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Collective spontaneous emission in coupled quantum dots: Physical mechanism of quantum nanoantenna

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We investigate the collective spontaneous emission in a system of two identical quantum dots (QDs) strongly coupled through the dipole-dipole (d-d) interaction. The QDs are modeled as two-level quantum objects, while the d-d interaction is described as the exchange of a virtual photon through the photonic reservoir. The master equation approach is used in the analysis. The main attention is focused on antenna characteristics of the two-QD system—the radiation intensity dependence on the meridian and azimuthal angles of observation. We show that the radiation pattern of such a system is nonstationary and its temporal behavior depends on the initial quantum state. In particular, for entangled initial states the radiative pattern exhibits oscillations on the frequency which corresponds to the d-d interaction energy. We also analyze spectral properties of the directional diagram. The comparison of radiation patterns is carried out for two QDs and two classical dipoles. The concept of quantum nanoantenna is proposed based on collective spontaneous emission in QD ensembles.

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I. INTRODUCTION

Spontaneous radiation of excited systems is a fundamental physical effect which has been in the focus of the research for a long time. The properties of the spontaneous radiation are strongly influenced by many factors, such as energy spectrum of the emitter and the emitter-electromagnetic field coupling constant. The simplest model is an isolated two-level emitter in the vacuum weakly interacting with the field (the Wigner-Weisskopf theory of spontaneous emission). In this model, the decay is exponential and the decay rate is proportional to the partial density of photonic states (PDOS) of the emitter and to the coupling factor. A possibility to control the decay rate arises when a two-level oscillator is placed in a transparent dielectric medium or into an electrodynamic system such as microwaves, photonic crystal, or nanoantenna. In all mentioned cases, the modification of the PDOS takes place resulting in the decay rate change. This effect is commonly referred to as the Purcell effect. In the strong coupling regime the Wigner-Weisskopf approach becomes invalid: the exponential decay law becomes violated and an oscillating behavior appears due to the multiple emission and reabsorption of photons by the emitter (the so-called vacuum Rabi oscillations). The Rabi frequency is proportional to the coupling factor.

The spontaneous emission is strongly affected by the interaction between the emitters in an ensemble. If the separation between the emitters, exemplified, e.g., by quantum dots (QDs), is small, there is a probability that the excitation can be transferred via charge tunneling or long-range radiative interaction. If the interemitter distance is a little bit longer, they are coupled by a near-field quasielectrostatic dipole-dipole (d-d) interaction, mostly referred to as the Förster energy transfer. It should be noted that in actual experimental conditions, one of the mechanisms prevails over the others, allowing one to study them separately.

The interaction of multiple emitters in an ensemble changes the energy spectrum of the system, introducing additional energy levels which correspond to correlated multiparticle states of the system, i.e., entangled states. As a consequence, the temporal dynamics of the spontaneous emission is strongly modified even in the weak-coupling regime. The superradiance effect illustrates this statement. It should be noted that Dicke’s model is restricted to systems with linear extension much less than the wavelength. The increase of the system size makes the phase relations between emitters important. This enables not only the control of the temporal behavior but also the control of the spatial structure of the spontaneous emission. The latter appears in, e.g., directed spontaneous emission from an extended ensemble, correlated emission of a single photon, quantum interference in cooperative Dicke emission, collective Lamb shift in single photon superradiance, and finite time disentanglement via spontaneous emission.

In the strong-coupling regime the cooperative effects manifest themselves in essentially nonmonotonous oscillating behavior of the spontaneous emission. Obviously, theoretical modeling of such systems gets significantly more complex: a solution of the quantum many-body problem is required. That is why, e.g., collective effects in the strong-coupling regime are commonly modeled by a reduced two-particle system placed in a free space, in a waveguide, in a microwaves, and in the vicinity of a plasmon nanowire. The main attention is paid to the evaluation of the total irradiated power and its spectral characteristics. The latter is of importance for the interpretation of the photoluminescence measurements. Temporal
correlations of the spontaneous emission are also studied,\textsuperscript{31} while the role of the spontaneous emission spatial structure seems to be underestimated. Meanwhile, by analogy with classical optical nanoantennas,\textsuperscript{32-35} the spatial inhomogeneity of spontaneous emission leads to the concept of a quantum nanoantenna—a device converting quantized near field of a source into the radiation far field.\textsuperscript{33,36} A quantum antenna proposed in Ref. 33 is based on the spontaneous emission of an isolated emitter weakly coupled to a metal-dielectric structure of a special type. In Ref. 36, the excitation source is a terahertz tunneling current induced by an optical Rabi wave propagating along the chain of tunneling-coupled QDs. In the present paper we propose a quantum antenna utilizing collective spontaneous emission of strongly coupled oscillators. A system of two d-d coupled QDs is the simplest realization of such a system.

In order to experimentally observe the modified decay dynamics due to strong coupling in semiconductor quantum dots, one should realize a set of requirements: (i) the excitonic dipole moments in all QDs should be mutually aligned, (ii) the QDs should spectrally overlap, and (iii) one needs full QD position control in order to reproducibly change the distance between the QDs. In addition, it would be highly recommended when (iv) the QDs could be embedded in a low refractive index material which will substantially enhance the coupling and (v) that the QD-QD detuning should, when possible, be variable in a reproducible manner.

It should first be stated that in almost all photoluminescence experiments on, e.g., Stranski-Krastanov InAs/GaAs QDs, one employs nonresonant excitation into the barrier material, followed by incoherent relaxation of the electron-hole pair into the semiconductor quantum dot. Since in such samples, the bases of QDs are usually elongated in a certain direction, we might be able to observe the effect of coupling even by employing nonresonant excitation into the barrier. But for the QDs which are circularly shaped in the plane perpendicular to the growth direction, then the in-plane excitonic dipole moment averages out to zero and a modified decay dynamics can only be observed in the case of resonant excitation\textsuperscript{37} in which a laser beam is exactly resonant with the QDs, resulting in a net alignment of the excitonic dipoles.

As compared to the frequently studied Stranski-Krastanov InAs/GaAs QDs, the nanowire quantum dots, which are, e.g., thin InAs segments inside narrow GaAs nanowires, look much more promising to study the effects of strong coupling and thus the modified decay dynamics. Indeed, nanowire QDs feature an accurate and full position control using either nanoimprint\textsuperscript{38} or electron beam lithography\textsuperscript{39} for positioning the Au nanoparticles which catalyzes the nanowire growth. The effects of strong coupling are probably most easily observed in zinc-blende/wurtzite crystal phase QDs\textsuperscript{40} which feature perfectly atomically flat interfaces, thus providing a good opportunity to spectrally overlap different QDs, either in the same nanowire, or in different nanowires. Moreover, nanowire quantum dots can be embedded into a low refractive index material like PDMS,\textsuperscript{41} SiO\textsubscript{2}, or Si\textsubscript{3}N\textsubscript{4} for increasing the coupling and getting rid of the substrate-based photoluminescence. Finally, it has already been shown that a nanowire can be individually contacted\textsuperscript{42} thus in principle allowing one to tune the mutual QD-QD detuning by using the quantum confined Stark effect. In view of this recent progress in the preparation of nanowire quantum dots, we feel that a study towards the effects of strong coupling between two QDs is presently particularly timely as we expect that the (i)–(v) criteria mentioned above can soon be realized in nanowire quantum dots.

In this work, we have extensively simulated the spontaneous emission decay dynamics, spectrum, and spatial structure of an InAs double-QD system embedded in GaAs material in the strong-coupling regime with a great emphasis on the photoluminescence emission pattern from such a sample. In Sec. II, we state the model and specify the corresponding master equation formalism. A general solution of the master equation and observable antenna characteristics are also presented in this section. Based on this theory, in Sec. III we investigate the dynamics, spatial structure, and spectrum of the two-QD spontaneous emission for different initial excitations and we check the influence of QD-QD distance. The summary and concluding remarks are contained in Sec. IV.

II. BASIC FORMALISM

Consider a system of two identical nonoverlapping QDs embedded into a dielectric medium with relative permittivity \(\varepsilon_b\). We assume the host medium to be homogeneous, nondispersive, and nonabsorptive. The QDs are modeled as atomlike emitters\textsuperscript{43–46}—two-level systems, with excited state \(|e\rangle\), ground state \(|g\rangle\), transition frequency \(\omega\), and transition dipole moments \(\mu\). Since the size of the QD is much smaller than the wavelength of the emitted radiation, a QD can be approximated by a transition dipole moment which only has in-plane components. We assume that the QDs are located at positions \(r_1\) and \(r_2\). The QD structure interacts with the quantum electromagnetic (EM) field regarded as a reservoir.\textsuperscript{47-50} The field is described by the operator

\[
\hat{\mathbf{E}} = \hat{\mathbf{E}}^{(+) + \text{H.c.}},
\]

where

\[
\hat{\mathbf{E}}^{(+)}(r, t) = i \sum_{\mathbf{k}, s} \frac{\hbar \omega_k}{2 \varepsilon_0 \varepsilon_b V} \mathbf{e}_k \hat{a}_k \mathbf{e}^{i \mathbf{k} \cdot \mathbf{r}}.
\]

The abbreviation H.c. denotes Hermitian conjugation. A single mode is characterized by the wave vector \(\mathbf{k}\), frequency \(\omega_k\), and the unit polarization vector \(\mathbf{e}_k\) with orthogonal polarization directions \((s = 1, 2)\). \(V\) is the normalization volume. Operators \(\hat{a}_k\) and \(\hat{a}_k^\dagger\) are the annihilation and creation operators of this field mode which satisfy bosonic commutation relations. In the electric dipole approximation, the Hamiltonian of the system “double-QD + EM field” can be written as a sum of two terms,

\[
\hat{H}_0 = \sum_{i=1}^{2} \hbar \omega_i \hat{S}_i^z + \sum_{\mathbf{k} s} \hbar \omega_k \left( \hat{a}_k \hat{a}_k^\dagger + \frac{1}{2} \right),
\]

\[
\hat{H}_I = -i\hbar \sum_{\mathbf{k} s} \sum_{i=1}^{2} [\mu \cdot \mathbf{g}_{ki}(r_i)(\hat{S}_i^+ + \hat{S}_i^-)\hat{a}_{ki} - \text{H.c.}],
\]

where \(\hat{H}_0\) corresponds to the bare systems and \(\hat{H}_I\) is the interaction Hamiltonian. Hereafter one should distinguish the index \(i\) from the imaginary unit. The quantities \(\hat{S}_i^+ = |e_i\rangle\langle g_i|\), and \(\hat{S}_i^- = |g_i\rangle\langle e_i|\).
and \( \hat{S}_i^+ = |g_i\rangle \langle e_i| \) are the dipole raising and lowering operators and \( \hat{S}_i^- = (|e_i\rangle \langle e_i| - |g_i\rangle \langle g_i|)/2 \) is the population operator of the \( i \)th QD. The coupling constant

\[
\mathbf{g}_k(r_i) = \left( \frac{\alpha_k}{2 \epsilon_0 \varepsilon_\infty \hbar \mathbf{V}} \right)^{1/2} \mathbf{e}_k e^{i \mathbf{k} \cdot \mathbf{r}_i},
\]

is the mode function of the three-dimensional vacuum field, evaluated at the position \( \mathbf{r}_i \) of the \( i \)th QD. Our model will be valid for both Stranski-Krastanov pyramidal InAs/GaAs QDs and dislikewise nanowire QDs in which the QD height is much smaller than its diameter.

In the further analysis we will follow the master equation approach in which the time evolution of the collection of QDs interacting with the electromagnetic field is considered in terms of the density operator \( \hat{\rho}_\text{QD} \) characterizing the statistical state of the combined system of the QDs and the field. In the Born approximation,\(^{31}\) the interaction between the QDs and the field is supposed to be weak, and there is no back response of the QDs on the field. So the state of the EM field does not change in time, and we can write the density operator \( \hat{\rho}_\text{QD}(t) = \hat{\rho}_0 \rho(t) \hat{\rho}_0^\dagger \), where \( \hat{\rho}_0 \) and \( \hat{\rho}_0^\dagger \) are the density operators of the QDs and the electromagnetic field, respectively. In the master equation approach, the mean value of any QD-based operator \( \hat{Y} \) can be obtained by \( \langle \hat{Y} \rangle = \text{Tr}[\hat{\rho} \hat{Y}] \) where \( \hat{\rho} = \hat{\rho}_0 \) and, in the rotating wave approximation, the density matrix obeys the equation of motion,

\[
\frac{d\hat{\rho}}{dt} = -i \sum_{i=1}^{3} \omega_i \left[ \hat{S}_i^+, \hat{\rho} \right] - i \sum_{i \neq j} \Omega_{ij} \left[ \hat{S}_i^+ \hat{S}_j^-, \hat{\rho} \right] - \frac{1}{2} \sum_{i,j=1}^{3} \Omega_{ij} \left( \hat{S}_i^- \hat{S}_j^+ + \hat{S}_i^+ \hat{S}_j^- - 2\hat{S}_i^- \hat{S}_j^- \rho \right), \quad (6)
\]

where

\[
\Omega_{ij} \equiv \Gamma = \frac{\epsilon^{1/2} \alpha_0^3 \mu^2}{3 \pi \epsilon_0 \hbar \varepsilon_\infty}
\]

is the spontaneous emission rate of a single QD and \( \mu \) is the QD dipole moment.

