The science of making more torque from wind: diffuser experiments and theory revisited
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The science of making more torque from wind: Diffuser experiments and theory revisited.

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Abstract. History of the development of DAWT’s stretches a period of more than 50 years. So far without any commercial success. In the initial years of development the conversion process was not understood very well. Experimentalists strived at maximising the pressure drop over the rotor disk, but lacked theoretical insight into optimising the performance. Increasing the diffuser area as well as the negative back pressure at the diffuser exit was found profitable in the experiments. Claims were made that performance augmentations with a factor of 4 or more were feasible, but these claims were not confirmed experimentally. With a simple momentum theory, developed along the lines of momentum theory for bare windturbines, it was shown that power augmentation is proportional to the mass flow increase generated at the nozzle of the DAWT. Such mass flow augmentation can be achieved through two basic principles: increase in the diffuser exit ratio and/or by decreasing the negative back pressure at the exit. The theory predicts an optimal pressure drop of 8/9 equal to the pressure drop for bare windturbines independent from the mass flow augmentation obtained. The maximum amount of energy that can be extracted per unit of volume with a DAWT is also the same as for a bare wind turbine. Performance predictions with this theory show good agreement with a CFD calculation. Comparison with a large amount of experimental data found in literature shows that in practice power augmentation factors above 3 have never been achieved. Referred to rotor power coefficients values of \( C_{P,\text{rotor}} = 2.5 \) might be achievable according to theory, but to the cost of fairly large diffuser area ratio’s, typically values of \( \beta > 4.5 \).

1. Nomenclature

- \( V_0 \): velocity far in front of diffuser
- \( V_1 \): velocity at wind turbine inside diffuser
- \( V_3 \): velocity at diffuser outlet
- \( V_e \): velocity in the wake of DAWT
- \( p_0 \): pressure in front of diffuser
- \( p_1 \): pressure at nozzle before wind turbine
- \( p_2 \): pressure at nozzle after wind turbine
- \( p_3 \): pressure at diffuser exit
2. DAWT history

The first assessments on “ducted windmills” were performed by Lilley an Rainbird in 1956 [1]. In the seventies a significant amount of wind tunnel experiments were carried out by Foreman et al [3,5] from Grumman Aerospace Dept. USA, and by Kogan, Seginer and Igra [2,4] from the Ben-Gurion University in Israel. Conclusions from these early studies varied significantly. The Lilley and Rainbird study [1] concluded that no performance improvements were possible, when performance is referred to the exit area, where Foreman et al [3] conclude that “…the DAWT produced 4.25 times the power of the same turbine operating as a conventional turbine” and Igra [4] even concluded that “…the shrouded wind turbine is superior to any similar horizontal axis wind turbine of the same diameter”. Apart from experiments these researchers also developed basic theoretical models to analyse their experiments. These models however lacked a coherent and complete description of the major phenomena in the flow field of a DAWT.

Diffuser augmented wind turbine (DAWT) geometries were a hot topic at the Wind Energy Innovative Systems Conference 1979 in the USA. One of the conclusions from this conference in general was that interesting power augmentations are possible, but economic application of DAWT’s seemed not feasible because of the high costs of the configuration. Many research groups shared this opinion and with the rapid development of horizontal axis wind turbine technology in the second part of the eighties and the nineties the R&D of DAWT’s disappeared from the research agenda’s.

About 20 years after the initial activities of Forman et al [3] and Igra [4] the DAWT concept revived, through an intensive development, promotion and public relation campaign by a company called Vortec Energy from New Zealand. This company claimed power coefficients of above 2 (referred to the rotor swept area) which claim was based on former wind tunnel experiments of Grumman Aerospace (Forman et al [3]) backed by wind tunnel experiments performed by Flay and Nash from the University of Auckland NZ. Vortec Energy realized a 7.3 m diameter DAWT in 1997 and started experimenting with this full scale configuration. In an article of Flay et al published on the former website of Vortec Energy one of the conclusions is “…field measurements on the full scale DAWT gave performance which was lower than predicted in the Grumman publications”. From CFD calculations performed by Flay et al it was found that the maximum power coefficient was found of around 1 referred to the rotor swept area. Compared to the diffuser exit area the power coefficient was significantly below the Betz value.

Recently DAWT windturbines have regained new attention: Several papers are published by Abe et al on a flanged diffuser geometry; Mertens [15] published a PhD thesis in 2006 where he addresses diffuser like concentrator effects that building may have on urban wind turbines and several new small DAWT wind turbines have entered the market. Time to revisit the experiments as well as the theories for DAWT in more detail.

3. DAWT theoretical models

De Vries [6] was one of the first to develop a consistent theory for DAWT’s. He distinguishes simple diffuser theory from shrouded turbine theory. In the simple diffuser theory he models the 1

\[ a \] axial induction factor
\[ \beta \] diffuser area ratio (ratio of exit area to nozzle area)
\[ \gamma \] back pressure velocity ratio
\[ c_{pi} \] pressure coefficient at location \( i \)
\[ C_{T,\text{diffuser}} \] thrust coefficient of diffuser (non dimensionalised with swept rotor area)
\[ C_{T,\text{total}} \] total thrust coefficient of diffuser plus rotor
\[ C_{T,\text{rotor}} \] thrust coefficient of rotor
\[ C_{P,\text{rotor}} \] performance coefficient of rotor (non dimensionalised with swept rotor area)
\[ C_{P,\text{exit}} \] performance coefficient of diffuser (non dimensionalised with DAWT exit area)
dimensional flow through a diffuser, but anticipated on the fact that the exit pressure should be equal to ambient. He incorporated negative backpressure values as found in the earlier experiments by introducing an empirical exit pressure coefficient. In his shrouded turbine approach he makes an attempt to model the radial forces on the shroud in a momentum approach. That results into the conclusion that “…optimum values of about $C_P = 2$ can be obtained”. What was not found by De Vries was the fact that the optimal pressure drop over the rotor equals the optimal pressure drop over a bare windturbine.

In 1999 Hansen et al [9,10] showed by means of CFD computations “…that the Betz limit can be exceeded with the ratio corresponding to the relative increase in mass flow through the rotor”.

A simple momentum theory for DAWT’s was also derived, at that time by the author [12] using a number of straightforward assumptions. It is assumed that there is no viscous wake mixing process behind the diffuser but the effect of negative backpressures are taken into account in the performance prediction. From this DAWT momentum theory it can be seen that the achievable power is comparable with the power of a normal HAWT (horizontal axis wind turbine) having a diameter equal to the exit diameter of the diffuser. But from this momentum model it can also be seen that larger performances are possible when a substantial low “back pressure level” can be achieved at the diffuser exit.

4. Pressures and velocities inside an empty diffuser

In one dimensional momentum theory the velocity and pressure relations in an empty diffuser are directly related to its geometry, i.e. to the variation of cross sectional area. Figure 1 shows the indexes used at the various locations. The cross sectional area of the exit of the diffuser is used as reference. Ambient pressure $p_0$ is found far in front of the diffuser and in the far wake behind the diffuser. Throughout the diffuser the velocity and pressure relations can found by application of the continuity equation and by Bernoulli’s law for the total pressure:

$$p_{tot} = p_0 + \frac{1}{2} \rho V_0^2 = p_1 + \frac{1}{2} \rho V_1^2 = p_3 + \frac{1}{2} \rho V_3^2 = p_a + \frac{1}{2} \rho V_e^2$$

(1)

![Figure 1: Pressure and velocity relations in an empty diffuser](image)

Using the continuity equation it can be seen easily that the relation between the velocity at the nozzle and the velocity at the diffuser exit is proportional to the diffuser area ratio $\beta$:

$$V_1 = \beta V_3$$

(2)
The total pressure at the nozzle, using the above relation can thus be written as:

\[ p_{\text{tot}} = p_1 + \frac{1}{2} \rho (\beta V_0)^2 \]  

(3)

At first it is assumed that there is no back pressure at the diffuser exit, thus \( p_3 = p_0 \) and \( V_0 = V_3 \). Then the pressure at the nozzle equals:

\[ p_1 = p_0 + (1 - \beta^2) \frac{1}{2} \rho V_0^2 \]  

(4)

This shows that an under pressure will exist at the nozzle, whenever the diffuser area ratio \( \beta \) is larger than 1. This is evidently the case when the exit area is larger than the nozzle area, and no flow separation is present.

\[ V_3 = \gamma V_0 \]  

(5)

At the diffuser exit however a back pressure may exist, e.g. when, through the Kutta condition, the flow is forced to deflect in radial direction. Then the velocity at the exit will differ from the undisturbed velocity \( V_0 \) in front of the diffuser. Writing the back pressure relation as a relation between \( V_0 \) and the velocity \( V_3 \) at the diffuser exit yields the following expression for the pressure at the nozzle:

\[ p_1 = p_0 + (1 - \beta^2 \gamma^2) \frac{1}{2} \rho V_0^2 \]  

(6)

Thus for a diffuser area ratio \( \beta \) larger than 1 and for zero or negative back pressure (\( \gamma \geq 1 \)), there will be an under pressure at the nozzle.

The velocity relations inside the diffuser are rather simple once its geometry is known. Starting from the velocity \( V_3 \) at the exit of the diffuser the velocities at other locations can be directly calculated from the local area to exit area ratio, with application of the continuity equation, and under the assumption of a uniform velocity distribution at each diffuser section.

With respect to the axial forces on the empty diffuser, the situation is in fact quite straightforward. Applying the axial momentum conservation law upon the volume of air in a surrounding stream tube at large distance from the diffuser will show that there is no net force in axial direction on the diffuser. An equivalent way of stating this is that Bernoulli’s law can be applied along all streamlines and that this shows that far in front of the diffuser as well as far behind the diffuser, both undisturbed pressure \( p_0 \) as well as undisturbed velocity \( V_0 \) are established again. Of course this is only valid for an empty diffuser, where no energy extracting device is present. In several references dealing with diffusers for wind turbines, e.g. De Vries [6] and Dick [8] this result is also presented.

The ratio of the velocity found in the nozzle of the diffuser (without wind turbine) and the inflow velocity \( V_0 \) far in front of the diffuser is very often the cause for erroneous predictions of the achievable power from a DAWT. Combining the equations (2) and (5) does show that the nozzle velocity is equal to \( \beta \gamma V_0 \). The maximum achievable power however is not equal to \( \beta \gamma^3 C_{p_{\text{max}}} \), where \( C_{p_{\text{max}}} \) is the maximum achievable power of the wind turbine without diffuser. The actual maximum will be significantly lower, as will be derived below.

5. Velocity and pressure relations within DAWT’s

When a windturbine is located in the diffuser, things start to become more complex. The velocity relations within a DAWT are still rather straightforward since they are, in this one dimensional theory, determined by the shape of the diffuser. Although the shape of the inlet should be adequate for achieving a smooth inflow and for prevention of flow separation, the precise shape is not
very important for the understanding of the DAWT energy conversion process. The velocity at the diffuser exit $V_3$ will, in general, be lower than $V_0$. Behind the diffuser exit the velocity might reduce further until the final velocity in the wake $V_e$ is established at ambient pressure $p_0$. In the present momentum theory approach the exact location of the wind turbine inside the diffuser is not relevant for understanding the energy extraction process. The effect of the wind turbine will be a drop in total pressure, somewhere inside the diffuser, which can be represented by a reduced total pressure level at the exit of the diffuser. The most suitable location for the wind turbine from a constructional point of view is inevitable the nozzle of the diffuser. Here the smallest cross-sectional area is found and thus use can be made of the smallest rotor diameter.

