y) \) for the magnetic shear and \( q \)-profile scans carried out in this work:

\[
\langle k_x \rangle(k_y) = k_y s \sqrt{q^2} + 0.4 e^{-2|s|} + \frac{3}{2} (k_y - 0.2) H (k_y - 0.2) \quad (5.5)
\]

Where \( H \) is a Heavyside function. The first term on the RHS is the effective \( k_x \) due to the magnetic shear, which also appears in the previous \( k^2\perp \) formulation. The second term approximates the value of the non-linear contribution to \( \langle k_x \rangle \) in the long wavelength, transport relevant region. The third term on the RHS was set to fit the non-linear \( \langle k_x \rangle \) at higher \( k_y \), and only plays a minor role at transport relevant \( k_y \). The first term dominates at high magnetic shear, while the second term dominates at low magnetic shear. The QuaLikiz mixing length rule formulation for \( k^2\perp \) was thus set in accordance with equation 5.5. The results for \( k_y|_{\text{max}} \), using the QuaLikiz \( \gamma_k \) and the new \( k^2\perp = k^2_y + \langle k_x \rangle^2 \),
are seen in Fig. 5.9, where the results for the newly formulated $k_{\perp}^2$ are compared with the previously formulated $k_{\perp}^2$. The new formulation leads to a much improved reproduction of the non-linear $k_y|_{\text{max}}$. The absolute value $|s|$ in the exponent in the $\langle k_x \rangle$ formulation was set to avoid a non-physical exponential increase of $\langle k_x \rangle$ at large absolute values of negative magnetic shear. While the continuing decrease of the correlation length at negative magnetic shear is then not captured, we note that in general the QuaLiKiz predictions at negative magnetic shear deviate from the non-linear predictions due to an underprediction of the slab ITG branch. This is seen in Fig.5.9, for $s < -0.3$, where the high values predicted for $k_y|_{\text{max}}$ is due to mode stabilization at lower $k_y$, and not due to $k_{\perp}^2$. The present formulation is thus relevant for low magnetic shear, while improved
agreement between QuaLiKiz and non-linear flux predictions at negative magnetic shear is left for future work.

In addition to improving the $k_\perp$ formulation, we can consider also introducing a shearing effect of zonal flows on large wavelength turbulence [25]. This can be done by an ExB quench rule similar to that employed in the quasilinear models GLF23 [26, 27] and TGLF [28] for external toroidal velocity. Indeed, the non-linear $k_y |_{\text{max}}$ was also recovered by calculating $\max(\gamma_{\text{eff}}/k_y^2)$, where $\gamma_{\text{eff}}$ is a quenched growth rate taken as $\gamma_{\text{eff}} = \max(0, \gamma_k - 0.3\omega_{\text{ExB}})$. $\omega_{\text{ExB}}$ is the effective zonal flow ExB shear rate taken from the GENE non-linear simulations, including the effect of the autocorrelation time on the effective shearing time [29]. 0.3 is a tuned coefficient which provided a best fit to the

Figure 5.9: Reproduction of the non-linear flux spectra maximum from the maximum of the $\gamma_k / k_\perp^2$ (nl) spectrum, where $\gamma_k$ is from QuaLiKiz, and $\langle k_x \rangle^2$ is taken from a formula found to reproduce GENE non-linear simulations.
non-linear flux spectra maxima throughout the set of simulations. $k_{\perp}$ was replaced by $k_y$ due to the approximate $k_x$ and $k_y$ isotropy. However, the simultaneous inclusion of $\gamma_{\text{eff}}$ and the improved $k_{\perp}$ formulation does not change the calculation of $k_y|_{\text{max}}$ compared to the sole inclusion of the improved $k_{\perp}$ formulation. This is because the role played by the zonal flow shearing - the quenching of long wavelength modes - is effectively played by the non-isotropy of $k_x$ and $k_y$ at low $k_y$. Due to the challenge in parameterizing the zonal flow shear rate $\omega_{ExB}$ throughout parameter space within QuaLiKiz, and given the minor effect the zonal flow shearing of low $k_y$ modes has on determining $k_y|_{\text{max}}$ if the non-linear ($k_x$) is included in the model, it was decided not to include a zonal flow growth rate quench model in QuaLiKiz.

A possible mechanism for the dependence of magnetic shear on the zonal flow impact on $(k_x)$ is now discussed. In non-linear mode coupling, the amount of energy that is injected into the primary mode is balanced by the coupling to other modes at the same scale or nearby scales. In general this coupling must satisfy a frequency matching condition as well as a wavenumber matching condition ($\omega_p + \omega_q - \omega_k = 0, p + q - k = 0$). At high magnetic shear, the modes are more strongly ballooned (more localized around $\theta = 0$), and in addition the effective local radial wavenumber varies poloidally, since $k_{\text{eff}}^p = k_y \hat{s} + k_x$. This makes it rather difficult for the dominant ballooning eigenmode to couple effectively to a zonal mode, which is poloidally symmetric, with a direct interaction. However, in the case of low shear, the ballooning structure of the primary mode is much less pronounced, and in addition the local $k_{\text{eff}}^p$ is a much weaker function of $\theta$. This makes it easier for the primary mode to couple directly to a zonal mode. This dependency of magnetic shear on zonal flows is well known and has been observed in a number of different contexts, for example in ETG gyrofluid simulations [30]. In the direct simulations carried out in this work, the relative amplitude of zonal flows is indeed found to be greater at low magnetic shear, as seen in Fig.5.10 for the set A (GA-standard case) $s = 1$ and $s = 0.1$ cases. The saturated electrostatic potential amplitudes are normalized, thus the lower normalized amplitude of the $s = 0.1$ case reflects the higher relative amplitude of the $s = 0.1$ $k_y > 0$ modes (zonal flows) compared with the $s = 1$ case.

Since in the case of wave interactions it takes at least three waves to interact, when the direct interaction with the zonal flow is allowed, another drift mode with non-zero $k_x$ also extracts energy from this interaction (in fact enstrophy and not energy is exchanged, but for the sake of current argument this is not of essential importance). This has two effects on turbulence. First, the zonal flows isotropize turbulence, since they can shear streamers apart. They also reduce the degree of ballooning in the mode structure. This isotropization can make the non-linearly formed $k_x$, important in the computation of the flux or the mixing length saturation level. At high magnetic shear, the 'non-linear' $k_x$ is less important, since the zonal flow drive is lessened due to effective $k_x$ being a function of $\theta$. But at low magnetic shear, the turbulence is isotropized due to zonal flows, so a substantial amount of the energy is found in $k_x \neq 0$ fluctuations.

In summary, the previous $k_{\perp}^2$ formulation in QuaLiKiz led both to an incorrect estimate of the non-linear flux spectrum maximum, and to the wrong dependency on magnetic shear in the $k_{\perp}^2$ description itself at low magnetic shear. From analysis with non-linear
5.4 Non-linear effects at low magnetic shear

and linear GENE simulations, it was found that improving the $k^2_\perp$ formulation in the QuaLiKiz mixing length rule, taking into account the $\langle k_x \rangle (k_y)$ found from non-linear simulations, leads to an improved estimate of $k_y^2/|max$ and the magnetic shear and $q$-profile dependence of $k^2_\perp$. The increased values of $\langle k_x \rangle (k_y)$ at low magnetic shear at low, transport relevant, $k_y$ values may be related to the observed greater relative amplitude of zonal flow modes. This may rise due to the less pronounced ballooning structure of the low magnetic shear eigenmodes. It is also possible that linearly unstable modes with $k_x$-center $\neq 0$ - which are more prevalent at low magnetic shear - further contribute to the $k_x$ spectrum.

Frequency broadening

In the QuaLiKiz model the width of the broadened electrostatic field frequency function due to stochastic scattering is taken as the linear growth-rate, $\gamma_k$. This assumption can be checked by comparison to the non-linear frequency spectra from GENE. The width of the non-linear frequency spectra, calculated for each $k_y$, is compared to the linear growth rate from linear GENE. In Fig.5.11 we see the examples of the frequency broadening data and Lorentzian fit from set A (GA-standard case) $s = 1$ and $s = 0.1$ cases for $k_y = 0.2$. The GENE calculated linear frequency ($\omega_{lin}$) and growth rate ($\gamma_{lin}$) are compared with the non-linear frequency spectrum center ($\omega_{fit}$) and width ($\Delta\omega_{fit}$) as defined by the Lorentzian fit. We note that in analysis of GYRO frequency spectra, both Lorentzian and Gaussian fits to the frequency spectra were satisfactory in recovering the FWHM, while

Figure 5.10: Relative amplitude of zonal flow modes ($k_y = 0$) compared with the $k_y > 0$ modes for the GA-standard $s = 1$ and $s = 0.1$ cases. $k_x$ has been averaged over at each $k_y$. 

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a T-model was in fact the best fit to the data [10]. In Fig. 5.12 the comparison between the non-linear frequency spectra width (from the Lorentzian fit) and the linear growth rate is shown for all the sets at $s = 1$, $s = 0.6$, and $s = 0.1$. The assumption generally holds for $s = 1$ and $s = 0.6$ in the transport-relevant range of $k_y < 0.3$. However, it is very interesting to note that for $s = 0.1$, the frequency width does not equal the linear growth rate for any value of $k_y$. In the transport relevant range $0.15 < k_y < 0.5$, the non-linear frequency width is significantly higher than the linear growth rate estimation. This is consistent with the generally lower autocorrelation times observed at low magnetic shear, as seen in table 5.3. This must be taken into consideration in the quasi-linear model.

This increased frequency broadening at low magnetic shear may be a result of non-linear decorrelation mechanisms, such as zonal flows, which play a stronger role at low magnetic shear as seen in the previous correlation length discussion. Also in the set with lowest flux, set D, where the impact of zonal flows is also expected to be stronger due to the vicinity to the instability threshold, the frequency broadening is greater than the linear growth rate estimation even at $s = 0.6$ and $s = 1$. The existence of zonal flows affects the response function. This can be seen via a renormalization of the wave-kinetic equation. In the presence of zonal flows, the basic wave-kinetic response function takes the form:

$$ R = \frac{R_0}{1 + R_0^2 \left( \frac{\partial}{\partial r} \left( V_E(r) k_y \right) \right)^2 \frac{\gamma_{lin}}{\Delta \omega^2}} $$

where $N$ is the wave action. Using this form instead of the linear response function, we
5.4 Non-linear effects at low magnetic shear

Figure 5.12: Non-linear frequency spectra fitted Lorentzian width and linear growth rates are compared for a range of $k_y$, for all 4 sets studied, for $s = 1$ (left column), $s = 0.6$ (middle column), and $s = 0.1$ (right column).
5 Quasilinear transport modelling at low magnetic shear

obtain:

$|\Phi_{k,\omega}|^2 \propto \frac{|\Phi_k|^2}{(\omega - \omega_k)^2 + \gamma_k^2 + \alpha \left( \frac{\partial}{\partial r} V_E(r) \right)^2}$

where $V_E(r)$ is the zonal flow velocity, and $\alpha \equiv \frac{k^4}{2} \phi(\kappa y)|^{2}$. This equates to increased frequency broadening, beyond the linear growth rate assumption. Such a reduction of the fluxes due to perpendicular flow shear was also employed in a transport model by Hinton and Staebler [31].

We define the ratio between the frequency broadening from the non-linear simulations and the linear growth rate, at each magnetic shear value, as the weighted average $\langle \Delta \omega_k / \gamma_k \rangle$, with the weight set by the $\phi(k_y)$ amplitude spectrum. For the entire collection of simulations, $\langle \Delta \omega_k / \gamma_k \rangle_{s=0.1} = \frac{1}{2} \langle \Delta \omega_k / \gamma_k \rangle_{s=1} = 2.5$, averaging over all four sets. This reflects the increased frequency broadening at low magnetic shear, which is highest for set A (GA-standard case) and lowest for set D. In the absence of more rigorous theory, we have simply included an additional shear dependent normalization factor to the fluxes calculated by QuaLiKiz, in the form of $2.5(1 - |s|)$ for $|s| < 0.6$. This normalization factor roughly captures the effect of the decreased autocorrelation time due to increased frequency broadening (possibly due to zonal flows) at low magnetic shear.

The increase in the frequency broadening is also correlated with a reduction in the effective non-linear transport weight. This is seen by comparing the ratio of the quasilinear transport weight (flux over spectral intensity as a function of $k$) with the non-linear transport weight, defined as an average [32]. These averages were found to be $\sim 1.4$ - at transport relevant $k$ values - for a variety of different cases. When calculating this average for the $s = 0.1$ GA-standard case, as seen in Fig.5.13, we however obtain an increased average compared to the $\sim 1.4$ value found for the $s = 1$ case. This over-}

age increase, corresponding to the deviation of the quasilinear flux from the non-linear flux due to the non-constancy of the transport weight, corresponds well with our ad-hoc correction factor from the frequency broadening.

