The benefits of FEM-SS-BES (Finite Element Method, State-Space, Building Energy Simulation) modeling exchange for building physics

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The benefits of FEM-SS-BES (Finite Element Method, State-Space, Building Energy Simulation) modeling exchange for building physics

A.W.M. (Jos) van Schijndel, R.P. (Rick) Kramer
Department of the Built Environment, Eindhoven University of Technology, Netherlands

ABSTRACT: An overall objective of energy efficiency in the built environment is to improve building and systems performances in terms of durability, comfort and economics. In order to predict, improve and meet a certain set of performance requirements related to the indoor climate of buildings and the associated energy demand, numerical simulation tools are indispensable. In this paper we consider three types of numerical simulation tools: Finite Element Method (FEM), Building Energy Simulation (BES) and State-Space (SS) together. It is concluded that one of the main benefits of FEM-SS-BES modeling exchange is the possibility to simulate building energy performances with high spatial resolution and low computational duration times.

1 INTRODUCTION

An overall objective of energy efficiency in the built environment is to improve building and systems performances in terms of durability, comfort and economics. In order to predict, improve and meet a certain set of performance requirements related to the indoor climate of buildings and the associated energy demand, numerical simulation tools are indispensable. In this paper we consider three types of numerical simulation tools: Finite Element Method (FEM), Building Energy Simulation (BES) and State-Space (SS). For each tool separately, there exist a vast number of references. Also on two tools combined, i.e. FEM-BES, BES-SS, FEM-SS, there is quite a lot of literature. However there is lack of research on an overall evaluation of the three tools FEM-SS-BES together. In this paper we present benefits of the FEM-SS-BES modeling exchange for building physics. The main reasons for converting models in each other are summarized in Table 1.

Table 1. The main reasons for converting models in each other.

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>FEM</th>
<th>BES</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEM</td>
<td></td>
<td></td>
<td>Global effects</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Lumped results</td>
<td>Computation Speed</td>
</tr>
<tr>
<td>BES</td>
<td>Local effects</td>
<td></td>
<td></td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>High resolution results</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS</td>
<td>Inverse Modeling</td>
<td></td>
<td></td>
<td>*</td>
</tr>
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</table>

In this work FEM is just a method of solving Partial Differential Equations (PDEs), like Finite Volume methods (FVM) or Finite Difference methods (FDM).

We start with two combinations that are quite obvious and already commonly used.

BES to FEM – BES is used to simulate the energy performance of buildings, using lumped parameter modeling. If local effects are important, FEM can be used to obtain high resolution results based on distributed parameter models and using BES simulation results as boundary values.

FEM to SS – FEM based simulations can easily become computational time consuming. One of the methods to improve the computing time is to reduce the mathematical model to a lower order model by using for example a State-Space (SS) approximation. One of the main benefits of SS models is, that very efficient computation algorithms exist, that are able to almost completely reduce the computation time. If such a reduced order SS model is accurate enough, this method can be used for improving computation speed.

This paper comprehends an investigation of the remaining combinations of Table 1. Each combination is presented in a separate Section, including background information and case studies. After these Sections the overall conclusions are provided.
2 FEM TO BES

Commonly used within BES tools are zonal approaches of the volumes, assuming uniform temperatures in each zone, and 1D modeling of the walls. Due to the rapid development of Finite Element Method (FEM) software and Multiphysics approaches, it should be possible to build and simulate full 3D models of buildings regarding the energy demand. Moreover, the 3D models would also provide detailed (i.e., high resolution) results of the indoor climate and the constructions. The main problem regarding the use of FEM for BES is how to compare a distributed parameter model (FEM) with a lumped parameter model (BES)? Because BES and FEM have quite different approaches, we used the following method: Step 1, start with a simple reference case where both BES and FEM tools provide identical results. Step 2, add complexity and simulate the effects with both tools. Step 3, compare and evaluate the results. For step 1, a suitable reference case was found at the current International Energy Agency Annex 58. It concerns a test box with overall dimension 120x120x120 cm³. Comsol was used to build a 3D model of the test box. In order to compare the Comsol 3D FEM model with the HAMBase (de Wit 2006 & HAMLab 2012) lumped model, an equivalent heat conduction of the air is used in Comsol instead of CFD.

