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A singularity solution of shear faulting in swelling ionised porous media

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Introduction
Shales, clays, gels and biological tissues all endure swelling associated with ionisation of the porous medium [2]. We will study intervertebral disc herniation, which includes numerical simulations of crack formation and crack propagation during shrinkage [3.,4.]. Analytical solutions are a must for validation of numerical codes.

Methods
Shear faulting
A dislocation on an existing crack is considered as verification (fig 1):

\[
\left. u \right|_{z=0^+} - \left. u \right|_{z=0^-} = f(x) \text{H}(t).
\]

Figure 1: Shear fault: \( f(x) \) block function.

Model
Lanir’s biphasic theory describes the saturated porous medium by assuming incompressible constituents and infinitely fast ion flow. Perturbation on an equilibrium gives a strongly coupled system (1), with \( u \) displacement, \( \mu \) chemical potential and \( \mu_c \) and \( K \) physical constants.

\[
\begin{align*}
\mu \nabla^2 u + (c - \mu) \nabla \text{tr}(\epsilon) - \nabla \mu_c &= 0, \\
\frac{\partial \mu}{\partial t} - K \nabla^2 \mu_c &= 0.
\end{align*}
\]

(1)

Analytical solution
Decoupling the systems of equations by means of stress functions \( S \) and \( E \) [1.] results in equation (2). The system is solved using linear transformations (Fourier and Laplace). The shear stress is calculated at the crack surface. Using Simpson rule for inverse Fourier the chemical potential is computed.

\[
\begin{align*}
\nabla^2 S &= 0, \\
\frac{\partial^2 e_{\text{eff}}}{\partial t^2} - cK
\nabla^2 E &= 0.
\end{align*}
\]

(2)

Numerical simulation
Calculations are performed in Abaqus. The mesh is created from two symmetrical meshes, \( z \geq 0 \) and \( z \leq 0 \). Nodes outside the cracksurface are tied and the cracksurface is made a sliding contact to ensure fluid flow.

Results and Discussion
Shear stress
Analytically the shear stress (red) shows a high singularity in \( x \). Simpson’s rule for Fourier integration cannot cope with dislocation at \( z = 0 \). (fig 3a). Numerically we have same singularity pattern, but high mesh-dependency is seen (fig 3b).

Figure 3: High singularity in shear stress. a) Analytical versus Simpson’s rule. b) Mesh dependency at left crack-tip.

Shear stress at \( t = 0 \) is the result of initial incompressible behaviour of the medium. Fluid flow results in relaxation of the stresses. This is consolidation. Shear stress at equilibrium decreases with decreasing osmotic pressure. (fig 4)

Figure 4: Shear stress in crack: consolidation.

Chemical potential
The chemical potential is high at compression areas (\( xz > 0 \)) and fluid flow is initiated to lower areas (\( xz < 0 \)). Chemical potential rapidly decreases further from the crack. (fig 5)

Figure 5: Chemical potential: fluid transport

Same trend was seen numerically.

Remarks
A quantitative comparison has not yet been performed.

References: