Inventory models with lateral transshipments : a review

Inventory Models with Lateral Transshipments: A Review

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Abstract

Lateral transshipments within an inventory system are stock movements between locations of the same echelon. These transshipments can be conducted periodically at predetermined points in time to proactively redistribute stock, or they can be used reactively as a method of meeting demand which cannot be satisfied from stock on hand. The elements of an inventory system considered, e.g. size, cost structures and service level definition, all influence the best method of transshipping. Models of many different systems have been considered. This paper provides a literature review which categorizes the research to date on lateral transshipments, so that these differences can be understood and gaps within the literature can be identified.

1 Introduction

Inventory systems often account for a large proportion of a business' costs. This makes it crucial to manage them efficiently. The 'traditional' design of an inventory system is hierarchical, with transportation flows from one echelon to the next, i.e. from manufacturers to wholesalers and from wholesalers to retailers. More flexible systems also allow *lateral transshipments* within an echelon, i.e. between wholesalers or retailers. In this way, members of the same echelon *pool* their inventories, which can allow them to lower inventory levels and costs whilst still achieving the required service levels.

Two main strands of literature on lateral transshipments can be identified that differ in the timing of transshipments. Lateral transshipments can either be restricted to take place at predetermined times before all demand is realised, or they can take place at any time to respond to stockouts or potential stockouts. We will refer to these two types as *proactive transshipment* and *reactive transshipment*. In proactive transshipment models, lateral transshipments are used to redistribute stock amongst all stocking points in an echelon at predetermined moments in time. This can be arranged in advance and organized such that the handling costs are as low as possible. Since handling costs are often dominant in the retail sector, this type of lateral transshipment is most useful in that environment. Reactive transshipments respond to situations where one of the stocking points faces a stock out (or the risk of a stock out) while another has sufficient stock on hand. This kind of lateral

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transshipment is suitable in an environment where the transshipment costs are relatively low compared to the costs associated with holding large amounts of stock and with failing to meet demands immediately. This is often the case in a spare parts environment. For example, Kranenburg (2006) discusses a semi-conductor company ASML with such a cost profile, and shows that using lateral transshipments can save the company up to 50% of annual inventory related costs for spare parts.

Lateral transshipments are graphically illustrated in Figure 1 for a simple inventory system with a single central warehouse (echelon 1) that supplies a number of stock points (echelon 2) between which lateral transshipments are allowed. Each arrow represents a possible transshipment route.

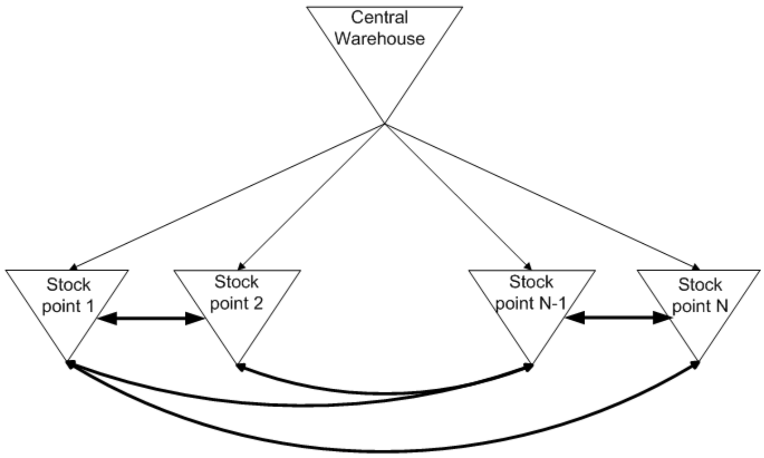

Figure 1: Lateral transshipments

Note from Figure 1 that lateral transshipments must take place between stock points of the same echelon. Both the transshipment of actual products and the transshipment of demand, where that demand is directly satisfied from a different location, are considered in the literature and included in this review. Literature on multi-echelon distribution networks where items in the replenishment pipeline are reallocated before the items physically arrive at the location are excluded from this review. Contributions on emergency shipments from a different echelon or outside supplier are also excluded, unless there are lateral transshipments as well. We remark that these excluded types are sometimes referred to as transshipments in the literature. Also, many alternative terms have been used to describe lateral transshipments, such as lateral resupply, reallocation of stock, substitutions and stock transfers.

Obviously, the added flexibility of allowing lateral transshipments implies that an inventory system is more difficult to control and optimize. Besides deciding when and how much to order from the 'regular' supplier, decisions on when, how much, and from where to transship are needed. Because of this added complexity, the literature is mainly restricted to the two echelon systems similar to the one depicted in Figure 1, and some contributions limit the system further by not considering the central warehouse and/or allowing only a limited number of stocking points in the second echelon.

Nevertheless, optimal control of lateral transshipments has been researched in many different settings. As already mentioned, some authors consider a single echelon whereas others consider two. Models also differ in the number of stocking locations, types of ordering, and so on. One key feature is whether a transshipment policy is using *complete pooling* or *partial pooling*. The former is a general term attached to policies where the transshiping location is willing to share all of its stock, the latter is used when part of the stock is held back to cover future demand. In the next section, we will provide a list of characteristics, and corresponding tables that can be used to quickly compare the various contributions to the literature. This is followed by a detailed review of the contributions over a number of sections. The organization of those sections will be clarified at the end of the next section, after discussing the key characteristics. In the final section, we end with conclusions and identify opportunities for future research.

2 Classification

As discussed in the previous section, an important distinction is the one between proactive transshipments that occur at fixed points in time and reactive transshipments that can happen at any time. The contributions to the literature are further classified by a number of characteristics related to the inventory system, the ordering policy and to the modeling of transshipments in particular. These characteristics are listed in Table 1, with the type of transshipment as proactive or reactive included for completeness.

Number of items	1, 2 or any number M
Number of echelons	1, 2 or P
Number of locations (Depots)	2, 3 or any number N
Identical locations?	Yes, (Identical) Costs or No
Unsatisfied demands	Backorder or Lost Sales
Timing of regular orders	Continuous review or Periodic review
Order Policy	(R, Q), (s, S), $(S-1, S)$, General or Other
Type of transshipments	Proactive or Reactive
Pooling	Complete or Partial
Decision making	Centralized or Decentralized
Transhipment cost structure	Per Item, Per Transhipment, Both or None

Table 1: Key characteristics for classifying the literature, related to the inventory system, ordering and transshipments

In Tables 2, 3 and 4, we use these characteristics to compare the different models that have been analyzed with proactive transshipments, reactive transshipments under periodic order review and reactive transshipments under continuous order review, respectively. The contributions are listed in alphabetical order based on the first author's name (and in increasing order based on the year of publication for multiple contributions by the same first author).