The parameters \( \Omega_{12} \) and \( \Gamma_{12} \) are respectively the diagonal and off-diagonal matrix elements of the interaction energy operator \( \mathbf{V} \),

\[
\hat{V} = \hbar \left( \begin{array}{cc} \Omega_{12} & i \Gamma_{12} \\ i \Gamma_{12} & \Omega_{12} \end{array} \right).
\]

(8)

The quantity \( \Gamma_{12} \) is shown to be\(^{27,47-50,52,53}\)

\[
\Gamma_{12} = \frac{3}{2} \beta \left\{ \alpha \left( \frac{\sin(k_0 r_{12})}{k_0 r_{12}} \right) + \beta \left( \frac{\cos(k_0 r_{12})}{(k_0 r_{12})^2} - \frac{\sin(k_0 r_{12})}{(k_0 r_{12})^2} \right) \right\},
\]

where \( \alpha = (\mathbf{e}_{\mu_1} \cdot \mathbf{e}_{\mu_2}) - (\mathbf{e}_{\mu_1} \cdot \mathbf{e}_{\mu_2}) (\mathbf{e}_{\mu_1} \cdot \mathbf{e}_{\mu_2}) \) and \( \beta = (\mathbf{e}_{\mu_1} \cdot \mathbf{e}_{\mu_2}) - (\mathbf{e}_{\mu_1} \cdot \mathbf{e}_{\mu_2}) \). Here \( \mathbf{e}_{\mu_1}, \mathbf{e}_{\mu_2} \) are unit vectors along the QD transition dipole moments and \( \mathbf{e}_{\mu_2} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{r_{12}}, \beta = \sqrt{\varepsilon_\infty \varepsilon_0} / \epsilon_0 c \).

The parameter \( \hbar \Omega_{12} \) in Eq. (6) is the dipole-dipole interaction energy between quantum dots coupled through the vacuum field. When \( k r_{12} \neq 0 \), this coherent coupling potential is\(^{27,47-50,54,55}\)

\[
\Omega_{12} = \frac{3}{4} \Gamma \left\{ -\alpha \cos(k_0 r_{12}) \left( \frac{k_0 r_{12}}{k_0 r_{12}} \right)^2 + \beta \left( \frac{\sin(k_0 r_{12})}{(k_0 r_{12})^2} + \frac{\cos(k_0 r_{12})}{(k_0 r_{12})^2} \right) \right\},
\]

and can be understood as the exchange of a virtual photon between an empty and an excited QD mediated by the quantized electromagnetic field. The coupling parameters \( \Gamma_{12} \) and \( \Omega_{12} \) determine the correlation properties of the multi-QD system and strongly depend on the inter-QD separation.

Calculation of the Lamb shift is possible based on Eq. (10) in the limit \( r_{12} \to 0 \). The divergence appearing in this case requires a special renormalization\(^{56}\) of the frequency shift. We will not consider this technique here, but assume that the renormalized shift is included in the transition frequency \( \omega_0 \) in Eq. (6).\(^{28}\)

For obtaining Eqs. (6) to (10), we have used the Markov approximation in which the dynamics of the QD system is slow as compared to the much faster decay of correlation functions of the photonic reservoir.\(^{57}\) In other words, the Markov approximation holds when there is no memory in the system. It is clear from Eqs. (9) and (10) that when the QD dipole moments are perpendicular to each other, the collective parameters will vanish and there will be no coupling between the QDs. By contrast, if the QD dipole moments are parallel or antiparallel, the collective parameters attain their maximal values, with opposite signs.

To obtain a better understanding of the nature of a double-QD system at different initial excitation conditions, we should calculate the eigenstates of the combined system “QDs + photonic reservoir,” which correspond to the eigenstates of Eq. (6). This system behaves as a single four-level system with eigenstates \(|g\rangle = |g_1\rangle|g_2\rangle\) and \(|e\rangle = |e_1\rangle|e_2\rangle\) energies \( E_s = -\hbar \omega_0 \) and \( E_e = \hbar \omega_0 \), respectively, as well as the entangled states \(|s\rangle = \frac{1}{\sqrt{2}} (|e_1\rangle|g_2\rangle + |g_1\rangle|e_2\rangle)\) and \(|a\rangle = \frac{1}{\sqrt{2}} (|e_1\rangle|g_2\rangle - |g_1\rangle|e_2\rangle)\) with energies \( E_s = \hbar \Omega_{12} \) and \( E_a = -\hbar \Omega_{12} \). The ground state \(|g\rangle\) and the fully excited state \(|e\rangle\) are not influenced by the dipole-dipole interaction, but the energy of the symmetric \(|s\rangle\) and the antisymmetric \(|a\rangle\) states are shifted from their unperturbed energies by \( \pm \hbar \Omega_{12} \). The symmetric and antisymmetric transitions are uncorrelated and decay with the rates \( \Gamma_s, \Gamma_a \), respectively, \( \Gamma_{s,a} = \Gamma_{12} \pm \Gamma_{12} \).

The origin of the Dicke-type\(^{58}\) cooperative spontaneous emission comes from the entanglement of the QD system with the electromagnetic field modes. This means that the interference of the radiation patterns from each of the individual oscillating dipoles should be taken into account. The constructive interference leads to faster decay of the system (superradiance) while the destructive interference leads to the slower decay (subradiance). We are mostly interested in the time profile of the spontaneous emission intensity \( I(t) \) as well as in spatial profile in meridianal and azimuthal planes. The time variation of the population can be directly probed in time resolved transmission or differential reflectivity\(^{28}\) measurements. The dynamics of the emission intensity can be recorded in a time resolved photoluminescence measurement.
where \( A \) is the following matrix:

\[
A = \begin{pmatrix}
-\Gamma & 0 & \xi^* & \xi \\
0 & -\Gamma & \xi & \xi^* \\
\xi & \xi^* & -\Gamma & 0 \\
\xi^* & 0 & -\Gamma & 0
\end{pmatrix},
\]

and \( \mathbf{f}(t) = 2\Gamma_{12} x_3(0) e^{t_0} \exp(-2\Gamma t) \), \( t_0 = (0,0,1,1) \), and \( \xi = -i\Omega_{12} - \Gamma_{12}/2 \). Due to the special symmetry properties of the matrix the general exact solution of Eq. (11) can be written in the simple analytical form. It reads

\[
x(t) = e^{-\Gamma t} Q(t) \begin{pmatrix}
1 \\
1 \\
1 \\
1
\end{pmatrix} \begin{pmatrix}
y(0) - \gamma b \\
0 \\
0 \\
0
\end{pmatrix} - \gamma \mathbf{a} e^{-2\Gamma t},
\]

where \( \gamma = 2\Gamma_{12}^2 x_3(0)/\Gamma_{12} \) and symbols \( Q(t) \) and \( U \) respectively denote the matrices

