Periods of vibration of braced frames with outriggers

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Abstract

This paper presents simple equations for the natural lateral and rotational periods of vibration for high-rise steel structures comprising braced frames with outrigger trusses. The structural floor plan can be symmetric or asymmetric combinations of identical bents. Each braced frame is modelled by a cantilever with a bending stiffness and racking shear stiffness. The outriggers are represented by a rotational spring. The stiffness of the outrigger spring is dependent on three specific modes of behaviour: bending and racking shear in the outrigger truss and axial lengthening or shortening of the exterior columns. This yields a simple calculation of the maximum horizontal deflection of the structure. For the rotational frequencies, the bending, racking shear and spring stiffness parameters of the steel structure each are combined to yield a torsional stiffness, a warping stiffness and a warping spring stiffness which allow an assessment of the rotation at the top. The approximate method of analysis relates the natural frequencies of the structure to the top deflection and rotation when the self-weight is taken as a distributed horizontal load. In the calculations a braced frame is represented by a rigid stick model with its overall flexibility assigned to a rotational spring at the base. Results obtained by the simplified method are compared to those from a finite element analysis. The approximate method gives acceptable conservative results for preliminary analysis.

Keywords: braced frames; outrigger trusses; rotational spring; warping spring.

1. Introduction

In the early design stages of a tall building, there are some criteria that need to be satisfied in order to assure that the chosen structural system is an optimal one. The maximum lateral displacements are controlled by employing the drift index which is generally accepted to be 1/500. The natural period of free vibration is also an important factor and is used for obtaining the design base shear. Several empirical estimates are given by different seismic codes. The ASCE 7-10 [1], UBC-97 [2], and Eurocode 8 [3] give an equation for estimating...
the natural vibration period of a braced frame: \( T = C_t H^{0.75} \) where \( C_t = 0.0488 \) and \( H \) is the height of the building in meters. Other codes, like ATC 3-06 [4], give a different formula: \( T = 0.09 H/D^{0.5} \) where \( D \) is the dimension of the building, parallel to the applied force. Ellis [5] considered the frequencies of 163 buildings and recommended the following equation for the translational frequency: \( T = H/46 \) and for the fundamental torsional mode \( T_\phi = H/72 \). Variations on these frequencies can be found in the literature, for steel structures: \( T = H/45, T = H/78 \) or \( T_\phi = H/108 \) (for truss bracing) [6]. The dynamic behavior of outrigger structures was first studied by Rutenberg [7] who presented graphical solutions for the first three natural periods of vibration for high-rise shear walls with infinitely rigid outriggers. The free vibration of outrigger braced shear wall structures was also studied by Moudarres and Coull [8] with the help of a transfer matrix technique. The suggested solutions do not allow a rapid assessment of the vibration periods. The aim of this study is to present simple approximate equations for the fundamental lateral and rotational periods of vibration for tall buildings with a symmetric or an asymmetric structural floor plan comprising identical steel braced frames with trussed outriggers.

2. Modeling of outrigger structure

The method of analysis is based on an earlier presented method for the calculation of the optimum location of the outrigger up the height of a braced frame structure [9] where the structure as shown in Figure 1(a) is represented by seven stiffness parameters. The central braced frame has a uniform bending stiffness \( EI_b \) and a racking shear stiffness \( GA_b \). The outrigger structure consists of a truss on both sides of the frame with a bending stiffness \( EI_o \) and a racking shear stiffness \( GA_o \). The axial stiffness of the exterior columns below the outrigger level is combined to form a global bending stiffness \( EI_c \). The outriggers and exterior columns are replaced by a single rotational spring at outrigger level, as shown in Figure 1, with stiffness:

\[
K_o = \left[ b \left( 2\alpha^2 EI_o \right) + 1/ \left( \alpha^2 h GA_o \right) + z_o/ EI_c \right]^{-1}
\]  

where \( H \) is the total height of the structure; \( z_o \) is the location of the outrigger measured from the base of the structure; \( \alpha = \ell/b \) is a non-dimensional parameter where \( b \) is the length of the outrigger and \( \ell \) is the distance between an exterior column and the centroid of the braced frame and \( h \) is the height of the outrigger. When the structure is subjected to a horizontal uniform load the restraining moment in the spring and the lateral deflection at the top of the structure then are:

\[
M_t = w \left( z_o H^2 - z_o^2 H + z_o^3 / 3 \right) / 2 EI_b + w (1 - z_o / \alpha GA_b) \left( 1/K_o + z_o/ EI_b + 1/ \left( \alpha^2 h GA_b \right) \right)
\]

\[
\delta_{\text{max}} = w H^3 / 8 EI_b + w H^2 / 2 GA_b - M_t (z_o (1 - z_o / 2)/ EI_b - 1/ \alpha GA_b)
\]