	System					Ordering		Transhipments			
	No. item	No. echelon	No. depot	Identical depots?	Backorder or Lost sales	Timing	Policy	Type	Pooling	Decision making	Cost
Agrawal et al. 2004	1	1	N	Yes	Lost Sales	Period	General	Proactive	Partial	Cent	Item
Allen 1956	1	1	N	No	Lost Sales	Period	General	Proactive	Partial	Cent	Item
Allen 1961	1	1	N	No	Lost Sales	Period	General	Proactive	Partial	Cent	Trans
Allen 1962	1	1	N	No	Lost Sales	Period	General	Proactive	Partial	Cent	Trans
Banarejee et al. 2003	1	2	N	No	Backorder	Period	(S-1,S)	Both	Partial	Cent	Item
Bertrand et al. 1998	1	2	N	No	Backorder	Period	(S-1,S)	Proactive	Partial	Cent	Item
Burton et al. 2005	1	2	N	No	Backorder	Period	(S-1,S)	Both	Partial	Cent	Item
Das 1975	1	1	2	No	Both	Period	(S-1,S)	Proactive	Partial	Cent	Item
Diks et al. 1996	1	2	N	Costs	Lost Sales	Period	(S-1,S)	Proactive	Partial	Cent	Item
Diks et al. 1998	1	2	N	Costs	Lost Sales	Period	(S-1,S)	Proactive	Partial	Cent	Item
Gross 1963	1	1	2	Costs	Backorder	Period	(S-1,S)	Proactive	Partial	Cent	Item
Hoadley et al. 1977	1	2	N	Yes	Backorder	Period	(S-1,S)	Proactive	Partial	Cent	Item
Jönsson et al. 1987	1	2	N	Costs	Backorder	Period	(S-1,S)	Proactive	Partial	Cent	Item
K'markar et al. 1977	1	1	N	Costs	Backorder	Period	(S-1,S)	Proactive	Partial	Cent	Item
K'markar 1980	1	1	N	Costs	Backorder	Period	(S-1,S)	Proactive	Partial	Cent	Item
Gross 1963	1	1	2	Costs	Backorder	Period	(S-1,S)	Proactive	Partial	Cent	Item
Lee et al. 2002	1	1	N	Yes	Lost Sales	Period	(S-1,S)	Proactive	Partial	Decent	None
Lee et al. 2007	1	1	N	Costs	Backorder	Period	(S-1,S)	Proactive	Partial	Cent	Item
Mercer et al. 1996	1	1	3	No	Lost Sales	Period	General	Proactive	Partial	Cent	None
Tagaras et al 2002	1	1	2	Costs	Backorder	Period	(S-1,S)	Proactive	Partial	Cent	Item

Table 2: Key characteristics for the literature on proactive lateral transshipments

	System					Ordering		Transhipments			
	No. it-em	No. ech-elon	No. de-pot	Iden-tical depots?	Backorder or Lost sales	Timing	Policy	Type	Poo-ling	Deci-sion making	Cost
Anupindi et al. 2001	1	1	N	No	Lost Sales	Period	Genereal	Reactive	Complete	Decent	Item
Archibald et al. 1997	M	1	2	No	Lost sales	Period	(S-1,S)	Reactive	Partial	Cent	Item
Archibald 2007	1	1	N	No	Lost sales	Period	(S-1,S)	Reactive	Partial	Cent	Item
Archibald et al. 2008	1	1	N	No	Lost sales	Period	(S-1,S)	Reactive	Partial	Cent	Item
Archibald et al. 2009	1	1	N	No	Lost sales	Period	(S-1,S)	Reactive	Partial	Cent	Item
Bendoly 2004	1	2	N	Yes	Backorder	Period	(S-1,S)	Reactive	Both	Cent	None
Chang et al. 1991	1	1	N	no	Backorder	Period	General	Reactive	Complete	Decent	Both
Dong et al. 2004	1	2	N	Costs	Lost Sales	Period	General	Reactive	Complete	Cent	Item
Granot et al. 2003	1	1	N	No	Lost Sales	Period	General	Reactive	Partial	Decent	Item
Herer et al. 1999	1	1	2	No	Backorder	Period	General	Reactive	Both	Cent	Item
Herer et al. 2001	1	1	2	No	Lost Sales	Period	General	Reactive	Partial	Cent	Both
Herer et al. 2002	1	1	N	No	Backorder	Period	(S-1,S)	Reactive	Partial	Cent	Item
Herer et al. 2003	1	1	N	No	Lost Sales	Period	General	Reactive	Partial	Cent	Both
Herer et al. 2006	1	1	N	No	Backorder	Period	(S-1,S)	Reactive	Partial	Cent	Item
Hu et al. 2005	1	1	N	Costs	Backorder	Period	(S-1,S)	Reactive	Complete	Cent	Item
Hu et al. 2007	1	1	2	No	Lost Sales	Period	Other	Reactive	Complete	Decent	Item
Hu et al. 2008	1	1	2	No	Lost Sales	Period	Other	Reactive	Complete	Decent	Item
Köchel 1990	1	1	N	No	Lost Sales	Period	General	Reactive	Partial	Cent	Item
Krishnan et al. 1965	1	1	2	Costs	Backorder	Period	(S-1,S)	Reactive	Partial	Cent	Item
Nonås et al. 2007	1	1	N	No	Both	Period	(S-1,S)	Reactive	Partial	Cent	Item
Ozdemir 2006	1	1	N	No	Backorder	Period	(S-1,S)	Reactive	Partial	Cent	Item
Robinson 1990	1	1	N	No	Both	Period	(S-1,S)	Reactive	Partial	Cent	Item
Rudi et al. 2001	1	1	2	No	Lost Sales	Period	General	Reactive	Partial	Decent	Item
Slikker et al. 2005	1	1	N	No	Lost Sales	Period	General	Reactive	Partial	Decent	Item
Sosic 2006	1	1	N	No	Lost Sales	Period	General	Reactive	Partial	Decent	Item
Tagaras 1989	1	1	2	No	Backorder	Period	(S-1,S)	Reactive	Both	Cent	Item
Tagaras et al. 1992	1	1	2	No	Backorder	Period	(S-1,S)	Reactive	Both	Cent	Item
Tagaras 1999	1	1	3	No	Backorder	Period	(S-1,S)	Reactive	Complete	Cent	Item
Wee et al. 2005	1	2	N	Cost	Lost Sales	Period	(S-1,S)	Reactive	Partial	Cent	Item
Yang et al. 2007	1	1	2	No	Backorder	Period	Other	Reactive	Partial	Cent	Item
Zhang 2005	1	2	N	Costs	Lost Sales	Period	General	Reactive	Complete	Cent	Item

