Comparison of supply chain planning concepts for general multi-item, multi-echelon systems

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Ton G. de Kok
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Comparison Of Supply Chain Planning Concepts
For General Multi-Item, Multi-Echelon Systems

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Comparison Of Supply Chain Planning Concepts

For General Multi-Item, Multi-Echelon Systems

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Abstract

In this paper we present two alternative concepts to control general multi-item, multi-echelon systems under stochastic stationary demand for end items. Such systems consist of items that are assembled from other items and in turn are assembled into other items. Each assembly process involves a planned lead time. For such systems optimal control policies are unknown to-date. Therefore we resort to control concepts that at least enable an exact computation of the control parameters. The two alternative concepts represent two fundamentally different modeling concepts: an LP-based concept representing application of deterministic mathematical programming models in a rolling schedule context (the common practice in so-called Advanced Planning Systems), and modified base stock policies representing application of classical multi-echelon inventory models. The parameters of the LP-based concept can be determined by discrete event simulation. The parameters of the modified base stock policies can be determined analytically. We compare the two concepts based on the required supply chain capital investment required to guarantee target end item service levels. Surprisingly, the modified base stock policies outperform the LP-based concept. We provide managerial insights as well as a deeper understanding into a number of fundamental issues related to supply chain planning and supply chain design.
1. Introduction

As a result of companies concentrating on their core competencies the mutual dependence of companies on each other is stronger than ever. This mutual dependence affects all primary business functions, i.e. development, manufacturing and sales and service. It effects strategic, tactical and operational decisions. Through this mutual dependence individual companies have become more aware than ever that they are part of a network through which we can identify flows of material, information and money. This awareness has lead to the notion of Supply Chain Management. Supply Chain Management encompasses all management activities that aim at satisfying needs of customers according to predefined customer service targets at minimal costs for all companies involved in the delivery of the product or service to these customers. Whether a company is considered to be involved in the delivery depends on the definition of the Supply Chain or Supply Network. E.g. it is quite common to define the supply chain of automotive companies as the network consisting of the automotive company and its first and second tier suppliers. Another example is the PC supply chain, which typically consists of suppliers of modules like hard disks, CD-ROM players, monitors and main boards, PC assembly plants and dealer networks.

In the context of supply chain management we focus in this paper on Supply Chain Planning (SCP), i.e. the tactical and operational control of material flows through a network consisting of nodes that transform materials, after which the materials are transported to other nodes. Such networks are known as multi-item, multi-echelon networks. In this paper we assume that capacity restrictions of transformation processes at the nodes are translated into constant lead times. I.e. we assume that
capacity management at each node is such that input materials released at the node are transformed into output materials after a fixed time. Shipment lead times are assumed to be fixed as well. Although we are aware of the relevance of explicitly incorporating capacity restrictions into supply chain management models, we claim that the issues we would like to raise in this paper are not affected by the absence of capacity restrictions. Notice that it is most common to assume constant lead times for co-coordinating material flows through a supply chain (cf. Silver, Pyke and Peterson[1998], ch. 12).

The supply chain management control issue considered in this paper is the choice for a particular control concept. We distinguish between two supply chain control concepts:

- Mathematical programming models applied within a rolling schedule context
- Echelon stock (or base-stock) control concepts.

We refer to Silver, Pyke and Peterson[1998] for a state-of-the-art bibliography and an introduction to the two different concepts. The first concept is applied in recently introduced software packages commonly known as Advanced Planning Systems (APS). The pioneering work by Blackburn and Millen[1982] has already provided some insight into the effectiveness of the mathematical programming approach in the context of Multi Level Lot sizing problems. In this paper we formulate a standard LP model that applies in the context of SCP (cf. Billington et al [1983] and Belvaux and Wolsey[2001]). The second concept, proposed by Magee[1958], is mostly considered to be a theoretical approach. Recently, De Kok and Visschers[1999] proposed a class of multi-item
multi-echelon policies that allow for an exact analysis of such systems under stationary stochastic demand. They show that for specifically structured supply networks this policy performs well compared to pure base stock policies integrated with some rationing mechanism, like run-out time prioritization, in case of item shortages. In this paper we apply this class of policies without a restriction on the structure of the supply chain, thereby providing a deeper insight into the effectiveness of these policies for the control of general supply networks.

The objective of the paper is to compare both concepts on the basis of their associated supply chain inventory capital investment required to realize a target customer service level. This comparison is enabled by a procedure that uses discrete event simulation to determine safety stock parameters required as an input to the LP-based rolling schedule concept in order to guarantee the required performance. Discrete event simulation is used as well to confirm that the modified base stock policies derived from De Kok and Visschers[1999] indeed satisfy the required performance.

The contribution of the paper to existing literature is that it presents to our knowledge a first rigorous treatment of control policies for general multi-item multi-echelon models under stationary demand and constant lead times. For such systems the structure of the optimal policy is unknown, whereas in practice virtually any supply network structure is "general", i.e. both exhibits divergence and convergence. Furthermore the paper compares two fundamentally different SCP concepts, viz. a deterministic-demand-model-based concept and a stochastic-demand-model-based concept, of which the former is the currently dominating paradigm for SCP. We arrive at results that may have important implications for the future of Supply Chain Planning.
The paper is organized as follows. In Section 2 we define in more detail the class of supply chain control problems we deal with in this paper, which is the class of general assembly networks without lot sizing restrictions. In Section 3 we focus on mathematical programming models in a rolling schedule context as applied to supply chain control problems. Section 4 is dedicated to a discussion of stochastic multi-echelon models as a tool for supply chain control. In Section 5 we present the results from our computational study based on the comparison of two of the three concepts for a problem set. From our assessment of the various control concepts we derive conclusions and directions for future research in Section 6.