In the one-dimensional momentum theory developed by Van Bussel [12] it is strived to develop a theory having the closest equivalency with momentum relations for ordinary wind turbines. Hence it is assumed that at the exit of the diffuser the same conditions apply as just after an ordinary wind turbine (assuming no extra back pressure). Thus the following expression is introduced:

$$V_3 = (1-a)V_0$$ (7)

So the axial induction factor $a$ is defined at the exit of the diffuser. Just as in ordinary wind turbine momentum theory this induction is half the induction factor found in the far wake behind the DAWT (thus $V_e = (1 - 2a)V_0$). From the continuity equation in then follows, using the diffuser area ratio $\beta$, that the velocity at the wind turbine in the nozzle of the DAWT equals $V_1 = \beta V_3$, and thus:

$$V_1 = \beta (1-a)V_0$$ (8)

The velocities at other locations inside the diffuser can, as in the situation without wind turbine, be determined by application of the continuity equation.

When an extra back pressure at the exit of the diffuser is present, the expression for the velocity at the exit yields:

$$V_3 = \gamma (1-a)V_0$$ (9)

analogous to equation (5) for the empty diffuser. The velocity at the nozzle then equals

$$V_1 = \beta \gamma (1-a)V_0$$ (10)

The pressure relations can easily be obtained by application of Bernoulli’s law in the flow in front of the rotor and similarly in the flow behind the rotor. When it is assumed that the rotor is located in the nozzle this leads to:

$$p_1 = p_0 + [1 - \beta^2 \gamma^2 (1-a)^2] \frac{1}{2} \rho V_0^2$$ (11)

for the pressure in front of the rotor and to:

$$p_2 = p_0 + [(1 - 2a)^2 - \beta^2 \gamma^2 (1-a)^2] \frac{1}{2} \rho V_0^2$$ (12)

for the pressure behind the rotor.

Thus the following expression for the pressure jump over the rotor can be written down:

$$p_2 - p_1 = 4a(1-a) \frac{1}{2} \rho V_0^2$$ (13)
which is the classical relation for the pressure drop over a normal wind turbine. 
In other words: the pressure drop over the wind turbine is independent of the diffuser area ratio, the 
back pressure ratio and even the location inside the diffuser, as can be shown easily!
This means that the amount of energy extracted per unit of volume is identical to the amount of energy 
extracted for an unaugmented turbine. The amount of air passing the turbine in the diffuser has 
however been increased with a factor \( \beta \gamma \) compared to a normal wind turbine of the same size. And as 
the case with momentum theory for ordinary wind turbines optimal values are found for for \( a = 1/3 \).

![Figure 2: Optimal velocity and pressure relations in a DAWT obtained from momentum theory](image)

Left: diffuser without extra backpressure (\( \gamma =1 \))
Right: diffuser with extra backpressure (\( \gamma >1 \))

6. Power and thrust expressions for a DAWT.
With the expressions in the previous section it is easy to write down the equations for the power 
obtained from a DAWT. Based upon the area of the wind turbine the power coefficient of the DAWT 
follows from the equations (8) and (13):

\[
C_{P,\text{rotor}} = \beta \gamma 4a(1-a)^2
\]  

(14)

And consequently the power coefficient based upon the diffuser exit area:

\[
C_{P,\text{exit}} = \gamma 4a(1-a)^2
\]  

(15)

From the expressions above an interesting conclusion can be drawn with regard to the distribution 
of the total thrust acting upon the DAWT. The thrust on the rotor inside the diffuser is exactly the 
same as the thrust on a bare rotor, as can be seen from equation (13):

Application of the principle of conservation of momentum for the flow through the DAWT leads to 
the following expression:

\[
C_{T,\text{total}} = \beta \gamma 4a(1-a)
\]  

(16)

Where, for the present purpose, the rotor area is used for non-dimensionalising the total thrust.