Table 3: Key characteristics for the literature on reactive lateral transshipments under periodic order review

	System					Ordering		Transhipments			
	No. item	No. echelon	No. depot	Identical depots?	Backorder or Lost sales	Timing	Policy	Type	Pooling	Decision making	Cost
Alfredsson et al. 1999	1	2	N	No	Lost sales	Cont.	(S-1,S)	Reactive	Complete	Cent	None
Axsäter 1990	1	2	N	No	Backorder	Cont.	(S-1,S)	Reactive	Complete	Cent	Item
Axsäter 2003a	1	1	N	No	Backorder	Cont.	(R,Q)	Reactive	Partial	Cent	Both
Axsäter 2003b	1	1	N	No	Backorder	Cont.	(R,Q)&(S-1,S)	Reactive	Partial	Cent	Item
Ching et al.	1	2	2	Yes	Lost Sales	Cont.	(R,Q)	Reactive	Complete	Cent	None
Chiu et al.	1	P	N	Yes	Lost Sales	Cont.	(R,Q)	Reactive	Complete	Cent	Both
Dada 1992	1	2	N	Costs	Lost Sales	Cont.	(S-1,S)	Reactive	Complete	Cent	None
Evers 1996	1	1	N	Yes	Backorder	Cont.	(R,Q)	Reactive	Partial	Cent	None
Evers 1997	1	1	N	Yes	Backorder	Cont.	(R,Q)	Reactive	Partial	Cent	None
Evers 2001	1	1	N	Yes	Lost Sales	Cont.	(R,Q)	Reactive	Partial	Cent	Item
Grahovac et al. 2001	1	2	N	No	Backorder	Cont.	(S-1,S)	Both	Complete	Both	Item
Hadley et al.	1	1	N	No	Backorder	Cont.	(R,Q)	Reactive	Partial	Cent	Trans
Huo et al. 2007	1	1	N	No	Backorder	Cont.	(R,Q)	Reactive	Complete	Cent	Item
Jung et al. 2003	1	2	N	No	Backorder	Cont.	(S-1,S)	Reactive	Complete	Cent	Item
Köchel 1996	1	1	2	No	Backorder	Cont.	(S-1,S)	Reactive	Both	Cent	Item
Kukreja et al. 2001	1	1	N	No	Backorder	Cont.	(S-1,S)	Reactive	Complete	Cent	Item
Kukreja et al. 2005	1	1	N	No	Backorder	Cont.	(s,S)	Reactive	Complete	Cent	Item
Kuntanoglu 2008	1	2	N	No	Lost Sales	Cont.	(S-1,S)	Reactive	Complete	Cent	Item
K'oglu et al. 2009	1	2	N	No	Lost Sales	Cont.	(S-1,S)	Reactive	Complete	Cent	Item
Lee 1987	1	2	N	Yes	Backorder	Cont.	(S-1,S)	Reactive	Complete	Cent	Item
Liu et al. 2007	1	1	N	No	Backorder	Cont.	(S-1,S)	Reactive	Partial	Cent	Item
Mahar et al. 2009	1	2	N	Costs	Backorder	Cont.	(S-1,S)	Reactive	Partial	Cent	Item
Minner et al. 2003	1	1	N	Yes	Lost Sales	Cont.	(R,Q)	Reactive	Partial	Cent	Both
Minner et al. 2005	1	1	N	Yes	Lost Sales	Cont.	(R,Q)	Reactive	Complete	Cent	Both
Needham et al. 1998	1	2	3	Yes	Lost Sales	Cont.	(R,Q)	Reactive	Both	Cent	Item
Olsson 2008	1	1	2	Yes	Both	Cont.	(R,Q)	Reactive	Complete	Cent	Item
Sherbrooke 1992	1	2	N	Yes	Backorder	Cont.	(S-1,S)	Reactive	Complete	Cent	Item
U'beeck et al. 2009	1	1&2	N	No	Backorder	Cont.	(S-1,S)	Reactive	Complete	Cent	Item
Wong et al. 2005	M	1	N	No	Lost sales	Cont.	(S-1,S)	Reactive	Complete	Cent	Item
Wong et al. 2006a	1	2	N	No	Backorder	Cont.	(S-1,S)	Reactive	Complete	Cent	Item
Wong et al. 2006b	M	1	2	No	Lost sales	Cont.	(S-1,S)	Reactive	Complete	Cent	Item
Wong et al. 2007	1	1	N	No	Backorder	Cont.	(S-1,S)	Reactive	Both	Both	Item
Xu et al. 2003	1	1	2	Costs	Backorder	Cont.	(R,Q)	Reactive	Partial	Cent	Item
Yanagi et al. 1992	1	1	2	Costs	Backorder	Cont.	(S-1,S)	Reactive	Complete	Decent	Item
Zhao et al. 2005	1	2	N	No	Backorder	Cont.	(S-1,S)	Reactive	Both	Cent	Item
Zhao et al. 2006	1	1	2	Costs	Backorder	Cont.	(S-1,S)	Both	Both	Decent	Item

Table 4: Key characteristics for the literature on reactive lateral transshipments under continuous order review

The organization of the remainder of this paper is based on the characteristics listed in Table 1, and is also in line with Tables 2, 3 and 4. We first review proactive lateral transshipments, then consider reactive transshipments under periodic ordering, and end with reactive transshipments under continuous ordering. There are additional subdivisions based on the characteristics that are most relevant for these three types of models. These divisions and subdivisions are shown in Table 5.

<table>
<tr><td rowspan="2">Proactive lateral transshipments (3)</td><td>Ordering not considered (3.1)</td><td></td></tr>
<tr><td>Ordering considered (3.2)</td><td></td></tr>
<tr><td rowspan="5">Reactive lateral transshipments (4)</td><td rowspan="3">Periodic ordering (4.1)</td><td>1 Echelon, Centralized (4.1.1)</td></tr>
<tr><td>2 Echelon, Centralized (4.1.2)</td></tr>
<tr><td>Decentralized (4.1.3)</td></tr>
<tr><td rowspan="2">Continuous ordering (4.2)</td><td>Complete pooling (4.2.1)</td></tr>
<tr><td>Partial pooling (4.2.2)</td></tr>
</table>

Table 5: Organisation of the review (Section numbers between brackets).

3 Proactive lateral transshipments

In periodic order review models, the start or end (or some other point) of an order period provide ‘natural’ opportunities for redistributing the stock over all locations. This explains why, to the best of our knowledge, all research on proactive lateral transshipments is done in a periodic review setting.