2. Modeling general supply chain structures

In this paper we concern ourselves with general supply chain structures from a material co-ordination point of view. In that case the structure of a supply chain is defined by parent-child relationships between items. Let us consider a supply chain where at the control level we deal with N items. For each item we define

\[ L_i \] throughput time between time of release of an order for item \( i \) and time at which the ordered items are available for usage in other items and/or delivery to customers

\[ a_{ij} \] number of items \( i \) required to produce one item \( j \)

\[ D_i(t) \] exogenous demand for item \( i \) in period \( t \), i.e. demand in period \( t \) for item \( i \), that is...
not derived from demand for items in \( E_i \setminus \{i\} \)

\( P \) set of products with exogenous demand, i.e. \( \{i \mid \exists t \geq 1, D_i(t) > 0\} \)

\( E \) Set of end products, i.e. \( \{i \mid \forall j, a_{ij} = 0\} \)

\( I \) Set of intermediate items, i.e. \( \{i \mid \exists j, a_{ij} > 0\} \)

We assume that the incidence graph defined through \((a_{ij})\) is acyclic. In the sequel we assume w.l.o.g. that only end products exhibit exogenous demand, i.e. \( P = E \).

We assume that \( L_i \) is constant. This assumption is motivated by the fact that at supply chain level we set conditions for lower levels, one of which is the planned throughput times. It is the responsibility of lower levels, such as the shopfloor level, to ensure that the throughput times are kept constant. It is clear that capacity constraints effect the throughput times, yet we assume that such constraints are reflected in the throughput times. The lower levels, such as shopfloor control, are able to provide a high due date reliability, because of different kinds of flexibility. Such flexibility is unknown to the supply chain control level and therefore cannot be modeled at that level (cf. Bertrand et al[1990]).

A supply chain control concept decides on the release of items, both in quantity and timing. One requirement a supply chain concept must satisfy is feasibility. Feasibility can be defined along the following lines. Let
\( r_i(t) \) quantity of item \( i \) released at the start of period \( t, t \geq 0, \forall i \)

\( J_i(t) \) net inventory of item \( i \) at the start of period \( t \) immediately before quantity released at the start of period \( t-L_i \) is available, \( t \geq 0. \forall i \)

\( I_i(t) \) physical inventory of item \( i \) at the start of period \( t \) immediately before quantity released at the start of period \( t-L_i \) is available, \( t \geq 0. \forall i \)

\( B_i(t) \) backlog of item \( i \) at the start of period \( t \) immediately before quantity released at the start of period \( t-L_i \) is available, \( t \geq 0. \forall i \)

Then \( r_i(t) \) must satisfy the following equations,

\[
J_i(t+1) = J_i(t) - \sum_{j=1}^{N} a_{ij} r_j(t) - D_i(t) + r_i(t - L_i), \forall i, \ t = 0, 1, 2, ..., T
\]

(1)

which is the inventory balance equation for general assembly networks. Furthermore \( r_i(t) \) must satisfy the following inequalities,

\[
\sum_{j=1}^{N} a_{ij} r_j(t) \leq \max(0, I_i(t) - B_i(t)) + r_i(t - L_i), \forall i, \ t = 0, 1, 2, ..., T
\]

(2)

\[
r_i(t) \geq 0, \forall i, \ t = 0, 1, 2, ..., T
\]

It can be shown that (2) is equivalent with

\[
B_i(t+1) - B_i(t) \leq D_i(t), \forall i, \ t = 0, 1, 2, ..., T - 1,
\]
which states that the backlog from the start of a particular period to the start of the next period cannot increase more than the exogenous demand during this period. This implies that at intermediate items without exogenous demand we do not allow for backorders. At items with exogenous demand this demand may cause backorders.

In order to compare different supply chain planning concepts we define a cost structure and a performance criterion. We define $C(t)$ as the cost incurred at the start of period $t, t \geq 0$,

$$C(t) = \sum_{i=1}^{N} h_i I_i(t),$$

where

$$h_i \quad \text{value of item } i \quad \forall i$$

Notice that $C(t)$ is not really a cost function but represents the total supply chain inventory capital investment at the start of period $t$. We are interested in the long-run average value of $C(t)$,

$$\bar{C} = \lim_{t \to \infty} \frac{1}{t} \sum_{s=1}^{t} C(s).$$

The long-run average supply chain inventory holding cost can be derived from multiplying $\bar{C}$ by the interest rate. As performance criterion we choose $P_{ij}, \forall i \in P$, defined as
For each supply chain planning concept \( P \) we want to solve the following problem

\[
\min \ C (P)
\]

\[\text{s.t.} \]
\[
P_{1,i} (P) \geq P_{1,i}^*, \ i = 1, 2, \ldots, N
\]

Here we express the dependence of both \( C \) and \( P_{1,i} \) on \( P \). It is important to state here that until now we do not know the optimal policy for the multi-item order release problem we discuss in this paper. This compels us to define a (suboptimal) supply chain planning concept first and then determine the solution to the problem. We have chosen to compare two concepts that have been discussed extensively in literature:

- an LP-based rolling schedule concept, where each period an LP is solved to optimality, from which the immediate order release decisions are derived.
- base stock policies that have been modified to ensure that the feasibility conditions are satisfied.

The LP-based concept is discussed in Negenman[2000] and is similar to the concept proposed by Billington et al[1983]. Belvaux and Wolsey[2001] discuss solution methods to this problem when in addition lot sizing constraints are incorporated. We emphasize here that our focus is not on the
analysis of the LP-instances to be solved each time period, but on the performance of the LP-based concept under stationary stochastic demand.