An additional possible explanation for the non-unity overage could be the non-linear transfer of energy from unstable eigenmodes to stable eigenmodes. This is proposed to lead to a similar factor $\sim 1.4$ between quasilinear and non-linear fluxes [33]. This is because the construction of the quasilinear flux assumes that the unstable mode contributes fully to the electrostatic potential amplitude. However, in the non-linear case, linearly damped modes take up a given proportion of the total electrostatic potential amplitude. The net contribution of these linearly damped modes to the flux is typically much reduced compared with the dominant unstable mode. Thus, since the transport weight (following a $k_y$ decomposition) is defined as the flux divided by the electrostatic potential, the non-linear weight is expected to be lower than the quasilinear weight due to the existence of the low-flux-contributing linearly damped modes which do contribute to the saturated electrostatic potential amplitude. An increased proportion of linearly damped modes in the low magnetic shear system would then be consistent with increased transport weight overage. A broadened frequency spectrum could also be a signature of an increased spec-
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The increase of zonal modes - as observed at low magnetic shear - is expected to lead to increased coupling to $n > 0$ linearly damped modes. Indeed, as seen in Fig.5.14, singular value decompositions of the saturated electrostatic potential of the GA-standard $s = 1$ and $s = 0.1$ cases show that the $s = 0.1$ case has a much broader spectrum of linearly stable modes beyond the dominant unstable mode. This method of analysis is outlined in Hatch et al [34]. The existence of a broader spectrum of linearly damped modes is consistent with a higher transport weight overage. However, we also note that at the current level of analysis it is not possible to separate in the singular value decomposition linearly damped modes from linearly unstable modes with midplane $k_{x0} \neq 0$ (related to the conventional ballooning angle through $k_{x0} = k_y \theta_{0s}$).

While the $k_{x0} = 0$ modes are dominant for both $s = 0.1$ and $s = 1$, the $k_{x0}$ growth rate spectrum is broader for $s = 0.1$, when compared with $s = 1$. These modes could in principle also contribute to the frequency broadening. More analysis is necessary in this respect.

In summary, at low magnetic shear the non-linear frequency broadening is observed to be larger than the linear growth rate width assumption. This broadening is correlated with an increase in the quasilinear to non-linear transport weight overage. An ad-hoc magnetic shear dependent normalization factor, based on the degree of increased non-linear frequency broadening at low magnetic shear, has been introduced into QuaLiKiz. The physical explanation of the frequency broadening and associated transport weight overage increase may be related to the relative increase in the zonal flow amplitude at low magnetic shear. A direct renormalization of the linear response function due to zonal flows is suggested. An additional explanation may be the increased non-linear coup-

Figure 5.13: Ratio of quasilinear to non-linear transport weights for the GA-standard $s = 1$ and $s = 0.1$ cases.
Quasilinear transport modelling at low magnetic shear

Figure 5.14: Singular value spectrum of mode amplitudes calculated from the GA-standard $s = 1$ and $s = 0.1$ saturated electrostatic potentials.

ling (via zonal flows) to linearly damped modes. More theoretical work is necessary to more fully uncover the mechanism of the increased frequency broadening and transport weight overage increase at low magnetic shear. However, on a more pragmatic level, the QuaLiKiz and non-linear flux prediction comparison is much improved through the combination of the $k_\perp$ formulation improvement and the ad-hoc magnetic shear dependent normalization. This is shown in the following section.

5.4.4 Comparison of QuaLiKiz and non-linear flux predictions

Having taken into account both modifications discussed above, the $k_\perp^2$ alteration and frequency broadening renormalization, we now compare the particle and heat flux predictions from the modified QuaLiKiz and the non-linear simulations, for all GENE runs carried out, as well as GYRO for the GA-standard magnetic shear, $q$-profile, and $R/L_T$ scans. The results are displayed in Fig. 5.15. The agreement with the GA-standard case $s$-scan is much improved compared to the previous mixing length formulation shown in Fig. 5.1. Simultaneously, the agreement with the $q$-scan and $R/L_T$ scan is maintained. When exploring additional areas in parameter space, i.e. the B, C, and D sets, reasonable agreement is also observed. These results are summarized in table 5.4, where the RMS error between the particle and heat fluxes between QuaLiKiz and the non-linear GENE and GYRO simulations are listed. The RMS error is defined as $\sigma = \sqrt{\frac{\sum_i \epsilon_i^2}{\sum_j \chi_j^2(NL)}}$, where $\epsilon_i$ is the diffusivity or flux difference between QuaLiKiz and the non-linear prediction for a given simulation, and $\chi_j(NL)$ the diffusivity or flux from the non-linear simulation. The RMS errors obtained using the previous QuaLiKiz version, correcting only $k_\perp^2$, and both correcting $k_\perp^2$ and including the frequency broadening renormalization are all compared. Both modifications are of comparable importance.
5.4 Non-linear effects at low magnetic shear

Table 5.4: RMS errors between QuaLiKiz and non-linear simulation predicted fluxes, for all GENE and GYRO runs shown in Fig. 5.15.

<table>
<thead>
<tr>
<th>Run</th>
<th>$\sigma_{\chi_i}$</th>
<th>$\sigma_{\chi_e}$</th>
<th>$\sigma_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>QuaLiKiz09</td>
<td>1.13</td>
<td>1.53</td>
<td>1.84</td>
</tr>
<tr>
<td>With new $k_2^\perp$ only</td>
<td>0.52</td>
<td>0.77</td>
<td>1.24</td>
</tr>
<tr>
<td>With new $k_2^\perp$ and frequency broadening renormalization</td>
<td>0.26</td>
<td>0.33</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 5.15: Comparison of the altered QuaLiKiz model with the 4 sets of GENE runs, and with the GYRO GA-standard case s-scan, q-scan, and R/L $T$ scan.
5 Quasilinear transport modelling at low magnetic shear

5.5 Discussion and conclusions

Quasilinear flux predictions in the previous version of QuaLiKiz did not agree with GYRO non-linear flux predictions at low magnetic shear. This work has systematically examined the assumptions underlying the QuaLiKiz model, to uncover the physics behind this discrepancy. Improvements in the model were implemented, guided and validated by a further set of non-linear simulations carried out with GENE. This led to a significant improvement in the comparison between QuaLiKiz and non-linear simulations.

It was found that the previous formulation of \( k_\perp^2 \) in the QuaLiKiz mixing length rule for the saturated potential did not reproduce the magnetic shear dependence observed in non-linear simulations, neither in the \( k_y |_{\text{max}} \) of the flux spectrum, nor in the relation between \( \langle k_x \rangle \) and \( k_y \). Furthermore, the frequency broadening at low magnetic shear was observed in the non-linear simulations to be greater than the linear growth rate broadening assumption. This is correlated with an increase in the ratio between the quasilinear and non-linear transport weights (transport weight overage), which must be taken into account in the model.

The QuaLiKiz mixing length rule was thus improved by introducing a formula for \( \langle k_x \rangle/k_y \) in the \( k_\perp \) formulation, which captures the magnetic shear and \( q \)-profile dependence from the non-linear simulations. This successfully reproduces the non-linear \( k_y |_{\text{max}} \) from the maximum of the \( \gamma k/k_\perp^2 \) mixing length spectrum, and also captures the degree of isotropy between \( \langle k_x \rangle \) and \( k_y \), setting the precise value of the correlation length. Furthermore, a renormalization based on the weighted average of the increased frequency broadening was implemented, which was consistent with the observed degree of increase in the transport weight overage. These modifications led to a much improved agreement between the new version of QuaLiKiz, and non-linear GYRO and GENE magnetic shear scans for various values of \( R/L_T, q \), and \( T_i/T_e \).

We note that the reproduction of \( k_y |_{\text{max}} \) can also be achieved by invoking zonal flow quenching of large wavelength turbulence in the mixing length rule. This results in reduced correlation lengths at low magnetic shear due to the stabilization of the toroidal ITG branch at combined low \( s \) and \( k_y \), which leads to reduced growth rates at low \( sk_y \). This stabilization was discussed when summarizing linear analytical and numerical results at low magnetic shear. In spite of the lower ITG threshold at low magnetic shear, the \( \gamma \)-stiffness is also lower at low magnetic shear. At low \( k_y \) this is particularly the case, due to toroidal ITG branch stabilization at low \( sk_y \). Since the \( \langle k_x \rangle \) dependence on \( k_y \) also depends on zonal flows, these two approaches for altering the mixing length rule stem from the same physics.

As detailed in Appendix 5.B and Appendix 5.C, the intrinsic assumptions in the QuaLiKiz model were also examined. It was found that locality holds down to \( s = 0.1 \). The lowest order ballooning representation is also valid down to \( s = 0.1 \), following comparisons of the QuaLiKiz fluid eigenmode assumption and GENE linear eigenfunction solutions. The quasilinear approximation also holds at low magnetic shear. This was verified by calculating the Kubo numbers from the entire set of non-linear GENE simulations. It was found the \( K < 1 \) also at low magnetic shear. This strengthens the claim that the quasilinear/non-linear discrepancy at low magnetic shear is purely due to the formula-
5.A Analytic fluid limit

The analytic fluid limit is obtained by considering mode frequencies far above the resonance, i.e. \( \omega \gg \omega_d, k_\parallel V_\parallel \). This corresponds to driving gradients far above the linear thresholds. We assume an ordering of the form:

\[
\kappa_{\perp} \rho_e \ll 1, \kappa_{\perp} \rho_i \ll 1, \kappa_{\perp} \delta_e \ll 1, \kappa_{\perp} \delta_i \ll 1
\]

as well as adiabatic passing electrons, large-aspect-ratio, \( \alpha = 0 \), and strongly ballooned eigen-...
functions centered around the equatorial midplane. The fundamental linear gyrokinetic dispersion equation can then be simplified to the following [35]:

\[
[1 + \frac{n^2\omega_{pe}^2}{f_p\omega^2} - \frac{n\omega_{pe}}{\omega} + \frac{n\omega_{di}}{\omega} + (-\frac{k^2c_{eff}^2}{2s^2} + \frac{k^2 d_{eff}^2}{2}) (1 + \tau \frac{n\omega_{pe}}{2L_{te}})\phi] = 0 \tag{5.6}
\]

Where \(\rho_{pe}\) and \(\delta_{pe}\) are the particle Larmor radii and banana widths, \(n\) is the toroidal wave number, \(\omega_{pe} = -\frac{eB}{m_ee}\cos \theta + s\theta \sin \theta\) is the vertical drift frequency, \(\omega_{di} = -\frac{eB}{m_ee}\cos \theta - s\theta \sin \theta\) are the diamagnetic frequencies associated with the density and pressure gradient lengths respectively, \(V_{\parallel}\) is the ion parallel velocity, \(\tau = \frac{c_{L}}{c_{Te}}\), \(Z\) is the effective ion charge, \(f_{t,p}\) are the trapped and passing particle fractions respectively, \(c_{eff}^2 \equiv \frac{k\epsilon}{m_ee^2}\), and \(d_{eff}^2 \equiv \frac{k\epsilon}{m_ee^2}\sum_i \frac{m_i^2\delta_i}{\epsilon_i} + \frac{4\tau \epsilon m_i}{\epsilon_i^2}\). In the limit where \(q(r) \approx q(r_0) + q'(r_0)x\), we can write \(k_0 = k_0 L = k_0 L_s\), where \(x \equiv (r - r_0)\) (the distance from the rational surface) and \(L_s \approx Rq/s\). Transforming the radial coordinate from Fourier space to real space, we can also write \(k_0^2 = k_0^2 - d^2/dx^2\). Equation 5.6 is then a second order differential equation for \(\phi(x)\). A Gaussian solution, \(\phi = \phi_0 e^{-\frac{x^2}{2\epsilon^2}}\), with \(\epsilon^2 = -\frac{\omega_{pe} + \omega_{di}}{k_0^2d_{eff}^2}\), implies that \(\phi'' = -\left(\frac{1}{\epsilon^2} + \frac{x^2}{\epsilon^2}\right)\phi\), and thus imposes a quadratic equation for the solution of the growth rate \(\gamma = n\Omega m(\Omega')\), with \(\Omega = \omega/\rho\):

\[
\Omega^2(1 + \frac{k_0^2}{4d_{eff}^2}) + \Omega(-\omega_{pe} + \omega_{di} + i\frac{k_0}{nL_s} c_{eff}a d_{eff}^2 - \frac{k_0^2}{4d_{eff}^2} \tau \frac{\omega_{pe}'}{2}) = 0 \tag{5.7}
\]

To recover the interchange limit, where curvature dominates, we take the ordering \(f_p, f_t, \frac{\Omega}{\epsilon}, \sim O(1), k_0 d_{eff}, L_p/R, \frac{\omega_{pe}'}{\epsilon^2} \sim O(\delta), L_p/L_s \sim O(\delta^2)\), where \(\epsilon\) is the inverse-aspect-ratio. In this limit, we obtain for the growth rate:

\[
\gamma^2 = \frac{\left(f_t + \frac{\Omega}{\epsilon}\right) n^2\omega_{pe}^2}{f_p} \tag{5.8}
\]

While for the slab limit, the relevant ordering is \(f_p, f_t, \frac{\Omega}{\epsilon}, \sim O(1), k_0 d_{eff}, L_p/R, \frac{\omega_{pe}'}{\epsilon^2} \sim O(\delta), L_p/L_s \sim O(\delta^2)\), which leads to the following expression for the growth rate:

\[
\gamma^2 = \frac{\tau \frac{n\omega_{pe}^2}{Z} k_0 d_{eff}^2 c_{eff} \tau}{2L_s} \tag{5.9}
\]

Let us examine the magnetic shear dependence in each of the above limits. For the interchange limit, the \(s\)-dependence can be found in \(\omega_{pe}\), where \(\omega_{pe} \propto \cos \theta + s\theta \sin \theta \Rightarrow \gamma^2 \propto \cos \theta + s\theta \sin \theta\). In the slab limit, since \(L_s \approx Rq/s\), we have \(\gamma^2 \propto |s|\). In both cases, the growth rates increase with magnetic shear. The fact that the growth rates increase with \(s\) in the limit far above the linear instability threshold, while simultaneously the ITG instability threshold also increases with \(s\), points towards reduced growth rate stiffness at lower magnetic shear.
5.B Locality approximation at low magnetic shear

The QuaLiKiz model is a local code (as are the versions of non-linear GYRO and GENE used here in this work), and thus assumes constant profile gradient lengths. This can be formulated as $\max(w, d) < \min(L_{\text{w}(e,i)}, L_{T(\nu)})$, where $w$ is the mode width, $d$ the distance between rational surfaces, and $L_{\text{w}}, L_{T}$ the electron or ion density and temperature gradient lengths respectively. This assumption must fail at a sufficiently low value of $s$ as the distance between rational surfaces grows further and further apart, since $d = 1/sk_{\rho}$.