Some authors have analysed redistribution in isolation as a first step towards understanding its effect. We will discuss their contributions first, before continuing with studies on more complex situations, where redistribution is considered alongside replenishment decisions.

3.1 Standalone redistribution

Articles which study redistribution on its own either ignore ordering completely or assume a specific ordering policy with arbitrarily chosen parameters values. The research in this area has established ideas on when it is best to redistribute and whether acting proactively is beneficial.

The basic redistribution model examines one period with a single opportunity for redistribution. These models generally assume that all retailers are located in an area close together, so that the transportation times for redistribution are negligible. There are three possible strategies for the timing of the redistribution of stock: at the beginning of the period (Allen 1958, 1961, 1962), at a prespecified point in time during the period (called a static policy in Agrawal et al. (2004)), or determined dynamically according to the observed demand (Agrawal et al. (2004)).

Determining the redistribution moment dynamically offers the largest flexibility but it is more difficult to implement. In a numerical study, Agrawal et al. (2004) illustrate that a dynamic policy often outperforms a static policy with respect to costs, saving up to 30%. However, the application of such a policy necessitates that each period is subdivided and the decision to redistribute is computed and assessed before each sub-period begins. In order to do this, all future effects have to be taken into account. The dynamic programming formulation as suggested in Agrawal et al. (2004) can be used to determine the optimal decisions but computations may be cumbersome. Therefore, as an alternative, the authors suggest a myopic heuristic that only compares redistribution now or one sub-period later, and illustrate that it works well.

The benefit of acting proactively is considered by Hoadley and Heyman (1977) who investigate if redistribution of stock at the beginning of a period is beneficial if there is additionally the option to transship items at the end of a period in order to react to stockouts. Under the assumption of identical cost parameters for all retailers, they provide a closed form condition deciding if redistribution is cost effective.

Continuing on a similar idea, Banerjee et al. (2003) and Burton and Banerjee (2005) compare the performance of a proactive redistribution policy to a simple reactive transshipment method. In these studies it is assumed that replenishment orders are placed according to a periodic base-stock policy, where the period is set equal to 20 days and the order-up-to level is equal to the average demand during lead time plus review period. A redistribution policy is introduced called Transshipment Inventory Equalisation (TIE), which levels the number of days supply at each location, alongside a reactive policy termed Transshipment Based on Availability (TBA) where a transshipment is initiated in case of shortages. In a series of simulation studies TIE and TBA policies are compared. Whilst Banerjee et al. (2003) find that the TBA policy is slightly more effective in preventing stockout incidents, Burton and Banerjee (2005) show that the TIE policy generally achieves lower overall system costs. This highlights how the overall system objectives can influence the suitability of transshipment policies.

The comparison is extended in Lee et al. (2007) by including an additional proactive heuristic, called the Service Level Adjustment (SLA) policy. At the beginning of each sub-period, the probability of no stockout during the lead time of the next replenishment order is computed and compared with given upper and lower bounds. The retailer with the largest surplus stock (that is not needed to achieve the upper bound) replenishes the retailer with the highest requirement (to reach the lower bound). A simulation study illustrates that this policy leads to a better cost performance than both the TIE and TBA policies when transportation costs are sufficiently low.

A case study looking at a periodic review transshipment policy is Mercer and Tao (1996). In this paper one aspect of a major supermarket's distribution system is examined. Transshipments are shown to lower the number of sales lost. However, some practical risks of using transshipments are also mentioned, such as the reduced forecast accuracy frequent stock rebalancing may cause and the increased occurrence of human error in a more complex system.

3.2 Redistribution combined with replenishments

When replenishment and redistribution decisions are considered together the complexity of the problem increases. Therefore only static point redistribution has so far been considered. A number of authors assume that redistribution only takes place at reorder moments, and we will discuss their contributions first. For two locations with negligable leadtime, Gross (1963) derives the policy that minimizes average shortage, inventory, and transshipment costs. He shows how the optimal transshipment and replenishment policy depends on the starting inventory levels and the cost parameters. The starting inventory levels divide the plane in six regions and each region has a corresponding optimal policy whose structure is derived for different cost parameters. For more than two locations an analytical approach is too cumbersome, and a numerical iterative procedure is provided. Karmarkar and Patel (1977) expand upon this model by looking in more detail at the multilocation case. For a large number of locations they propose a linear programming based technique which they find to be robust. Both of the previous models only examine the single period problem. Karmarkar (1980) provide the multiperiod expansion and find that the characteristics of the optimal solutions are similar to the single period case.

For multiple locations with lead times Diks and de Kok (1996, 1998) propose and compare several heuristics for determining the amounts to transship when distributing at reorder moments (after the arrival of a replenishment order but before a new order is placed). In Diks and de Kok (1996) it is assumed that a so-called Consistent Appropriate Share Rationing (CAS) policy is used for the redistribution as well as for the allocation at the central warehouse in case of a shortfall of stock. This policy balances the system stock by trying to keep each location's fraction of total projected inventory constant. The fraction allocated to each stocking location is chosen such that predetermined service levels are achieved. The authors develop approximations to compute the policy parameters, system-order-up-to-level and allocation fractions, such that the target service levels can be met at minimal cost.

A similar analysis is done in Diks and de Kok (1998) for a Balanced Stock (BS) rationing policy. This policy is a more general form of the CAS. While both redistribution strategies need much less safety stock than a system without redistribution, the BS policy outperforms the CAS policy. Moreover, redistribution is shown to be most beneficial in cases with a large number of retailers, a high service level, or a long replenishment lead time. It should be noted that a crucial assumption in their analysis is that all retailers observe the same replenishment lead time from the warehouse.

Next, we consider proactive redistribution between reorder moments, which is especially suitable in situations with long order cycles. In general only one predetermined redistribution opportunity per period is considered in the literature. Tagaras and Vlachos (2002) find that the redistribution point does have a large influence on the performance of the transshipment policy but its timing is dependent on the system characteristics such as demand. Jönsson and Silver (1987) argue that backorders are most likely to occur at the end of a replenishment cycle, which is when redistribution should take place. However, in general the optimal point in time for redistribution is currently unknown and both Das (1975) and Lee and Whang (2002) do not comment on the length of the two sub-periods that they use.

The optimal structure of the transshipment and ordering policy is only known under very restrictive assumptions. For two locations, negligible lead times and a pre-fixed redistribution time during the ordering cycle, Das (1975) derives the optimal redistribution policy minimizing inventory, shortage and transportation costs. This 'Base Stock Conserving Policy' transships in such a way that the resulting stock levels are as close as possible to the base stock levels.