The modified base stock control concept has been proposed by De Kok and Visschers[1999]. This concept allows an exact analysis of general supply chain structures, i.e. multi-item, multi-echelon with both convergent and divergent relationships between items. In conjunction with the results in Diks and De Kok[1999] for divergent N-echelon inventory systems it is possible to compute near-optimal policies within this class of modified base stock policies.

We emphasize here that the objective of the paper is to compare these essentially different concepts in order to derive insights into their fundamental differences. This especially implies that we do not extensively explain all the details of the analysis that underlie the computation of the base stock levels used in the modified base stock control concept. For details of that analysis we refer to Diks and De Kok[1999] and De Kok and Visschers[1999].

3. Mathematical programming models for Supply Chain Management

Currently commercial SCP software assumes a rolling schedule concept, where each planning cycle a mathematical program is solved, either to optimality or some heuristics are applied. For uncapsicatated SCP problems without lot sizing restrictions it is rather straightforward to formulate an LP model that fits in this rolling schedule context. In fact, the set of equations given in Section 2 constitute the basis for this formulation of a mathematical programming model. A fundamental
problem with the application of rolling schedule concepts to situations with stochastic demand is to
determine decisions such that predefined performance targets are met. The customer service level
restrictions defined in Section 2 require additional decision variables. Following the inventory
management literature (cf. Silver et al. [1998]) we introduce the concept of safety stocks in order to cope with short-term exogenous demand uncertainty,

\[ v_i \] safety stock parameter of item \( i, i \in P \)

The safety stock parameters are used to control the end-item service levels.

Another issue to be dealt with is the mutual dependence of release decisions for different items. We can derive that a planning horizon \( T \) should be at least equal to the maximum cumulative planned lead time as defined by the product structure (cf. Orlicky [1975] for the definition of the maximum cumulative lead time, see also the next Section). For uncapacitated systems is suffices to have \( T \) exactly equal to the maximum cumulative lead time.

In order to derive the systemwide order release decisions at the start of period \( t \) we need to forecast exogenous demand until period \( t+T-I \). The solution to the MP problem not only provides us with the immediate order release decisions, but in addition provides us with planned order release decisions. Therefore we define the following variables,

\[ \hat{D}_i(t, t+s) \] forecast of exogenous demand for item \( i \) in period \( t+s \) as decided on at the start
of period $t$, $t \geq 0$, $s \geq 0$, $\forall i$

$\hat{B}_i(t, t+s)$ forecast of backlog of item $i$ at the start of period $t+s$ as determined at the start of period $t$, $t \geq 0$, $s \geq 0$, $\forall i$

$\hat{r}_j(t, t+s)$ forecast of quantity of item $i$ released at the start of period $t+s$ as determined at the start of period $t$, $t \geq 0$, $s \geq 0$, $\forall i$

Note that $\hat{r}_j(t, t+s)$, $s \geq 0$, can be considered as the decision variables of the supply chain planning problem at the start of period $t$. In the sequel we assume that there is a time origin $0$ at which the SCP problem is solved first and the initial state of the system at time $0$ is known. This is important because some equations formulated below are formulated in terms of decisions taken from time $0$ onwards, i.e. from period $1$ onwards.

Now using the feasibility constraints derived in Section 2 and the objective function defined there we can formulate a Linear Programming model that can be solved by standard algorithms, such as the simplex method.

$$
\min \sum_{i=1}^{N} \left( \sum_{s=1}^{T} h_i(\hat{I}_i(t, t+s) - v_i)^+ + \sum_{s=1}^{T} \theta h_i(v_i - \hat{I}_i(t, t+s))^+ \right) \quad (O_1)
$$

such that
\[
\hat{I}_i(t,t+s+1) - \hat{B}_i(t,t+s+1) = \hat{I}_i(t,t+s) - \hat{B}_i(t,t+s) \\
- \sum_{j=1}^{N} a_{ij} \hat{r}_j(t,t+s) - \hat{D}_i(t,t+s), \quad s=0,\ldots,T-1 \\
+ \hat{r}_i(t,t+s-L_i), \quad \forall i
\]

\[
\hat{B}_i(t,t+s+1) - \hat{B}_i(t,t+s) \leq \hat{D}_i(t,t+s), \quad \forall i, \quad s=0,\ldots,T-1
\]

\[
\hat{r}_i(t,t+s) \geq 0, \quad \forall i, \quad s=0,\ldots,T-1
\]

\[
\hat{I}_i(t,t+s) \geq 0, \quad \forall i, \quad s=0,\ldots,T-1
\]

\[
\hat{B}_i(t,t+s) \geq 0, \quad \forall i, \quad s=0,\ldots,T-1
\]

Notice that the model formulation includes decisions variables with decisions taken before period \(t\). Obviously we must assume that

\[
\hat{r}_i(t,t+s) = r_i(t+s), \quad s < 0, \quad \forall i, \quad t \geq 1
\]

The decisions implemented at the start of period \(t\) are given by \(\{r_i(t)\}\), which are derived from the equations below.

\[
r_i(t) = \hat{r}_i(t,t), \quad \forall i, \quad t \geq 1
\]

Furthermore notice that the objective function we propose, reflects the trade-off between holding costs and shortage costs and takes into account the safety stocks. Assuming that expensive items are
more important than cheap items we have assumed that the cost per item \(i\) backlogged at the end of a period is proportional to \(h_i\), the cost per item held on stock at the end of a period. We assume that \(\theta \gg 1\). At first glance the objective function \((O_t)\) does not represent the real inventory holding costs and backorder costs. However, we should keep in mind that the MP problem formulation is only an attempt to model the supply chain planning problem under stochastic exogenous demand. In that sense any such formulation results into a heuristic with respect to the original optimization problem. Still objective function \((O_t)\) reflects the trade-off between inventory holding and backorder costs. On top of that the safety stock parameters control the service levels. This becomes even more evident from the following lemma.