This leads to the conclusion that the thrust on the diffuser is dependent upon the thrust of the rotor:

\[
C_{T,\text{diffuser}} = C_{T,\text{total}} - C_{T,\text{rotor}} = (\beta \gamma - 1)4a(1-a)
\]  

(17)
Note again that in (17) the diffuser thrust coefficient is non-dimensionalised with the rotor area!! Hence the thrust on the diffuser is proportional to the extra mass flow obtained in the DAWT, where the thrust on the rotor is not!!

Optimal power coefficients are obtained for $a = 1/3$, just as in the situation of ordinary windturbines. In his approach Van Bussel [12] stresses that the developed theory does not cover the determination of the maximum achievable back pressure ratio $\gamma$. Values might be obtained however by examining experiments.

7. Comparisons with Grumman Aerospace experiments
In the seventies researchers from Grumman Aerospace, performed an extensive series of model experiments in their free jet wind tunnel facility which can be used for comparison with the theoretical expressions presented above. The first experimental results were published for slotted diffusers with a half angle of $20^\circ$, [3]. In these experiments the rotor was replaced by a screen, see fig 3 left.

![Figure 3: Comparison of centre line pressure distributions](image)

Left: experimental results from Foreman et al slotted diffuser 20° half angle
Right: predicted distribution from DAWT momentum theory by Van Bussel

A comparison is shown in figure 3 between the experimental results obtained with one of the Foreman diffusers and the basic momentum theory model. The measured back pressure value $c_{p3} = -0.42$ gives, according to the above DAWT momentum theory, rise to an augmentation of 12%. Hence the theoretical (inviscid) upper value for this diffuser yields a $C_P$ value of 0.66 (referred to the diffuser exit area). Yet the experiments revealed an experimental value of only 0.35.

In a survey paper published in some year’s later [5] they present the results of more experiments in a fairly concise way.

<table>
<thead>
<tr>
<th>Diffuser Area Ratio: 2.78</th>
<th>30 degree diffuser</th>
<th>40 degree diffuser</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity at nozzle/screen</td>
<td>1.36 1.30 1.26 1.20 0.99</td>
<td>1.28 1.23 1.17 1.10 0.96</td>
</tr>
<tr>
<td>Pressure coefficient at exit</td>
<td>-0.575 -0.58 -0.57 -0.535 -0.51</td>
<td>-0.615 -0.63 -0.58 -0.545 -0.49</td>
</tr>
<tr>
<td>Disk loading Coefficient</td>
<td>0.68 0.80 0.88 0.92 0.91</td>
<td>0.61 0.71 0.76 0.77 0.86</td>
</tr>
<tr>
<td>Rotor Power Coefficient</td>
<td>0.90 1.04 1.12 1.10 0.92</td>
<td>0.78 0.88 0.88 0.85 0.84</td>
</tr>
<tr>
<td>Diffuser Power Coefficient</td>
<td>0.32 0.38 0.40 0.40 0.33</td>
<td>0.28 0.32 0.32 0.31 0.30</td>
</tr>
</tbody>
</table>

Table 1: overview of experimental results obtained by Oman et al [3].
In table 1 the results from the measurements using slotted diffusers with half angles of 30 and 40 degrees and an area ratio of 2.78 are summarised.

It must be noted that the quoted values for the disk loading in table 1 are different from the values given in Gilbert et al [5]. Like most of the experimentalists they defined the non dimensional disk loading with respect to the local velocity, where the present theory always refers to the undisturbed wind velocity $V_0$. From table 1 it can be seen that the 30° half angle diffuser was the most successful in achieving a high power coefficient. The maximum performance can be found at a disk loading of 0.88, as is predicted by the present theory.