At low shear $d$ and $w$ are comparable in the QuaLiKiz model (see Appendix A in [9]).

We can estimate this $s$ value as follows: firstly since $d$ and $w$ are comparable at low shear, we find the $s$ for which $d = L_{T}$, $\frac{d}{w} = \frac{L_{T}}{w} \Rightarrow sk_{\rho}R = \frac{L_{T}}{w} \Rightarrow s(k_{\rho}R) = \frac{L_{T}}{w}$. Taking $k_{\rho}R_{s} = 0.1$, which is approximately the value above which unstable modes are predicted to contribute significantly to the fluxes, and taking a typical $R/L_{T}$ value of 9, we obtain: $s = 0.001$. For mid-sized tokamaks, a typical value of $\frac{L_{T}}{w}$ is 0.001, thus a typical $s$ for which the locality assumption fails is 0.1, higher for smaller tokamaks and lower for larger tokamaks.

5.C Analytical fluid eigenmode assumption

In this appendix we show, in comparisons with linear-GENE, that the analytical fluid Gaussian eigenmode assumption made in QuaLikiz, and the strong ballooning representation inherent therein, are valid for $|s| > 0.1$.

The scale separation of the microinstabilities, where $k_{\parallel} \ll k_{\perp}$, is captured in the eikonal representation for the electrostatic potential perturbation: $\Phi_{n}(r, \theta, \phi) = e^{-i(n-\theta)(\rho_{n}/\rho)}\phi_{n}(r, \theta)$, where $\phi_{n}$ has slow varying $\theta$-dependence along the field line. To first order $q(r)\approx q(r_{0}) + x\nu_{n}$, with $r_{0}$ corresponding to the resonating surface where $q(r_{0}) = m_{n}/n$, and $x = r - r_{0}$, the radial distance from the resonating surface. The poloidal harmonic of $\Phi_{n}$ is thus:

$$\tilde{\phi}_{n}(r) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \phi_{n}(\theta) e^{i\nu_{n}\frac{\theta}{2\pi}} d\theta \quad (5.10)$$

With $\nu = m - m_{n}$, $d = 1/nq_{n}$, and where we have assumed radial dependence only through $q(r)$. In the QuaLiKiz model, $\phi(r)$ is not solved, but prescribed from the fluid limit solution, which is a Gaussian: $\tilde{\phi}(r) = \phi_{0}e^{-r^{2}/2\sigma_{\theta}^{2}}$, with $w$ the Gaussian width [8].

The fluid limit solution is outlined in appendix 5.A. If $\phi_{n}(\theta)$ vanishes sufficiently quickly at $\pm\pi$, then equation 5.10 describes a Fourier transform. $\phi_{n}(\theta)$ is then the conjugate of $\phi_{0}(r)$, with $\theta = k_{\rho}d$. However, for low shear and long wavelengths, this assumption may fail since the prescribed $\phi(k_{\rho})$ solution may not be compatible with $\phi(\theta)$ periodicity since then $\phi(\theta)$ may not necessarily vanish at $\pm\pi$. Indeed, it is known that as $s \to 0$ the full ballooning representation solution of $\phi(\theta)$ tends to a Mathieu function [36], which is not localized is the vicinity $\theta = 0$. Therefore we expect that below a particular value of $s$, the QuaLiKiz Gaussian eigenmode assumption must break down.
The validity of the QuaLiKiz assumption that the eigenmode is a Gaussian and vanishing at $\pm \pi$, i.e. $\phi_n(\theta = \pm \pi) \to 0$, is examined versus the solution of $\phi_n(\theta)$ from linear GENE, displayed in the ballooning representation [37]. The QuaLiKiz/GENE comparison is seen in Fig.5.16 for the GA standard case at both $s=0.1$ and $s=1$. We examine the cases with the transport relevant $k_y = 0.1, 0.15, 0.2$. The GENE calculated modes tend to be more extended in the ballooning angle for the combination of low $s$ and low $k_y$. The QuaLiKiz assumption holds for all cases apart from $s = 0.1, k_y = 0.1$. However, since the peak of the non-linear flux spectrum for the GA-standard $s = 0.1$ case falls between $k_y = 0.15 - 0.2$, the linear calculations for $k_y = 0.1$ are less critical in the low shear case (see section 5.4.3). We can conclude that the lowest order ballooning representation assumption made in QuaLiKiz holds down to $s = 0.1$.

![Graphs showing GENE linear eigenfunctions compared with the QuaLiKiz strong ballooning assumption for the mode envelope, for $s=1$ (upper panels) and $s=0.1$ (lower panels) GASTD case parameters.](image)

Figure 5.16: Calculated GENE linear eigenfunctions compared with the QuaLiKiz strong ballooning assumption for the mode envelope, for $s=1$ (upper panels) and $s=0.1$ (lower panels) GASTD case parameters.
References

5 Quasilinear transport modelling at low magnetic shear


6 Ion temperature profile stiffness: non-linear gyrokinetic simulations and comparison with experiment

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Abstract

Recent experimental observations at JET show evidence of reduced ion temperature profile stiffness at low magnetic shear ($\hat{s}$) in the presence of flow shear. Non-linear gyrokinetic simulations are performed, aiming to reproduce the experimental results and discover the physical mechanism behind the observations. The sensitivity of profile stiffness to various parameters are assessed. It is found that non-linear electromagnetic effects, even at low $\beta_e$, can significantly reduce the profile stiffness, although not by a degree sufficient to explain the experimental observations. The effect of flow shear itself is not predicted by the simulations to lead to a significant reduction in flux. Agreement between the simulations and experiment is only attained when ignoring parallel velocity gradient destabilisation and assuming a degree of perpendicular flow shear above the experimental values. Finally, a new method of calculating linear growth-rates in the presence of rotation is proposed, where the growth-rates are calculated over an assumed non-linear correlation time.

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6 Ion temperature profile stiffness: non-linear gyrokinetic simulations and comparison with experiment

6.1 Introduction

It is well established that one of the primary limitations of tokamak energy confinement is ion-Larmor-radius scale turbulent transport driven by background pressure gradients [1]. The ion-temperature-gradient (ITG) instability in particular has been long identified as an ubiquitous unstable mode in tokamak plasmas [2–5], and is primarily responsible for ion heat losses. The instability saturates in a non-linear state in conjunction with non-linearly excited zonal-flows, forming a self-organised turbulent system which sets the transport fluxes [6].

In addition to self-organised zonal-flows, the application of external flow shear is predicted to suppress turbulence through two broad mechanisms: decorrelation of the turbulent structures in the non-linear phase once the shearing rate is comparable with or exceeds the inverse non-linear autocorrelation time; suppression of the driving linear modes by coupling the mode at the most unstable spatial scale to nearby, more stable spatial scales [7, 8]. Flow shear has been observed experimentally to be correlated with tokamak transport barriers [9, 10], and with improved confinement in hybrid scenarios [11]. Non-linear gyrofluid simulations have predicted turbulence quenching above $\gamma_E/\gamma_{\text{max}} = 1 \pm 0.3$ [12, 13], where for purely toroidal rotation the normalised $E \times B$ shear rate $\gamma_E \equiv \frac{\partial v}{\partial r} / (v_{th} R)$, and $\gamma_{\text{max}}$ is the maximum linear growth rate in the absence of rotation. Later gyrokinetic simulations predicted that quenching occurs at somewhat higher, but similar flow shear compared to the gyrofluid simulations, at $\gamma_E/\gamma_{\text{max}} = 2 \pm 0.5$ [14–16]. The seeming robustness of the transport quench has motivated formulations of effective growth rate reduction due to the flow shear in the mixing length rule of quasilinear transport models such as GLF23 and TGLF [17, 18]. When including parallel velocity gradient (PVG) destabilisation [19, 20] in the system, simulations have shown that they can limit the transport quench [12, 15, 21]. For pure toroidal rotation, the degree of the PVG destabilisation depends on the magnetic geometry since then $\gamma_p = \frac{3}{2} \gamma_E$, where $\gamma_p$ is the PVG shear rate, and $\epsilon \equiv \frac{E}{R}$. It has been recently observed in dedicated experiments at JET that ion temperature profile stiffness can be reduced at low normalised radii ($r/a < 0.5$), to a level significantly below the expectation from ITG turbulence [22, 23]. This has been hypothesised to be related to the correlation between low magnetic shear ($\hat{s}$) and increased flow shear in the low stiffness discharges. The term ‘stiffness’ refers to the sensitivity of gyro-Bohm normalised ion heat flux to the driving $R/L_{Ti}$ (normalised inverse gradient length). The observations concentrated on $x = 0.33$ and $x = 0.64$ (where $x$ is the normalised toroidal flux coordinate). While stiffness was observed to be low at $x = 0.33$ when the flow shear was increased, no significant difference in $R/L_{Ti}$ was observed at $x = 0.64$. A previous non-linear gyrokinetic study based on the recent JET discharges, as detailed in Ref. [23], has reported only a ITG threshold shift with rotation, as opposed to a decrease in stiffness as observed.

In this paper, we extend this previous work and perform systematic and extensive linear and non-linear simulations with the GENIE [24] non-linear gyrokinetic code to determine whether the experimental observation can be reproduced by gyrokinetic simulations. Three JET discharges have been selected: 70084, 66130, and 66404. Discharges 66130
and 66404 are situated on the ‘low-stiffness curve’ at \( x = 0.33 \) seen in Fig.1 in Ref. [23]. Discharge 70084 is a low flux, low rotation discharge selected to provide a data point near the turbulence threshold. These discharges have been reanalysed with the CRONOS suite of integrated modelling codes [25] to identify any differences in parameters apart from rotation within the chosen discharge set - such as \( R/L_n, \beta_e \), and fast particle content - which may lead to decreased stiffness. The sensitivity to the ion heat flux and stiffness to each of these parameters was tested with GENE in dedicated \( R/L_T \) scans. Finally, complete simulations - i.e. collisional, electromagnetic, multi-species, and with rotation - were carried out at both \( x = 0.33 \) and \( x = 0.64 \) to determine whether the gyrokinetic simulations can reproduce the experimental fluxes.

The rest of the paper is organised as follows. In section 6.2 the GENE gyrokinetic code is briefly reviewed, as are the base parameters of the simulated discharges. In section 6.3 the results at \( x = 0.33 \) are reviewed. In section 6.4 the results at \( x = 0.64 \) are reviewed. In section 6.5 we present a comparison of sub-critical turbulence predictions for adiabatic and kinetic electrons, and also introduce a novel method of growth rate calculation in the presence of flow shear. In section 6.6 the conclusions are presented.

### 6.2 GENE simulations

GENE solves the gyrokinetic Vlasov equation, coupled self-consistently to Maxwell’s equations, within a \( \delta f \) formulation [26]. Computational efficiency is gained by solving in field line coordinates, \( x \) is the radial coordinate, \( z \) is the (poloidal) coordinate along the field line, and \( y \) is the binormal coordinate. Both an analytical circular geometry model (derived in Ref. [27]) as well as numerical geometry are used in this work. The analytical model differs from the often applied \( s - \alpha \) model, which has inconsistencies of order \( \epsilon = a/R \). For the numerical geometry, the FINESSE code was used to solve the extended Grad-Shafranov equation including toroidal rotation [28]. All simulations are local. Both linear and non-linear simulations are performed. In the linear mode, an eigenvalue solver may be used to compute multiple modes for each point in parameter space [29]. In the presence of rotation, when no time-independent eigenmodes can form, a complementary initial value solver is used.

Three discharges from the data-set presented in Ref. [23] were analyzed in more detail at \( x = 0.33 \) and \( x = 0.64 \), where \( x \) is the normalised toroidal flux coordinate. The discharges are 70084, 66130, and 66404. 70084 corresponds to a representative low rotation, low flux discharge and 66130 and 66404 are discharges further up on the ‘high rotation, decreased stiffness’ curve as seen in Fig.1 in Ref. [23]. The kinetic profiles were spline fitted and interpretative runs were carried out with the CRONOS integrated modelling suite of codes [25] for the equilibrium calculations and q-profile calculations. The kinetic profiles were then averaged over 1 s centered around 50/50/47 s respectively for calculations of the gradient lengths and other quantities such as \( \beta_e \). The parameters are shown in table 6.1. In the table, the two quoted \( \langle Z_{eff} \rangle \) values correspond to the charge-exchange (CX) and Bremsstrahlung measurements respectively, which are not consistent with each other. The CX measurements provide the lower quoted values. The CX \( \langle Z_{eff} \rangle \)
measurements for discharge 66404 were not available. The sensitivity of the transport to the range of \( \langle Z_{11} \rangle \) spanned by the diagnostics is explored in section 6.3.7. \( \nu^* \) is the normalised collisionality: \( \nu^* = \frac{\nu \epsilon}{\nu_{\text{cr}}^*} \), with \( \epsilon = a/\rho \) and \( \nu_{\text{cr}}^* = \sqrt{\frac{q}{\rho}} \). In previous gyrokinetic simulations based on 70084 parameters, the values for \( \nu^* \) were taken as 0.57/1.3 [23]. This is not fully consistent with the \( \nu \) values from the CRONOS modelling shown here. However, uncertainty still exists regarding the validity of neoclassical current diffusion in setting the values of the \( \nu \)-profile, particularly at inner radii [30]. Therefore, to be consistent with the previous works, we assume \( \nu = 1.3 \). As seen in section 6.3, we also carry out most of the parameter scans at \( x = 0.33 \) with \( \delta = 0.2 \) to ascertain that we are in a 'low-\( \delta \) regime', in spite of the contradiction with the CRONOS current diffusion modelling predictions.