For the three dimensional model of the structure the salient planar properties \( EI_b, GA_b \) and \( K_o \) will be used to obtain the rotational stiffness parameters of the structure. The warping stiffness \( EI_o \), torsional stiffness \( GJ \) and a warping spring stiffness parameter \( K_o \) can quite easily be obtained as follows, where \( c_i \) is the distance between an outrigger bent and the elastic neutral axis of the structure.

\[
EI_o = \sum EI_b c_i^2; \quad GJ = \sum GA_b c_i^2; \quad K_o = \sum K_o c_i^2
\]
It can quite easily be shown that the restraining bi-moment and the maximum rotation at the top are:

\[ B_{\omega_0} = \frac{m_t (z_0 H^2 - z_0^2 H + z_0^3/3)}{2EI_{\omega_0} + m_t (1 - z_0) / \alpha GJ} \left\{ 1 + \frac{1}{K_{\omega_0} + z_0 / EI_{\omega_0} + 1/\alpha^2 h GJ} \right\} \]  

(5)

\[ \varphi_{\text{top}} = m_t H^4 / 8EI_{\omega_0} + m_t H^2 / 2GJ - B_{\omega_0} \left( z_0 (1 - z_0 / 2) / EI_{\omega_0} - 1 / \alpha GJ \right) \]  

(6)

where \( m_t \) is the horizontal load \( w \) multiplied by the eccentricity of the loading. The above equations allow a rapid assessment of the maximum lateral displacement of the elastic axis and rotation about this axis.

### 3. Natural periods of vibration

For slender lateral load resisting structures where the first mode shape can be taken to be similar to that of a flexural cantilever beam with a uniformly distributed stiffness and mass, the first natural frequency \( \omega \) can be approximated by the generally accepted equation [10] given in expression (7a), where \( \rho \) is the mass per unit height of the building in kg/m and \( EI \) is the flexural stiffness of the structure. Rewriting equation (7a) by applying the mass of the building as a horizontal load, the horizontal period of vibration can be expressed as a function of the top deflection of the flexural cantilever \( \delta_{\text{max}} \) (the gravitational acceleration is 9.81 m/s\(^2\)):

\[ \omega = 1.875^2 \sqrt{EI / \rho H^4} \rightarrow T = 1.614 \sqrt{\delta_{\text{max}}} \]  

(7a), (7b)

The horizontal deflection configuration of a typical braced frame with a single outrigger deviates significantly from that of a pure flexural cantilever as shown in Figure 2. The bending deflection displays a single curvature over the height of the structure, while a braced frame typically shows a double curvature deflected shape. The horizontal displacement pattern of a braced frame with an outrigger structure has a triple curvature. In order to select a simplified deflected configuration, closer to the triple curvature, it is suggested to adopt a straight line which can be represented by a rigid cantilever. The flexibility of the structure can then be completely represented by a single rotational spring at the base of the structure. For this system the equation of motion is:

\[ \frac{\rho H^3}{3} \frac{d^2 \theta}{dt^2} + K_{\theta} \theta = 0 \]  

(8)
In equation (8) $K_0$ is the rotational stiffness representing the structure and $\rho H^3/3$ is its mass moment of inertia. The angular frequency is expressed as a function of the maximum displacement $\delta_{\text{max}}$ of the structure when the mass of the building is horizontally applied to the structure (see eqn. (9a)). Rewriting Equation (9a) yields the approximate simple expression (9b) for the natural period of vibration for a braced frame with an outrigger truss. This gives a 1.5% longer period than the more familiar equation.

$$\omega = \sqrt{3K_0/\rho H^3} = \sqrt{1.5g/\delta_{\text{max}}} \rightarrow T = 1.638\sqrt{\delta_{\text{max}}} \quad (9a), (9b)$$

For the rotational natural period of vibration of a structure comprising identical braced frames with outrigger trusses the angular frequency can analogously be approximated by:

$$\omega_\psi = 1.875^2 \sqrt{EI_0/r^2 \rho H^4} \quad \text{where} \quad r^2 = \left(a^2+b^2\right)/12 \quad (10)$$

where $EI_0$ is the warping stiffness along the height and $r$ is the radius of gyration of the building, in which $a$ and $b$ are the width and depth of the building. The period of torsional vibration then becomes:

$$T_\psi = 1.614\sqrt{r\varphi_{\text{max}}} \quad (11)$$

where $\varphi_{\text{max}}$ is the maximum angle of twist of the structure subjected to a uniformly distributed torque of magnitude $\rho g r$.