**Sample Path Lemma**

Suppose a sample path \(\{D_i(t)\}\) of the demand process and a sample path \(\{D_i(t, t+s)\}\) of the forecasting process are given. Furthermore assume that for all end-items

\[
I_i(0) = v_i, \quad i \in P.
\]

Then the solution to the problem expressed in terms of the material order releases \(\{r_t(t, t+s)\}\) with objective function \((O_t)\) is the same for each value of \(v_i\) for all \(t \geq 1\) and for all \(s \geq 0\).

The proof of the sample path lemma is based on induction. Given the initial inventory levels for items with independent demand, it is clear that the objective function \((O_2)\) implies an optimal
solution \( \{r_i(1,1+s)\} \) that is the same for any value of \( v_i \). This implies that \( \{v_i(1)\} \) are the same for any value of \( v_i \). But then \( I_i(1) - B_i(1) - v_i \) is the same for any value of \( v_i \). This argument can be repeated for any value of \( t \). Formally, one uses the statement in the lemma as induction assumption.

The sample path lemma allows us to apply the following procedure for computation of the safety stock parameters.

i. Run a discrete event simulation of the system with \( v_i=0 \), where we solve each period the LP that follows from the set of linear constraints given above and the linear objective function \((O_i)\) that results if \( v_i=0 \).

ii. From the discrete event simulation compute the empirical distribution function

\[
\text{of } P\{J_i(t) - v_i \leq x\}.
\]

iii. Given this empirical distribution function compute \( v_i^* \), such that the required end-item service level is achieved.

iv. Run another simulation with \( v_i^* \) in order to compute \( \bar{C}(P) \).

We note here that step (iii) can be executed for most well-known performance measures, such as non-stockout probability at the end of a period, fill rate and average backlog. In this paper we apply this procedure with service criterion \( \alpha \), the probability of a non-negative stock at the end of an arbitrary period. We notice here that the above approach can be applied to many other MP problems, so that alternative rolling schedule approaches for specific stochastic planning and scheduling problems can be compared.
4. Echelon stock policies for supply chain management

In Magee[1958] so-called base-stock policies are introduced to control the replenishment of item inventories. Later Clark and Scarf[1960] showed that these base-stock policies are optimal for serial systems, i.e. the supply chain consists of items that have at most one parent and at most one child. Clark and Scarf assumed that ordering costs were negligible or implicitly taken into account through the review period used. Clark and Scarf use the notion of echelon stock policies instead of base-stock policies.

In order to describe a base-stock policy or, equivalently, a periodic review echelon order-up-to-policy, we define,

- **Echelon stock of item i**: physical stock of item i plus the physical stock of and the order in transit to all items in which item i is used, either directly or indirectly, minus the backlog of the end products in which item i is used.

- **Echelon inventory position of item i**: echelon stock of item i plus all scheduled receipts for item i.

Given these definitions we can define the following variables,
$S_j$  

echelon order-up-to-level for item j

$Y_j(t)$  

echelon inventory position of item j at the start of period $t$

Using the above definitions we can determine the quantity of item $i$ released at the start of period $t$ according to the echelon order-up-to-policy,

$$r_j(t) = S_j - Y_j(t), \forall j$$

Clearly, this policy is extremely simple, but for assembly networks there is a major drawback. It may be possible that the quantity released cannot be met due to lack of material. In that sense pure base-stock policies as proposed by Magee[1958] do not satisfy the feasibility property.

As shown by Rosling [1989] and Langenhof and Zijm [1990] the echelon-order-up-to-policies for pure assembly systems, i.e. each item has at most one parent, can easily be modified by taking into account the availability of the child items.

Let us derive this modified base stock policy for pure assembly systems. First of all note that for pure assembly systems we have exactly one end product and each item $i$ has exactly one successor $suc(i)$. Thus we can uniquely define the cumulative lead time $L_i^c$ of item $i$,

$$L_i^c = L_i, \ i \in E,$$
\[ L_i^e = L_i + L_{\text{cum}(t)}, \quad i \in I. \]

Given the definition of \( L_i^e \), we can state that \( Y_i(t) \) represents the coverage by item \( i \) of the end product demand from the start of period \( t \) until the start of period \( t + L_i^e \) just before releasing the item ordered at the start of period \( t \). Now notice that for all items with a longer cumulative lead time that at time \( t \) we know exactly their coverage of end product demand from the start of period \( t \) until the start of period \( t + L_i^e \). Define

\[ Z_{ij}(t) \text{ coverage of end product demand by item } j \text{ from the start of period } t \text{ until the start of period } t + L_i^e, \quad L_j^e > L_i^e. \]

Given the state information \( \{Y_i(t), \{Z_{ij}(t)\}\} \) we can define the modified base stock policy as follows,

\[ r_j(t) = \max\{0, \min\{S_j, \min\{Z_{ij}(t)\}\} - Y_j(t)\}. \]

It is shown in Rosling [1989] and Langenhof and Zijm [1990] that the policy described through the above equation is cost-optimal. In De Kok and Seidel [1990] and Van Houtum and Zijm [1991] simple computational schemes are given to determine the optimal echelon order-up-to-levels.

However, in general this policy cannot be used if items have multiple parents, i.e. when the product structure defined is divergent for at least one item. This is because of the fact that in case such an
item has not sufficient stock available, then it has to be decided how much stock to allocate to which parent item. In such a situation it is to date unknown what policy is cost-optimal.

