Abstract: Optical microwave beam-steering (OMBS) is of interest for broadband wireless communication and phased array radar. Traditionally the optical all-pass filter (OAPF) based on micro-ring resonator (MRR) can only be used to manipulate group delay. Moreover, the requirement of lossless condition of the OAPF limits its potential of monolithic integration on InP platform. To overcome this limitation and also to provide freedom for amplitude manipulation, a novel loss-control MRR is proposed for both group delay and amplitude manipulation. A mathematical model is built up for analysis and then a simulation is carried out to study the proposed design.

I Introduction
Optical microwave beam-steering (OMBS) is of interest for the broadband phase antenna array due to its wide bandwidth, low loss and immunity to electromagnetic interference. In previous research, the most efforts are made for military application like broadband phase array radar. Recently, the utilization of OMBS for broadband fibre wireless communication attracts lots of interests. The key enable technique is the suitable optical delay unit (ODU). The micro-ring resonator (MRR) based ODU are gaining high interests in terms of integration capability, design flexibility and small footprint [1-3]. MRRs require no cleaved facets or grating couplers to realize optical feedback and are therefore particularly suited for monolithic integration with other components. These applications of MRRs are based on the concept of the optical all-pass filter (OAPF). Traditionally the optical all-pass filter (OAPF) based on micro-ring resonator (MRR) can only be used to manipulate group delay. Moreover, the requirement of lossless condition of the OAPF limits its potential of monolithic integration on Indium-Phosphide (InP) platform. To overcome this limitation and also to provide freedom for amplitude manipulation, a novel loss-control MRR is proposed for both group delay and amplitude manipulation. A mathematical model is built up for analysis and then a simulation is carried out to study the proposed design.

II. Theoretical Model, Analysis and Simulation

Fig. 1. The principal scheme of lattice double chain
The structure of OAPF based on MRR is depicted in Fig.1. A straight waveguide is coupled to the micro-ring. The coupling ratio of the directional coupler formed by the straight waveguide and part of the ring can be controlled by adjusting the gap and the coupling length between the straight waveguide and the ring. The phase shift can be realized by tuning the refractive index of the ring waveguide by thermal-optic or electro-optic methods. The loss inside the MRR can be adjusted by using the in-ring semiconductor optical amplifier. In previous publications, the loss in the ring is usually neglected for the lossless platform like SiN. In this case all input optical power will eventually arrive at the output port. This structure is so called optical all-pass filter because of this constant amplitude of the transfer function. The loss cannot be neglected for some platforms like InP. It is therefore important to understand the role of loss. In the following sections, the impact of loss will be modelled and analyzed for transmission function and group delay function.

A. General modelling of OAPF with loss

The transfer function of OAPF can be modelled based Z-Transform as proposed in ref.[]. In the Z-domain, the input and output optical signal can be expressed as X and Y, respectively. As shown in Fig.1, the through and cross transmissions between the straight waveguide and ring are noted as c and s. The delay, phase shift and loss inside the ring is modelled as ζ as shown in Fig.1. Thus the relationship between input and output can be expressed as:

\[ Y = cX + X(-js)ζ(-js) + X(Z^{-1})(-js)ζ(-js)cζ(-js) + X(-js)ζ(-js)cζ(-js) + ... \]

It can be further simplified as: \[ Y = X(e^{-jζ})(1-cζ) \]. Thus the H transfer function can be expressed as below:

\[ H = Y / X = -γe^{-jφ}(-cγ^{-1}e^{jθ} + Z^{-1})/(1-cγe^{-jθ}Z^{-1}) \]

In the following, the impact of loss to the square magnitude response and the group delay response will be discussed based on Eq.2.

B. Square magnitude response of OAPF with loss

The power response is useful for optical system design and easy to measure. Thus the square magnitude response is important for optical filter design. The square magnitude response can be expressed as below:

\[
\left| H(e^{jΩT}) \right|^2 = -γe^{-jφ}(-cγ^{-1}e^{jθ} + e^{-jΩT})/(1-cγe^{jθ}e^{-jΩT})^2 = γ^2\left|(-cγ^{-1}e^{jθ} + e^{-jΩT})\right|^2 / (1-cγe^{jθ}e^{-jΩT})^2 = γ^2[ -cγ^{-1}\cos(φ) + \cos(ΩT) ]^2 + γ^2[cγ^{-1}\sin(φ) + \sin(ΩT)]^2
\]

\[
(3) \quad \frac{1-cγ\cos(φ+ΩT)}{1-cγ\cos(φ+ΩT)+1+c^2γ^2} = \frac{c^2 + γ^2 - 2cγ\cos(φ+ΩT)}{-2cγ\cos(φ+ΩT)+1+c^2γ^2}
\]

where Ω and T are normalized angular frequency and round trip time of the ring. Now we investigate into the impact of loss. If the loss is neglected, the value of γ is equal to 1, the square magnitude response can be expressed as below:
\[ |H(e^{j\Omega T})|^2 = \frac{c^2 + 1 - 2c \cos(\phi + \Omega T)}{-2c \cos(\phi + \Omega T) + 1 + c^2} = 1 \] (4)