For the application of the redistribution rule described in Das (1975), a central decision maker is needed. In contrast, Lee and Whang (2002) study a system under decentralized control. At the end of the first sub-period a secondary market is opened where retailers trade units at a uniform price, possibly via the Internet. For an infinite number of retailers who optimise their profits, the optimal stock level after the trading and redistribution is given by a news-boy equation if order sizes and clearing price are given. The optimal equilibrium market clearing price is independent of the demand realizations in the first sub-period, and smaller than the price a retailer has to pay for a unit at the beginning of the whole selling period.

Whereas Das (1975) and Lee and Whang (2002) assume that redistribution is instantaneous, Jönsson and Silver (1987) allow positive transhipment times. They divide an order cycle into H sub-periods, and time redistribution so that the units arrive at the beginning of the sub-period $H-1$. They study a one warehouse, multiple retailer system where the warehouse uses a base-stock policy for ordering and the entire order is allocated to the retailers after its arrival. They derive approximations for average inventories and backorders which can be used to determine the system order-up-to level that satisfies a system service level requirement. Given an upper bound for the ratio of shipment to holding cost for which redistribution is economically justified, they illustrate that redistribution becomes more advantageous in situations with high demand variability, long order cycles, many retailers, high service levels and short lead times. Bertrand and Bookbinder (1998) extend the model of Silver and Jönsson by allowing for non-identical cost parameters, focusing on cost reduction rather than service level criteria.

The final paper which considers non-negligible transshipment times is Tagaras and Vlachos (2002). Whilst a lot of transshipment research looks at long term average costs, Tagaras and Vlachos focus on the sensitivity of the policy based on the variability within the demand distribution. Considering a two location system, with a redistribution point that is optimized using simulation for each specific case, they find that preventive transshipments are generally beneficial. However, major benefits are only obtained when demand is highly variable.

4 Reactive lateral transshipments

Reactive lateral transshipments have been analyzed in both a periodic review and a continuous review setting. The results will be reviewed in Sections 4.1 and 4.2, respectively.

4.1 Continuous lateral transshipments under periodic review

We structure this subsection by first considering centralized systems with a single echelon (Section 4.1.1), followed by centralized systems with two echelons (Section 4.1.2), and finally decentralized systems (Section 4.1.3) where each stock point (retailer) aims to maximize its own profit.

4.1.1 One echelon centralized systems

The first research to consider a reactive mode of transshipment was that by Krishnan and Rao (1965). They consider a model that is similar to the periodic lateral transshipment model of Gross (1963) which has negligible transshipment times, but they aim to minimize cost through transshipments once all demand is known. This type of model is continued through the work of Robinson (1990), who provides an optimal solution for a multi-location, multi-period model. However, this solution can only be determined for networks with either two nonidentical locations or any number of identical locations. For more than two non-identical locations, a LP based heuristic solution procedure is proposed and shown to perform well for a number of scenarios. Nonås and Jörnsten (2007) are able to determine an alternative 'greedy transshipment policy' which optimally solves the three location case but the multilocation case remains only solvable under certain conditions.

Yang and Qin (2007) look at a similar model in a different way, by considering 'virtual' transshipments. This research is set in the power generating industry and transshipments can also take place when inventory at both locations is negative, thus reallocating backorders. This highlights one of the many ways that transshipment models can be diversified. Another such element is considered by Köchel (1990) who look at the possibility of selling stock outside of the system before demand is realised and then transshiping after the demand is observed.

Returning to the multilocation problem of Robinson, a similar model is that of Herer et al. (2006). They consider a more general cost structure and use LP and a network flow framework to produce a method which is shown to be more robust that that of Robinson. Further developments are done by Özdemir et al. (2006) who look at putting capacity constraints on the transport network and observe that these restrictions change the system's inventory distribution and increase the total cost.

An alternative approach is taken by Hu et al. (2005) to develop the multilocation problem. They calculate a simplified model which can be used to approximate ordering policies under certain conditions. These conditions are that the system contains a small number of stocking locations, and that it has a transshipment cost which is small when compared to holding and stockout costs. The simplifications within the model come from assuming that transshipments are free and instantaneous.

A similar style of post demand reactive transshipment is observed in Tagaras (1989). He considers a two location model with negligible lead times and transshipment times, which is later extended to positive lead times by Tagaras and Cohen (1992). These papers consider both partial and complete pooling, whilst stocking decisions are computed using an approximating heuristic. It is shown that complete pooling offers the best performance within this particular system. Tagaras (1999) takes a more detailed look at a related three-location network and finds that the precise form of transshipment does not effect the results greatly under complete pooling. More significantly he goes on to show that the benefit of transshipments increases with the number of locations.

However, complete pooling is not the best option for every system. Herer and Rashit (1999) show that in a system with positive ordering costs, partial pooling can outperform complete pooling. Partial pooling can take many forms and Herer and Rashit (1999) consider transshipping only a preset fraction of the available inventory in any period. The best fraction depends on the setup and cost structure of the system and varies a lot depending on these parameters.

The final model which considers transshiping after demand is realised is by Hu (2008). This research differs from the majority of other research on transshipments in that it focuses on a system with uncertain production at each location rather than central resupply. For a two location system an optimal policy is computed and further analysis examines when this policy finds most benefit in transshiping.

Whilst results for systems which tranship after all demand in the period has been observed give useful insights into the transshipment problem, in practice it is often more likely that continuous demand will be observed and each instance may trigger a reactive transshipment. For such a problem Archibald et al. (1997) consider a two location system where both experience Poisson demand. Demand that is not met from stock on hand at a location or by lateral transshipments is lost to the system (and can be emergency ordered). Archibald et al. show that an order-up-to policy is optimal. Moreover, they prove that there exist threshold times dependent on the inventory level, such that a location should only fulfill a lateral transshipment request if the time until the next ordering opportunity is less than the threshold time. Archibald (2007) continues this line of research for a multilocation setting. He examines the performance of three proposed heuristics. The results show that all three partial pooling heuristics outperform both no pooling and complete pooling. Out of the three, the least conservative appears to work best over the range of test settings. This heuristic determines for each location whether there is at least one other location that benefits from transshiping to it and, if so, fulfills transshipment requests from not only that location but any location. A useful extention of this work is that by Archibald et al. (1997), who look at the multiproduct case where each location only has a fixed capacity.

Further work on multilocation systems is undertaken in Archibald et al. (2008) and Archibald et al. (2009). These papers consider the real world situation of a tyre retailer with a large network of locations. Archibald et al. (2009) look to mitigate the problem of dimensionality with this type of system by approximating the dynamic programming value function. This is done by a pairwise decomposition, which considers two locations at a time and has been shown to improve upon the previous heuristics proposed in Archibald (2007) and also upon complete pooling. One restriction on this model is that all locations must have the same review period. Archibald et al. (2008) relaxes this restriction by using a two step heuristic that first calculates a static policy for determining which location meets a demand, and then applies dynamic programming policy improvement.