In De Kok and Visschers [1999] a class of policies is proposed that introduces uniquely defined state variables similar to $Z_i(t)$ and allocation mechanisms derived from the analysis of divergent systems (cf. Van der Heijden et al. [1997]), so that it is possible to generate feasible item order releases in a straightforward way. Furthermore within this class of policies it is possible to characterize the optimal policy under i.i.d. exogenous demand and near-optimal policies can be found numerically. For pure assembly systems this class of policies coincides with the modified base-stock policies described above. The approach proposed in De Kok and Visschers [1999] is based on an artificial hierarchy derived from the structure of the general supply network. This structure is determined by the BOM and the planned lead times. Below we restrict ourselves to the main ideas behind this hierarchical planning concept. The artificial hierarchy enables to define state variables that unambiguously define the item order releases. Without loss of generality we assume that $E=P$.

For each item we can define its associated end products,

$$P(i) = \{i\}, \ i \in P$$

$$P(i) = \bigcup_{j \in V_i} P(j), \ i \in I$$

Furthermore we define
\[ L_i^c = L_i, \; i \in P, \]
\[ L_i^c = L_i + \max_{j \in V_i} L_j, \; i \in I. \]

Now we define the root node \( s \) as

\[ s = \arg \{ \max_i L_i^c \}, \]

i.e.

\[ L_i^c \geq L_i, \; i \in I \cup P. \]

Without loss of generality we assume that \( s \) is unique and that all cumulative lead times are different. Now we develop a hierarchical procedure that decides on all item order releases related to the end products in \( P(s) \). The hierarchy is derived from the cumulative lead times \( L_i^c \). Define the set of items \( C_i \) as follows,

\[ C_i = \{ j | L_j^c > L_i^c, P(j) \cap P(i) \neq \emptyset \}. \]

We assume without loss of generality that

\[ P(j) \cap P(i) = P(i), \forall j \in C_i, \]
i.e. all items that are used in the same end products as item \( i \), but are ordered earlier than item \( i \), are common to end products in \( P(i) \). In case this restriction does not hold, we can find a partition of \( P(i) \) and a one-to-one related partition of \( C_i \) for which the above holds for each related couple of subsets and apply the principles below to each of the subsets. Finally we define

\[
P(C_i) = \bigcap_{j \in C_i} P(j).
\]

The first decision in the hierarchy is to order item \( s \) at the start of period \( t \) according to a pure basic stock policy, i.e.

\[
r_s(t) = S_s - Y_s(t).
\]

Let us consider item \( i \) to be ordered at the start of period \( t \). In principle we would like to order according to a base stock policy, i.e. bring the echelon inventory position to \( S_i \), to cover future end product demand. Notice that decisions have already been taken in the past for items \( j \in C_i \) that affect this decision. In fact we assume that our decision hierarchy in the past determined

\[
Z_{C(i)}(t) \quad \text{coverage of future end product demand for all items in } P(C_i) \text{ at the start of period } t.
\]
Given our assumptions stated above we have that $P(i)$ is a subset of $P(C_i)$. Therefore we distinguish between two situations:

(i) \quad P(i) = P(C_i)

(ii) \quad P(i) \neq P(C_i).

In situation (i) we have that $Z_{C(i)}(t)$ is fully dedicated to future demand of end products in $P(i)$. Our target coverage equals $S_i$, but it does not make sense to increase the coverage above $Z_{C(i)}(t)$. Thus we release an order for item $i$ as follows,

$$r_i(t) = \max\left(0, \min\{S_i, Z_{C(i)}(t)\} - Y_i(t)\right).$$

In situation (ii) $Z_{C(i)}(t)$ is intended to cover future demand for other end products than those in $P(i)$. The problem is that we must decide how much to order for item $i$, yet it is well possible that we need not order yet any other items related to $P(C_i) \setminus P(i)$. In this case we maintain our hierarchy in decision making by introducing an artificial base stock level $S_{P(C_i) \setminus P(i)}$ that relates to end products in $P(C_i) \setminus P(i)$. This implies that the target coverage of future demand for all end products in $P(C_i)$ equals $S_i + S_{P(C_i) \setminus P(i)}$. In case $Z_{C(i)}(t)$ is below this target level, then we must decide about the rationing of the deficit. This yields the following order release policy for item $i$,

$$r_i(t) = \max\left(0, S_i - q_i \left(S_i + S_{P(C_i) \setminus P(i)} - Z_{C(i)}(t)\right) - Y_i(t)\right).$$
Here we use a linear rationing policy, where \( q_i \) is the fraction of the deficit allocated to end products in \( P(i) \). Notice that situation (i) is a special case of situation (ii) with \( q_i = 0 \) and \( S_{P(C),P(i)} = 0 \).

We have stated above that the procedure above holds for any supply network. Informally speaking, the above approach creates a number of divergent systems of decision nodes. Each decision node relates to a unique combination of a set of (intermediate) items and a set of end products.

We note here that multiple divergent systems emerge when an order must be released for an item that is not contained in all the end products for which we know the limits on future coverage due to earlier decisions. In that case this item becomes the root node for another divergent tree of decision nodes and we can apply a pure base stock policy for this item in relation to the end products not considered before.

Due to the fact that the general assembly network is translated into a set of divergent systems we can apply the algorithms proposed by Diks and De Kok [1999] in order to determine order-up-to-levels and rationing fractions that satisfy the service level constraints. Diks and De Kok [1999] show that the policies derived are close to optimal, since these policies approximately solve the set of generalized Newsboy equations that are associated with the optimal order-up-to-policies. This implies that given the concept of modified base stock policies as defined above we can derive close-to-optimal policies for this concept even for large-scale systems. In De Kok [2001] extremely
efficient algorithms are proposed for determining these close-to-optimal policies based on the relationship between generalized Newsboy equations and generalized ruin probabilities.