2.1 Modeling

Equation (1) shows the PDE:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot k \nabla T$$  \hspace{1cm} (1)

where T is temperature (K), t is time (s), \(\rho\) is density (kg/m³), c is specific heat (J/kgK) and k is heat conduction coefficient (W/mK). Equation (2) shows the boundary values:

$$q_{\text{boundary}} = h(T - T_e) + q_{\text{irrad}}$$  \hspace{1cm} (2)

where \(q_{\text{boundary}}\) is the heat flux at a specific boundary (W/m²), h is the heat transfer coefficient (W/m²K), T_e is the external air temperature (K) and q_{irrad} is the net radiation from the sun and sky to the surface (W/m²). The temperature distribution in the test box is simulated using Dutch weather data.

As mentioned above the model was implemented and solved using Comsol. The default second-order Lagrange element type was used. The mesh contained 4414 tetrahedral elements and with average element quality of 0.7512. The number of degrees of freedom solved for was 6679 using the PARADESO algorithm with absolute and relative tolerances of 0.001. The temporal convergence error was less than 10⁻⁵ for each time step. After the solution was obtained with these settings, the grid dependency was evaluated by a grid refinement study. The latter showed no significant changes in the solution.

2.1.2 Results

Figure 2 shows the 3D snapshots of the isosurfaces in simulated by the FEM software.

![Figure 2: 3D snapshots of the isosurfaces simulated by the FEM software.](image-url)
The main challenge now is how to match the high resolution distributed temperature results of Comsol with the lumped temperature results of the BES model. For this reference case (opaque test box, heavy weight, ACH=0) we were able to get a very good match by using a so-called equivalent heat conduction coefficient for the air inside the box in Comsol.

\[ k_{eq} = \frac{d}{R} = \frac{1}{0.34} = 2.9 \]  

Figure 3 shows the comparison of the simulated mean indoor air temperature using Comsol (blue line) and HAMBase (green line) during the first month. The verification result is satisfactory.

From figure 3, two important facts can be concluded: Firstly, these results can be used as an additional verification benchmark for both Comsol as well as HAMBase. And secondly, it seems to be possible to accurately reproduce a BES simulation using a relative simple heat conduction based FEM model with an equivalent heat conduction coefficient for the indoor air, but so far without CFD and internal radiation. The next steps are to study effect of including windows and CFD.

### 2.2 Step 2. including windows

The achievements of the first step i.e. the reference case were quite successful. Therefore we started to add more complexity in the form of solar irradiation.

#### 2.2.1 Modeling

A window was modeled at the south wall of the test box by the use of surface-to-surface radiation module of Comsol. We refer to the appendix for more details.

#### 2.2.2 Results

A preliminary detailed result is presented in Figure 4. The solar irradiation inside the box is clearly visible. This figure shows the 3D temperature distribution in the test box with a window at a specific time. This was also modeled in HAMBase.
2.3 Step 3, including CFD

Currently we are working on the best way to compare these high resolution distributed temperature results of Comsol with the lumped temperature results of the BES model. This is by far the most challenging part of this part of the research.