A separate direction for research is the study of dynamic deterministic demand systems. Herer and Tzur (2001) develop a solution for a two location problem. Looking at determining optimal ordering and transshipment decisions over a finite horizon, they examine the key properties of the system. These properties form a framework that allows this type of model to be solved in polynomial time. This problem is later extended by Herer and Tzur (2003) to a multilocation system.

Finally, Herer et al. (2002) look more generally at the usefulness of transshipments under the term 'legility' which looks to provide a lean and agile inventory system. By looking at some of the previously discussed models they show that transshipments help to improve system performance under these two criteria and produce a way of analyzing this information.

4.1.2 Two echelon centralized systems

In a system with two echelons there are several ways in which stockouts can be satisfied through emergency stock movements. Lateral transshipments are one possibility but there could be situations where it is beneficial also to perform emergency shipments from the central warehouse. Wee and Dada (2005) consider this problem with five different combinations of transshipments, emergency shipments and no movements at all and devises a method for deciding which setup is optimal under a given model description. This research allows the structure of the emergency stock movements to be established.

Dong and Rudi (2004) examine a different aspect by looking at the benefits of lateral transshipments for a manufacturer that supplies a number of retailers. They compare the case where the manufacturer is the price leader to the case of exogenous prices. For exogenous prices, it is found that retailers benefit more when demand across the network is uncorrelated. For the endogenous price case, modeled as a Stackelberg game, the manufacturer exploits his leadership to increase his benefits, leaving retailers worse off if they use transshipments. These results are restricted to demand that follows a normal distribution, but Zhang (2005) extends them to general demand distributions.

In a more retail case study based approach, Bendoly (2004) studies a model with internet and store based customers. They utilize lateral transshipment ideas to show how partial pooling of goods can improve a system's performance. The examined model considers a modern retail environment where stores are operated alongside internet channels and is an example of the practical uses of lateral transshipments.

4.1.3 Decentralized systems

A decentralized system is one in which each stocking point operates to meet its own goals. Chang and Lin (1991) consider when it is beneficial for such a system to actually operate as a more centralized system by using transshipments. They compare a decentralized model with a centralized model and deduce some properties that, if met, show that costs will be reduced if the operation shares resources. A related model to this is that of Slikker (2005) who studies whether independent vendors benefit by co-operating as a grand coalition. The problem is modeled as a general newsvendor situation with N retailers. Using a game theoretic approach, it is shown that if retailers cooperate then they can always achieve a higher profit. This shows that no retailer has an incentive to leave the grand coalition and illustrates that centralized ordering and transshipments can also be beneficial for decentralized systems. A limitation of this study is that transshipment costs are not included.

If it is beneficial for a system to include more co-operation, the next stage is to establish how co-operation can be established. Two papers which consider this are Rudi et al. (2001) and Hu et al. (2007). Both consider newsvendor type models with a manufacturer and two retailers. In Rudi et al. (2001) transshipment prices are determined in advance by an accepted authority, for example by the manufacturer who would like to stimulate stock sharing and is also willing to invest in an information system to provide accurate stock level data. Rudi et al. (2001) show that there exists a Nash equilibrium for the ordering quantities, and that the joint profit is generally not maximized at this equilibrium. In Hu et al. (2007), necessary and sufficient conditions are derived for the existence of transshipment prices that induce retailers to make jointly optimal decisions. The research focuses on finding linear transshipment costs which will induce co-operation but it is shown that these do not always exist. This highlights an area of future research which could consider more complex pricing structures

An extension of this type of model to N retailers is discussed by Anupindi et al. (2001). They drop the assumption of predetermined transshipment prices, and instead apply a rule for allocating the additional profits from transshipments. This rule uses a price that is based on the dual of the transshipment problem. It is shown that this rule is always in the core of the corresponding transshipment game. However, Granot and Sosic (2003) show that if retailers can decide how much to share (i.e. partial pooling), then it may happen that no residual inventory is distributed and hence no additional profit is gained. Granot and Sosic (2003) also identify a class of allocation rules that results in complete pooling, but that is not in the core of the corresponding transshipment game. While the Shapley value is also not in the core of this transshipment game (Granot and Sosic (2003)), it can be shown that this allocation rule leads to a farsighted stable grand coalition for symmetric retailers (see Sosic (2006)).

4.2 Reactive lateral transshipments under continuous review

Reactive lateral transshipment models which look to transship whenever there is a stockout or potential stockout can apply both partial and complete pooling. We consider complete pooling first which is reviewed in Section 4.2.1, followed by partial pooling in Section 4.2.2.

4.2.1 Complete Pooling

Complete pooling is often applied in a spare parts environment, where holding and back-ordering costs are typically large compared to transshipment costs. The basic model for repairable spare parts is called METRIC, developed by Sherbrooke (1968). In this model, items can be repaired at a central base-depot which supplies the individual locations with the repaired items. These locations use a one-for-one ordering system to replenish their stocks.

Lee (1987) considers lateral transshipments in such a model. He divides the stock locations into pooling groups, and focuses on one such group. All locations are identical and face Poisson demand. Failed parts are repaired at the central repair facility, which has infinite repair capacity but repair times are positive and probabilistic. Lee tests three rules for selecting from where to tranship: random selection, maximum stock on hand, and smallest number of outstanding orders. For all rules, Lee derives service level (fill rate) approximations that are used to minimize costs under service level constraints. Lee finds that for all rules, the use of emergency lateral transshipments leads to significant savings as less stock is needed at the bases. He also finds no significant difference in the performance of the three rules, but one has to keep in mind that all locations are assumed to be identical. Axsäter (1990) relaxes this restrictive assumption of identical locations, and also includes stock holding at the central depot. He presents improved methods for approximating service levels.

Kukreja et al. (2001) study a similar model, but only consider the lower echelon. They use a different selection rule: transship from the location with the lowest transshipment cost that has stock on hand. Kukreja et al. find approximations for the fill rates and apply those in a heuristic to find optimal base stock levels for the locations. Kukreja and Schmidt (2005) extend this model for compound Poisson demand processes and (s, S) replenishment policies, where they select the sending location by a dynamic programming rule each time a lateral transshipment is requested. A simulation based method is proposed to find optimal values for s and S. Another paper which considers an alternative order policy in this setting is that of Huo and Li (2007). In a similar fashion, they focus on the lower echelon under an (R, Q) order policy and determine approximations comparable to those of Axsäter (1990).