In the GENE simulations, typical grid parameters were as follows: perpendicular box sizes \([L_{\theta}, L_z] = [170, 125]\) (in units of \( \rho \)), perpendicular grid discretisations \([n_{\theta}, n_z] = [192, 48]\), 24 point discretisation in the parallel direction, 32 points in the parallel velocity direction, and 8 magnetic moments. Extensive convergence tests were carried out for representative simulations throughout the parameter space spanned in this work. The lack of convergence of the heat fluxes with increasing \( n_y \) as reported for GYRO [31] simulations of discharge 70084 in Ref. [23] - associated with increasing zonal flows - was not encountered here. In our cases the convergence with \( n_y \) is well behaved. The difference may stem from the different treatment of the radial boundary conditions in the GENE and GYRO simulations. Further investigation is necessary to ascertain this. The heat fluxes shown in the following sections are in gyroBohm normalised units, \( T_{\text{gyrBohm}}^{-5/2} / e^2 B^2 R^2 \), \( k_B \) is in units of \( 1/\rho \). These heat fluxes correspond to time averaged values over the saturated state of the GENE simulations. The statistical flux variations due to intermittency are for clarity not explicitly shown as error bars. This variation is typically 5 – 10% for our parameters. \( \gamma \) and \( \gamma_E \) are in units of \( c_s/R \) where \( c_s \equiv \sqrt{T_e/\rho} \). All rotation is considered to be purely toroidal unless specifically mentioned. For the low/high rotation discharges, \( \gamma_E = 0.1/0.3 \) was assumed for both \( x = 0.33 \) and \( x = 0.64 \), which are representative values for the low and high stiffness discharges from the dataset in Ref. [23].
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In this section, we first isolate the effect of various parameters on ion profile stiffness, at \( x = 0.33 \) (the low shear region). These parameters are: \( \hat{s} \), rotation, effect of rotation on the magnetohydrodynamic (MHD) equilibrium, \( R/L_n \), \( \beta_e \), and fast particle content. We then proceed to realistic simulations of 70084, 66130, and 66404 for a full comparison between the gyrokinetic predictions and the experimental heat fluxes. These simulations simultaneously include: numerical geometry, collisions, electromagnetic effects, \( Z_{eff} \) (by including a carbon species), and realistic \( T_e/T_i \).

6.3.1 Stiffness sensitivity to \( \hat{s} \)

While the ITG turbulence threshold increases with \( \hat{s}/q \) \([32]\), the stiffness decreases with both decreasing \( \hat{s} \) and decreasing \( q \). For decreasing \( \hat{s} \), the reduced stiffness has been shown to be correlated with increased coupling to zonal flows \([33]\). For decreasing \( q \), this is due to an increased downshift in the peak wavenumber of the turbulence spectrum, indicating increased correlation lengths \([34]\). These sensitivities are seen in Fig.6.1. For example, we can see that for both the \( \hat{s}/q = 0.6/1.3 \) and \( \hat{s}/q = 1/2 \) cases the turbulent threshold is similar while the stiffness is lower for the \( \hat{s}/q = 0.6/1.3 \) case.

The precise value of \( \hat{s} \) from the integrated modelling is uncertain, in light of inconsistent results regarding agreement between \( q \)-profile motional Stark effect (MSE) measurements and \( q \)-profile interpretative modelling \([30]\). Thus, for the remainder of this work, we assume a ‘reference’ \( \hat{s} \) of 0.2 instead of 0.57 at \( x = 0.33 \), to ascertain that we are studying a low-\( \hat{s} \) regime. For the numerical geometry cases, the current profile input into CRONOS was modified such that at \( x = 0.33 \), values of \( \hat{s}/q = 0.2/1.3 \) were obtained following the solution of the Grad-Shafranov equation.

6.3.2 Stiffness sensitivity to rotation

In this section we isolate the effect of rotation on stiffness, assuming pure toroidal rotation. Collisionless, electrostatic simulations based on 70084 parameters (with \( \hat{s}/q = 0.2/1.3 \)) are carried out, applying analytical circular geometry \([27]\). The predicted gyroBohm normalised ion heat fluxes from the \( R/L_{Ti} \) scans are shown in Fig.6.2. The sensitivity to \( \gamma_E \) is examined. Even for \( \gamma_E = 0.6 \), double the highest level of flow shear achieved in the reference data set from Ref. \([23]\), the simulated level of reduced stiffness is significantly less than the experimental observation. However, interesting effects related to the competition between stabilising \( E \times B \) shear and destabilising parallel velocity gradient (PVG) modes - particularly in the vicinity of the threshold - are observed. At low \( R/L_{Ti} \), the PVG destabilisation can dominate over the ITG turbulence, reducing stiffness in that region of parameter space. Due to the PVG destabilisation, the fluxes do not continue to decrease towards the ITG instability thresholds. This is seen in Fig.6.2a by examining the various curves at fixed \( R/L_{Ti} \). At low \( R/L_{Ti} \), the fluxes rise with \( \gamma_E \) due to PVG drive. However at higher \( R/L_{Ti} \), the fluxes decrease with \( R/L_{Ti} \) due to the ITG stabilisation by perpendicular ExB flow shear dominating over the PVG destabilisation.
In Fig. 6.2b the parallel velocity gradients were artificially removed from the system, and the picture reverts to a threshold shift. Note that particularly for the (red) $\gamma_E = 0.3$ and (black) $\gamma_E = 0.6$ curves the apparent reduced slope near threshold is not necessarily indicative of reduced stiffness in that regime, since the actual effective non-linear threshold may lie between the precise values of the $R/L_T$ values chosen for the simulations.

For pure toroidal rotation, the relative importance of PVG destabilisation versus $E \times B$ stabilisation is sensitive to the geometric parameter $q/\epsilon$ (where $\epsilon \equiv r/R$) [35]. As $q/\epsilon$ increases, the field lines are increasingly projected onto the toroidal direction. In Fig. 6.3 a $q/\epsilon$ scan is carried out by varying $\epsilon$ in the various $R/L_T$ scans. Simulations with $\epsilon = 0.11, 0.15$ assuming circular geometry were performed, as well as an $\langle \epsilon \rangle \equiv \langle r \rangle / R = 0.13$ case from the flux surface averaged minor radius at $x = 0.33$ using numerical geometry (in this case from the HELENA [36] equilibrium in the CRONOS simulations). The $R/L_T$ values in the plots corresponding to numerical geometry are defined here with respect to the averaged midplane minor radius. The relative strength of the PVG destabilisation is seen to weaken as expected with decreasing $q/\epsilon$, until an almost pure threshold shift case is reached with $q/\epsilon = 8.7$. While this qualitative PVG dependence on $q/\epsilon$ is well known, we here make quantitative predictions with kinetic electrons for $R/L_T$ values in the vicinity of the turbulence threshold. For our particular case, the 70084 $x = 0.33$ parameters are very close to the boundary of a 'zero-turbulence-manifold' as defined in Ref. [37]. That study was carried out with adiabatic electrons, may decrease the predicted $\gamma_E$ necessary for turbulence quenching compared with the kinetic electron case due to the reduced trapped electron drive.

The interplay between PVG destabilisation and $E \times B$ stabilisation demands that PVG modes are correctly accounted for in reduced transport models - such as in gyrokin-
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Figure 6.2: Non-linear GENE \( R/L_{T_i} \) scans based on 70084 parameters at \( x = 0.33 \) \((q/\epsilon = 11.8)\) and various levels of \( \gamma_E [c_s/R] \). Runs including PVG destabilisation are seen in (a). Runs ignoring PVG destabilisation are seen in (b). All runs were electrostatic, collisionless, and with circular geometry.

Figure 6.3: \( q/\epsilon \) sensitivity of the PVG destabilisation as seen in \( R/L_{T_i} \) scans of ion heat flux. As \( q/\epsilon \) is progressively raised, the \( \gamma_E \) induced stabilisation can not only be reduced but can even be reversed in the region of the instability threshold. Runs were electrostatic, collisionless, and with circular geometry.

etic or gyrofluid based quasilinear models. It is insufficient in such formulations to include a growth rate quench model due to \( E \times B \) stabilisation without simultaneous self-consistent and validated modelling of the PVG modes. Correct modelling near the turbulent thresholds is particularly critical for high temperature tokamaks, such as ITER. This is because the normalised fluxes are expected to be in the vicinity of the turbulence thresholds due to the \( T_{5/2}^{1/2} \) normalisation dependence.

Finally, we note that the observed Dimits shift \cite{38} in these cases is only \( \Delta R/L_{T_i} \approx \)
0.5, or alternatively \( \frac{\Delta (R/L_{T_i})}{R/L_{T_{crit}}} \approx 15\% \). These values are similar to the \( \frac{\Delta (R/L_{T_i})}{R/L_{T_{crit}}} \approx 20\% \) shifts observed in previous realistic simulations [39]. The linear threshold was calculated by extrapolation to zero-growth-rate of linear \( R/L_{T_i} \) scans with GENE. The linear threshold in the numerical geometry case is nearly identical to the circular geometry case. Thus we can conclude that a significant departure from the Dimits shift paradigm is so far not experimentally observable with great certainty due to the experimental error in extrapolating to the instability thresholds.

### 6.3.3 Effect of rotation on the equilibrium

The effect of rotation on stiffness through its influence on the equilibrium was examined. An extended Grad-Shafranov equation including toroidal rotation was solved with the FINESSE code, using the 70084 pressure and F profiles as input, where \( F \equiv B_{tor} R \). For the rotation profiles, scaled profiles from 66404 were used such that static (\( \gamma_E = 0 \)), \( \gamma_E = 0.3 \) and \( \gamma_E = 0.6 \) cases were studied. All values correspond to \( x = 0.33 \). The different equilibria are seen in Fig. 6.4. The sensitivity of the equilibria to these levels of rotation are found to be small, as expected due to the Mach number squared scaling of the ‘rotation pressure’. Only a 10\% increase in the Shafranov shift was observed between the static and \( \gamma_E = 0.6 \) case. The non-linear predicted flux sensitivity to this different Shafranov shift is also minimal, with only a 6\% decrease in ion heat flux when the \( \gamma_E = 0.3 \) equilibrium is used compared with the static equilibrium for a run with \( R/L_{T_i} = 6.9 \). We can thus conclude that the effect of rotation on the equilibrium itself does not play a role in setting the profile stiffness.

![Figure 6.4: Flux surfaces in the vicinity of the magnetic axis from a solution of the generalised Grad-Shafranov equation using the kinetic profiles of 70084 and scaled rotation profiles from 66404. Three cases are shown: static (red solid curves), \( \gamma_E = 0.3 \) (blue dashed curves) and \( \gamma_E = 0.6 \) (black dashed-dotted curves).](image)
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6.3.4 Effect of $R/L_n$

In the limited experimental data set studied, there is a wide variation in $R/L_n$, from 1.4 in the 70084 case to 3.8 in the 66404 case (which corresponds to the highest $R/L_{Te}$ in the data set). The sensitivity of the turbulence to the $R/L_n$ value was examined. In particular, the possibility that non-linear ITG-TEM (trapped electron mode) interplay takes place which can reduce the level of turbulence and thus the stiffness, as reported in Ref. [40], was examined. In Fig.6.5 these linear scans are shown. For $R/L_n = 1$, the dominant mode propagates in the ion diamagnetic direction (ITG mode). However, for $R/L_n = 3.8$ the mode at low $R/L_{Te}$ propagates in the electron diamagnetic direction. This is most probably a density gradient driven TEM mode, which is stabilised by $R/L_{Te}$ (which would correspond to low stiffness) until it switches to an ITG mode at $R/L_{Te} \approx 5$. At that point we would expect turbulence stabilisation according to Ref. [40]. However, for higher $R/L_{Te}$, the growth-rate stiffness is similar to the $R/L_n = 1$ case, as a pure ITG regime is reached. For $R/L_n = 5$ the TEM-dominated regime is maintained for much higher $R/L_{Te}$. However, the highest experimental $R/L_n$ in the data set of Ref. [23] is $R/L_n \approx 4$. Furthermore, at the experimental high $R/L_{Te}$ values the transport is ITG dominated and stiff even for $R/L_n = 5$. Thus it is unlikely that $R/L_n$ is responsible for reduced profile stiffness. Furthermore, even if the stiffness is low, the actual growth rates themselves are high, and we may expect a high degree of transport.

These results are maintained in the non-linear scans, seen in Fig.6.6. While at lower $R/L_{Te}$, stiffness is indeed reduced in the TEM regime for the high $R/L_n$ case, at higher $R/L_{Te}$ the growth-rate stiffness is similar to the $R/L_n = 1$ case. The actual growth rates are shown in (a), and frequencies in (b). Runs were electrostatic, with collisions, and with circular geometry.

![Figure 6.5: Linear $R/L_{Te}$ scans based on 66404 parameters at $x = 0.33$ with varying $R/L_n$. Growth rates are shown in (a), and frequencies in (b). Runs were electrostatic, with collisions, and with circular geometry.](image-url)
$R/L_{Ti}$ values the difference in stiffness between the $R/L_n = 1$ and $R/L_n = 3.8$ cases becomes negligible. We can conclude that the variance of $R/L_n$ in the data set is not responsible for the observed difference in stiffness.