Analogously to the analysis of planar structures the same holds true for the rotational analysis of the structure, i.e. the improved period of torsional vibration becomes:

$$T_\psi = 1.638\sqrt{r\varphi_{\text{max}}} \quad (12)$$

Equations (13) and (16) can be used for a rapid assessment of the natural lateral and rotational periods of vibration of the building.
4. Accuracy of the simplified method

In order to determine the accuracy of the suggested equations, a limited error analysis was performed on six typical high-rise structures: one symmetric and one asymmetric floor plan as shown in resp. in Fig. 3 (a) and Fig. 3 (b); three building heights of 100 m, 160 m and 200 m; outrigger locations at the top (\(z_o=H\)), at mid-height (\(z_o=H/2\)) and at three quarters up the height of the structure (\(z_o=3H/4\); two sets of sectional properties for the structural elements which are given in Table 1. The two 100 m high buildings have identical one-bay X-braced frames, while the other structures have identical two-bays X-braced frames, resp. Fig 3(c) and (d). Each outrigger truss is 12 m long and has a single storey height of 4 m with four X-braced frame segments for the 100 m high buildings, while for the 160 m and 200 m high buildings the outrigger trusses are two storeys high (8 m) with 8 X-braced frame segments. The elastic modulus of steel is taken: \(E = 2.1 \times 10^8\) kN/m². The structural parameters for the braced frames with outriggers are computed using expressions given earlier [9]. The mass per unit volume for the dynamic analysis is 200 kg/m³.

Table 1 Cross sectional areas of the structural elements

<table>
<thead>
<tr>
<th>Building Height m</th>
<th>Braced Frames</th>
<th>Exterior Frames</th>
<th>Outrigger beams</th>
<th>Outrigger diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Columns m²</td>
<td>Diagonals m²</td>
<td>Max. m²</td>
<td>Min. m²</td>
</tr>
<tr>
<td>H</td>
<td>A1</td>
<td>A1</td>
<td>A1</td>
<td>A1</td>
</tr>
<tr>
<td>100</td>
<td>0.08</td>
<td>0.03</td>
<td>0.08</td>
<td>0.10 0.004 0.004</td>
</tr>
<tr>
<td>160</td>
<td>0.30</td>
<td>0.10</td>
<td>0.30</td>
<td>0.10 0.003 0.003</td>
</tr>
<tr>
<td>200</td>
<td>0.70</td>
<td>0.30</td>
<td>0.70</td>
<td>0.10 0.004 0.005</td>
</tr>
</tbody>
</table>

The natural lateral and torsional periods of vibration of the structures, given by equations (13) and (16), are presented in resp. Table 2 and Table 3. The results from finite element analyses done with the SAP2000 F.E. package are given in brackets.

5. Conclusions

The proposals for the natural periods of vibration given by equations (13) and (16) give results that are within 4% of those obtained from finite element analyses. Also, the percentage absolute maximum and average errors for the rigid stick model (eqns. 13 and 16) are smaller than those obtained for the flexural cantilever beam model (eqns. 10 and 15).
Table 2 Natural lateral periods of vibration (from equation (13) and from the F.E.M. analysis, in brackets)