\[
Q(t) = \begin{pmatrix}
e^{\Gamma_{12} t} & e^{-\Gamma_{12} t} & e^{2\Omega_{12} t} & e^{-2\Omega_{12} t} \\
e^{\Gamma_{12} t} & e^{-\Gamma_{12} t} & -e^{2\Omega_{12} t} & -e^{-2\Omega_{12} t} \\
e^{-\Gamma_{12} t} & e^{\Gamma_{12} t} & -e^{2\Omega_{12} t} & -e^{-2\Omega_{12} t} \\
e^{-\Gamma_{12} t} & e^{\Gamma_{12} t} & e^{2\Omega_{12} t} & e^{-2\Omega_{12} t}
\end{pmatrix},
\]

\[
U = \begin{pmatrix}
1 & 1 & -1 & -1 \\
1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 \\
1 & -1 & 1 & 1
\end{pmatrix},
\]

while \( \mathbf{a}^T = (1,1,\Gamma/\Gamma_{12},\Gamma/\Gamma_{12}) \) and \( \mathbf{b}^T = (\Gamma_{12}/\Gamma_{12}, -\Gamma_{12}/\Gamma_{12},0,0) \). Expression (12) is the basis for the subsequent analysis.

Following the same method as introduced in Ref. 49, the far-field spontaneous emission intensity at a distance \( r \) from the origin of the double-QL system (Fig. 1) is actually the radial component of the Poynting vector at that point. The positive-frequency part of the outgoing electric field operator for the system of two identical QDs can be written similar to Ref. 2 as

\[
\hat{\mathbf{E}}^{(+)}(r,t) = \frac{k_0^2 \mu_0}{r} \sin \theta \times (\hat{S}_1^+ (t - \sqrt{\epsilon_1} r_1/c) + \hat{S}_2^+ (t - \sqrt{\epsilon_2} r_2/c)) \tag{14}
\]

where \( r_{1,2} = d = (d/2) \sin \theta \sin \phi \) and \( r, \theta, \phi \) are the spherical coordinates with zero at the center of the QD connecting axis. The observed value of intensity is defined as \( I(r,t) = \langle \mathbf{E}^+(r,t) \mathbf{E}^{(+)*}(r,t) \rangle \). For the intensity calculations we need the correlator \( \langle \hat{S}_1^+(r,t) \hat{S}_2^+(r,t+\tau) \rangle \), which is reduced to \( \langle \hat{S}_1^+(r,t) \hat{S}_2^+(r,t) \rangle \) in the quasistationary approximation.

For the sake of a better analysis and comparison, we write the final result of the emission intensity in a form similar to

\[
\frac{dx}{dt} = A x + f,
\]

FIG. 1. Schematic indication of the position of the observation point with respect to the QD positions and dipole orientations in circular coordinates. In this picture, the QD dipoles are perpendicular to the QD-QD axis. The z axis is along the QD dipole directions. The y axis is along the line connecting the QDs and the x axis is perpendicular to y and z.

The macroscopic antenna theory59 as

\[
I(\mathbf{r},t) = \frac{k_0^2 \mu_0^2}{r^2} g(\theta) f(\theta, \varphi, t - t_r),
\]

is the emission pattern originating from the interaction between the two QDs; \( \Theta(t) \) is the Heaviside step function equal to zero at \( t \leq 0 \) and equal to unity at \( t \geq 0 \). As is depicted in the schematic picture of Fig. 1, in deriving Eq. (15), we have assumed that the QD dipole moments are parallel and we are in the far-field regime \( (r \gg r_{1,2}) \). In the antenna theory, the quantity \( f(\theta, \varphi, t) \) is conventionally referred to as an array factor. Generally, the array factor characterizes antenna arrays with an arbitrary number of elements but can also be applied to arrays with two elements as we deal with our case. Hereafter we shall follow this fixed terminology.

The total emitted luminescence power can be represented by

\[
I_\Sigma(t) = -dP(t)/dt,
\]

where \( P(t) = h\omega_0(x_1 + x_2) + h\Omega_{12}(x_3 + x_4) \) is the total energy of the QD system. Using the general solution (12), we can consider the temporal behavior of \( I_\Sigma(t) \) for different initial states. Note that the quantity \( I_\Sigma(t) \) is an integral characteristic and thus does not comprise spatial variables. The relation

\[
I_\Sigma(t) = \lim_{r \to \infty} \left( r^2 \int I(\mathbf{r},t)dO \right)
\]

couples this characteristic with the far-field spontaneous emission intensity \( I(\mathbf{r},t) \) at a space point \( \mathbf{r} \) (\( dO = \sin \theta d\theta d\varphi \) is the solid angle).
III. SIMULATIONS AND DISCUSSION

A. Time resolved spontaneous emission

As demonstrated in the previous section, in the absence of any external laser field the master equation (6) is reduced to a close set of coupled equations, Eq. (11), which fully describes the population dynamics and time decay rate of the spontaneous emission. By replacing the correlated terms \( \langle S_i^+ S_j^- \rangle \) presented by Eq. (12) into Eq. (15), we arrive at

\[
\begin{align*}
\langle \Xi_e \rangle = & \quad \frac{2}{\Gamma_a} \left( \Gamma_a e^{-\Gamma_a t} - 2 \Gamma_a e^{-2\Gamma_a t} \right) \cos^2 \psi \left( \frac{\Psi}{2} - \frac{\Psi}{2} e^{-\Gamma_a t} \right) \\
& + \frac{2}{\Gamma_r} \left( \Gamma_r e^{-\Gamma_r t} + 2 \Gamma_r e^{-2\Gamma_r t} \right) \sin^2 \psi \left( \frac{\Psi}{2} - \frac{\Psi}{2} e^{-\Gamma_r t} \right),
\end{align*}
\]

(18)

for the array factor \( \langle \Xi_e \rangle \) of the initially double-excited state \( |\psi_{ee}\rangle = |e_1\rangle |e_2\rangle \). Analogously, for the array factor \( \langle \Xi_{ent} \rangle \) of the initially entangled state \( |\psi_{ent}\rangle = \cos \Phi |e_1\rangle |g_2\rangle + \sin \Phi |g_1\rangle |e_2\rangle \) with the excitation phase shift \( \Phi \), we obtain

\[
\begin{align*}
\langle \Xi_{ent} \rangle = & \quad -\sin \psi \cos 2\Phi \sin(2\Omega_{12}t) e^{-\Gamma_1 t} \\
& + \left( 1 + \sin 2\Phi \right) \cos^2 \psi \frac{\Psi}{2} e^{-\Gamma_1 t} \\
& + \left( 1 - \sin 2\Phi \right) \sin^2 \psi \frac{\Psi}{2} e^{-\Gamma_1 t}.
\end{align*}
\]

(19)