6.3.5 Effect of $\beta_e$

In this section the sensitivity of the stiffness on $\beta_e$ is examined. The simulations carried out take discharge 66404 parameters as a reference. Linear (at $k_y = 0.4$) and non-linear $\beta_e$ scans are shown in Fig.6.7. From the linear scans, it is clear that the range of experimental $\beta_e$ values ($0 - 0.5\%$) are significantly below the kinetic ballooning mode (KBM) thresholds, characterised in the plot by the sharp upturn in growth rates at $\beta_e \approx 1.5 - 2.4$. Below the KBM threshold, $\beta_e$ stabilises the ITG mode [41, 42]. For our parameters, this leads to a growth rate reduction of $\approx 25\%$ at $\beta_e = 0.5\%$. This is at the upper range of our experimental $\beta_e$ values. The KBM threshold varies with $R/L_{Ti}$, since the threshold is determined by the total pressure gradient through $\alpha_{MHD}$. This $25\%$ growth rate stabilisation factor is not exceeded when repeating the linear simulations for $k_y = 0.1 - 0.3$. However, a striking observation is that the non-linear $\beta_e$ ITG stabilisation significantly exceeds the linear stabilisation. A decrease in ion heat flux by a factor of $65\%$ is seen in Fig.6.7b for the $\gamma_E = 0$, $R/L_{Ti} = 9.2$ case between $\beta_e = 0 - 0.48\%$. Simultaneously, while the ion heat flux is reduced by $\beta_e$ in the $\gamma_E = 0$, $R/L_{Ti} = 4.6$ case, it is not totally quenched. This is indicative of a decrease in stiffness as opposed to a threshold shift. We can thus conclude that electromagnetic effects play a significant role in stiffness reduction for our parameters, even at relatively low values of $\beta_e$. While this
stiffness reduction is not sufficient to fully explain the experimentally observed stiffness reduction, it is a factor which must be taken into consideration. Additionally, from linear gyrokinetic analysis, $\beta_e$ stabilisation of ITG modes has been invoked as a possible factor in improved hybrid scenario confinement at ASDEX Upgrade and DIII-D, particularly at outer radii [43]. The increased non-linear $\beta_e$ stabilisation reported here may point to an even greater importance of this effect than previously recognised.

Simulations in which significant non-linear stabilisation of ITG modes at $\beta_e$ values below the KBM limit have been previously reported [44]. There, an increase of the ratio between the zonal flow shearing rate to unstable mode growth rates ($\omega_{ZF}/\gamma_k$) was observed with $\beta_e$. A possible physical mechanism for this relative increase in zonal flow activity, based on increased coupling between Alfvénic modes and drift waves, has been suggested [45]. In Fig.6.8 we plot the mode amplitude spectra for the $\gamma_E = 0$, $R/L_{Te} = 9.2$ $\beta_e$-scan shown in Fig.6.7. The amplitude spectra have been normalised to the zonal flow ($k_y = 0$) amplitudes. Indeed a relative increase in the $k_y = 0$ modes is seen for the electromagnetic cases, which may be related to the ITG $\beta_e$ stabilisation. We note that in Ref. [44] a significant instability threshold shift was observed for $\beta_e >\sim 0.48\%$.

With our parameters, no such shift is seen across our spanned range of $\beta_e = 0 - 0.48\%$.

It is interesting to note that the PVG drive seems to be strengthened by finite $\beta_e$, and at higher $\beta_e$ no stabilisation at all with rotation is seen. The deduction that the PVG drive is increased is from spot checks where the parallel velocity gradients were artificially suppressed. In those cases, $E \times B$ stabilisation remains strong at finite $\beta_e$.

In Fig.6.9 we can see, from the entire data set in Fig.1 of Ref. [23] the correlations between $R/L_{Te}$ and $\beta_e$ for $x = 0.33$ and $x = 0.64$. There is a generally positive correlation between $\beta_e$, and $R/L_{Te}$, at $x = 0.33$, consistent with the reduced stiffness. At $x = 0.64$, $\beta_e$ is generally much lower than at $x = 0.33$. This is expected to further increase the stiffness at $x = 0.64$, beyond the increase due to the higher $\hat{s}$ and $q$ values.
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Figure 6.8: Amplitude spectra from the $\gamma_E = 0$, $R/L_{Ti} = 9.2$, non-linear $\beta_e$ scan displayed in Fig.6.7b.

Figure 6.9: Correlation between $R/L_{Ti}$ and $\beta_e$ at $x = 0.33$ (a) and at $x = 0.64$ (b) from the entire data-set presented in Ref. [23].
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6.3.6 Inclusion of fast particles

The discharges studied have relatively low density, and thus for the higher rotation cases where significant NBI is employed, it is possible that a large fast ion fraction is present. This is predicted to reduce the turbulent drive, since the fast ion species behaves mostly adiabatically due to the higher temperatures, and the density of the main ion species is reduced through dilution. In ASDEX Upgrade strong evidence has pointed to a fast ion dilution mechanism for ITB formation [46]. Monte Carlo simulations of the NBI injection and subsequent fast ion slowing down were carried out for discharge 66404 with NEMO/SPOT [47] within the CRONOS integrated modelling framework. The average fast particle temperature ($\approx 35$ keV) at $x = 0.33$ was calculated. A linear GENE scan of fast particle densities (relative to $n_e$) can be seen in Fig.6.10. The scan is carried out for various $k_y$ values in Fig.6.10a, assuming $R/L_{T_{fast}} = 0$. The $R/L_{T_{fast}}$ sensitivity is examined in Fig.6.10b at $k_y = 0.4$. A suppression of the growth rates is observed with increasing $n_{fast}/n_e$. However, for our parameters the fast ion fraction is predicted by NEMO/SPOT to be only $\sim 10\%$, and thus insufficient to strongly affect the growth rates. While a fast ion fraction has been proposed to be responsible for some mismatch between gyrokinetic simulations and experiments [48], in our case it seems that the fast ion fraction may be too low to play a significant role.

![Figure 6.10: Linear fast particle density scans on 66404 parameters at $x = 0.33$ at various values of $k_y$. Runs were electromagnetic, with collisions, and with numerical geometry.](image-url)
6.3.7 Full simulations including all effects

Simulations of all three discharges in the data set at \( x = 0.33 \) were carried out. These simulations assumed \( s = 0.2 \), included rotation, the effect of rotation on equilibrium, \( R/L_n \), finite \( \beta \), collisions, experimental \( Z_{eff} \), and \( T_e/T_i \). The effect of \( Z_{eff} \) - which is stabilizing for ITG turbulence - was modelled by lumping all impurities into a kinetic fully stripped carbon ion species. The carbon temperature and \( R/L_T \) were taken as identical to the main ion species. Simulations with \( Z_{eff} \) at values corresponding to both the CX and Bremsstrahlung values were carried out to test the sensitivity. The growth rate sensitivity to \( Z_{eff} \) and \( T_e/T_i \) for linear GENE runs based on discharge 66404 can be seen in Fig.6.11. For our cases, the \( Z_{eff} \) stabilisation is more than offset by the \( T_e/T_i > 1 \) destabilisation. From spot checks the error in simulated fluxes due to the statistical \( T_e/T_i \) error is less than 10%. Assuming \( R/L_{T_e} = 0 \) for the carbon species instead of \( R/L_{T_C} = R/L_{T_i} \) altered the bulk ion heat flux by less than 2%.

The comparison between the GENE non-linear simulations and the experimental heat fluxes from power balance is shown in Fig.6.12. In (a), the GENE simulations with both \( Z_{eff} = 1.4 \) and \( Z_{eff} = 1.9 \) are shown. In (b), the \( T_e/T_i = Z_{eff} = 1 \) simulations are shown. The predicted reduced degree of stiffness is still significantly higher than the experimental observation. Only by not including PVG destabilisation and simultaneously assuming a \( \gamma_E \) value significantly higher than the experimental value (as seen in Fig.6.12b), can simulated heat fluxes comparable to the experimental fluxes be obtained for high \( R/L_{T_C} \). However, the simulation for the low rotation, low flux case - discharge 70084 - does agree with the experiment.

![Figure 6.11: Sensitivity of growth rates to \( Z_{eff} \) (a) and \( T_e/T_i \) (b) from linear GENE runs based on 66404 parameters at \( x = 0.33 \). Runs were electromagnetic, with collisions, and with numerical geometry.](image-url)
6.3 Low magnetic shear case

Figure 6.12: Comparison of non-linear simulations and experimental results for the three separate discharges at \( x = 0.33 \). The comparison with the experimental range of \( Z_{\text{eff}} \) and actual \( T_e/T_i \) (1.08/1.25/1.14 for 70084/66130/66404 respectively) is seen in (a), while in (b) the \( T_e/T_i = Z_{\text{eff}} = 1 \) case is shown, both with and without the inclusion of PVG modes and for various values of \( \gamma_E \). Runs were electromagnetic, with collisions, and with numerical geometry.

The dramatic decrease in flux seen in the 66404 case when transitioning from \( \gamma_E = 0.3 \) to \( \gamma_E = 0.6 \) in the no-PVG case in Fig.6.12b can be understood as a consequence of suppression of lower \( k_y \) modes. In Fig.6.13, a 2D linear \( R/L_{Ti} \) and \( k_y \) scan of growth rates for discharge 70084 (including realistic parameters) is shown. 70084 is shown instead of 66404 for this illustration since the ITG behaviour is similar but the TEM driven modes at low \( R/L_{Ti} \) in the 66404 case complicates the picture without changing the qualitative message. No rotation was included in the scan. The contours for \( \gamma = 0 \) (linear threshold), \( \gamma = 0.15 \) and \( \gamma = 0.3 \) are highlighted in white. Modes with growth rates up to \( \gamma = 0.15, 0.3 \) can be expected to be suppressed by \( \gamma_E = 0.3, 0.6 \) respectively. Low \( k_y \) modes drive a significant amount of flux, as represented by the \( \gamma/k^2 \) mixing length argument. In Fig.6.13, the estimated low \( k_y \) cutoff due to a flow shear of \( \gamma_E = 0.6 \) is thus \( k_y \sim 0.3 \). In that regime, the flux should be significantly reduced since the high-flux-inducing low \( k_y \) modes are sheared out of the system. Indeed, when comparing the flux spectra maxima for the \( \gamma_E = 0.3 \) and \( \gamma_E = 0.6 \) no-PVG non-linear cases, the maxima are at \( k_y = 0.25 \) and \( k_y = 0.35 \) respectively, in line with the intuition gained from the linear runs.
6 Ion temperature profile stiffness: non-linear gyrokinetic simulations and comparison with experiment

![Diagram](image_url)

Figure 6.13: 2D plot of growth rates as a function of $R/L_{Ti}$ (x-axis) and $k_y$ (y-axis) for discharge 70084 with realistic parameters - identical to the non-linear case in Fig.6.12. The growth rates are in units of $[c_s/R]$. The three white contours correspond to $\gamma = 0$, 0.15, 0.3 from left to right respectively.

### 6.4 High magnetic shear case

In this section we compare the gyrokinetic simulations to the experimental results for the three discharges at $x = 0.64$. The experimental observations were that the transport was much stiffer at $x = 0.64$ compared with $x = 0.33$, and that no significant departures from high stiffness were observed at higher rotation. In Fig.6.14 a non-linear $R/L_{Ti}$ scan with various levels of $\gamma_E$ is shown. The scan is based on discharge 70084 parameters, but applies circular geometry, $\hat{s}/q = 3/2$, and is collisionless and electrostatic. The simulated stiffness is indeed greater than the $x = 0.33$ case as can be seen by comparison with the $\gamma_E = 0$ curve in Fig.6.2a. Moreover, the degree of experimental $\gamma_E$ variation between the discharges (between $\gamma_E = 0.1 - 0.3$) is also not sufficient to lead to a significant difference in $R/L_{Ti}$ for the same level of flux.

In Fig.6.15, the full comparison between the simulations and the experiments is shown. As in the $x = 0.33$ case, these gyrokinetic simulations are electromagnetic, collisional, with numerical geometry, include a carbon species at a density consistent with $Z_{eff} = 1.4$, and include the experimental $T_e/T_i$. The red curve in Fig.6.15 corresponds to a $R/L_{Ti}$ scan with the 70084 parameters to gauge the stiffness level. For the high rotation cases 66130 and 66404, the experiment and the predictions differ by approximately a factor of 2. This difference can be explained within the uncertainties of the input modelling parameters. For example, an additional 66404 simulation carried out with $Z_{eff} = 2.4, T_e/T_i = 1.1$ and $R/L_{Ti} = 7.8$ led to agreement with the experimental flux within the experimental error-bars. However, the discrepancy between the model-
6.5 Characteristics of linear simulations with flow shear

In this section we discuss some fundamental characteristics of local linear simulations when including flow shear. This includes: the differences between predictions of sub-critical turbulence when assuming either adiabatic or kinetic electrons; the presentation of a novel growth rate calculation method in the presence of rotation, which inhibits the formation of stationary eigenmodes. The second point is of particular relevance for the validation of effective growth rate models applied in the mixing length rule of quasilinear transport models in the presence of flow shear.

Figure 6.14: Non-linear $R/L_Ti$ scan for various levels of $\gamma_E$, based on the 70084 parameters at $x = 0.64$.

ling and simulation for the low rotation 70084 discharge cannot be easily explained by the possible variation in the input parameters.

An $R/L_{Te}$ sensitivity check for discharge 66130 was carried out, due to the large $R/L_{Te}$ error bar. It was found from the dedicated non-linear simulations that within the possible $R/L_{Te}$ range the impact on ion transport is minimal.