With a back pressure coefficient of $c_{p3} = -0.57$ that leads to a back pressure velocity ratio of 1.24, and hence, according to the theory, a maximum rotor performance coefficient $C_{P_{MAX, Rotor}} = 2.04$. This is again substantially more than the 1.12 that was found in the experiment.

In figure 4 the experimental results for the 30° diffuser are compared to the values from the theoretical model.

![Figure 4: Comparison of experimental results from Gilbert et al [5] with results from momentum theory of Van Bussel [12].](image)

Left: using the geometrical diffuser area ratio
Right: using the reduced mass flow rate found in the experiment

When looking to the graph in the left hand side of figure 4 there is a fairly good similarity in shape. The most striking difference between experiments and theory is seen in the values for the velocity ratio at the location of the rotor (screen). With a geometrical diffuser area ratio of 2.78 the optimal velocity ratio at the rotor disk should be around 1.85 for a diffuser without extra back pressure, and even to 2.3 with a backpressure velocity ratio of 1.24, where actual (rotor disk averaged) values of only 1.26 were experienced.

The reason might be found by considering the mass flow through the slotted diffuser. Unfortunately the survey paper does not quote values for the added mass which is injected in the diffuser in order to prevent boundary layer separation in the diffuse, but in the earlier publication of Oman [3] it was stated that, for the 20° diffuser, the exit mass flow was 1.86 times the mass flow through the screen. And evidently the mass flow injected in the diffuser behind the screen does not directly contribute to the power (it does in an indirect way through establishing a lower back pressure at the diffuser exit).

When the comparison of the experimental results is recalculated with an effective reduced mass flow rate, fig 4 right, it can be seen that momentum theory does describe DAWT flow conditions quite well. As can be seen the experimental diffuser shows a performance break down for disk loadings above $C_{T, rotor} = 0.92$, where theory extends up to the theoretical maximum $C_{T, rotor} = 1.0$. 


8. Comparisons with CFD calculations

In 1999 Hansen et al [10] showed by means of CFD computations “…that the Betz limit can be exceeded with the ratio corresponding to the relative increase in mass flow through the rotor”. This is in complete agreement with the momentum theory as can be seen with the equations (10) and (14). Moreover the comparison with CFD calculation of Hansen [10] shows a good agreement of the calculated rotor power coefficients over the whole range of disk loadings. Only at the higher loads there are discrepancies, most probably caused by the fact that viscous effects, which are evidently not taken into account in the momentum theory calculations.

![Figure 4: Comparison of momentum theory results with CFD calculations by Hansen et al.](image)

Unfortunately no other basic parameters were presented in the Hansen’s, such as the diffuser exit pressure and velocities at the rotor disk, which could have been used for further validation of the momentum theory.

9. Further comparisons with experiments

Apart form the Grumman aerospace experiments [3,5] a number of other researchers performed model experiments. Most of them are however only partially documented in freely accessible literature. In 1997 the company Vortec Energy realized a 7.3 m diameter DAWT based upon the experiments performed by the Grumman Aerospace diffuser team. Unfortunately the experienced performance was disappointing. For further analysis of this diffuser Flay et al performed a number of wind tunnel tests. These seemed at first instance suitable for comparison with the momentum theory predictions. In comparison however it was found, by determination of the non dimensional disk loading with the undisturbed wind velocity \( V_0 \) in stead of with the local velocity \( V_1 \) that values were measured up to \( C_T = 1.67 \). Since this is highly beyond what can be expected the experimental conditions were re-examined. This led to the conclusion that the actual performance of the model diffuser tested is highly affected by tunnel blockage. The experimental set up used a 500 mm diameter screen in a diffuser with an area ratio \( \beta = 2.6 \). This means an exit area of slightly more than 0.5 m\(^2\). With a 2 x 3 m closed wind tunnel section this implies a blockage factor of 0.085. But under optimal operating conditions the area of the wake of a DAWT can become more than twice the DAWT exit area. In such cases the dynamic pressure in the external flow along the test section may increase with some 40%. And that evidently shows that blockage effects have strongly influenced the Flay
measurements, as was already seen from the extreme $C_T$ values. Thus these measurements could not be used for detailed comparison with the momentum theory.