Four further papers in this area expand on work already discussed but in distinct areas. Jung et al. (2003) consider a model similar to that of Axsäter (1990), but allow finite repair capacities at the depot and at the bases. They derive approximations for the fill rates and use them in an algorithm to find optimal stocking levels. Sherbrooke (1992) considers the model of Axsäter as well, but evaluates expected backorders. He determines the stocking levels via the so-called VARI-METRIC model (Sherbrooke (1986)) and evaluates the reduction of backorders when lateral transshipments are applied. Sherbrooke concludes that the impact of lateral transshipments in this model is largest for parts with low demand rates. Finally, both Yanagi and Sasaki (1992) and Wong et al. (2006a) evaluate the downtime of a single echelon system due to backorders or waiting for a lateral transshipment and thus specifically consider non-zero lead times. Wong et al. (2006a) derive exact expressions for the service level (fraction of demands satisfied without backordering), the expected downtime caused by lateral transshipments, and the expected number of lateral transshipments. An approximation is developed for the expected downtime caused by backorders. Yanagi and Sasaki (1992) focus on developing approximations for the mean number of failed items and the probability that a backorder occurs.

The standard METRIC model and all papers discussed so far in this subsection assume backordering of demands that cannot be satisfied immediately. Instead, Dada (1992) considers the METRIC model with lost sales. He shows that this model can be represented by a Markov chain. However, the complexity of this chain implies extensive computational times. Therefore, an approximation algorithm is proposed to calculate various system performance measures. A disadvantage is that the algorithm does not always converge. Alfredsson and Verrijdt (1999) find approximations that do converge for a similar model by simplifying the shipment options.

Building on a single item model in Wong et al. (2005a), Wong et al. (2005b) and Wong et al. (2006b) deviate from the METRIC model by using a multi-item approach where machine uptime matters rather than availabilities of individual parts. They consider a single echelon system with the former restricted to two locations while the latter looks at the wider multi location setting. A request for a spare part that cannot be satisfied from stock on hand or a lateral transshipment, is satisfied from an outside source and is lost for the system. The evaluation of the uptime is exact, and is incorporated in heuristic procedures for optimizing stocking levels for all parts. Wong et al. (2007) take the spare parts transshipments research into a new area by considering a decentralized system. In their work they consider both competative and cooperative aspects to find a suitable method of allocating costs in such a system. This has practical applications as it is not uncommon for distinct companies to pool together their risk within a spare parts system.

Another practical feature which is considered within a spare parts model is time-based service levels. Kutanoglu (2008) and Kutanoglu and Mahajan (2009) consider this type of system. Time-based service levels are proportions of demand which need to be satisfied within a certain time period. The former article adapts two of the previously discussed models to examine cost and service levels in this setting within two and three location networks. This allows time-based service levels to be achieved while it is noted that there is particular sensitivity to changes in demand. The later paper considers how suitable stocking levels can be obtained so as to minimize cost. This is achieved in the form of an enumeration based algorithm.

While many of the previously discussed models on spare parts are similar there are subtle differences between the systems which can critically alter which policy is best to apply. Utterbeeck et al. (2009) consider this problem and propose a method for deciding which, out of six possible network structures, is most efficient for a given system. One and two echelon networks are considered with both lateral transshipments and emergency shipments as available options. Using guided local search in a simulation optimization setup the best structure can be determined. This provides a new aspect of the transshipment problem that can be considered and optimized alongside the transshipment and ordering policies.

The model of Minner and Silver (2005) differs from those in previously mentioned articles in that retailers apply an (R, Q) policy for replenishing stock. The paper analyzes when complete pooling is better than no pooling. This work is related to that on partial pooling by Minner et al. (2003), which will be discussed further in the next subsection. Another (R, Q) based model is that of Needham and Evers (1998). Rather than focusing on what policy is best, they look at the cost structure of a system and when using transshipments would be beneficial. This is achieved through a metamodel which can be utilized as a management tool in making this decision. Also using an (R, Q) policy, Ching et al. (2003) look at a two location model which includes returns. Only a proportion of these returns can be repaired and it is assumed that the system has limited capacity. By applying lateral transshipments they are able to decreases the amount of returns that are rejected.

Olsson (2008) has looked at an optimal (R, Q) ordering policy under complete pooling. By assuming that the lead time for an order to arrive is exponentially distributed, the steady state nature of the system can be analysed. However, due to the problem complexities the optimal solution is restricted to systems with only two locations. An interesting finding of this work is that optimal reordering policy is not always symmetrical even if the two locations have identical attributes.

Restricting analysis to two locations of a single echelon is a common necessity when looking at complex transshiping decisions. It is for this reason that the paper by Chiu and Huang (2003) is this only known research that considers a system with more than two echelons. However, they then restrict focus to a single location of each echelon and assume that unmet demand can be satisfied through transshipments if required. In a system concerned with 'just in time' mananagement it is shown that transshipments can help alleviate the problems caused by random delivery times.

The final paper which considers complete pooling is Grahovac and Chakravarty (2001). They consider a two echelon system with an emergency ordering option. They propose a two parameter (S, K) policy, where S is the base stock level for the replenishments and K the threshold for triggering transshipment requests. When the inventory level of a retailer drops below K, they first tries to receive an emergency order with a shorter lead time from the upper echelon, and only uses a lateral transshipment if this is not possible. The authors develop an efficient and accurate approximate evaluation method and show that in case of identical retailers, the optimal K levels are often (in 95% of their cases) equal to -1, which implies that anticipation of future demand is not beneficial. However, for all considered cases with non-identical retailers, the optimal value for K is nonnegative.

4.2.2 Partial Pooling

Systems with partial pooling do not automatically transship to meet all demand. Items can be reserved for future local demand. Such systems are more difficult to control and optimize than systems with complete pooling, as there is the additional managerial decision of how much inventory to reserve.

The first research which included a partial pooling style of transshipments was Hadley and Whitin (1963). In a system where the whole system ordered as one (using an (R, Q) policy), each location has a critical inventory level which will trigger a transshipment from another location which is above their own critical level. These levels are calculated using preset service levels. Under this setup an optimal order and redistribution policy is determined using dynamic programming. An interesting feature of this model is that two types of transshipments are included. The transshipments have non-negligable lead times so both a regular 'slow' transshipment and a 'faster' option are available.

A simpler model is studied by Köchel (1996). In this two location queueing model, transshipments are seperate entities within the system. An initial model that uses complete pooling is advanced to partial pooling by including an parameter of arbitrary value to dictate how much stock to hold back.