Examining the differences in experimental parameters for all 3 discharges between $x = 0.33$ and $x = 0.64$ in table 6.1, we can see that both $\hat{s}$ and $q$ are higher at $x = 0.64$, and $\beta_e$ is lower. All of these differences are expected to lead to higher stiffness. These qualitative differences in $q$-profile and $\beta_e$ between low and high radii are generic (apart from special cases such as in ITB discharges), and should hold in general in tokamak discharges.
6 Ion temperature profile stiffness: non-linear gyrokinetic simulations and comparison with experiment

Figure 6.15: Comparison between gyrokinetic simulations and experiment at \( x = 0.64 \) for all three discharges. The experimental values (with the error bars) are shown for 70084 (red marker), 66130 (green marker) and 66404 (blue marker). The simulated values are shown with the same colour coding and marker style for all three discharges. In addition, simulations identical to the 70084 simulation but with reduced \( R/L_{T_i} \) were also carried out to gain a measure of the stiffness at realistic parameters. Runs were electromagnetic, with collisions, and with numerical geometry.

6.5.1 Sub-critical turbulence with kinetic vs adiabatic electrons

In Ref. [49], the analysis of sub-critical turbulence for Cyclone Base Case (CBC) parameters (circular geometry, \( \hat{s}/q = 0.8/1.4, \epsilon = 0.18, R/L_{n_i} = 2.2 \) assumed adiabatic electrons. This has motivated an examination of whether the qualitative behaviour of the system changes when studied with kinetic electrons. In Fig.6.16, \( \gamma_E \) scans performed by linear-GENE with CBC parameters taking \( R/L_F = 6.9 \) and \( R/L_T = 8.75 \) with kinetic and adiabatic electrons are shown. \( R/L_{T_i} = R/L_{T_e} \) for the kinetic electron cases. The growth rates were calculated by a time average, as in Ref. [49], for comparison. The methodology of growth rate calculation in a rotating system is further discussed in section 6.5.2. The simulations were performed with the initial value solver and thus calculate only the fastest growing mode. When including kinetic electrons, the static (i.e. without rotation) growth rates are significantly increased. This corresponds to the well known effect of added trapped electron drive to the ITG mode. A striking additional observation is that with the inclusion of kinetic electrons the modes are not quenched even at high \( \gamma_E \), seemingly undermining the sub-critical turbulence paradigm.

This lack of mode quench turns out to be the result of a secondary mode in the system: an electron temperature gradient driven trapped electron mode. This influence of trapped electrons is evident from collisional simulations, shown in Fig.6.17. As colli-
6.5 Characteristics of linear simulations with flow shear

Figure 6.16: Comparison of Cyclone Base Case linear-GENE simulations with $R/L_T = 8.75$ (red curves) and $R/L_T = 6.9$ (blue curves) with kinetic (solid curves) and adiabatic (dashed curves) electrons. $R/L_{T_i} = R/L_{T_e}$ for the kinetic electron cases.

sionality increases, the kinetic electron curve converges to the adiabatic electron curve, as the trapped electrons are progressively detrapped. The mode structure of the two leading modes as found by the eigenvalue solver for the kinetic electron $R/L_T = 8.75, \gamma_E = 0$ case is shown in Fig.6.18. The leading mode (propagating in the ion diamagnetic direction) is narrow in ballooning space, typical of ITG modes. However, the secondary mode (propagating in the electron diamagnetic direction) is extremely extended in ballooning space. This explains the relative lack of sensitivity of this mode to rotation, which induces a linearly time-dependent $k_x$ shift. Quenching occurs when the mode structure is shifted to stable $k_x$ values faster than the growth rate in the unstable region. If the unstable region is extended in $k_x$ space, then rotation is less effective to quench the mode.

The onset of this mode for CBC parameters is at $R/L_{T_e} \approx 5$, as seen in a $R/L_{T_e}$ scan shown in Fig.6.19. The growth-rates and frequencies for the two dominant modes are plotted (as calculated by the eigenvalue solver). The subdominant mode switches from an ion mode to an electron mode at $R/L_{T_e} \approx 5$, following which the secondary mode growth rate rises sharply with $R/L_{T_e}$.

However, in spite of the non-negligible growth-rate, we note that the secondary electron mode does not induce significant ion fluxes, as established by the low value of the $T_i\cdot\phi$ crossphase. Additionally, non-linearly the effect of the mode also seems to be negligible and does not drive significant electron flux. This was established by non-linear CBC simulations with $\gamma_E = 1.6$ and $R/L_{T_i} = 8.75$, comparing the cases with $R/L_{T_e} = 4$ and $R/L_{T_e} = 8.75$. In the linearly stable $R/L_{T_e} = 4$ case sub-critical turbulence is ignited in the non-linear simulation, as in Ref. [49]. The $R/L_{T_e} = 8.75$ case, which
6 Ion temperature profile stiffness: non-linear gyrokinetic simulations and comparison with experiment

![Graph showing comparison of adiabatic electron linear γ_E scans with kinetic electron scans with various levels of normalised collisionality.](image)

**Figure 6.17**: Comparison of adiabatic electron linear γ_E scans (CBC parameters with R/L_T = 8.75) with kinetic electron scans with various levels of normalised collisionality.

![Graph showing mode structure for the two leading modes in the kinetic electron R/L_T = 8.75 case.](image)

**Figure 6.18**: Mode structure for the two leading modes in the kinetic electron R/L_T = 8.75 case. The dominant mode (blue curve) propagates in the ion diamagnetic direction (note the positive mode frequency), while the subdominant mode (red curve) propagates in the electron diamagnetic direction.
linearly has an unstable electron mode, had both ion and electron fluxes that matched the \(R/L_{Te} = 4\) case within the statistical fluctuations of the simulation. In summary, in spite of the seeming breaking of the sub-critical turbulence paradigm due to the existence of sub-dominant electron modes when including kinetic electrons at high \(R/L_{Te}\), the impact of this mode is minimal. For the single non-linear case examined, the sub-critical turbulence conclusion derived from adiabatic electron studies remain pertinent. Further study in more extended regions of parameter space are necessary to gain increased confidence in this statement.

### 6.5.2 Novel growth-rate calculation method

In the cases with rotation discussed above, the growth rates were calculated from an average of the fluctuating (due to flow shear) growth rate. However, the timescale of this averaging is much greater than the non-linear decorrelation time, which raises doubts whether this averaging leads to a physically relevant quantity. In Ref. [16], an alternative method was proposed whereby the effective growth rate is calculated over the time during which the mode amplitude is increased by a predefined amplification factor (100 was suggested). In this work, we propose an additional physics-based method for effective growth rate calculation. The underlying assumption is that the relevant timescale for growth rate calculations is the non-linear decorrelation time. We then make the further assumption that the non-linear decorrelation time at each spatial scale is \(1/\gamma_k\), the inverse linear growth rate of the most unstable mode at that spatial scale. From analysis of frequency broadening from previous non-linear simulations, this assumption has shown
to hold in many cases at low, transport relevant $k_y$ values [33]. The proposed method is thus to iteratively calculate the growth rate using an adaptive time window until the width of the time window converges to the inverse growth rate calculated. This is illustrated by an example given in Fig. 6.20, where a mode amplitude timetrace for the CBC kinetic electron $R/L_T = 8.75$ case with $\gamma_E = 0.3$ is shown. A diagnostic was written in which a time window scans over the amplitude timetrace. At each point in the scan $\gamma_k(t_1, \Delta t)$ is defined as $\gamma_k(t_1, \Delta t) = \ln \left( \frac{n(t_1 + \Delta t)}{n(t_1)} \right) / (\Delta t)$. $t_1$ is the starting point of the time window, $\Delta t$ is the time window width, and $n$ is the density fluctuation level. For a given $\Delta t$, $\gamma_k(\Delta t)$ is then defined as an average over the $\gamma_k(t_1, \Delta t)$ peaks in the $t_1$ scan over the timetrace, which smooths out any variations in the peaks which may arise due to the discrete $\Delta k_x$ shifts present in the implementation of the flow shear in GENE [50]. The time window displayed in Fig. 6.20 is at a position corresponding to one of the $\gamma_k(t_1, \Delta t)$ peaks in the $t_1$ scan for a given $\Delta t$. $\gamma_k$ is then obtained by iterating this procedure and adapting $\Delta t$, until convergence is achieved when $|1 - (\Delta t) \cdot \gamma_k| < \delta$. The convergence criteria $\delta$ has been chosen as 0.02 for the particular examples shown in this work. We will from henceforth refer to this growth rate calculation method as the ‘$\tau_{ac}$ method’.

Growth rate calculations using the $\tau_{ac}$ method are displayed in Fig. 6.21 for the Cyclone Base Case at $R/L_T = 8.75$ and for both adiabatic and kinetic electrons. In the kinetic electron case, $R/L_{Te} = 4$ such that we avoid the sub-dominant electron mode which masks the ITG quenching at high $\gamma_E$ as discussed in section 6.5.1. For both adiabatic and kinetic electrons, the fundamental quench behaviour is very similar, with the quench occurring at $\gamma_E \approx 2\gamma_0$, where $\gamma_0$ is the static growth rate. However, for low $\gamma_E$
6.6 Conclusions

Observations at JET have shown evidence of reduced ion temperature profile stiffness correlated with low magnetic shear and increased flow shear. This has motivated extensive non-linear gyrokinetic simulations, using the GENE code, with parameters based on a subset of these JET discharges. Transport sensitivity scans of various parameters that

Figure 6.21: Comparison of growth rate calculations with long time-scale averaging and the new $\tau_{ac}$ method for CBC parameters with $R/L_{Te} = 8.75$ for both adiabatic and kinetic electrons.

the initial sharp drop in growth rate previously seen is now mitigated, and much smoother profiles result. At higher $\gamma_E$, the growth rates are sufficiently low that the time window in the $\tau_{ac}$ method averages over a number of cycles, and is effectively equivalent to the previous long time-scale averaging method.

In reduced transport models, the growth rate quenching due to flow shear may be modelled through an effective growth-rate in a mixing length rule. However, it is vital that simultaneously the PVG destabilisation is included in the linear model itself, to not overestimate the transport reducing effect of flow shear. The parametric dependencies of both the flow shear quenching and PVG destabilisation can be explored in large-scale linear scans with codes such as GENE, and the new $\tau_{ac}$ growth-rate calculation method may offer higher fidelity with physical growth-rate values. In future work, GENE linear growth rate calculations as presented here are to be applied for validation of the gyrokinetic quasilinear model QuaLiKiz [51]. Comparison with non-linear results can be carried out in selected locations in parameter space.

6.6 Conclusions

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differed between the discharges - aside from rotation - were carried out, to assess potential mechanisms that may lead to the observed decreased stiffness regime. Full simulations including electromagnetic effects, numerical geometry, $Z_{eff}$, experimental $T_e/T_i$, and rotation were then performed at $x = 0.33$ (in the low stiffness zone) and $x = 0.64$ for the discharges studied, and the predictions were compared with experiment. Finally, a discussion of differences between adiabatic and kinetic electron formulations of flow shear quenching of linear modes was presented. A novel physics-motivated method of growth rate calculation in the presence of flow shear was introduced. The conclusions can be summarised as follows:

(1) According to the gyrokinetic simulations, the measured values of toroidal flow shear are insufficient to explain the transition to the low stiffness regime. While the competition between parallel velocity gradient (PVG) destabilisation and $E \times B$ stabilisation can reduce the stiffness in the vicinity of the turbulence threshold, the predicted flux levels themselves are still significantly higher than the experimental values. Improved agreement between the simulations and observation is however obtained when ignoring PVG destabilisation, and assuming $\gamma_{E}$ values above the measured values. These rotation settings are more consistent with poloidal rotation, which we have assumed to be negligible due to neoclassical damping. However, poloidal rotation values significantly above neoclassical values have been observed within internal transport barriers (ITBs) [9]. While highly unlikely that in our regime any significant poloidal rotation could be maintained in the presence of the neoclassical damping, it would still be of great interest to measure the poloidal rotation in the low-stiffness-regime discharges to examine whether nonetheless any anomalous poloidal rotation is observed.

(2) The predicted Dimits shift for the discharges simulated is $\Delta \left( \frac{R}{L_{T_i}} \right) \approx 0.5$. A significant departure from the Dimits shift paradigm is thus not experimentally observable with great certainty due to the experimental error in extrapolating to the instability thresholds.

(3) The transport sensitivity to $R/L_n$, variations, fast particle content, and the effect of rotation on the equilibrium, were all examined. It was established that none of the above factors are mechanisms for the transition to the reduced stiffness regime.

(4) The sensitivity of the transport to $\beta_e$ was examined. It was established that even for the relatively low $\beta_e$ values present in these discharges, the non-linear electromagnetic ITG stabilisation is significant. This stabilisation, at least for $\beta_e < 0.48\%$, is a stiffness reduction as opposed to a threshold shift. The non-linear stabilisation is significantly greater than the linear $\beta_e$ stabilisation, and may be related to an increased relative amplitude of zonal modes. Further investigations of the parameterisation of this effect is important for incorporation into the ‘mixing length rule’ of quasilinear transport formulations. It is expected that this effect would, using such formulations, lead to more optimistic predictions for the energy confinement in future devices such as ITER and DEMO.
6.6 Conclusions

(5) The gyrokinetic predictions and experimental fluxes were also compared at \( x = 0.64 \) for the three discharges. The experimental variation in flow shear between the discharges is not predicted to be sufficient to lead to a discernible difference in \( R/L_T \). While the comparison for the high rotation discharges 66404 and 66130 agreed within the experimental uncertainties, it was not possible to reconcile the prediction and observation for the low rotation discharge 70084. The situation is thus the opposite of the comparisons at \( x = 0.33 \), where only the 70084 comparison agreed within the experimental uncertainties.

(6) A comparison of flow shear quenching of linear modes assuming either adiabatic or kinetic electrons was carried out for Cyclone Base Case (CBC) parameters. Above an \( R/L_T \) threshold, a sub-dominant electron mode can arise which is not quenched by flow shear due to the mode being widely extended in ballooning space. However, this mode drives negligible ion heat flux, and a limited non-linear exploration has shown that the contribution of this mode to the electron heat flux is minimal. For the case examined, the predictions of the onset of sub-critical ion heat flux driving turbulence in the presence of high flow shear are maintained with kinetic electrons.