5. Managerial insights from comparison of SCP concepts for general supply chains

In this Section we compare the LP(-based SCP) concept presented in Section 3 with the MBS concept presented in Section 4. The LP concept represents the class of deterministic optimization models in a rolling schedule setting, while the MBS concept represents the class of stochastic models for SCP. Given the structural generality of the supply networks under consideration we have chosen to develop a simplified case situation inspired by the High Volume Electronics supply chain that enables us to gain managerial insights from our comparative study. In addition we compare both concepts for a real life case study from High Volume Electronics to see if our generic insights obtained are confirmed.

A simplified case

In figure 5.1 we present our simplified case example consisting of 4 end products (items 1,2,3,4) that each are assembled from 3 purchased items: a specific item (items 5,6,7,8), a semi-common item (items 9,10) shared with one other item and an item common to all end product (item 11). The planned lead times of these purchased items are denoted by $L_s, L_{sc}$ and $L_c$, respectively. The planned lead time of final assembly is denoted by $L_f$. 

We have fixed $L_f$ equal to 1. In table 5.1 below we vary $(L_s, L_{sc}, L_c)$ in order to see the impact of different lead time structures. We note here that the planned lead time structure impacts the divergent structures that emerge when applying the MBS concept. In Appendix 1 we present the divergent structures associated with each of the three planned lead time structures.

The demand for the end products in consecutive periods is i.i.d. with mean 100. We also assume that the demand processes for different end products are uncorrelated. We define
To get insight into the impact of demand variability on the choice of an SCP concept we vary $c_i^2$ as 0.25, 0.5, 1 and 2. Unless stated otherwise, we assume identical demand parameters for all end products.

Based on the cost structure in high volume electronics supply chains we developed a base case cost structure as follows. Analogously to the definition of planned lead times we define the added value of final assembly by $h_f$, the purchase value of specific items by $h_s$, the purchase value of semi-common items by $h_{sc}$ and the purchase value of the common item by $h_c$. In the base case we assume that

$$(h_f, h_s, h_{sc}, h_c) = ($10, $10, $30, $50).$$

Hence the common component is expensive, while the added value of assembly is only 10% of the total value of the end product. An example of such a situation is the manufacturing of TV’s. Typically the Cathode Ray Tube is 50% of the total cost, a Printed Circuit Board may account for 30% of the cost, while additional materials such as a housing account for another 10% of total cost. In our base case comparisons the customer service objective is to achieve a non-stockout probability $P_i$ of 95%.
The above case description has been the basis for a numerical study where discrete event simulation was used to compute the performance of the two SCP concepts after applying the computational procedures described in Section 3 and 4. The run-length of all simulation runs was chosen to ensure accurate point estimates of supply chain capital investments and customer service levels.

For our base case we generated twelve different SCP scenarios by combining three planned lead time structures and four squared coefficients of variation. The results are given in Table 5.1.

<table>
<thead>
<tr>
<th>$c_2^2$</th>
<th>$(L_L, L_S, L_{sc}, L_{ct})$</th>
<th>Supply Chain Inventory capital</th>
<th>Customer service</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$MBS_{ana}$</td>
<td>$MBS_{sim}$</td>
</tr>
<tr>
<td>0.25</td>
<td>(1,1,2,4)</td>
<td>72188</td>
<td>71682</td>
</tr>
<tr>
<td>0.25</td>
<td>(1,4,2,1)</td>
<td>76154</td>
<td>76476</td>
</tr>
<tr>
<td>0.25</td>
<td>(1,1,4,2)</td>
<td>74162</td>
<td>73550</td>
</tr>
<tr>
<td>0.5</td>
<td>(1,1,2,4)</td>
<td>105114</td>
<td>104448</td>
</tr>
<tr>
<td>0.5</td>
<td>(1,4,2,1)</td>
<td>112226</td>
<td>112316</td>
</tr>
<tr>
<td>0.5</td>
<td>(1,1,4,2)</td>
<td>108079</td>
<td>107616</td>
</tr>
<tr>
<td>1</td>
<td>(1,1,2,4)</td>
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</tr>
<tr>
<td>1</td>
<td>(1,4,2,1)</td>
<td>165264</td>
<td>165328</td>
</tr>
<tr>
<td>1</td>
<td>(1,1,4,2)</td>
<td>157294</td>
<td>157034</td>
</tr>
<tr>
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<td>218551</td>
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<td>246637</td>
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</tr>
<tr>
<td>2</td>
<td>(1,1,4,2)</td>
<td>228967</td>
<td>228789</td>
</tr>
</tbody>
</table>

*Table 5.1 Supply chain stock capital comparison, identical end product demand*

From Table 5.1 we conclude that the analytical results for the supply chain inventory capital obtained with the MBS concept, found in the column with heading $MBS_{ana}$, coincide with the results obtained with discrete event simulation, found in the column with heading $MBS_{sim}$. As expected the
procedure described above for the LP concept yields the target $P_I$-levels ($P_{I,LP}$). Likewise the MBS policies computed yield the required customer service levels ($P_{I,MBS}$).

Furthermore we find that the MBS concept considerably outperforms the LP-based concept. This in itself is an important and striking result. We should realize ourselves that most commercial software for Supply Chain Planning currently available is at best employing the LP-based concept of Section 3 and most likely LP is embedded in a set of heuristics to generate a feasible solution to the deterministic SCP model described in Section 3. This statement is in line with the findings of Stadtler et al [2001].