Here \( \Psi = k_0 r_{12} \sin \theta \sin \varphi \). In the particular case of \( \Phi = 0 \), the entangled state \( |\psi_{ent}\rangle \) is transformed into the initially single-QD excited state \( |\psi_{ee}\rangle = |e_1\rangle |g_2\rangle \). In that case Eq. (19) reduces to an expression for the array factor \( \langle \Xi_{ee} \rangle \):

\[
\begin{align*}
\langle \Xi_{ee} \rangle = & \quad \sin \psi \sin(2\Omega_{12}t) e^{-\Gamma_1 t} + \cos^2 \psi \frac{\Psi}{2} e^{-\Gamma_1 t} \\
& + \sin^2 \psi \frac{\Psi}{2} e^{-\Gamma_1 t}.
\end{align*}
\]

(20)

In the case of \( |\psi_{ee}\rangle \), where both QDs are initially excited, there is a channel decaying with the emission rate \( 2\Gamma_1 \). This channel is attributed to the two-photon emission (TPE) phenomena and is a consequence of exciton-exciton coupling. For the case of the biexciton which is actually a system of two coupled excitons in a single QD, the TPE process has been experimentally observed. In our case, the difference is that the coupled excitons belong to two separate QDs.

Let us compare the array factors for the system of two QDs with the array factors for two classical dipoles. Following Ref. 59, the latter quantity can be presented as

\[
\langle \Xi_{cl} \rangle = B \sin \psi + B_+ \cos \psi \left( \frac{\Psi}{2} - \frac{\Psi}{2} e^{-\Gamma_1 t} \right) + B_- \sin^2 \psi \left( \frac{\Psi}{2} - \frac{\Psi}{2} e^{-\Gamma_1 t} \right),
\]

(21)

where \( B = -2 J_1 J_2 \sin \delta \varphi \), \( B_+ = J_1^2 + J_2^2 \pm 2 J_1 J_2 \cos \delta \varphi \), \( J_{1,2} \) are the dipole currents, and \( \delta \varphi \) is the mutual phase shift. It should be noted that the dipole currents are induced by independent sources. That is why both the dipole currents and phases are a priori independent quantities. The factor (21) completely determines the directional emission pattern of a two-dipole antenna. For in-phase and opposite-phase dipoles \( B_+ \gg B_- \) and \( B_- \approx B_+ \), respectively.

The factors analogous to all three terms in Eq. (21) are also presented in Eqs. (18)–(20). The essential difference is that these factors decay in time for the system of QDs. This means that the directional emission pattern of two coupled QDs becomes time dependent since different decay channels are dominant for different emission directions. Indeed, for the entangled state the correlation \( \{ 1 \pm \sin(2\Phi) \exp\left[ -\Gamma_1 t \right] \} \) is obvious. Consequently, the second term in Eq. (21) corresponds to a symmetrical (superradiant) mode while the third one corresponds to an asymmetrical (subradiant) mode. Thus the time dependence of the directional emission pattern is a consequence of the \( d-d \) interaction, i.e., of the process which has no analogs in classical antennas. If one of the dipoles in the classical antenna is unloaded, for example \( J_2 = 0 \), we arrive at \( \langle \Xi_{cl} \rangle = J_1^2 \). This means that the second dipole does not manifest itself and the antenna radiates as a single dipole. In a system of QDs such a situation corresponds to the state \( |\psi_{ee}\rangle \); see Eq. (20). The difference is that the aforementioned nonstationarity provides the array factor dependence on \( \Psi \). As a result, the radiation directivity converges with the decay. The value \( \Phi = \pi/4 \) in Eq. (19) corresponds to the excitation of \( |a\rangle \) mode in the QD system. In this case \( \langle \Xi_{ent} \rangle \sim \cos^2(\Psi/2) \), which corresponds to the in-phase excitation of classical dipoles by identical currents: \( B_+ = B_0 = 0 \) if we assume in Eq. (21) \( J_1 = J_2 \) and \( \delta \varphi = 0 \). Analogously, one can find that \( \Phi = 3\pi/4 \) corresponds to the excitation of the \( |a\rangle \) mode and thus corresponds to the antiphase excitation of classical dipoles: letting \( J_1 = J_2 \) and \( \delta \varphi = \pi \) in Eq. (21) we arrive at \( B = 0 \).

For the double-excited state \( |\psi_{ee}\rangle \), from Eq. (18), the relations \( 2(\Gamma_a e^{-\Gamma_a t} + 2\Gamma_1 e^{-2\Gamma_1 t})/\Gamma_a \rightarrow B_+ \) and \( 2(\Gamma_a e^{-\Gamma_a t} - 2\Gamma_1 e^{-2\Gamma_1 t})/\Gamma_a \rightarrow B_- \) are followed. It can easily be found that \( B_+ \gg B_- \) at small \( t \) and thus \( \langle \Xi_{cl} \rangle = \text{const} \), i.e., in that case the system radiates as a single dipole. The situation is changed at large \( t \) when contributions of the symmetrical mode and two-photon channel becomes negligible. As a result, the radiation gains directionality [at large \( t \), \( B_+ \gg B_- \) providing \( \langle \Xi_{cl} \rangle \sim \cos^2(\Psi/2) \)].

It should be emphasized that the discussed nonstationarity of the array factors, which is inherent to quantum emitters, is not the only difference to classical dipoles. The second important factor is the oscillations in the first term in Eqs. (19) and (20) originated from quantum correlations between states in different QDs. This term is odd with respect to \( \Psi \) and corresponds to coupling of dipoles in classical antennas. Moreover, this term describes oscillations of the array factor with the frequency \( \Omega_{12} \). A strong \( d-d \) coupling between the QDs induces this process [the terms \( \sim O(\Omega_{12}) \) cannot be treated as small perturbations].

Thus, in the strong-coupling regime, the dipole-dipole interaction qualitatively changes the emission pattern as compared with macroscopic antennas. As has been shown above, in two-QD antennas the emission pattern becomes nonstationary. Moreover, from Eqs. (18) and (19) it follows that, in the general case, the spatial and temporal behavior of the spontaneous emission are not factored. This is because every eigenstate of the system (symmetric, antisymmetric, and two-photon state) is characterized by its own emission rate (\( \Gamma_a, \Gamma_{1,2} \)), respectively) and its own emission pattern.

From now on, we focus on the observation points located at \( (\theta = \pi/2, \varphi = 0) \), \( (\theta = \pi/2, \varphi = \pi/4) \), and \( (\theta = \pi/2, \varphi = \pi/2) \), which are respectively denoted by 1, 2, and 3 in Fig. 2.
The time profile of the spontaneous emission from a two-QD system into different angular directions has been indicated in Fig. 3. It is observed that at $\varphi = 0$, both $|\psi_{ee}\rangle$ and $|\psi_{ee}\rangle$ decay with the same emission rate, but in other directions, each channel decays with its own rate. The oscillations due to the first term of Eqs. (19) and (20) with the frequency $\Omega_{12}$ are also observable.