(7) A new method of growth rate calculation in the presence of flow shear has been presented. This method is based on the assumptions that the relevant time-scale for growth-rate calculation is the non-linear decorrelation time, and that this time can be represented by \( 1/\gamma_k \) at each length-scale. \( \gamma_k \) can thus be calculated iteratively with an adaptive time window. It is planned to apply this method for aiding in the validation of effective growth rates in quasilinear transport models.

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References

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7 Evaluation and future prospects

In this section we revisit our original research questions and evaluate the progress made in addressing them. The underlying motivation behind the research was to increase the understanding of the turbulent regime typical of tokamak hybrid scenarios, which have the potential to extrapolate to a long-pulse reactor scenario. We have focused primarily on the effect of the characteristic hybrid scenario $q$-profile and magnetic shear (derived from the current profile) on ITG turbulence. In section 7.1, the specific research sub-questions posed in section 1.3 are answered. In section 7.2, the outlook for follow-up work and new potential research directions is surveyed.

7.1 Conclusions

What is the optimum application of external current drive in the ITER hybrid scenario for obtaining the current profile shape that minimises turbulent transport? (Chap.3 and Ref. [1])

The CRONOS suite of integrated modelling codes [2] coupled to the GLF23 transport model [3] was applied for optimising the current drive application in the ITER hybrid scenario under certain constraints. These constraints were the maintenance of $q > 1$, achieving $P_{\text{fus}} \sim 350 \text{ MW}$, and reaching $Q > 5$. The primary instability which leads to turbulent transport in the scenario was the ITG mode. In GLF23 the critical threshold of the instability has a dependence on the current profile in line with the $\hat{s}/q$ parameterisation in analytical formulations [4]. This led to a clear correlation between the volume averaged $\langle \hat{s}/q \rangle$ and $P_{\text{fus}}$ in the simulated discharges.

Maximising $\langle \hat{s}/q \rangle$ by application of current drive, while still satisfying the $q > 1$ constraint, was shown to be key to optimising the scenario, according to the GLF23 transport model. The ideal current drive combination for optimum $q$-profile shaping was seen to be the combination of off-axis neutral beam injection (NBI) and electron cyclotron current drive (ECCD). NBI provides high current drive efficiency (defined as [MA]/[MW]), increasing the non-inductive current fraction necessary for $q > 1$. While the NBI current deposition shape is not flexible, the general position of the off-axis NBI deposition is beneficial for the desired $q$-profile shaping. ECCD has reduced current drive efficiency compared with NBI, but the ECCD current profiles are focused and tunable, allowing for more flexibility in $q$-profile shaping. Strong sensitivity of the fusion power to the precise ECCD settings was observed, with up to 20% difference of $P_{\text{fus}}$, when adjusting the simulated ECCD launcher mirror angles by only a few degrees. The application of ion cyclotron resonance heating (ICRH) was not predicted to be beneficial to the scenario due to the stiffness of the GLF23 transport model and the lack of ICRH current drive. The application of lower hybrid current drive (LHCD) was predicted to be detrimental due its high current drive efficiency combined with an unfavourable deposition radius.
The current deposition location of LHCD is too far off-axis, which strongly decreases \( \langle \dot{\varepsilon}/q \rangle \), and is predicted to lead to reduced fusion power.

The importance of current profile shaping during the ramp-up phase of the discharge was stressed. Current diffusion following current ramp-up tends to increase \( \langle \dot{\varepsilon}/q \rangle \). Thus to maximize the average \( Q \) gain factor during a discharge, the \( q \)-profile should reach its final stationary-state optimum shape immediately after the L-H transition.

**What are the limits for defining an ITER hybrid scenario operational window given the scenario constraints? (Chap.3 and Ref. [1])**

One of the strongest sensitivities of the ITER hybrid scenario predictions was the pedestal height assumption. The optimisation exercise was carried out for \( T_{\text{ped}} = 3, 4, 5 \) keV. For \( T_{\text{ped}} = 3 \) keV no satisfactory scenario was found for any current drive mix, with \( x(q = 1) = 0.44 \) at 1200 s. For \( T_{\text{ped}} = 4 \) keV a satisfactory hybrid scenario was predicted. This scenario however demands an available ECCD power of up to 37 MW, more than the ITER baseline level of 20 MW. This strongly informs the considerations for potential ITER heating and current drive system (H/CD) upgrades for advanced scenario operation. For \( T_{\text{ped}} = 5 \) keV, the operational margins are much wider, and a satisfactory ITER hybrid scenario is predicted within the bounds of the ITER baseline H/CD system.

The strong sensitivity of the pedestal height in setting the operational windows of the scenario suggests that integrated modelling including self-consistent pedestal predictions is vital. Nevertheless, the simulations carried out do show that for \( T_{\text{ped}} \geq 4 \) keV a scenario is predicted that satisfies the primary ITER hybrid scenario goal of \( Q > 5 \) for \( t > 1000 \) s. Furthermore, the trends predicted for the scenario improvement with \( q \)-profile shaping hold regardless of the various modelling assumptions.

**Is the predicted \( q \)-profile impact on the turbulent transport validated by present-day experimental discharges? (Chap.4 and Ref. [5])**

This impact of increased \( \dot{\varepsilon}/q \) on core confinement was assessed on JET and ASDEX Upgrade (AUG) hybrid scenario discharges. This was carried out by integrated modelling with CRONOS coupled to the GLF23 transport model as in the ITER hybrid scenario study. For both machines, discharge pairs were analysed displaying similar pedestal confinement yet significant differences in core confinement and \( q \)-profiles.

For the JET pair, it was found that isolating the effect of the \( q \)-profile could explain \( \sim 60\% \) of the observed confinement difference. The bulk of the remaining difference could be explained by the variations in the experimental edge values serving as the GLF23 boundary conditions.

For the AUG pair, the isolated impact of the \( \dot{\varepsilon}/q \) effect in explaining the confinement difference was \( \sim 35\% \) - still significant but not the dominating factor. A further \( \sim 25\% \) of the confinement difference could be explained by the experimental edge condition variations between the pair. Two possible factors were identified in the reduced confinement AUG discharge which may play a role in setting the remaining 40% confinement difference: the existence of an NTM, and a flattened core rotation profile. These differences hamper the isolation of the \( \dot{\varepsilon}/q \) effect in AUG pair compared to the JET pair.
The absolute predictions of the core energy content in all discharges depend on the degree of rotational flow shear turbulence suppression assumed. When including the full GLF23 flow shear suppression model with $\alpha_{ExB} = 1.35$, significant overprediction of the core energy content was obtained for all discharges. With $\alpha_{ExB} = 0$, satisfactory agreement with the experimental values was obtained. A number of suggestions can be raised as to why this core energy content overprediction occurs. Firstly, the inclusion of real geometry, which is not taken into account in GLF23, is predicted to reduce the ITG critical thresholds [6]. This would diminish the degree of overprediction. This was also observed with QuaLiKiz [7] critical threshold modelling of these discharges. Furthermore, $\alpha_{ExB} = 1.35$ may be an unrealistically high value. More recent non-linear gyrokinetic simulations point towards appropriate values of $\alpha_{ExB} \sim 0.5$ [8–10]. Additionally, parallel velocity gradient (PVG) destabilisation may play a role in reducing the effectiveness of the flow shear destabilisation [9, 11, 12]. It is important to validate the PVG destabilisation of quasilinear models against more complete physical models. Nevertheless, the $s/q$ effect on core confinement is independent of the flow shear suppression model. The encouraging results regarding the validation of the importance of $q$-profile shaping for increasing ITG critical thresholds in hybrid scenarios hold, regardless of the absolute predictions.

While the work in Chap.3-4 concentrated on the beneficial effect of high $s/q$ at outer radii on the ITG critical thresholds, Chap.5-6 concentrated on the beneficial effect of low magnetic shear at inner radii on the stiffness. The following questions were asked:

**What is the non-linear physics at low magnetic shear ($s$) that leads to reduced stiffness in ITG turbulence? (Chap.5 and Ref. [13])**

From extensive simulations with the non-linear gyrokinetic code GENE [14], it was found that the changes in the spatio-temporal scales of ITG turbulence at low-$s$ results in flux reduction compared to high-$s$, in spite of the linear critical threshold not decreasing at low-$s$. This constitutes a stiffness reduction.

It was shown that at low-$s$ the turbulence radial correlation length decreases compared to high-$s$. This was postulated to be due to the observed increased zonal flow activity at low-$s$, which leads to increased eddy shearing. The increased zonal flows are possibly due to the increased poloidal symmetry of the linear ITG modes at low-$s$, which facilitates non-linear mode-coupling between the drift waves and the zonal flows. This effect is combined with increased non-linear frequency broadening around the linear ITG frequencies at the spatial scale of each $k_y$ mode at low-$s$. This led to a tendency of the autocorrelation time to decrease at low-$s$. Since the Kubo numbers are below (although close to) unity in the system, this is also expected to lead to a reduction in diffusivity [15]. The frequency broadening may also be a signature of increased coupling to damped linear modes in the system, which would reduce the flux. It was postulated that the increased zonal flows at low-$s$ may facilitate the coupling to damped linear modes.

**Can the QuaLiKiz model be improved such that increased agreement with non-linear ITG turbulence predictions is obtained at low magnetic shear? (Chap.5 and Ref. [13])**
The knowledge acquired from the non-linear simulations was applied for improving the QuaLiKiz gyrokinetic quasilinear transport model. It was first established that the discrepancy between QuaLiKiz and non-linear simulations at low-\(\hat{s}\) resulted from the mixing length rule, and not from other fundamental approximations in the QuaLiKiz model. It was, however, seen that for negative-\(\hat{s}\) the QuaLiKiz growth rate predictions are not accurate, most probably due to the eigenmode width ansatz made in the model. This has repercussions for using QuaLiKiz for steady-state scenario modelling.

The QuaLiKiz mixing length rule was improved by introducing a new formula for capturing the non-linear \(\langle k_x \rangle\) values at each \(k_y\) scale-length, based on the non-linear observations. This improved the accuracy of the assumption made for reproducing the non-linear flux spectrum maxima in the QuaLiKiz model, and also more accurately captures the non-linear correlation length trends. The increased frequency broadening effect was captured in an *ad-hoc* manner by simply dividing the QuaLiKiz predicted fluxes by a \(\hat{s}\) dependent factor corresponding to the average degree of frequency broadening observed in the non-linear simulations at each \(\hat{s}\). This roughly captures the effect of decreased correlation time in the diffusivities.

The combined effect of these two improvements led to significantly improved agreement between QuaLiKiz and a database of non-linear \(\hat{s}\)-scans. This increases our confidence in applying QuaLiKiz for hybrid scenario modelling in the future.

Can the observed reduced stiffness observed in JET discharges at low-\(\hat{s}\) and high rotational flow shear be understood through non-linear simulations? (Chap.6 and Ref. [16])

Gyrokinetic simulations with *GENE* were carried out based on specific JET discharges where reduced ion temperature stiffness was observed at combined low-\(\hat{s}\) and high rotational flow shear [17]. It was found that, according to the simulations, the measured values of toroidal flow shear cannot explain the transition to the low stiffness regime. For the low radii cases where lowered stiffness was observed, the heat flux value from a low rotation, low flux discharge was successfully predicted by the simulations. However, the simulations for the high rotation discharges produced flux values ~ 4 times higher than the experimental values. The competition between stabilising \(E\times B\) shear and destabilising parallel velocity gradient (PVG) destabilisation did lead to decreased stiffness with rotation, but significantly less than the experimental observation.

However, many additional insights were gained from the simulations. The sensitivity to the stiffness to the various parameters varying between the discharges was assessed. The sensitivity was found to be low for all cases apart from \(\beta_e\) (the ratio between electron and magnetic pressure). The non-linear ITG stabilisation with \(\beta_e\) was seen to be significantly higher than the linear stabilisation, and is manifested as stiffness reduction as opposed to a critical threshold shift. This was correlated with an increase in zonal flow activity. While the degree of stiffness reduction is not sufficient to fully explain the experimental observation, it does play a significant role and cannot be ignored. With regard to hybrid scenarios, linear \(\beta_e\) stabilisation has already been invoked to partially explain improved core confinement [18]. Since the non-linear stabilisation is stronger, the future investigation of this effect is highly desirable.
Finally, it was possible for the simulations to reach the experimental flux values for the high rotation cases when simultaneously assuming a flow shear level higher than the experiment, and ignoring the PVG drive. This may be consistent with significant poloidal rotation, even though it is commonly considered to be negligible due to neoclassical damping. This stimulates the experimental investigation of the poloidal rotation in this class of discharges.

7.2 Outlook

Many avenues for continued research based on the work covered in this research remain open. These consist both of the generalisation and completion of the research carried out, as well as new potential fruitful research directions that can be taken. These ideas are listed below, sorted by the chapter in which the original research was expounded.

7.2.1 Chap.3

Given the sensitivity of the results to the pedestal assumption, it would clearly be of great benefit to self-consistently include a prediction for the pedestal height in the integrated modelling. Initial work in this direction has already been carried out [19], where ITER hybrid scenario predictions with CRONOS were carried out with pedestal heights consistent with predictions from the EPEDI pedestal model [20], based on the work carried out in Chap.3. The next step would be to fully integrate the EPEDI model into CRONOS, for fully self-consistent simulations. Additional steps for increased modelling self-consistency could be to include particle and angular momentum modelling. The application of advanced transport models such as TGLF [21] and QuaLiKiz would be advantageous in this regard, since they incorporate improved formulations for trapped electrons compared to GLF23, which is important for particle transport. TGLF also has dependence on plasma shaping, as opposed to GLF23, which could be important for more accurate predictions of the scenario.

While the inclusion of ICRH was not predicted to be beneficial for the hybrid scenario due to a lack of current drive, it would be interesting to examine direct ion heating with He3 minority heating. While both the lack of current drive for $q$-profile shaping, and the dilution of the main D and T species by He3 may reduce the fusion power, this could be offset by a strong effect of direct ion heating, depending on the stiffness of the transport model used. This question has not yet been addressed by integrated modelling.

It was shown that to maximise the average fusion gain $\langle Q \rangle$ in the discharge, it is important to optimise the $q$-profile shaping during the ramp-up phase of the discharge. In this work the ramp-up phase was not fully optimised. Additional work has been started in this direction [22]. Additional optimisation work is projected to continue with a fast current profile solver [23].

We have insisted on maintaining the constraint $q > 1$ for a ‘satisfactory’ hybrid scenario. This constraint was based on the avoidance of deleterious NTMs seeded by sawteeth. However, there is empirical evidence that the $\beta$ value for NTM triggering by
sawteeth depends on the sawtooth period [24]. We can thus envisage a hybrid scenario with sufficiently low $\beta$ where the sawtooth period is reduced by maintaining a small $x(q = 1)$ radius and applying ECCD and ICRH for sawtooth control [25, 26]. A hybrid scenario with a small region of allowed $q < 1$ would greatly increase the operational window of the scenario, since the current profile tends to be peaked due to neoclassical resistivity and it is quite difficult to maintain $q > 1$ completely. Future work in this direction is highly recommended.

Finally, the sensitivity of the fusion power to the precise ECCD deposition opens up the opportunity for closed loop control of the hybrid scenario. In this work, the ideal location of the ECCD deposition was found by open loop optimisation, which is inefficient. It is envisioned that this can be carried out in a closed loop manner, optimising a cost function based on the ideal $q$-profile shape subject to constraints in the input power and $q > 1$. Initial work in this direction has already been carried out [27, 28], and continued work is recommended.

### 7.2.2 Chap.4

The main challenge of this work was the relatively small differences in $q$-profile between the discharges in each pair, which hampered achieving a high degree of certainty in the conclusions. Further $q$-profile shaping experiments aiming for larger differences would be extremely helpful for continued study. In addition, studies of recent JET hybrid scenarios with the ITER-like-wall (ILW) would be extremely beneficial for increased confidence in ITER extrapolations. It has been observed that with the ILW, impurity transport can play an important role in setting the electron temperature and resistivity during the rampup, which affects the $q$-profile evolution [29]. It would thus be important to improve the impurity transport capabilities of the integrated modelling codes to take this into account.

In addition, it was seen that the experimentally measured $q$-profiles were not always reproducible by current diffusion modelling. This discrepancy is particularly acute during the ramp-up phase. It has been suggested that the accuracy of the calculated neoclassical resistivity in the integrated modelling codes is dependent on the collisionality, and that the accuracy may be compromised at low collisionality [30]. It has also been suggested that using a free boundary solver for the equilibrium is important for the correct evaluation of the $q$-profile evolution during the ramp-up phase [31]. This matter should be investigated more carefully, since the scenario extrapolations depend strongly on the predicted $q$-profile evolution during the ramp-up phase.

### 7.2.3 Chap.5

Multiple improvements can still be made to the QuaLiKiz model. For increased efficiency in routine use as a transport model within the integrated modelling framework, the code itself can still be numerically optimised. In addition, plasma shaping should be taken into account in the model, which is seen to be important for calculation of ITG linear
7.2 Outlook

thresholds [6]. Additionally, incorporation into the model of parallel flow shear destabilisation and $E \times B$ shear stabilisation - validated against non-linear simulations - would then allow for the modelling of discharges with rotation, important for hybrid scenario modelling.

Further improvement in incorporating the effect of the increased non-linear frequency broadening at low-{$s$} can be made, beyond the ad-hoc normalisation factor currently applied. It may be possible to develop a relatively simple model for zonal flow generation, guided by non-linear simulations. The linear response could then possibly be renormalised based on the zonal flow mode amplitudes. A simpler improvement would be to incorporate the low-{$s$} parameterisation of the increased frequency broadening into the Lorentzian inserted into the frequency integration in the QuaLiKiz flux calculation itself.

7.2.4 Chap.6

The inclusion of rotation was only predicted to lead to ion heat fluxes close to the experimental values when PVG destabilisation was ignored, and when $\gamma_E$ was taken higher than the experimental values. This would be consistent with non-negligible poloidal rotation. The simulations assumed fully toroidal rotation. Experimental measurements of poloidal rotation for this class of discharges is thus recommended, in spite of the expectation of strong neo-classical damping.

Another possibility for obtaining reduced flux in the simulations, previously not taken into account, is $\alpha$-stabilisation due to the fast particle pressure [32]. While dilution due to the fast particles was ruled out as an important stabilising factor, self-consistent equilibria including the increased Shafranov shift due to fast particle pressure was not included in the calculations. This still needs to be examined.

The observed high degree of non-linear $\beta_e$ stabilisation is extremely interesting and deserves further study. Since this stabilisation is correlated with increased zonal flow activity, it would be interesting to examine in deeper detail any possible mechanisms for increased non-linear mode coupling to zonal flows. As a first step, any changes in the ITG mode structure which develop at finite $\beta_e$ should be studied, since this could facilitate coupling to zonal flows.
References

[29] E. Joffrin et al., 2012 24th *Int. Conf. on Fusion Energy* 2012 (San Diego, USA) (Vienna IAEA).
7 Evaluation and future prospects
Summary

Title: Turbulent transport in tokamak advanced scenarios

Nuclear fusion has the potential of providing a high baseline, environmentally friendly, and sustainable source of energy. A plasma consisting of the hydrogen isotopes deuterium and tritium, heated to temperatures of $\sim 10^8$ K, can sustain fusion reactions at a sufficiently high rate for energy production. In a reactor the plasma must be confined and insulated from the walls. The leading confinement concept is the tokamak, where the plasma is trapped in a toroidal chamber by helical magnetic fields. The confinement is then limited by turbulent transport, which leads to leakage of heat and particles at a rate higher than expected from collisional transport.

The poloidal component of the helical field is produced by toroidal current in the plasma itself. This current is in standard operation induced by a transformer in the centre of torus, where the plasma loop acts as the secondary circuit. The use of a transformer is not ideal for reactor operation; the discharge pulse times are limited since the transformer current cannot be infinitely ramped. The underlying question motivating the work in this thesis is thus: can an achievable tokamak operational scenario be developed which allows for significantly longer pulses, compatible with reactor requirements?

One such scenario developed in present-day tokamaks is the ‘hybrid-scenario’. The scenario operates at reduced plasma current compared with fully inductive scenarios, and has an increased non-inductive current fraction through increased external current drive. Importantly, hybrid scenarios display better confinement than expected from empirical scaling laws, which can compensate the confinement lost due to the reduced plasma current. This holds great promise for the extrapolation of this scenario to future machines, and may even pave the way towards a long-pulse reactor scenario. This thesis focuses on an a particular factor which may partially explain the improved confinement - the impact of the broad current profile which characterises hybrid scenarios, on the turbulent transport which limits plasma confinement. The particular instabilities studied are those driven by the ion temperature gradient (ITG). This the dominant source of turbulence in the tokamak regimes studied.

The current profile characteristics are represented by the following quantities derived thereof: the $q$-profile and magnetic shear ($\hat{s}$). Hybrid scenario $q$-profiles are characterised by high $\hat{s}/q$ at high radii and low-$\hat{s}$ at low radii. The beneficial effects of increased $\hat{s}/q$ at high radii for increasing ITG critical gradient thresholds are quantified in an extrapolation of the hybrid scenario to the ITER tokamak (currently under construction) by integrated modelling with the CRONOS suite of codes coupled to the GLF23 transport model for predicted heat transport. The confinement is optimised by tailoring the $q$-profile with external current drive sources. It is predicted that for a mix of off-axis neutral beam injection (NBI) and electron cyclotron current drive (ECCD), an ITER hybrid scenario satisfying $q > 1$, $Q = 5$, and $t > 3000$ s can be achieved for an edge transport barrier (pedestal) temperature of $T_{ped} > 4keV$. The fusion gain factor is defined as $Q = \frac{P_{fus}}{P_{input}}$. 

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It is also predicted that ion cyclotron resonance heating (ICRH) and lower hybrid current drive (LHCD) are not beneficial for the ITER hybrid scenario main burn phase, since they do not provide beneficial q-profile shaping.

These results are validated versus experimental discharges. Two pairs of hybrid scenario discharges from both the JET and ASDEX-Upgrade tokamaks are analysed. Each pair is characterised by similar pedestal heights but differences in q-profile and core confinement. The degree to which the differences in core confinement can be attributed to the \( \hat{s}/q \) effect is studied. CRONOS and GLF23 are used for predictive simulations of the discharges, as in the ITER extrapolation. These predictions are then compared to the actual experimental data. The effect of \( \hat{s}/q \) can be isolated in the simulations since all the various parameters and profiles in the simulations can be interchanged independently. Particularly for the JET pair, it is found that \( \hat{s}/q \) can indeed explain a major component of the core confinement difference. However, including the rotational flow shear turbulence suppression model in GLF23 leads to significant overprediction of the ion temperatures for all discharges studied. This raises questions regarding the importance of plasma shaping effects, and the validity of the parallel velocity gradient (PVG) destabilisation in the model.

The beneficial effect of reduced temperature profile stiffness at low-\( \hat{s} \) in inner radii is assessed with non-linear modelling using the GENE gyrokinetic code. It is found that at low-\( \hat{s} \) the turbulence correlation lengths are decreased and the non-linear frequency broadening increases compared to high-\( \hat{s} \) cases. Both these effects may be related to the observed increase in zonal flow activity, and leads to a reduction in the predicted flux compared to the high-\( \hat{s} \) cases. With these results, the validity of an advanced quasilinear transport model, QuaLiKiz, was extended to low-\( \hat{s} \) parameter space. The underlying assumptions of the model are examined, and the mixing length rule improved with guidance from the non-linear simulations. This work improves the confidence in using the QuaLiKiz transport model in the future for hybrid scenario predictions.

Finally, the experimentally observed stiffness reduction at low-\( \hat{s} \) and high flow shear was investigated with non-linear modelling. This topic is also of importance to hybrid scenarios, to further understand the relative importance of the various factors contributing to improved confinement. Hybrid scenarios on present-day machines also tend to be associated with high levels of flow shear. The simulations do not predict the degree of reduced stiffness as experimentally observed, and this question is thus still open. Experimental measurements of the poloidal rotation profile in this class of discharge may shed more light on the matter, as we assumed that the rotation is purely toroidal due to the expected neoclassical poloidal damping. Nevertheless, additional insights were gained into the non-linear nature of ITG turbulence. From electromagnetic simulations, the non-linear \( \beta_e \) (ratio of electron pressure to magnetic pressure) stabilisation of ITG turbulence is observed to be significantly greater than the well-known linear \( \beta_e \) stabilisation. This may be related to the observed increase in zonal flow activity within the \( \beta_e \) range studied. Beyond the increase in fundamental understanding, this effect could also be important for furthering the interpretation of transport in hybrid scenarios, which tend to operate at higher \( \beta_e \) than in inductive scenarios.
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Ever since the beginning of my physics studies, it has been my desire to join the ranks of those tackling the grand challenge of fusion energy development. The publication of this thesis represents a significant personal milestone towards that goal. However, the road has not been walked on alone. Over the last four years, both here in my adopted home in the Netherlands and during work trips to France, Germany, and the United Kingdom, I have had the fortune to meet and work with many wonderful people and scientists who have been more than instrumental in helping me begin to turn this desire into reality. While these few lines cannot be sufficient to convey the depth of my gratitude, I hope here to nevertheless try and express my deep thanks to all those who have made it possible – and enjoyable – to set forth on this adventure.

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Curriculum vitae

I was born on December 15, 1980, in Hong Kong. At age 14 I moved with my family to Israel. My initial education in Hong Kong was based on the British curriculum. In Israel I graduated in 1998 from the ‘Ort Yad-Giora’ high-school in Herzliya, having matriculated to the physics and computer science stream. Following a three-year compulsory national service stint in the Armoured Corps of the Israeli Defense Forces between 1999-2002, I began studying towards a B.Sc. degree in physics at Tel Aviv University in 2003. From the summer of 2005, I combined my studies with a research internship at the plasma physics laboratory at the Weizmann Institute of Science in Rehovot, Israel. Following my B.Sc. degree (graduated with Distinction), I was awarded with a Feinberg Graduate School Fellowship to attend graduate studies at the Weizmann Institute. I was conferred with a M.Sc. degree (awarded with the Dean’s Prize) in December 2008. The M.Sc. research, which resulted in a thesis entitled “High resolution measurements of a nearly collisionless plasma”, investigated the propagation of a magnetic shock on nanosecond timescales in a plasma-operating-switch (POS) device, using visible/UV spectroscopy.

In January 2009, I began my Ph.D. research with the high temperature computational plasma physics group at the FOM Institute DIFFER (formerly the FOM Institute for Plasma Physics “Rijnhuizen”) in Nieuwegein, the Netherlands. The work concentrated on the integrated modelling and gyrokinetic modelling of tokamak core plasma turbulence in regimes relevant for ‘hybrid’ tokamak operational scenarios. The work was mostly carried out at FOM DIFFER, together with multiple visits under EURATOM mobility to collaborators at CEA Cadarache in France, the Max Planck Institute for Plasma Physics in Germany, and to the Joint European Torus (JET) in the United Kingdom. The results of the work are presented in this thesis.