If the loss is too large to be neglected, the square magnitude response can be expressed as below:

\[ |H(e^{j\Omega T})|^2 = \frac{c^2 + \gamma^2 - 2c\gamma \cos(\phi + \Omega T)}{-2c\gamma \cos(\phi + \Omega T) + 1 + c^2\gamma^2} = 1 + \frac{(\gamma^2 - 1)(1 - c^2)}{1 + c^2\gamma^2 - 2c\gamma \cos(\phi + \Omega T)} \] (5)

According to Eq.5, the square magnitude response should be bell-shaped curve with maximum of 1. The minimum transmission point will appear in the resonating point \((\phi = \Omega T)\). The numerical simulation is carried out based on the Phoenix Software package. The basic parameters for the simulated OAPF with loss are listed in Table I. The simulated results are presented in Fig.2. We can see that the power is equally 0.8 to different frequencies in the 0dB loss case. The value of power response is 0.8 rather than 1 is mainly because the 1-dB insertion loss. For the lossy cases, the power transmission is lowest at the resonating point. We can also find that, the more the loss is, the lower the power transmission is.

### Table I: Basic simulation parameter

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<tr>
<th>Items</th>
<th>Value</th>
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<tr>
<td>c</td>
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<td>(\gamma)</td>
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</tr>
<tr>
<td>s</td>
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<tr>
<td>(\phi)</td>
<td>0</td>
</tr>
<tr>
<td>T</td>
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</tr>
</tbody>
</table>

C. Group delay function of OAPF with loss

The filter’s group delay is defined as the negative derivative of the phase of the transfer function with respect to the angular frequency as follow:

\[ \tau_g = \frac{d}{d\Omega} \tan^{-1} \left[ P(Z) \right]_{Z=e^{j\omega T}} = \frac{P'(e^{j\Omega T})}{1 + P^2(e^{j\Omega T})}, \quad P(e^{j\Omega T}) = \frac{\text{Im}(H(e^{j\Omega T}))}{\text{Re}(H(e^{j\Omega T}))} \] (6)

In the lossless case \((\gamma=1)\), the group delay response can be expressed as below:

\[ P(e^{j\Omega T}) = \frac{(1-c^2)\sin(\phi + \Omega T)}{2c - (1+c^2)\cos(\phi + \Omega T)}, \quad \tau_g = \frac{P'(e^{j\Omega T})}{1 + P^2(e^{j\Omega T})} = \frac{(1-c^2)T}{1 + c^2 - 2c\cos(\phi + \Omega T)} \] (7)

Now we look into the expression for the lossy case \((\gamma\neq1)\). The core function \(P\) is expressed as below:

\[ P(e^{j\Omega T}) = \frac{0.5 \cdot c\gamma(\gamma - 1)\sin[2(\phi + e^{j\Omega T})] + c^2(1-\gamma)\sin(\phi + e^{j\Omega T}) + (1-c^2)\sin(\phi + e^{j\Omega T})}{-c + 0.5 \cdot c\gamma(\gamma + 1) + 0.5 \cdot c\gamma(\gamma + 1)\cos[2(\phi + e^{j\Omega T})] + 2c - \gamma(1+c^2)\cos(\phi + e^{j\Omega T})} \] (8)
It is obvious that the additional terms $0.5 \gamma (\gamma - 1) \sin[2(\phi + e^{\alpha T})] + c^2 (1 - \gamma) \sin(\phi + e^{\alpha T})$ in numerator and $-c + 0.5 \gamma (\gamma + 1) + 0.5 \gamma (\gamma + 1) \cos[2(\phi + e^{\alpha T})]$ in denominator will introduce the variation for the final result. This variation will be investigated by simulation. The simulation is carried out based on the parameters presented in Table-I as well. As shown in Fig.3, the group delay responses for different cases (0dB to 1dB) are similar. This indicates that the variation induced by the additional terms do not contribute much to the group delay with the loss in that range. The curves for power responses and group delays are both presented in Fig.4 for comparison. We can see that the group delay values keep similar when the power responses vary from 0.8 to 0.2. This means that the amplitude can be independent in the range when the group delay is fixed to some values.

IV. Conclusion

In this paper, the optical all-pass filter with loss is theoretically investigated and numerically simulated. According to the theoretical analysis, the loss inside the ring will reduce the power transmission especially when the frequency is close to the resonating frequency. The simulated result goes well with our analysis. For the group delay, the loss induced variation is investigated by simulation. It shows that the group delays are quite similar when the loss varies from 0dB to 1dB. It means the amplitude can be controlled when the group delay is fixed. All the features show that the amplitude and group delay of the proposed optical all-pass filter based on loss-controlled micro-ring resonator can be controlled independently.

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References