The influence of the QD-QD separation on the decay time of the emission is demonstrated in Fig. 4. In each graph, you can compare the emission decay in the coupled case with the uncoupled case. Depending on the QD-QD separation, it is shown that the emission can be either faster or slower than the emission of the uncoupled system. The role of the coherent part of the dipole-dipole coupling, $\Omega_{12}$, becomes evident for very closely spaced QDs. This role is to introduce oscillations with frequency $\Omega_{12}$ on the decay dynamics of the emission from the $|\psi_{ee}\rangle$ and $|\psi_{ee}\rangle$ states.

**B. Spontaneous emission pattern**

The spontaneous emission pattern from a double-QD system can be measured by positioning detectors at different angular positions $(\theta, \varphi)$ with respect to the center of the QD-QD axis and keeping the radial distance fixed. The detected signal is actually the time integral of the intensity introduced in Eq. (15) and reads

$$f^{\text{int}}(r, \theta, \varphi) = \int_{t_i}^{\infty} f(\theta, \varphi, t - t_i) dt.$$  
(22)

By substituting the array factors given by Eqs. (18) and (19) into this equation, we obtain for the different initial states

$$f^{\text{int}}(\theta, \varphi) = \frac{2}{\Gamma},$$  
(23)

$$f^{\text{int}}(\theta, \varphi) = \sin \Psi \cos 2\Phi \frac{2\Omega_{12}}{4\Omega_{12}^2 + \Gamma^2},$$  
(24)

$$f^{\text{int}}(\theta, \varphi) = \frac{1 + \sin 2\Phi}{\Gamma_s} \cos^2 \frac{\Psi}{2} + \frac{1 - \sin 2\Phi}{\Gamma_s} \sin^2 \frac{\Psi}{2},$$  
(25)

It is evident from Eq. (23) that if both QDs are initially excited, the intensities measured at different angular positions are equal to the emission intensity in the uncoupled case. So, for the remainder, we only consider the spontaneous emission pattern in which only one QD is initially excited or the double-QD system is initially prepared in an entangled state. The angular emission pattern from a double-QD system is depicted in Fig. 5. It is clear that the emission is maximum at $\theta = \pi/2$ for all initial conditions. It is clear that if initially only one QD is excited or the system is prepared in an entangled state,
The emission pattern is not symmetric between $0 < \varphi < \pi$ and $\pi < \varphi < 2\pi$.

As follows from Eq. (15), the correlated spontaneous emission is a superposition of three wave packets propagating in the radial direction under different angles. The radial dependence of the intensity $I(\mathbf{r},t)$ is essentially different from the corresponding dependence of the spherical wave, $O(1/r^2)$, and has an additional coefficient which is a superposition of three exponents. Note that, for entangled states, this dependence contains also oscillations with period $2\Omega_{12}/\sqrt{\epsilon_0/c}$, which are due to the asymmetry of the array factor mentioned above.

As seen from Eqs. (19) and (20), these oscillations can be presented as a superposition of two counterpropagating spherical modes. These modes have complex-conjugated wave numbers that correspond to the interaction of two modes with oppositely directed energy fluxes. Such waves, in microwave electrodynamics conventionally referred to as complex waves, have been detected in plane waveguides with anisotropic walls and in some other waveguiding structures; see Ref. 61 for a brief overview. These waves are pairwise excited and do not transfer energy. In our case, instead of guided waves, we deal with spherical complex waves which also do not transfer energy.

The existence of such waves is mediated by the photonic reservoir and is governed by the spatial-temporal field correlations, being induced due to the dipole-dipole interaction between the QDs. Thus the contribution of the additional modes into the integrated intensity vanishes as a result of the integration in Eq. (17) which is in agreement with Ref. 28. As a result, the existence of complex spherical waves does not contradict the radiation conditions, which have a more general form for complex waves than the classical Sommerfeld radiation conditions.61

In order to investigate the effect of radiative coupling between QDs on the emission pattern of the system, we need to normalize the graphs in Fig. 5 with the emission pattern of the noninteracting systems with identical initial excitation when $\Gamma_{12} = \Omega_{12} = 0$. For a QD-QD separation of 100 nm, this is demonstrated in Fig. 6. For the case where only one QD is initially excited (a), there is a large asymmetry. The emission intensity is stronger for $0 < \varphi < \pi$, and weaker for $\pi < \varphi < 2\pi$, as compared to the uncoupled case. If the system is initially prepared in an entangled state (b), the angular pattern greatly depends on the phase shift $\Phi$ and, in general, the pattern is not symmetric.

The angular emission pattern of the entangled state is depicted in Fig. 7 for different initial entanglement. We observe that the emission pattern is only symmetric for a particular initial entanglement, but in general it is not symmetric. By changing the initial entanglement, we alter the spatial distribution of the incoming and outgoing components, leading to a large variety of emission patterns. In Fig. 6, inside the dotted circles, the emission intensity is stronger and outside the circles, the emission is weaker than the emission of uncoupled QDs.
the circles, it is weaker than for the uncoupled double-QD system with the same initial conditions.

The QD-QD distance is another important item in a double-QD system which governs the strength of radiative coupling between the QDs and also influences the emission pattern. The influence of this quantity is evident in coupling parameters \( r_{12} \) between the QDs and also influences the emission pattern. The QD system which governs the strength of radiative coupling system with the same initial conditions.

For larger QD-QD separations, \( r_{12} < \pi/2 \) it is reduced for other angles, as compared to the uncoupled double-QD system for \( |\psi_{eg}\rangle \) (solid line), \( r_{12} = 0.3\lambda \) (dashed line), \( r_{12} = 0.5\lambda \) (dash-dotted line), and \( r_{12} = \lambda \) (solid line). In this graph \( \theta = \pi/2 \).

In this section we have considered the spatial distribution of far-field intensity for the spontaneous emission of two QDs. The problem is of interest because the correlations governed by \( d-d \) interaction are able to essentially transform the spatial distribution. The ability to tune the directivity of the spontaneous emission, in fact, was noted previously\(^{19} \) if the initial state of an infinitely large and homogeneous medium of two-level atoms is prepared by the absorption of a photon, the emitted photon is directed along the absorbed one. As we have shown in our analysis, possibilities for the control of the spontaneous emission direction arise for the emitters with sizes which are in a particular direction comparable with the wavelength. One can conclude that such emitters can play the role of quantum-optical antennas, whereby we can generate an effective correlated spontaneous emission. Different nanosized objects can be utilized for the practical realization of such antennas, which can be referred to as quantum nanoantennas. As an example of such a two-element antenna we have considered the system of two identical QDs.