3 BES TO SS

Detailed modeling of the buildings itself may require much effort. Blueprints can be hard to find and destructive methods to obtain building material properties are not allowed. A simplified model with physical meaning is developed which is capable of simulating both temperature and moisture (Kramer 2012). The parameters of the model are derived by an inverse modelling technique which fits the output of the model to measured values of respectively temperature and relative humidity. The result is a combined 3rd-order thermal and a 2nd-order hygric model in State Space form (Linear Time Invariant). The inverse modelling with the developed simplified model is applied to four different case studies for validation purposes. Moreover, the case studies are used to compare the performance of the simplified hygrothermal model and HAMBase, an in-house developed Heat Air and Moisture simulation tool. This whole building model originates from the thermal indoor climate model ELAN which was already published in 1987 (de Wit et al. 1988). Separately a model for simulating the indoor air humidity (AHUM) was developed. In 1992 the two models were combined (WAVO) and programmed in the MATLAB environment (van Schijndel & de Wit 1999). Since that time, the model has constantly been improved using newest techniques provided by recent MATLAB versions. Currently, the hourly-based model named HAMBase, is capable of simulating the indoor temperature, the indoor air humidity and energy use for heating and cooling of a multi-zone building. The physics of this model is extensively described by de Wit (2006). Also, the simplified building models are coupled to a PI-controller which maintains the indoor temperature at 20°C, yielding the characterization of the energy performance.

3.1 State space (SS) model

One of many developed SS models is shown in Figure 5. The thermal model is a 3rd order model with 9 parameters. The hygric model is a 2nd order model with 5 parameters.

Figure 5. the developed thermal model (top) and hygric model (bottom).

Thermal model inputs:

i. Temperature outdoor;
ii. Solar irradiation on vertical plane oriented on NORTH;
iii. Solar irradiation on vertical plane oriented on EAST;
iv. Solar irradiation on vertical plane oriented on SOUTH;
v. Solar irradiation on vertical plane oriented on WEST;
vi. Fixed temperature node for modeling ground contact.

The reason for splitting up the solar irradiation is that the used model is represented in State Space form. The huge advantage of the State Space form is the very small calculation time. A State Space (SS) model belongs to the family of LTI-models (Linear Time Invariant), therefore the identified parameter fi (factor for solar gain) is time-invariant. Because the azimuth and elevation of the sun are not time-invariant, it is impossible to derive a good State
Space model with only one input signal of the solar irradiation: nevertheless, the solar gain into the building is not a constant factor \( f_I \) times Global Irradiation On Horizontal Plane. The solar model of Perez et al. (1987) is used to calculate the irradiance on a vertical surface.

Hygric model inputs:
1. Vapor pressure outdoor;
2. Fixed vapor pressure node.

3.2 Validation using a real building

The castle of Amerongen (See Figure 6) situated in Amerongen the Netherlands, is selected as reference building. It is a 17th century building, surrounded by a canal, with thick massive walls, varying from 0.7 to 1.5 m thick. The building covers five floors. The main building materials are brick, wood and slate roof covering. The main part of the castle is free floating, i.e. no climate conditioning, and some rooms have limited dehumidification (7 kg/day) and limited heating.

Figure 6. Surrounding area (left) and exterior (right) of castle of Amerongen.

The measurement data of the Grand Salon is used to identify the simplified model for this room. The identification procedure of the parameters of the state space is published in Kramer (2012). In this paper we show representative results and a summary of the parameter identification performance. Figure 7 presents the measured and the two simulated results i.e. HAMBase and SS for the indoor temperature and vapour pressure.

To be able to compare and rate how accurately the different models can reproduce a measured temperature or vapor pressure, three performance criteria are used: the MSE (Mean Squared Error), MAE (Mean Absolute Error) and FIT (goodness of FIT). The MSE is calculated according to,

\[
MSE = \frac{1}{N} \sum_{k=1}^{N} (y' - y)^2
\]

Figure 7. The indoor temperature (top) and vapor pressure (bottom), measured and simulated using HAMBase (BES) and SS.

where \( y' \) is the measured signal and \( y \) is the simulated signal. The MAE is calculated according to,

\[
MAE = \frac{1}{N} \sum_{k=1}^{N} |y' - y|
\]

The Goodness of Fit is calculated according to,

\[
FIT = 100 \cdot \left(1 - \frac{\text{norm}(y' - y)}{\text{norm}(y' - \bar{y}')}\right)
\]

\( \text{norm}(\cdot) \) is the Euclidean length of the vector \( y \), also known as the magnitude. The above equation therefore calculates in the numerator the magnitude of the error between measured and simulated signal. This is divided by the denominator, calculating how much the measured signal fluctuates around its mean. Table 2 shows the fit performance indicators for the BES (HAMBase) and SS models.
Table 2. The fit performance indicators for the BES and SS models.