In a multi location setting under a continuous review (R, Q) ordering policy Axsäter (2003a) considers a decision rule for partial pooling. The locations each face a compound Poisson demand process. Demand that cannot be fulfilled from either stock on hand or via a lateral transshipment are backordered. Axsäter derives a rule for determining how many units should be transshipped, depending on the complete state of the system. The rule is myopic, in that the decision is optimal under the assumption that no further transshipments can take place, and can be repeatedly used as a heuristic. Numerical investigations for small networks show that the rule substantially outperforms the no pooling and complete pooling policies.

Similar models are considered by Evers (2001) and Minner et al. (2003). They however examine the case of lost sales rather than backordering under (R, Q) policies. Evers (2001) restricts his analysis to two locations but with possible multi-location extentions. The rule proposed considers the direct costs incurred by transshipping compared to the savings incurred before making a transshipment decision. Minner et al. (2003) propose a more flexible heuristic for a multi-location setting and show numerically that it can significantly outperform the heuristic of Evers, as well as no pooling and complete pooling. Xu et al. (2003) develop a heuristic for the case with backorders, but assume that demand has to be satisfied only just before the new replenishment order arrives.

For (R, Q) ordering policies it is often difficult to optimise both ordering parameters and the transshipment policy. Related to this, Evers (1997) builds on initial work done in Evers (1996) by looking at how emergency reactive lateral transshipments alter the amount of required safety stock. Refered to as the 'portfolio effect model', the proportion of demand to transship between locations can be preset so that an adjusted safety stock level can be found. In general, the amount of stock required is lowered at the same service level or a higher service level can be achieved with the same amount of stock. However the method is found to be conservative in it's results, suggesting that further improvements can be found.

Axsäter (2003b) considers unidirectional transshipments. These are applicable in situations where backorder costs differ considerably between two stock points and can relate to product substitutions. This makes it detrimental to transship from a location with high backorder costs to a location with low backorder costs. For systems with three stock points that apply either a base stock policy or an (R, Q) policy, Axsäter derives approximations for the fill rate, the expected stock on hand, and the expected level of backorders. Liu and Lee (2007) extend the work of Axsäter (2003b) for the base stock policy and compound Poisson demand. They improve the approximations of Axsäter and find exact values for the two location model.

Product substitutions are one practical example of transshipments being used in an alternative way. Another is dual channel retail systems where the allocation of demand arriving from an internet based channel has to be determined. In a model considered by Mahar et al. (2009) the store closest to the delivery location would traditionally be allocated the demand to satisfy as well as their regular instore demand. A dynamic method of moving this demand to stores that are in a better position to satisfy it, is calculated and is shown to improve upon the traditional approach. A related model examining an internet based demand stream is that of Seifert et al. (2006), who consider whether lateral transshipments from real stores should be used to help satify demand to a new internet 'virtual' store within the same company. The research finds that integrating the 'virtual' store in the existing system generally outperforms it operating in isolation resulting in reduced costs and improved service levels.

Finally in relation to partial pooling, Zhao et al. (2006) derive the optimal replenishment and transshipment policy structure for a model where demand follows a Poisson process and unsatisfied demands are backordered in a decentralized system. A three parameter policy (S, K, Z) is shown to be optimal, where S is the base stock level for the replenishments, K the threshold for triggering transshipment requests and Z the threshold for accepting transshipment requests. This is a generalization of the policy proposed by Grahovac and Chakravarty (2001) for complete pooling that was discussed in Section 4.2.1. This policy covers many policy options. It applies a partial pooling policy when $Z > 0$ and complete pooling when $Z = 0$. Moreover, transshipments are triggered by shortages when $K \leq 0$ and by the risk of a shortage for $K > 0$. The model is further utilised in Zhao et al. (2005) where the scope of the investigation is expanded to consider the game theoretic aspects of such a system. Using these ideas they are able to study the sharing behaviour of the decentralized dealers.

5 Conclusion and some proposals for future research

This review has shown that lateral transshipments have been applied in many different types of inventory system in a varied range of industries. We have primarily classified the models based on whether they look to transship proactively, at a single point in time, or reactive once demand has been realised. However within each of these categories there are subtle differences between the models considered, such as whether order leadtimes are included, which often will change the complexity of the problem and the nature of the transshipment policy. As a whole these varying systems have shown that lateral transshipments are an important tool to be used in a supply chain as they help to reduce costs or increase service levels.

Whilst the many different supply chain subtleties provide numerous possibilities for further research and model development, it is important to highlight the key areas where research gaps exist. The use of proactive transshipments has been shown to lower overall system costs but determining when it is best to perform the redistribution when ordering decisions are also considered is still not fully understood. An analytical method to determine the best static redistribution point would be the first stage but the work on dynamic redistibution points by Agrawal et al. (2004) suggest that a more complex dynamic redistribution point would yield further savings.

It is also noted that the regular redistribution of stock helps to balance an inventory system, but it has not been considered in a continuous time setting. The nature of proactive transshipment means that they naturally fit with a periodic order review but a regular rebalancing of stock may still have benefits under continuous review. Related to this, reactive transshipments under continuous review generally only look to satisfy backorders. It is likely there could be a benefit of using a reactive trigger to proactively balance the stock of the two locations invovled. This benefit would most likely be observed in systems with a fixed cost per transshipment.

Optimal policies for lateral transshipment generally can only be found for a small number of locations due to the large dimensions involved in multiple locations. This leaves the possibility that current research on multi-location problem could be developed further using different ideas so that better transshipment policies can be found. An example of this is the research that has built upon the multilocation work of Robinson (1990) in the reactive periodic review setting.

Another avenue of development is to consider multiple products within the supply network. To date only a handful of papers, such as Archibald et al. (1997), have considered this problem. Incorporating multiple products opens up several possibilities and issues. For example, it may be cost effective to react at a location that is facing multiple potential stockouts rather than wait until a stockout happens. Ideas of stocking capacity can also be considered in such models.

The current literature is also focused on relatively compact networks with 1 or 2 echelons. A natural expansion would be to consider how transshipments effect larger network. Chiu and Huang (2003) considered a system with multiple echelons that have a single location on each that could source outside transshipments. However this leaves plenty of scope for additional research. Another idea that is only considered in one paper (Hadley and Whitin (1963)) is the possibility of multiple transshipment times. Modern distribution companies such as DHL often offer same day and next day services at differing costs so this could lead to additional decision options which could be explored.

Finally, advances in technology and business practices are opening up different areas that could apply lateral transshipment ideas and stimulate research. Several recent papers have looked at how e-business operates alongside standard retail channels and considered how demand can be moved within the system so as to improve performance. Lateral transshipments have also been considered in the return recycling systems and 'virtual' transshipments have been utilised in the power generating sector. This highlights how the increase in system wide information sharing coupled with a desire to make supply chains as lean as possible has produced more opportunities for the application of lateral transshipments.

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