Recently DAWT wind turbines have regained new attention: Several papers are published by Abe et al [14] on a flanged diffuser geometry. The large flange is positioned at the exit of the diffuser perpendicular to the undisturbed flow. The idea of applying this exit flange is to enhance the backpressure at the diffuser exit. Figure 5 shows the comparison of the different experiments reported with the maximum achievable performance based upon the present momentum theory. Apart from the maximum CP values based upon the DAWT exit area also the line including 25% rotor losses is shown.

![Figure 5](image)

**Figure 5** Comparison of theoretical achievable $C_p$-diffuser values as a function of the DAWT exit velocity with various experiments.

It can be seen from figure 5 that no experiments have led to power coefficients larger than 0.47 when referred to the exit area of the diffuser. It is also clear that large backpressure values can be realized, but so far not together with reasonable performances. With regard to the quoted experiments by Flay it must be noticed again that they are obtained in a wind tunnel where blockage effects most probably are too large, hence the large backpressure values.

Apart from the theoretical expression (15) depicted in the figure also a line is shown for the situation that rotor losses are as large as 25% hence leading to a value $C_p = 0.75 \times 0.593 = 0.45$ for a diffuser with a backpressure coefficient $c_{p3} = -0.333$. It can also be seen that the flanged diffuser as proposed by Abe et al did not really yield large (negative) values for the backpressure at the diffuser exit.

Figure 6 shows the relation between diffuser area ratio and the achievable rotor power coefficient $C_{P,\text{rotor}}$. 

Here it can be seen that the larger diffuser area’s indeed can lead to higher wind turbine performances. The diffuser area ratio used by Abe et al [13]: $\beta = 6.4$ is beyond the range of the other experiments. Despite this high area ratio, the resulting performance is poor as can be seen.

10. Conclusions
The focus in the earlier DAWT experiments was to try to achieve large values for the pressure drop over the rotor disc (most times simulated by screens). The intuitive target was apparently an optimal pressure drop equal to $8/9$ times the local dynamic pressure. Simple momentum theory has showed however that the optimal pressure drop equals $8/9$ times the undisturbed dynamic pressure.

With application of simple momentum theory it is shown that the amount of energy extracted per unit of volume with a DAWT is the same as for an ordinary bare wind turbine.

The increase in maximum performance of a DAWT is proportional to the mass flow. This proportionality goes with a geometric area ratio $\beta$ between nozzle and exit.

Without extra back pressure at the diffuser exit this means that the maximum power coefficient related to the exit area is equal to the Betz maximum 16/27.

In practice the maximum performance will be significantly lower, caused by viscous effects and possible early flow separation at the diffuser surface.

Reducing back pressure behind the exit of a DAWT can have a profitable effect on the performance. With respect to the maximum performance this enhancement factor is equal to the ratio between the diffuser exit velocity $V_3$ and $2/3V_0$.

The relation between disk loading and power coefficient, as found from simple momentum theory shows a good agreement with CFD calculations performed by Hansen et al.

Comparison with a large amount of experimental data found in literature shows that in practice power coefficients above $C_{p,\text{exit}} = 0.5$ have not yet been achieved. Momentum theory shows that values of $C_{p,\text{exit}} = 0.7$ or above can only be achieved with very significant back pressure reductions $c_{p3} < -1$.

Referred to rotor power coefficients values of $C_{p,\text{rotor}} = 2.5$ might be achievable, but to the cost of fairly large diffuser area ratio’s, typically values of $\beta > 4.5$.

So far a combination of a large value for the back pressure -$c_{p3}$, together with a significant mass flow augmentation has not yet been demonstrated.
References