<table>
<thead>
<tr>
<th></th>
<th>THERMAL</th>
<th>HYGRIC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSE</td>
<td>MAE</td>
</tr>
<tr>
<td></td>
<td>[°C]</td>
<td>[°C]</td>
</tr>
<tr>
<td>HAMBase</td>
<td>0.94</td>
<td>0.70</td>
</tr>
<tr>
<td>Simplified State Space</td>
<td>1.09</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Furthermore Table 3 and 4 show the numerical values of the parameters of the SS model.

Table 3. Numerical values of the thermal parameters.

<table>
<thead>
<tr>
<th>#</th>
<th>Par.</th>
<th>Value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gw/Cw</td>
<td>4.10E-07</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>2</td>
<td>Gi/Cw</td>
<td>2.60E-05</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>3</td>
<td>Gi/Ci</td>
<td>4.48E+01</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>4</td>
<td>Gfast/Ci</td>
<td>2.94E+00</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>5</td>
<td>Gint/Ci</td>
<td>3.47E-01</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>6</td>
<td>Gint/Cint</td>
<td>1.82E+02</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>7</td>
<td>fl-N/Cint</td>
<td>3.55E-05</td>
<td>m²K/J</td>
</tr>
<tr>
<td>8</td>
<td>fl-E/Cint</td>
<td>4.99E-05</td>
<td>m²K/J</td>
</tr>
<tr>
<td>9</td>
<td>fl-S/Cint</td>
<td>5.27E-05</td>
<td>m²K/J</td>
</tr>
<tr>
<td>10</td>
<td>fl-W/Cint</td>
<td>4.17E-05</td>
<td>m²K/J</td>
</tr>
<tr>
<td>11</td>
<td>Gfixed/Ci</td>
<td>1.52E+00</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>12</td>
<td>Tfixed</td>
<td>3.21E-05</td>
<td>°C</td>
</tr>
</tbody>
</table>

Table 4. Numerical values of the hygric parameters.

<table>
<thead>
<tr>
<th>#</th>
<th>Par.</th>
<th>Value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Gw/Cw</td>
<td>9.22E-10</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>2</td>
<td>Gi/Cw</td>
<td>1.62E-05</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>3</td>
<td>Gi/Ci</td>
<td>2.56E+01</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>4</td>
<td>Gfast/Ci</td>
<td>1.11E+01</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>5</td>
<td>Pfixed</td>
<td>-</td>
<td>Pa</td>
</tr>
<tr>
<td>6</td>
<td>Gfixed/Ci</td>
<td>-</td>
<td>s⁻¹</td>
</tr>
</tbody>
</table>

We summarize the main advantages of SS models:
(1) The computation time is extremely fast (~0.02 sec. for an hourly based period of a year).
(2) The parameters have, in contrast with black box approaches, physical meaning. This facilitates basic parameter studies using a single SS model.
(3) The parameters can be obtained without detailed knowledge of the building itself by using relative simple measurements, i.e. inverse modeling.
(4) The SS model can be used for BES purposes. The latter is presented in the next Section.

4 SS TO BES

In Section we show how a similar SS model obtained by the approach of the previous section (now called thermal building SS model), can be used to simulate energy performances.

A simple control strategy is designed to maintain the indoor temperature of a thermal building SS model at 20°C: a PI-controller is connected with the thermal model and a closed loop is established from the output Ti(ndoor) to the PI-controller, see Figure 8. By connecting the systems, a new model object is generated.

The fixed vapor node was not used here, because no vapour source was measured.
Figure 9. The power supply to maintain an indoor temperature of 20 °C all year.