To gain intuition for the surprising observation from table 5.1 we present for both concepts the allocation of supply chain inventory capital among stocks of common component, semi-common components, specific component and end product.
The results in table 5.2 indicate that the LP-based concept tends to withhold too much stock in (long-lead time) components. A possible explanation for this is that the LP-based concept aims at satisfying the end product demand forecast. After satisfying this forecast remaining stocks of components are not used for assembly of end products, because stocks of components are cheaper than stocks of assembled end products. Especially if exogenous demand is low in a number of consecutive periods, then the LP concept will tend to build up stock upstream. The base stock levels computed under the MBS concept are typically such that even during low demand periods inventory capital is pushed towards the locations where customer demand must be met. This follows from the fact that the sums of base stock levels at each echelon in the divergent structures underlying the MBS concept tends to increase only slightly upstream. If these sums had been equal at all echelons then no stocks would be held at all at upstream stages. The results in 5.2 seem to indicate that we are
close to that situation. Informally speaking, the base-stock policies have a *just-in-case* character, whereas the LP-based concept has a *just-too-late* character.

The rational behind the extremely low component stock levels with both concepts, and in particular for the base-stock policies, is that withholding stock at component level does not contribute to immediate customer service. Apparently this outweighs the so-called portfolio effect for common component stocks, i.e. demand for the common component is relatively more stable than demand for individual products.

Another interesting observation from our computational study is that under the LP concept the safety stocks for the identically distributed end products strongly differ. Our explanation is that for our base case with identical added values for all end products the LP-problem solved each period is strongly degenerate. Thereby it depends on the particular implementation of the algorithm (e.g. choice of tie-breaking rules), which end product is favored over the other with respect to the allocation of items. Apparently the CPLEX-solver used is not allocating these mismatches evenly over time among the end products. Of course, the base-stock policies are identical for all end products. Given this observation with respect to the allocation of stocks it is interesting to compare the average stock levels in case the values of end products are different. In table 5.3 we present some results that support our conclusion that the LP concept does not handle component shortages properly. In table 5.3 we define $\Delta_1$ as the standard deviation of the three different safety stocks for end products 1, 2 and 3 normalized by their mean, while $\Delta_2$ is defined as the relative difference
between the average safety stock for products 1, 2, and 3 and the safety stock of the lower cost end product 4.

<table>
<thead>
<tr>
<th>((c_1^2, c_2^2, c_3^2, c_4^2))</th>
<th>((h_1, h_2, h_3, h_4))</th>
<th>((L_1, L_2, L_3, L_4))</th>
<th>LP</th>
<th>MBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0.25, 0.25, 0.25, 0.25))</td>
<td>((20, 20, 20, 10))</td>
<td>((1, 1, 4, 2))</td>
<td>23%</td>
<td>48%</td>
</tr>
<tr>
<td>((0.25, 0.25, 0.25, 0.25))</td>
<td>((20, 20, 20, 10))</td>
<td>((1, 1, 2, 4))</td>
<td>11%</td>
<td>73%</td>
</tr>
<tr>
<td>((0.50, 0.50, 0.50, 0.50))</td>
<td>((20, 20, 20, 10))</td>
<td>((1, 1, 2, 4))</td>
<td>9%</td>
<td>58%</td>
</tr>
<tr>
<td>((0.50, 0.50, 0.50, 0.50))</td>
<td>((20, 20, 20, 10))</td>
<td>((1, 1, 4, 2))</td>
<td>17%</td>
<td>39%</td>
</tr>
<tr>
<td>((1, 1, 1, 1))</td>
<td>((20, 20, 20, 10))</td>
<td>((1, 1, 2, 4))</td>
<td>11%</td>
<td>39%</td>
</tr>
<tr>
<td>((1, 1, 1, 1))</td>
<td>((20, 20, 20, 10))</td>
<td>((1, 1, 4, 2))</td>
<td>16%</td>
<td>29%</td>
</tr>
</tbody>
</table>

Table 5.3. Safety stock differences for non-identical end products

It follows from Table 5.3 that the modified base-stock concept yields identical safety stocks for identical products \((\Delta_1=0)\), while the LP-concept yields considerably different safety stocks \((\Delta_1>0)\).

For different end products we find that the \(\Delta_2\), the difference between the low-cost end product safety stock and the high-cost end products safety stock, is much bigger for the LP concept than for the MBS concept.

The results in Tables 5.1-5.3 show that the LP-based concept rations shortages among items inappropriately. Identical products are not rationed similarly due to tie-breaking rules needed to deal with the degeneracy of the associated SCP problem. In case of different products LP rations shortages according to a priority list based on holding and penalty costs. The priority list implies that the item first on the list is satisfied first (if possible), after that the item second on the list is satisfied, etc. until no inventory is left. Lagodimos [1992] has shown that such a priority rationing mechanism
is suboptimal. The linear rationing rules used in the MBS concept ensure that shortages are shared among all products. Apparently this is superior. Informally speaking, LP is a greedy approach that is inferior to the balanced MBS approach.

A real-life case

Let us see whether our insights hold true for a real-life case study. Apart from the fact that this case illustrates the importance of the Supply Chain Planning concepts discussed in this paper, such a case study provides some idea of possible benefits of one Supply Chain Planning concept as compared with another one. We must keep in mind that the structural complexity of the problem discussed in this paper in terms of number of items, demand characteristics, lead times and costs is so huge, that simple cases like the ones presented above at best provide qualitative insights. Due to contingency insight into potential quantitative cost benefits can only be derived from real-life cases.

We consider the high volume electronics supply chain consisting of a consumer electronics products manufacturer, a semiconductor manufacturer, a major component manufacturer and the manufacturer of the housing. The end products are produced to stock. The customers of the supply chain are retailers and whole-salers. The supply chain consists of the following five distinct organizations.