<table>
<thead>
<tr>
<th>Building system</th>
<th>( H/2 ) Min.</th>
<th>Max.</th>
<th>Min.</th>
<th>Max.</th>
<th>Min.</th>
<th>Max.</th>
<th>0.05(H^{0.75} ) [1, 2, 3]</th>
<th>0.09(H^{-0.5} ) [4]</th>
<th>( H/46 ) [5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 ( H=100m )</td>
<td>3.83(3.73)</td>
<td>4.46(4.42)</td>
<td>3.76(3.82)</td>
<td>4.34(4.35)</td>
<td>3.98(4.11)</td>
<td>4.41(4.47)</td>
<td>1.58</td>
<td>1.5</td>
<td>2.17</td>
</tr>
<tr>
<td>A2 ( H=100m )</td>
<td>3.84(3.78)</td>
<td>4.74(4.69)</td>
<td>3.79(3.85)</td>
<td>4.60(4.66)</td>
<td>4.09(4.21)</td>
<td>4.67(4.71)</td>
<td>2.25</td>
<td>2.4</td>
<td>3.48</td>
</tr>
<tr>
<td>A1 ( H=160m )</td>
<td>3.53(3.47)</td>
<td>5.52(5.44)</td>
<td>3.65(3.80)</td>
<td>5.32(5.30)</td>
<td>4.19(4.37)</td>
<td>5.37(5.38)</td>
<td>2.66</td>
<td>3.0</td>
<td>4.34</td>
</tr>
<tr>
<td>A2 ( H=160m )</td>
<td>3.40(3.27)</td>
<td>5.49(5.34)</td>
<td>3.54(3.63)</td>
<td>5.29(5.20)</td>
<td>4.12(4.23)</td>
<td>5.34(5.28)</td>
<td>2.66</td>
<td>3.0</td>
<td>4.34</td>
</tr>
<tr>
<td>A1 ( H=200m )</td>
<td>3.91(3.80)</td>
<td>6.01(5.87)</td>
<td>4.03(4.15)</td>
<td>5.79(5.72)</td>
<td>4.61(4.75)</td>
<td>5.85(5.81)</td>
<td>2.66</td>
<td>3.0</td>
<td>4.34</td>
</tr>
<tr>
<td>A2 ( H=200m )</td>
<td>3.53(3.47)</td>
<td>6.01(5.87)</td>
<td>4.03(4.15)</td>
<td>5.79(5.72)</td>
<td>4.61(4.75)</td>
<td>5.85(5.81)</td>
<td>2.66</td>
<td>3.0</td>
<td>4.34</td>
</tr>
</tbody>
</table>

Table 3 Natural torsional periods of vibration (from equation (16) and from the F.E.M. analysis, in brackets)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 ( H=100m )</td>
<td>6.33(6.25)</td>
<td>7.37(7.30)</td>
<td>6.22(6.32)</td>
<td>7.17(7.19)</td>
<td>6.60(6.80)</td>
<td>7.29(7.38)</td>
<td>1.39</td>
<td>0.93</td>
</tr>
<tr>
<td>A2 ( H=100m )</td>
<td>3.29(3.23)</td>
<td>4.06(4.01)</td>
<td>3.24(3.29)</td>
<td>3.94(3.93)</td>
<td>3.50(3.60)</td>
<td>4.00(4.03)</td>
<td>2.22</td>
<td>1.48</td>
</tr>
<tr>
<td>A1 ( H=160m )</td>
<td>5.84(5.74)</td>
<td>9.13(8.98)</td>
<td>6.03(6.27)</td>
<td>8.80(8.75)</td>
<td>6.92(7.22)</td>
<td>8.88(8.90)</td>
<td>2.77</td>
<td>1.85</td>
</tr>
<tr>
<td>A2 ( H=160m )</td>
<td>2.91(2.80)</td>
<td>4.70(4.57)</td>
<td>3.03(3.10)</td>
<td>4.53(4.45)</td>
<td>3.52(3.62)</td>
<td>4.57(4.52)</td>
<td>2.77</td>
<td>1.85</td>
</tr>
<tr>
<td>A1 ( H=200m )</td>
<td>6.00(5.81)</td>
<td>9.35(9.10)</td>
<td>6.16(6.36)</td>
<td>8.97(8.87)</td>
<td>7.04(7.28)</td>
<td>9.05(9.00)</td>
<td>2.77</td>
<td>1.85</td>
</tr>
<tr>
<td>A2 ( H=200m )</td>
<td>3.35(3.26)</td>
<td>5.14(5.03)</td>
<td>3.45(3.55)</td>
<td>4.95(4.90)</td>
<td>3.94(4.06)</td>
<td>5.00(4.98)</td>
<td>2.77</td>
<td>1.85</td>
</tr>
</tbody>
</table>

The very simple expressions given by the various building codes yield much shorter periods of vibration with maximum errors larger than 50%. It can be concluded that the suggested simplified equations for the calculation of the first lateral and torsional frequencies of high-rise buildings with symmetric or asymmetric structural floor plans comprising combinations of identical braced frames with outrigger trusses yield good results for the preliminary stages of the structural design.

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References