Also HAMBase itself is used to control the indoor temperature of the Simple Building at 20°C by allowing unlimited heating and cooling powers (HAMBase determines the needed maximum power). The resulting power [W] supply of HAMBase is compared to the resulting specific power [K/s] supply of the Energy Model, see Figure 10 (please mind the logarithmic y-axis).

Figure 10. Necessary power to keep T_{\text{indoor}} of the thermal building SS model at 20°C.

The overall comparison of the HAMBase power with the Energy model’s specific power reveals a remarkably accurate similarity. A more detailed comparison is given in Figure 11: the Energy Model’s specific power, multiplied by a factor of 63 (J/K), provides the same excitation level as the HAMBase power. This factor 63 can independently be obtained during a stationary simulation for which the thermal capacities play no role.

Figure 11. Detailed view with specific power of Energy Model scaled (x 63).

Due to the feedback loop, the Energy Model supplies the power one sample later (1 hr) than HAMBase. If the Energy Model’s output is delayed one hour, the similarity increases as shown in Figure 12.

Figure 12. Idem as in Figure 11 but Energy Model output delayed 1 hour.

Figure 12 shows that the combination of the SS model of Figure 5 with a PI controller can accurately reproduce the simulating heating of the BES (i.e. HAMBase) model.

5 CONCLUSIONS

It is concluded that one of the main benefits of FEM-SS-BES modeling exchange is the possibility to simulate building energy performances with high spatial resolution and low computational duration times.

Regarding FEM to BES – Firstly, these results can be used as an additional verification benchmark for both Comsol as well as HAMBase. Secondly, it is seems to be possible to accurately reproduce a BES simulation using a relative simple heat conduction based FEM model with a equivalent heat conduction coefficient for the indoor air, but so far without CFD and internal radiation. The latter is left over for future research.

Regarding BES to SS - The paper presents case studies where SS models are successfully used for reducing computational times for BES models.

Regarding SS to BES – Using this so-called inverse modeling approach, it is possible to obtain building energy performances from SS models.

The FEM-SS-BES modeling exchange provides two alternative modeling approaches for each other. This may be beneficial if some specific limitations are encountered within one of the single FEM, BES, SS modeling methods.
6 NOMENCLATURE

A  state matrix, area [m²]
B  input matrix
C  output matrix
C_i indoor air capacitance [J/K]
C_int interior capacitance [J/K]
C_p specific heat capacity [J/kgK]
C_w envelope capacitance [J/K]
D transition matrix
d wall thickness [m]
fI effective irradiance [m²]
G_fast conductance from indoor air to outdoor air [W/K]
G_i conductance from indoor air to envelope [W/K]
G_int conductance from indoor air to interior [W/K]
G_w conductance from envelope to outdoor air [W/K]
Irrad solar irradiation [W/m²]
P power [W]
P_e outdoor air vapour pressure [Pa]
P_fixed fixed vapour pressure [Pa]
P_i indoor air vapour pressure [Pa]
P_sat saturation pressure [Pa]
P_vapour vapour pressure [Pa]
RH relative humidity [-] and [%]
SG solar gain factor of glazing [-]
T temperature [°C]
T_e outdoor air temperature [°C]
T_fixed fixed temperature [°C]
T_i indoor air temperature [°C]
T_int interior partitions temperature [°C]
T_sim simulated indoor air temperature [°C]
T_w temperature of walls [°C]
u input vector
x state vector

y output vector
ε emission factor [-]
k thermal conductivity [W/mK]
ξ hygric capacity [kg/m³]
ρ density [kg/m³]

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APPENDIX Radiative Heat Transfer

Equations:

\[ J = \rho G + \varepsilon \sigma T^4 \quad (A1) \]
\[ q = G - J = (1 - \rho)G - \varepsilon \sigma T^4 \quad (A2) \]
\[ \varepsilon = 1 - \rho \quad (A3) \]
\[ q = \varepsilon (G - \sigma T^4) \quad (A4) \]