### C. Spontaneous emission spectrum

Here we consider another interesting property of the correlated spontaneous emission which takes place for quantum states without a geometrical center of inversion (such as \( |\psi_{ent}\rangle \) and \( |\psi_{eg}\rangle \)). The spontaneous emission spectrum for such states is a triplet, a central line at the frequency \( \omega = \omega_0 \) and two additional lines appearing at the frequencies \( \omega = \omega_0 + \Omega_{12} \) and \( \omega = \omega_0 - \Omega_{12} \). The contribution of these emission lines to the field intensity is described by the first terms in Eqs. (24) and (25). Equations (18) and (19) are time averages over the oscillation period. But, in order to obtain the PL emission spectrum \( S(\theta, \varphi, \omega) \), we need to take a Fourier transform of the two-time correlation function \( f(\theta, \varphi, t_1, t_2) \), which is given by

\[
S(\theta, \varphi, \omega) = \frac{1}{\pi} \text{Re} \int_0^\infty \int_0^\infty f(\theta, \varphi, t_1, t_1 + \tau) e^{i\omega \tau} \, d\tau \, dt,
\]

where

\[
f(\theta, \varphi, t_1, t_2) = (\hat{S}_1^+(t_1)\hat{S}_1^-(t_2)) + (\hat{S}_2^+(t_1)\hat{S}_2^-(t_2)) + \cos \Psi[(\hat{S}_1^+(t_1)\hat{S}_2^-(t_2)) + (\hat{S}_1^-(t_1)\hat{S}_2^+(t_2))] + i \sin \Psi[(\hat{S}_1^+(t_1)\hat{S}_2^-(t_2)) - (\hat{S}_1^-(t_1)\hat{S}_2^+(t_2))].
\]

To evaluate the two-time correlation functions \( f(\theta, \varphi, t_1, t_2) \), we utilize a standard technique based on the Onsager theorem.\(^{2} \) As a result, the two-time correlation functions are expressed in...
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terms of the corresponding single-time correlators which are specified from the initial conditions. After some algebra we arrive at \( \tau > 0 \)

\[
\langle \hat{S}_s^+(t) \hat{S}_s^-(t + \tau) + \hat{S}_a^+(t) \hat{S}_a^-(t + \tau) \rangle
= [B_{es}(\tau) - B_{ea}(\tau)] \rho_{ee}(\tau) + C_{sg}(\tau) \rho_{sa}(\tau) + C_{ag}(\tau) \rho_{aa}(\tau),
\]

(28)

\[
\langle \hat{S}_s^+(t) \hat{S}_s^-(t + \tau) + \hat{S}_a^+(t) \hat{S}_a^-(t + \tau) \rangle
= [B_{es}(\tau) - B_{ea}(\tau)] \rho_{ee}(\tau) + C_{sg}(\tau) \rho_{sa}(\tau) - C_{ag}(\tau) \rho_{aa}(\tau),
\]

(29)

\[
\langle \hat{S}_s^+(t) \hat{S}_s^-(t + \tau) - \hat{S}_s^+(t) \hat{S}_s^-(t + \tau) \rangle
= C_{sg}(\tau) \rho_{sa}(\tau) - C_{ag}(\tau) \rho_{aa}(\tau).
\]

(30)

Here,

\[
C_{sg}(\tau) = \exp[-i \omega_+ \tau - \Gamma_s \tau / 2],
\]

(31)

\[
B_{es}(\tau) = (1 + \xi_1) \exp[-i \omega_- \tau - \Gamma_s \tau / 2] - \xi_1 C_{sg}(\tau),
\]

(32)

\[
\xi_1 = \Gamma_s / (2 \Omega_{12} + \Gamma), \xi_2 = \Gamma_s / (2 \Omega_{12} - \Gamma), \gamma_s,a = \Gamma + \Gamma_{s,a} / 2, \text{ and } \omega_{aa} = \omega_0 \pm \Omega_{12}.
\]

The equations for \( C_{ag} \) and \( B_{ea} \) are obtained from Eqs. (31) and (32) by the substitutions \( \omega_{aa} \rightarrow \omega_-, \Gamma_s \rightarrow \Gamma_\gamma, \) and \( \gamma_s \rightarrow \gamma_a.\)

The calculation of the frequency spectrum by Eq. (27) using (28)–(32) leads to very awkward final expressions. That is why we restrict ourselves to some partial cases of the initial conditions which are of the most physical interest. In particular, the initial conditions \( \rho_{0e} = 0, \rho_{1s,aa} = (1 \pm \sin 2 \Phi) / 2, \) and \( \rho_{0a} = \cos 2 \Phi / 2 \) in Eqs. (28)–(30) correspond to the entangled state. In that case, using (27) we obtain the following spectrum:

\[
S_{\text{ent}}(\omega) = \frac{1}{\pi} [S(\omega) + S(-\omega)],
\]

(33)

where

\[
S(\omega) = \frac{\rho_{0a}^0(1 - \cos \Psi)}{(\omega - \omega_+)^2 + \left(\Gamma_s / 2\right)^2} + \frac{\rho_{0a}^0(1 + \cos \Psi)}{(\omega - \omega_-)^2 + \left(\Gamma_s / 2\right)^2} - \frac{\rho_{0a}^0 \sin \Psi \Gamma_s (\omega - \omega) - \Gamma_s (\omega - \omega + \omega_0)}{[(\omega - \omega_-)^2 + \left(\Gamma_s / 2\right)^2][(\omega - \omega_+)^2 + \left(\Gamma_s / 2\right)^2]}.
\]

(34)

Analyzing (33) and (34) one can conclude that the entangled state spectrum comprises two resonant lines with frequencies \( \omega_+ \) and \( \omega_- \) as dictated by Eq. (34) and a nonresonant component induced by the second term in Eq. (33). It should be emphasized that resonant lines are non-Lorentzian. Their antisymmetrical component is proportional to \( \sin \Psi, \) i.e., is determined by the azimuthal asymmetry of the directional emission pattern. In the absence of the \( d-d \) interaction, these resonant lines merge to form a single symmetric line. Note that these peculiarities are the characteristics of the density of the energy flux into a particular angle. In the total radiation intensity the antisymmetrical components are averaged out due to integration over the space angle.