- consumer electronics product manufacturer
- major component manufacturer
- PCB manufacturer
- housing manufacturer
We consider a product family consisting of 22 end products (items 1-22), 22 specific housings (items 23-44), 6 major components (items 45-50), 1 PCB (item 51) that contains 6 IC's (items 52-57). In figure 5.2 the supply chain is presented. The (disguised) detailed data that define the supply chain models discussed in this paper, are presented in Appendix 2. In this appendix we also show that the customer service targets are met for both policies.

As before we compared the LP concept with the MBS concept given a target service level of 95%. The results of the comparison are presented in Table 5.4, where Δ equals the relative increase in
supply chain capital investments when applying the LP concept instead of the MBS concept. It follows from Table 5.4 that the MBS concept yields a substantial decrease of 20% in supply chain capital investment, while generating equal stockout probabilities. Major savings are achieved in capital investments in major components and PCB’s.

<table>
<thead>
<tr>
<th>MBS</th>
<th>LP</th>
<th>( \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3438721</td>
<td>3759679</td>
<td>9%</td>
</tr>
<tr>
<td>7513</td>
<td>17369</td>
<td>131%</td>
</tr>
<tr>
<td>62385</td>
<td>410974</td>
<td>559%</td>
</tr>
<tr>
<td>15391</td>
<td>50478</td>
<td>228%</td>
</tr>
<tr>
<td>44366</td>
<td>59212</td>
<td>33%</td>
</tr>
<tr>
<td>3568376</td>
<td>4297712</td>
<td>20%</td>
</tr>
</tbody>
</table>

Table 5.4. Comparing supply chain capital investments

Again we find that the LP-based concept tends to overstock upstream stockpoints. This is further illustrated in Table 5.5, where for each supply chain planning concept the percentage of supply chain capital investment for each item is given.

<table>
<thead>
<tr>
<th>MBS</th>
<th>LP</th>
<th>( \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>96%</td>
<td>87%</td>
<td>11%</td>
</tr>
<tr>
<td>0%</td>
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<tr>
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<td>10%</td>
</tr>
<tr>
<td>0%</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>1%</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 5.5. Distribution of supply chain capital investments

Some additional computations show that with the capital investment required by the LP-based concept, i.e. 4297712, the MBS concept would imply a non-stockout probability of 97%. In other words, in that case the same capital investment would reduce the stockout probability 5% to 3%, a decrease of 40%! 
This shows the potential of applying the MBS concept instead of the LP concept, while the latter concept is commonly applied in commercially available standard supply chain planning software. On top of that the logic enables to analytically obtain a near-optimal policy within this class of MBS policies, which is impossible for the LP concept. Finally the supply chain logic induced by the MBS concept is computationally extremely simple.

6. Conclusions and further research

In this paper we compared two concepts for supply chain planning: an LP-based rolling schedule concept and base-stock control for uncapacitated multi-item supply chains. We first derived necessary constraints that any SCP concept should satisfy in order to generate feasible material order releases. We derived an LP model that can be used for SCP in a rolling schedule context, similar to the models applied in currently available commercial SCP software. We derived a procedure based on simulation that can be applied to generate a set of safety stocks for final products that together with the LP-based concept ensure that customer service targets are met. We introduced a modified base-stock concept for general uncapacitated multi-item supply chains, that yields feasible plans and for which the optimal control parameters and the associated costs can be approximately determined analytically. We checked the concept with simulation showing the validity of the approximations.

A comparison between the two concepts surprisingly revealed that the MBS concept outperforms the LP concept considerably. A real-life case shows a decrease in capital investment of 20% with the
same customer service level. Also we identified some properties of the LP-based concept that may cause problems in practice.

Summarizing, we identify the modified base-stock concept as a promising alternative to current LP-based concepts used in practice. Further research includes the empirical validation of the MBS concept as well as extension of the concept to situations with lot sizing. Another important research challenge is the extension of the MBS concept to capacitated supply networks. This is one of the advantages of an LP-(MP-) based rolling schedule concept: such a concept can easily incorporate finite capacity constraints as is shown in Belvaux and Wolsey[2001].

7. References


Appendix 1. MBS decision node networks

(i) Decision node network for \((L_f, L_s, L_{sc}, L_d) = (1,1,2,4)\)

(ii) Decision node network for \((L_f, L_s, L_{sc}, L_d) = (1,1,4,2)\)
(iii) Decision node network for \((L_f, L_b, L_{sb}, L_d) = (1, 4, 2, 1)\)
Appendix 2. Detailed real-life case data

<table>
<thead>
<tr>
<th>end item $i$</th>
<th>$E[D_i]$</th>
<th>$\sigma(D_i)$</th>
<th>$L_i$</th>
<th>$\Delta h_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>121</td>
<td>121</td>
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<td>16.5</td>
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<td>84</td>
<td>1</td>
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</tr>
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</table>

Table A.1. Detailed data end products

- $E[D_i]$: expected demand per period for end item $i$
- $\sigma(D_i)$: standard deviation demand per period for end item $i$
- $P_{i,i}$: non-stockout probability target for end item $i$
- $L_i$: assembly lead time of an order for end item $i$
- $\Delta h_i$: added value of assembly of end item $i$
<table>
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<tr>
<th>component/subassy</th>
<th>L</th>
<th>Δh</th>
</tr>
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<td>housing 3</td>
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<td>9,3</td>
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*Table A.2. Detailed data components and subassemblies*

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Table A.3. End product service levels

$P_1^*$ target non-stockout probability

$P_{1,\text{MBS}}$ non-stockout probability under MBS concept

$P_{1,\text{LP}}$ non-stockout probability under LP concept

$P_{2,\text{MBS}}$ fraction of demand met directly from stock on hand under MBS concept

$P_{2,\text{LP}}$ fraction of demand met directly from stock on hand under LP concept