An even more important example is a mixed \(|a\rangle \) and \(|s\rangle\) initial state, which is described by the diagonal density matrix \( \rho_{0a}^0 = \rho_{0a}^s = 0 \) satisfying the normalization condition \( \rho_{0a}^s + \rho_{0a}^a = 1 \) with arbitrary \( \rho_{0a}^s. \) The spectrum for such an initial state can be obtained from Eq. (34) letting \( \rho_{0a}^s = 0. \) One can see that the spectrum is a superposition of two Lorentzian lines with different frequencies, directional emission patterns, and damping constants. Both directional diagrams are symmetrical with respect to the angle \( \Psi. \)

In a similar manner one can obtain the spectrum of the initially double-excited state \(|\Psi_{ee}\rangle. \) This spectrum can be represented as

\[
S_{ee}(\omega) = (1 + \cos \Psi)|S_s(\omega) + S_a(-\omega)|
+ (1 - \cos \Psi)|S_s(\omega) + S_a(\omega)|,
\]

(35)

where

\[
S_{s,a}(\omega) = \frac{1}{\pi} [G_s(\omega) - G_a(\omega)],
\]

(36)

and

\[
D(\omega) = \frac{\Gamma}{\Gamma^2 + 4 \Omega_{12}^2} [G_s(\omega) - G_a(\omega)]
- \frac{2 \Omega_{12}}{\Gamma^2 + 4 \Omega_{12}^2} [K_s(\omega) + K_a(\omega)],
\]

(37)

with

\[
G_s(\omega) = \frac{\Gamma_s \gamma_s / 2 + (\omega - \omega_+)^2}{[(\omega - \omega_-)^2 + \left(\Gamma_s / 2\right)^2][(\omega - \omega_-)^2 + \left(\Gamma_s / 2\right)^2]},
\]

(38)

\[
K_s(\omega) = \frac{\Gamma_s (\omega - \omega_+)}{[(\omega - \omega_-)^2 + \left(\Gamma_s / 2\right)^2][(\omega - \omega_-)^2 + \left(\Gamma_s / 2\right)^2]}.
\]

In Eq. (37), the parameters \( G_s \) and \( K_s \) can be obtained from \( G_t \) and \( K_t \) by exchanging \( \Gamma_s \leftrightarrow \Gamma_s, \gamma_s \rightarrow \gamma_s, \) and \( \omega_- \rightarrow \omega_. \)

The spectrum (35) is a superposition of two cascade processes \(|e\rangle \rightarrow |s\rangle \rightarrow |g\rangle \) and \(|e\rangle \rightarrow |a\rangle \rightarrow |g\rangle \) described by the terms \( S_s(\omega) \) and \( S_a(\omega), \) respectively. The corresponding nonresonant background is given by the terms \( S_{s,a}(\omega). \) The spontaneous emission spectra of a double-QD system for emission into different angular directions is indicated in Fig. 10. In the case

\[\text{FIG. 10. (Color online) Spontaneous emission spectra from } |\psi_{oo}\rangle \text{ (left), } |\psi_{oo}\rangle \text{ (middle), and } |\psi_{oo}\rangle \text{ (right) with } \Phi = \pi / 3 \text{ for } \varphi = 0 \text{ (dotted), } \varphi = \pi / 4 \text{ (solid), } \varphi = \pi / 3 \text{ (dashed), and } \varphi = \pi / 2 \text{ (dashed-dotted). The PL intensity in each graph is normalized to the emission of the two-QD system at } \omega = 0 \text{ with } \varphi = \pi / 2 \text{ and the same initial condition. In this graph } r_{12} = 50 \text{ nm and } \theta = \pi / 2.\]

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where both QDs are initially excited, the spectrum consists of three Lorentzian terms, each with amplitudes which are a function of the angular direction. That is why the emission spectra are different for the different emission directions.

D. Population dynamics

The total population probability of the double-QD system is equal to \( P(t) = (\hat{S}_1^+ \hat{S}_1^-) + (\hat{S}_2^+ \hat{S}_2^-) \), which is initially 0 if both QDs are empty, 1 if one of them is excited, and 2 if both QDs are excited, and \( P(t) \) varies between 0 and 2 for arbitrary populations. The total population dynamics can be directly probed in pump-probe reflectivity techniques by measuring the time profile of the reflected probe. It should be emphasized that the outgoing energy of the mode belonging to the first terms in Eqs. (19) and (20) for a given direction \((\theta, \phi)\) is exactly compensated by the incoming energy of this mode in the opposite direction \((\theta, \pi + \phi)\). Thus the contribution of these modes to the total emission intensity \( I_\Sigma(t) \) vanishes as a result of summation over all propagation angles.

IV. CONCLUDING REMARKS

In this paper, we have investigated the collective spontaneous emission in a system of two QDs strongly coupled via the dipole-dipole interaction. Resonant frequencies and dipole moments of QDs are assumed to be identical and oriented orthogonal to the QD-QD axis. The d-d interaction was described by the exchange of virtual photon through the photonic reservoir. The analysis was based on the master equation approach. The main conclusions of the paper are as follows.

1) There are three channels of the spontaneous decay in the two-QD system: superradiance, subradiance, and two-photon emission. Each of these decay channels is characterized by not only a differing radiative decay factor but also by its own directional emission pattern. The relative contribution of each of these channels to the total radiation pattern is determined by the initial state of the system. Thus the antenna characteristics of the two-QD system depends on its initial state.

2) The difference in the decay factors of each of these channels leads to a time-dependent total radiative emission pattern. Such an effect has no analogs in classical macroscopic antennas. For the entangled initial state, the total directional emission pattern shows oscillations with a frequency corresponding to the dipole-dipole interaction. The oscillating component is antisymmetric with respect to azimuthal angle. It should be emphasized that this oscillation effect is inherent to a particular directional emission diagram and vanishes for the total radiation intensity as a result of the integration of the radiation pattern over a solid angle. The oscillations are due to the strong coupling and cannot be described by a perturbation to the dipole-dipole interaction energy.

(3) The frequency spectrum of the spontaneous emission of the two-QD system is formed by a superposition of two resonant lines with frequencies \( \omega_{\pm} = \omega_0 \pm \Omega_{12} \) imposed on the nonresonant background, where \( \hbar \Omega_{12} \) is the d-d interaction energy. It should be noted that the amplitudes of resonant lines depend on the meridional and azimuthal observation angles. Generally, the resonant lines are non-Lorentzian—each of them is a superposition of two Lorentzian lines with the same frequency but with different amplitudes and widths. This property must be taken into account for the calculation of the spectral line shape.

The analysis carried out allows us to propose that the collective spontaneous emission of a QD ensemble provides an opportunity for the design of quantum nanoantennas whose radiative properties are dictated by the initial state of the system. Just by changing the QD-QD distance, we are able to increase (superradiance) or decrease (subradiance) the PL intensity as well as the spontaneous emission rate in a particular emission direction compared to those of the uncoupled system. A change of the QD-QD separation, \( r_{12} \), will also modify the amplitude and width of the emission spectrum. From the experimental point of view, the spontaneous emission spectrum (PL), and the emission dynamics can be easily measured by locating the detectors at different angular positions around the system.

The two-QD system considered in this paper is the simplest of this kind. As a next step, more complex systems can be considered, such as one-dimensional (1D) and 2D arrays of quantum dots, quantum dot rings, and also different combinations of emitting QDs with passive scattering elements (antennas of the Udo-Yagi type).

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